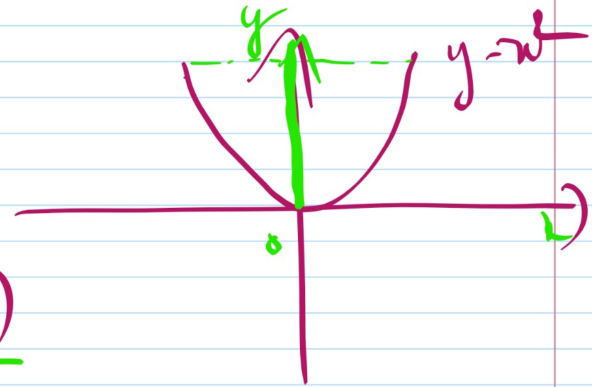
Range : Tập giá trị

VD: $y = x^2$ $\quad D = \mathbb{R}$

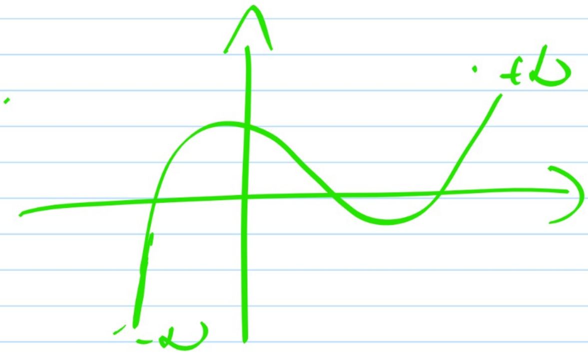
Range $= [0; +\infty)$



$y = 3x^3 + 2x^2 - 1$

$D = \mathbb{R}$

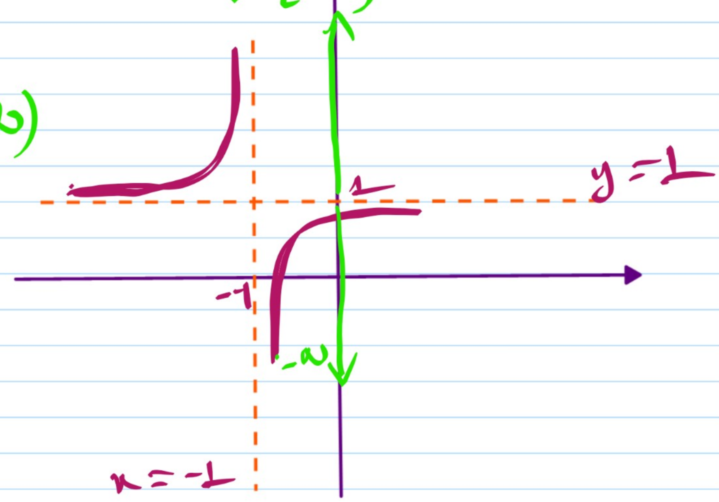
Range $= \mathbb{R}$



$y' = > 0$

$y = \dfrac{x-1}{x+1}$ $\quad D = \mathbb{R} \setminus \{-1\}$

Range $= (-\infty; 1) \cup (1; +\infty)$



$y = \dfrac{x}{1+x^2}$ $\quad D = \mathbb{R}$

Range =

$(1+x^2)y = x$

$\Leftrightarrow yx^2 - x + y = 0$ (Tìm n gtrị y sao cho pt x có ng.)

$\Delta \geq 0 \Leftrightarrow (-1)^2 - 4y^2 \geq 0$

$\Leftrightarrow \quad |y| \leq \frac{1}{2}$

$\text{Range} = [-1/2 ; \frac{1}{2}]$

$$y = \frac{x^2 - 2x + 4}{x^2 + 2x + 4}$$

$\Leftrightarrow (x^2 + 2x + 4) y = x^2 - 2x + 4$ (đưa về pt bậc 2 theo biến x)

$\Leftrightarrow (y-1)x^2 + 2(y+1)x + 4y - 4 = 0$

$\Delta' \geq 0 \Leftrightarrow (y+1)^2 - (y-1)(4y-4) \geq 0$

$\Leftrightarrow y^2 + 2y + 1 - 4y^2 + 8y - 4 \geq 0$

$\Leftrightarrow -3y^2 + 10y - 3 \geq 0$

$\text{Range} = [1/3 ; 3]$

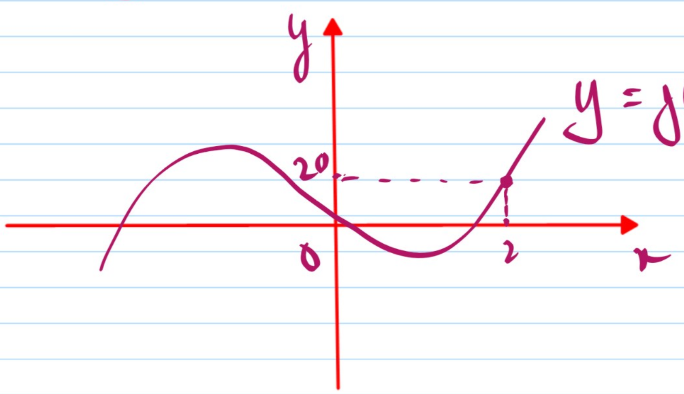
Limit function

LIMITS

(1) Limits

(2) Rules

(3) form $\frac{0}{0}$, $\frac{\infty}{\infty}$ ...



$y = f(x) = x^3 + 2x + 8$

$f(1) = 11$

$f(2) = 20$

$f(3) = \ldots$

$f(0,99999\ldots) = ?$ $\notin 1$

$f(1,0000\ldots 1\ldots) = ?$

$$\lim_{x \to 2} f(x) = \lim_{x \to 2} (x^3+2x+8) = 20$$

$$\lim_{x \to 2^-} (x^3+2x+8) = 20$$

$$\lim_{x \to 2^+} (x^3+2x+8) = 20$$

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = 20$$

So: $\lim_{x \to 2} f(x)$ Exists

$$\lim_{x \to 2^-} f(x) \neq \lim_{x \to 2^+} f(x)$$

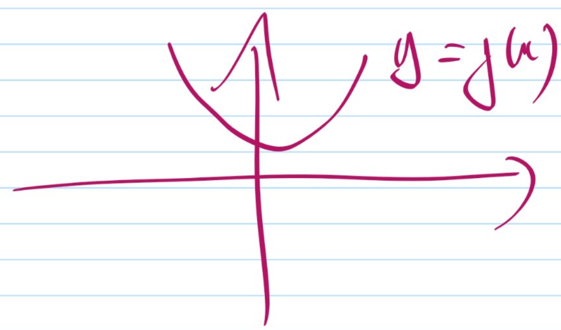
$\lim_{x \to 2} f(x)$ doesn't Exists

$$\text{Exists}\begin{cases} \lim\limits_{x \to 4^-} f(x) = a \\ \lim\limits_{x \to 4^+} f(x) = a \end{cases}$$

$$\lim_{x \to 4^-} f(x) \;\not\Rightarrow\; \lim_{x \to 4} f(x) \quad \text{Exists}$$

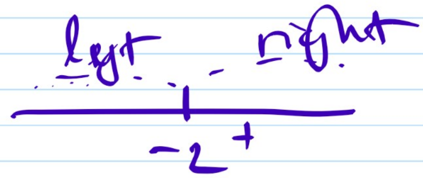
$$\lim_{x \to 30} f(x) = 100 \quad \begin{cases} \lim\limits_{x \to 30^-} f(x) = 100 \\ \lim\limits_{x \to 30^+} f(x) = 100 \end{cases}$$

Ex: $f(x) = x^2 + 4$

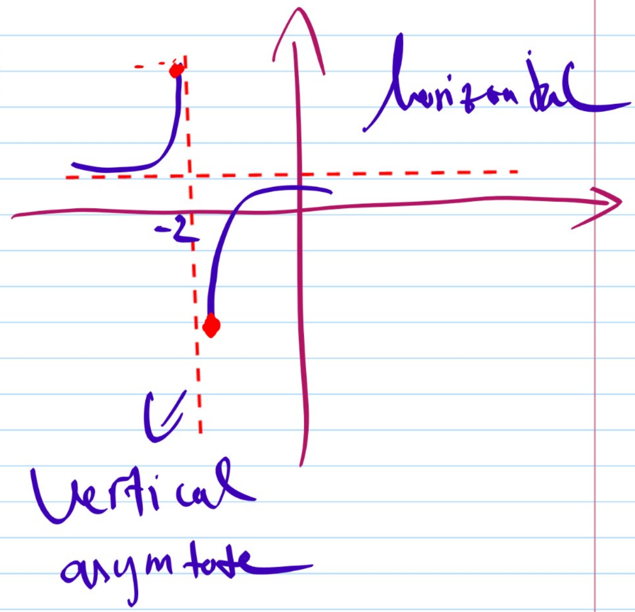


$$\lim_{x \to -1} f(x) = (-1)^2 + 4 =$$

$$\lim_{x \to 3} f(x) = (3)^2 + 4 \ldots$$



$$y = \frac{x-3}{x+2} \quad (x \neq -2)$$



$$\lim_{x \to -2^-} f(x) = +\infty$$

$$\lim_{x \to -2^+} f(x) = -\infty$$

$$\lim_{x \to 2} f(x) = \text{doesn't Exists}$$

$$\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) \to \lim_{x \to a} f(x) \text{ Exists}$$

$$\lim_{x \to a^+} f(x) \neq \lim_{x \to a^-} f(x) \to \text{Doesn't Exists}$$

$\frac{0}{0}$ form

Ex: $$\lim_{x \to 1} \frac{(x+3)\cancel{(x-1)}}{\cancel{x-1}} = \lim_{x \to 1} (x+3) = 4$$

Ex: $$\lim_{x \to -2} \frac{x^2-4}{x+2} = \lim_{x \to -2} \frac{(x-2)(x+2)}{x+2}$$
$$= \lim_{x \to -2} (x-2) = -4$$

Ex: $$\lim_{x \to 9} \frac{\sqrt{x}-3}{x-9} = \lim_{x \to 9} \frac{\sqrt{x}-3}{(\sqrt{x}-3)(\sqrt{x}+3)}$$
$$= 1/6$$

$\frac{\infty}{\infty}$ form

$$\lim_{x \to \infty} f(x)$$

$$\lim_{x \to \infty} \frac{1}{x} = 0$$

Ex: $$\lim_{x \to \infty} \frac{1x+1}{1x-1} = \lim_{x \to \infty} \frac{\cancel{x}(1+\frac{1}{x})}{\cancel{x}(1-\frac{1}{x})}$$
$$= \lim_{x \to \infty} \frac{(1+\frac{1}{x})}{(1-\frac{1}{x})} = 1$$

Ex: $$\lim_{x \to \infty} \frac{x^1-8}{x^2+5x} = \lim_{x \to \infty} \frac{x(1-\frac{8}{x})}{x^2(1+\frac{5}{x})}$$
$$= \lim_{x \to \infty} \frac{(1-8/x)}{x(1+5/x)} = \boxed{0}$$

Ex : $$\lim_{x\to\infty} \frac{x^2+8x+5}{x-1} = \lim_{x\to\infty} \frac{x^2\left(1+\frac{8}{x}+\frac{5}{x^2}\right)}{x\left(1-\frac{1}{x}\right)}$$

$$= \lim_{x\to\infty} \frac{x\left(1+\frac{8}{x}+\frac{5}{x^2}\right)}{\left(1-\frac{1}{x}\right)} = \infty$$

Ex: $$\lim_{x\to\infty} \frac{3x+5}{10x+1} = \frac{3}{10}$$

$$\lim_{x\to\infty} \frac{4x^2-3x+2}{1-x^2+4x} = -4$$

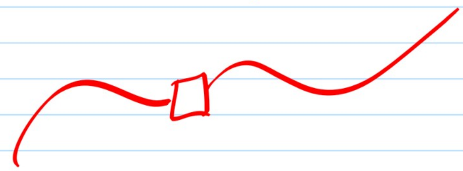
$$\lim_{x\to\infty} \frac{6-3x}{x+1} = -3$$

Ex: $$\lim_{x\to\infty} \frac{x+1}{x^2-3x} = 0$$

$$\lim_{x\to\infty} \frac{x^4+x^3-2}{x-1} = \infty$$

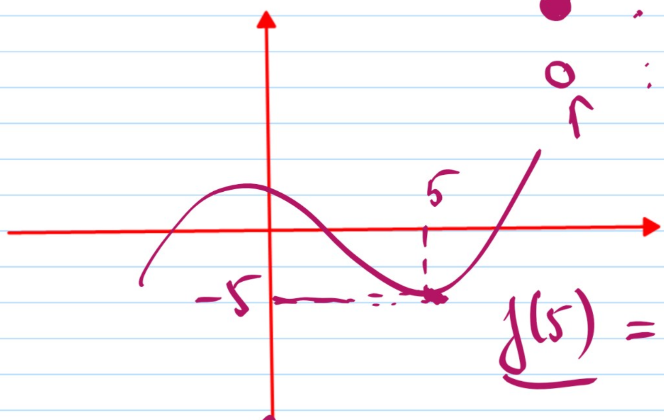
Continuity (Cont)

Continuous



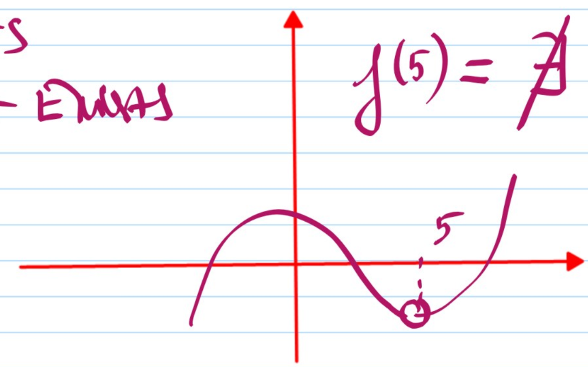
● : Exists

○ : doesn't Exists



$f(5) = -5$

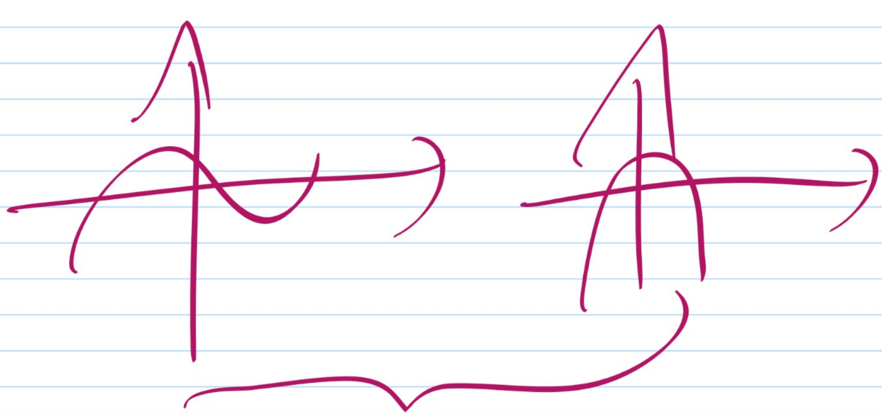
$f(5) = \nexists$



$$\lim_{x \to 5^-} f(x) = -5$$

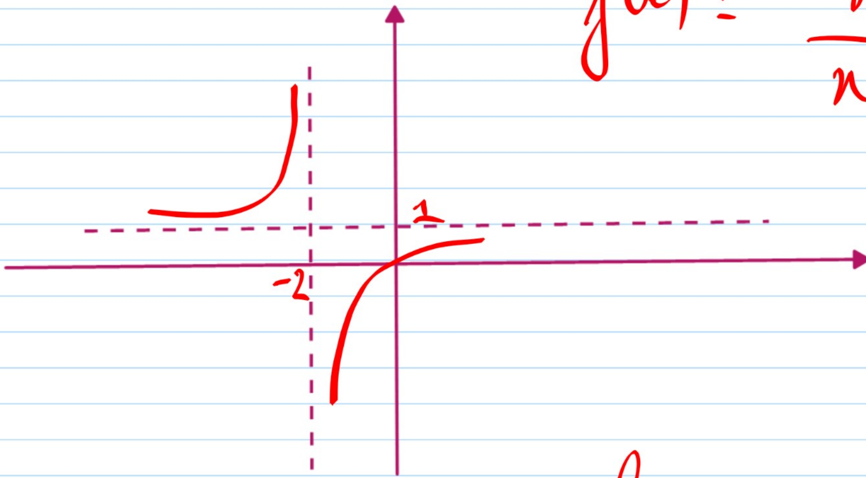
$$\lim_{x \to 5^+} f(x) = -5$$

$$\boxed{\lim_{x \to 5^-} f(x) = \lim_{x \to 5^+} f(x) = f(5) \Rightarrow \text{Continuous}}$$



Continuous

$$f(x) = \frac{x-3}{x+2}$$



$$\lim_{x \to -2^-} f(x) = +\infty \neq \lim_{x \to -2^+} f(x) = -\infty$$

$\Rightarrow$ doesn't exist