(+) Theorem : A: diagonalizable $\Leftrightarrow$ $P = (x_1\ x_2 \cdots x_n)$

$\downarrow$ eigenvectors

$$\Rightarrow \underline{P^{-1}AP} = \operatorname{diag}(\lambda_1\ \lambda_2 \cdots \lambda_n):$$

$\downarrow$ eigenvalues of A corresponding to x_i

Note : A: diagonalizable $\Leftrightarrow$ $\boxed{\#\lambda_i = \#x_i}$

Ex: Diagonalize the Matrix $A = \begin{pmatrix} 3 & 5 \\ 1 & -1 \end{pmatrix}$

$$C_A(x) = \begin{vmatrix} x-3 & -5 \\ -1 & x+1 \end{vmatrix} = (x-4)(x+2)$$

$\Rightarrow$ eigenvalues $\lambda_1 = 4$, $\lambda_2 = -2$

$\#\lambda = 2$

(+) $\lambda_1 = 4$

$(\lambda I - A)x = 0 \Rightarrow \begin{pmatrix} 1 & -5 \\ -1 & 5 \end{pmatrix} x = 0 \rightarrow x_1 - 5x_2 = 0$

$\rightarrow X_1 = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$

(+) $\lambda_2 = -2 \Rightarrow \begin{pmatrix} -5 & -5 \\ -1 & -1 \end{pmatrix} x = 0 \rightarrow x_1 + x_2 = 0$

$\Rightarrow X_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ / $\Rightarrow \#\lambda_i = \#x_i$

$\#x = 2$

$\Rightarrow A$: diagonalizable / $P^{-1}AP = \begin{pmatrix} 4 & 0 \\ 0 & -2 \end{pmatrix}$

EX2: Diagonalize the Matrix $A = \begin{pmatrix} 2 & 0 & 0 \\ 1 & 2 & -1 \\ 1 & 3 & -2 \end{pmatrix}$

$\lambda_1 = 2, \ \lambda_2 = 1, \ \lambda_3 = -1$ $\qquad \# \lambda = 3$

$X_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \ X_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \ X_3 = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}$

$\# x = 3$

$\longrightarrow \quad \# \lambda_i = \# x_i = 3$

$\Rightarrow$ A : diagonalizable.

$$\underline{P^{-1}AP} = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

* $A : n \times n$: λ : eigen value of A if $\boxed{Ax = \lambda x}$ $(x \neq 0)$

EX: $A : 2 \times 2$, with eigenvalues 1 and 3 and with corresponding eigenvectors $[1 \ 2]^T$ $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $[1 \ 1]^T$. Find the (1,2), (2,2) of A?

* Case 1 : $\lambda_1 = 1, \ x_1 = [1 \ 2]^T = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$Ax_1 = \lambda_1 x_1 \longrightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = 1 \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} a + 2b \\ c + 2d \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$\Rightarrow \begin{cases} a + 2b = 1 \\ c + 2d = 2 \end{cases}$ (1)

⊕ Case 2: $\lambda_2 = 3,\ x_2 = (1\ \ 1)^T = \begin{pmatrix}1\\1\end{pmatrix}$

$$Ax_2 = \lambda_2 x_2 \Rightarrow \begin{pmatrix}a & b\\ c & d\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix} = 3\begin{pmatrix}1\\1\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}a+b\\ c+d\end{pmatrix} = \begin{pmatrix}3\\3\end{pmatrix}$$

$$\Rightarrow \begin{cases} a+b = 3 \\ c+d = 3 \end{cases} \quad (2)$$

$$(1)\ (2) \Rightarrow \begin{cases} a = 5 \\ b = -2 \\ c = 4 \\ d = -1 \end{cases} \Rightarrow A\begin{pmatrix}5 & -2\\ 4 & -1\end{pmatrix}$$

(1,2)-entry of A

(2,2)-entry.

Ex: The eigenvalues of $A = \begin{pmatrix}3 & -2\\ 1 & 0\end{pmatrix}$ are 1 and 2

find an eigenvector for each eigenvalue.

$\boxed{Ax = \lambda x}$

⊕ $\lambda = 1$, $\Rightarrow \begin{pmatrix}3 & -2\\ 1 & 0\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix} = 1\begin{pmatrix}x_1\\x_2\end{pmatrix}$

$$\Rightarrow \begin{pmatrix}3x_1 - 2x_2\\ x_1\end{pmatrix} = \begin{pmatrix}x_1\\x_2\end{pmatrix}$$

$$\Rightarrow \begin{cases} 3x_1 - 2x_2 = x_1 \\ x_1 = x_2 \end{cases} \Rightarrow X_1 = \begin{pmatrix}1\\1\end{pmatrix}$$

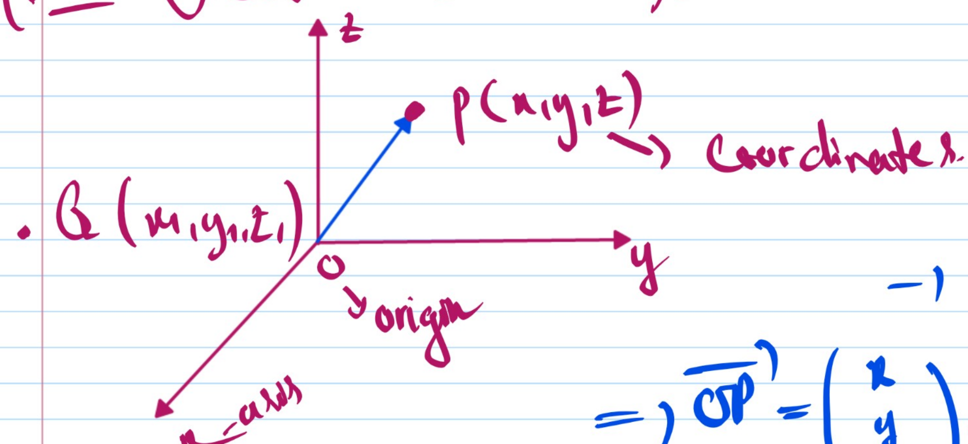
⊕ $\lambda_2 = 2 \longrightarrow \begin{pmatrix} 3 & -2 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 2 \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

$$\rightarrow \begin{cases} 3x_1 - 2x_2 = 2x_1 \\ x_1 = 2x_2 \end{cases}$$

$$\Rightarrow X_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

CHAPTER 4: VECTO GEOMETRY

4.1 Vectors and line.



$A(x_A, y_A, z_A)^T$

$B(x_B, y_B, z_B)^T$

$\rightarrow \overrightarrow{AB} = (x_B - x_A, y_B - y_A, z_B - z_A)^T$

$\Rightarrow \overrightarrow{OP} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \rightarrow$ Column

$$\overrightarrow{PQ} = \begin{pmatrix} x_1 - x \\ y_1 - y \\ z_1 - z \end{pmatrix}$$

⊕ Length and direction

$a \longrightarrow$ length $\|a\| \rightarrow$ chuẩn.

$$a = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow \|a\| = \sqrt{x^2 + y^2 + z^2}$$

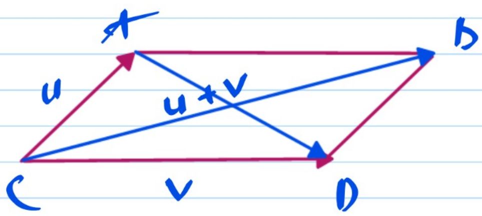
Ex: my $v = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} \rightarrow \|v\| = \sqrt{4+1+9} = \sqrt{14}$

$b = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \rightarrow \|b\| = \sqrt{9+16} = 5$

Ex 2: $A \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}$, $B = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$, find $\|\overrightarrow{AB}\|$

$\overrightarrow{AB} = \begin{pmatrix} -1 \\ -4 \\ -2 \end{pmatrix} \Rightarrow \|\overrightarrow{AB}\| = \sqrt{1^2+4^2+2^2} = \sqrt{21}$

The Parallelogram Law



$\overrightarrow{AB} + \overrightarrow{AC} = \overrightarrow{AD}$

$\overrightarrow{CA} + \overrightarrow{CD} = \overrightarrow{CB}$

$\overrightarrow{AB} = \overrightarrow{CD}$

$\overrightarrow{AB} \neq \overrightarrow{DC}$

$\overrightarrow{AB} = k\overrightarrow{AC} \Rightarrow \overrightarrow{AB} \parallel \overrightarrow{AC}$

parallel ($\parallel$)

$\overrightarrow{AB} \cdot \overrightarrow{AC} = 0 \Rightarrow \overrightarrow{AB} \perp \overrightarrow{AC}$

perpendicular ($\perp$)

Ex: $P(2,-1,4)$, $Q(3,-1,5)$, $A(0,2,1)$, $B(1,3,0)$

$\overrightarrow{PQ}$ and $\overrightarrow{AB}$ parallel?

$$\overrightarrow{PQ} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} , \quad \overrightarrow{AB} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

$$\frac{1}{1} \neq \frac{0}{1} \quad \frac{-1}{-1} \Rightarrow \text{not parallel}$$