MAX – MIN (Slot 6)

A rectangular garden is to be constructed using a rock wall as one side of the garden and wire fencing for the other three sides (Figure 4.62). Given 100 ft of wire fencing, determine the dimensions that would create a garden of maximum area. What is the maximum area?

→ 200 ft

$$100 = 2x + y \quad (*)$$

$A = x \cdot y$, Find x, y

$A \to$ Max

$$y = 100 - 2x$$

$$A = x \cdot y = x(100 - 2x) = -2x^2 + 100x$$

$$A' = -4x + 100 = 0$$

$$\Rightarrow x = 25 \to y = 50$$

$$\Rightarrow A_{max} = 25 \cdot 50 = 1250 \text{ ft}^2$$

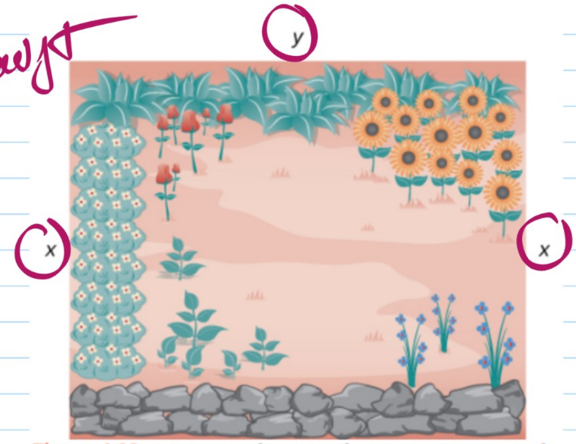


Figure 4.62 We want to determine the measurements x and y that will create a garden with a maximum area using 100 ft of fencing.

x	$-\infty$		25		$+\infty$
y'		$+$	0	$-$	
y		$\nearrow$	50	$\searrow$	

min

b) **Determine the maximum area if we want to make the same rectangular garden as in Figure 4.63, but we have 200 ft of fencing.**

$$200 = 2x + y$$

$$\Rightarrow y = 200 - 2x$$

$$A = x \cdot (200 - 2x) = -2x^2 + 200x$$

$$A' = -4x + 200 = 0$$

$$\Rightarrow x = 50 \Rightarrow y = 100$$

$$A = 50 . 100 = 5000 \text{ ft}^2$$

EX2: Find two number whose difference is 100 and whose product is a Min.

$$x - y = 100 \Rightarrow y = x - 100$$

$$A = x . y \Rightarrow A = x^2 - 100x$$

$$A' = 2x - 100 = 0 \Rightarrow x = 50$$

$$\Rightarrow y = -50$$

x	$-\infty$	50	$+\infty$
y'	$-$	0	$+$
y	$\searrow$	-50	$\nearrow$

EX3: Find Two number a and b whose Sum a+b is -3 and whose difference a-b is 7.

$$\begin{cases} a+b=-3 \\ a-b=7 \end{cases} \Leftrightarrow \begin{cases} a=2 \\ b=-5 \end{cases}$$

EX4: Find the point on the line

a: $y = 2x - 3$ that is closest to the origin.

$I(x; y) \in a$, $O(0; 0)$

$d(I; O) = Min$

$\Rightarrow d = \sqrt{x^2 + y^2}$

$\Rightarrow d = \sqrt{x^2 + (2x-3)^2}$

$\Leftrightarrow d = \sqrt{5x^2 - 12x + 9}$

$A(x_A; y_A)$

$B(x_B; y_B)$

$\underline{AB} = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$

$d' = \frac{1}{2\sqrt{5x^2 - 12x + 9}}(10x - 12) = 0 \Rightarrow \boxed{10x - 12 = 0}$

$\Rightarrow x = 6/5 \Rightarrow y = -3/5$

$\Rightarrow I(6/5; -3/5)$

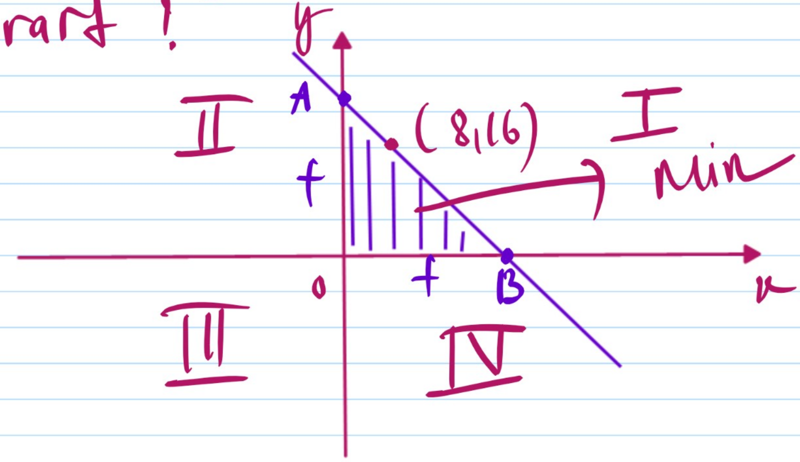
x	$-\infty$		$6/5$		$+\infty$
y'		$-$	0	$+$	
y		$\searrow$	$-3/5$	$\nearrow$	

$\underline{y\,min}$

$\boxed{5x^2 - 12x + 9 > 0}$

$\underline{\forall x \in \mathbb{R}}$

EX: find an equation of the line through the point (8, 16) that cuts of the least area of the point quadrant!



$y = m(x - x_0) + y_0$

$y = m(x-8)+16$

$d \; \boxed{y = mx - 8m + 16}$

find $m = ?$

$A \in Oy \Rightarrow x = 0 \Rightarrow y = -8m + 16$

$\Rightarrow A(0; -8m+16)$

$B \in Ox \Rightarrow y = 0 \Rightarrow x = \dfrac{8m-16}{m} \Rightarrow B\left(\dfrac{8m-16}{m}; 0\right)$

$S_{\Delta AOB} = \dfrac{1}{2}|OA|.|OB| = \dfrac{1}{2} OA.OB$

$= \dfrac{1}{2}(-8m+16)\left(\dfrac{8m-16}{m}\right)$

$= (-4m+8)\left(8 - \dfrac{16}{m}\right)$

$= -32m + 64 + 64 - \dfrac{128}{m}$

$S' = -32 + \dfrac{128}{m^2} = \dfrac{-32m^2 + 128}{m^2} \quad (m \neq 0)$

$f' = 0 \iff -32m^2 + 128 = 0$

$\Rightarrow m = \pm 2 \Rightarrow m = -2$

$\Rightarrow \boxed{y = -2x + 32}$

x	$-\infty$		-2		2		$+\infty$
y'		$-$	0	$+$	0	$-$	
y		$\searrow$	Min	$\nearrow$		$\searrow$	

+ Linear Approximation

(Xấp xỉ tuyến tính)

If x near a: $f(x) \approx f(a) + f'(a)(x-a)$

$L(x) = f(a) + f'(a)(x-a)$

linear approximation

or tangent line approximation $\quad (\sqrt{x})' = \frac{1}{2\sqrt{x}}$

EX: a) find the linear approximation.

$f(x) = \sqrt{x}$ at $x = 9$

Solution: $L(x) = f(9) + f'(9)(x-9)$

$= 3 + \frac{1}{6}(x-9)$

$\Rightarrow \boxed{L(x) = \frac{1}{6}x + \frac{3}{2}}$

$y = \sqrt{x}$

b) Use the approximation to the estimate $\sqrt{9.1}$

$\Rightarrow x = 9.1$

$$L(9.1) = f(9) + f'(9)(9.1 - 9)$$
$$\approx \frac{181}{60} = 3{,}016$$

EX2: Find the local linear approximation to $f(x) = \sqrt[3]{x}$ at $x = 8$ and use it to approximation $\sqrt[3]{8.1}$

$$f'(x) = \frac{1}{3} x^{-2/3}$$

$$L(x) = f(8) + f'(8)(x - 8)$$
$$= 2 + \frac{1}{12}(x-8)$$
$$= \frac{1}{12}x + \frac{4}{3}$$

$$L(8.1) = \frac{8.1}{12} + \frac{4}{3} \approx 2{,}0083$$