CHAPTER 2 : MATRIX ALGEBRA.

(Ma trận đại số)

1. Matrix addition, Scalar Multiplication and transposition

EX:
$$A = \begin{pmatrix} 1 & 3 \\ 0 & 5 \\ -1 & -2 \end{pmatrix}_{3\times 2} \qquad A = 3\times 2$$

$$B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}_{2\times 3} \qquad A = \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}_{2\times 2}$$

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}_{3\times 3}$$

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}_{m\times n}$$

(a_{ij}) → i = row, → j = Column

(i, j) – entry of A

EX:
$$A = \begin{pmatrix} 1 & 0 & 0 \\ 3 & -2 & 1 \\ 0 & 5 & 8 \end{pmatrix}$$

a_{22}

$(2,2)$ – entry of $A = -2$

$(2,3)$ " $A = 1$

$(1,2)$ " $A = 0$

* $A = B \iff \begin{cases} \text{Same Size} \\ a_{ij} = b_{ij} \end{cases}$

EX1: $A = \begin{pmatrix} a+1 & 0 \\ c-2 & d+1 \end{pmatrix}_{2\times 2}$, $B = \begin{pmatrix} 4 & b-5 \\ 2c-1 & 3 \end{pmatrix}_{2\times 2}$

find a, b, c, d "$A = B$"

$$\Rightarrow \begin{cases} a+1 = 4 \\ b-5 = 0 \\ c-2 = 2c-1 \\ d+1 = 3 \end{cases} \Rightarrow \begin{cases} a = 3 \\ b = 5 \\ c = -1 \\ d = 2 \end{cases}$$

* Square Matrix : $m \times m$: $A = \begin{pmatrix} 0 & 1 \\ 3 & 2 \end{pmatrix}_{2\times 2}$

(MT vuông)

Zero Matrix $O_{m\times n}$; $O_{2\times 2} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

Negative Matrix $A = (a_{ij})$; $-A = (-a_{ij})$

Identity Matrix (MT đơn vị) I_n → Square Matrix
→ 1 on the main diagonal, zero ...

$I_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}_{2\times 2}$, $I_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}_{3\times 3}$

* Main diagonal of a Matrix.

a_{ij} $(i = j)$

$$A = \begin{pmatrix} \underset{}{\overset{a_{11}}{3}} & -1 & 0 \\ 5 & 2\,a_{22} & 3 \\ -1 & 2 & \underset{a_{33}}{1} \end{pmatrix}$$

$$B = \begin{pmatrix} \overset{a_{11}}{-1} & 2 & 3 \\ 4 & \underset{a_{22}}{5} & 6 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 3 \\ 2 & 4 \\ 5 & 8 \end{pmatrix}$$

Triangular matrices, diagonal matrices

Upper triangular matrix $\begin{bmatrix} 3 & 13 & 7 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Lower triangular matrix $\begin{bmatrix} 3 & 0 & 0 \\ 11 & -1 & 0 \\ 0 & 3 & 2 \end{bmatrix}$

Diagonal matrix $\begin{bmatrix} 3 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

* Addition – Difference (A, B: Same size)

$$A \pm B = (a_{ij} \pm b_{ij})$$

$$kA = (k\,a_{ij})$$

Ex: $A = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 1 & 1 \\ 2 & 1 & -1 \end{pmatrix}$

Find $(2,3)$ – entry of Matrix $(4A - 2B)$

$$4A = \begin{pmatrix} 4 & 12 & 20 \\ 8 & 12 & 16 \end{pmatrix}$$

$$2B = \begin{pmatrix} 0 & 2 & 2 \\ 4 & 2 & -2 \end{pmatrix}$$

$$4A - 2B = \begin{pmatrix} 4 & 10 & 18 \\ 4 & 10 & 18 \end{pmatrix}$$

$(2,3)$- entry of $(4A-2B) = 18$

b) $C = \begin{pmatrix} 4 & -5 & 2 \\ 1 & 0 & 4 \\ 3 & -3 & 0 \end{pmatrix}$, $D = \begin{pmatrix} 2 & 1 & 1 \\ -1 & 0 & 2 \\ 3 & 6 & 5 \end{pmatrix}$

$(1,3)$- entry of Matrix $(-3C+5D) = ?$

$$-3C = \begin{pmatrix} -12 & 15 & -6 \\ -3 & 0 & -12 \\ -9 & 9 & 0 \end{pmatrix}$$

$$5D = \begin{pmatrix} 10 & 5 & 5 \\ -5 & 0 & 10 \\ 15 & 30 & 25 \end{pmatrix}$$

$$-3C+5D = \begin{pmatrix} -2 & 20 & -1 \\ -8 & 0 & -2 \\ 6 & 39 & 25 \end{pmatrix}$$

$(1,3)$ entry $(-3(-10)) = -1$

* Transpose of a Matrix
(MT chuyển vị)

$$A = (a_{ij})_{m\times n} \longrightarrow A^T = (a_{ji})_{n\times m}$$

row $\longrightarrow$ Column
Column $\longrightarrow$ row

$$A = \begin{pmatrix} 1 & 2 & 3 \end{pmatrix} \longrightarrow A^T = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix} \longrightarrow A^T = \begin{pmatrix} 1 & 3 & 5 \end{pmatrix}$$

$$B = \begin{pmatrix} 2 & 1 \\ 3 & 5 \end{pmatrix} \longrightarrow B^T = \begin{pmatrix} 2 & 3 \\ 1 & 5 \end{pmatrix}$$

$$C = \begin{pmatrix} 5 & 0 & 7 \\ 3 & 1 & 0 \\ -1 & -2 & 3 \end{pmatrix} \longrightarrow C^T = \begin{pmatrix} 5 & 3 & -1 \\ 0 & 1 & -2 \\ 7 & 0 & 3 \end{pmatrix}$$

$$D = \begin{pmatrix} 1 & 2 & 4 \\ -3 & 5 & 0 \end{pmatrix} \longrightarrow D^T = \begin{pmatrix} 1 & -3 \\ 2 & 5 \\ 4 & 0 \end{pmatrix}$$

$$E = \begin{pmatrix} 2 & 1 & 5 \\ 1 & 3 & 4 \\ 5 & 4 & -3 \end{pmatrix} \longrightarrow E^T = \begin{pmatrix} 2 & 1 & 5 \\ 1 & 3 & 4 \\ 5 & 4 & -3 \end{pmatrix}$$

If $\begin{cases} E^T = E \\ E: \text{Square Matrix} \end{cases} \Rightarrow E$: Symmetrix Matrix

(MT đối xứng)

EX: $A = \begin{pmatrix} 3 & a+2 & a+b \\ 1 & 3 & 2 \\ 3b & 2 & 2 \end{pmatrix}$ Find a, b?

$\downarrow$

Symmetrix Matrix

$$\longrightarrow \begin{cases} a+2 = 1 \\ a+b = 3b \end{cases} \Rightarrow \begin{cases} a = -1 \\ b = \end{cases}$$

* properties

1. $(A^T)^T = A$
2. $(kA)^T = kA^T$
3. $(A \pm B)^T = A^T \pm B^T$
4. $(A.B)^T = B^T A^T$

EX: Solve for A $\mathcal{M}_y$

$$\left(A + 3\begin{pmatrix} 1 & -1 & 0 \\ 1 & 2 & 4 \end{pmatrix}\right)^T = \begin{pmatrix} 2 & 1 \\ 0 & 5 \\ 3 & 8 \end{pmatrix}$$

$$\rightarrow \left[\left(A + 3\begin{pmatrix} 1 & -1 & 0 \\ 1 & 2 & 4 \end{pmatrix}\right)^T\right]^T = \begin{pmatrix} 2 & 1 \\ 0 & 5 \\ 3 & 8 \end{pmatrix}^T$$

$$\Rightarrow A + 3\begin{pmatrix} 1 & -1 & 0 \\ 1 & 2 & 4 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 3 \\ 1 & 5 & 8 \end{pmatrix}$$

$$\Rightarrow A = \begin{pmatrix} 2 & 0 & 3 \\ 1 & 5 & 8 \end{pmatrix} - 3\begin{pmatrix} 1 & -1 & 0 \\ 1 & 2 & 4 \end{pmatrix}$$

$$\Rightarrow A = \begin{pmatrix} -1 & 3 & 3 \\ -2 & -1 & -4 \end{pmatrix}$$

C_2: $$\left(A + 3\begin{pmatrix} 1 & -1 & 0 \\ 1 & 2 & 4 \end{pmatrix}\right)^T = \begin{pmatrix} 2 & 1 \\ 0 & 5 \\ 3 & 8 \end{pmatrix}$$

$$\Rightarrow A^T + 3\begin{pmatrix} 1 & -1 & 0 \\ 1 & 2 & 4 \end{pmatrix}^T = \begin{pmatrix} 2 & 1 \\ 0 & 5 \\ 3 & 8 \end{pmatrix}$$

$$\Rightarrow A^T + 3\begin{pmatrix} 1 & 1 \\ -1 & 2 \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 0 & 5 \\ 3 & 8 \end{pmatrix}$$

$$\Rightarrow A^T = \begin{pmatrix} 2 & 1 \\ 0 & 5 \\ 3 & 8 \end{pmatrix} - 3\begin{pmatrix} 1 & 1 \\ -1 & 2 \\ 0 & 4 \end{pmatrix}$$

$$\Rightarrow A^T = \begin{pmatrix} -1 & -2 \\ 3 & -1 \\ 3 & -4 \end{pmatrix} \Rightarrow A = \begin{pmatrix} -1 & 3 & 3 \\ -2 & -1 & -4 \end{pmatrix}$$

b/ $$\left(2A^T - 3\begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}\right)^T = \begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix}$$

$$\Rightarrow 2A - 3\begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}^T = \begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix}$$

$$\Rightarrow 2A - 3\begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix}$$

$$\Rightarrow 2A = \begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix} + 3\begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}$$

$$\Rightarrow 2A = \begin{pmatrix} 5 & 0 \\ 5 & 5 \end{pmatrix} \Rightarrow A = \frac{1}{2}\begin{pmatrix} 5 & 0 \\ 5 & 5 \end{pmatrix}$$

$$\Rightarrow A = \frac{5}{2}\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

c/ $$\left[\left(2A - 4\begin{pmatrix} 1 & 1 \\ -2 & 2 \end{pmatrix}\right)^T\right]^T = \left[\begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix}^T\right]^T$$

$$\Rightarrow 2A - 4\begin{pmatrix} 1 & 1 \\ -2 & 2 \end{pmatrix} = \begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix}$$

$$\Rightarrow 2A = \begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix} + 4\begin{pmatrix} 1 & 1 \\ -2 & 2 \end{pmatrix}$$

$$\Rightarrow 2A = \begin{pmatrix} 6 & 1 \\ -9 & 10 \end{pmatrix}$$

$$\Rightarrow A = \frac{1}{2}\begin{pmatrix} 6 & 1 \\ -9 & 10 \end{pmatrix}$$

$\Rightarrow$