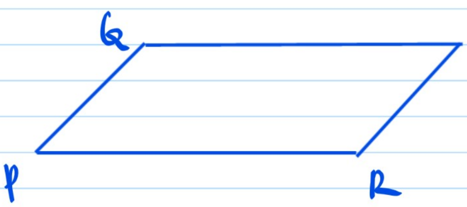
VECTOR

(E) The parallellogram law

EX: Let P, Q, R be the vertices (các đỉnh) of a parallelogram with adjacent sides (cạnh kề) PQ and PR. Find the vertex (đỉnh) S?

$P(3,-1,-1)$, $Q(1,-2,0)$, $R(1,-1,2)$



$S(x,y,z)$

$$\overrightarrow{QS} = \overrightarrow{PR}$$

$$\Leftrightarrow \begin{pmatrix} x-1 \\ y+2 \\ z \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ 3 \end{pmatrix}$$

$$\Rightarrow \begin{cases} x-1 = -2 \\ y+2 = 0 \\ z = 3 \end{cases} \Rightarrow \begin{cases} x = -1 \\ y = -2 \\ z = 3 \end{cases}$$

$$\Rightarrow S(-1,-2,3)$$

first coordinate of S, second, third

(F) Let $P_1(x_1,y_1,z_1)$ and $P_2(x_2,y_2,z_2)$

① $\overrightarrow{P_1P_2} = \begin{pmatrix} x_2-x_1 \\ y_2-y_1 \\ z_2-z_1 \end{pmatrix}$

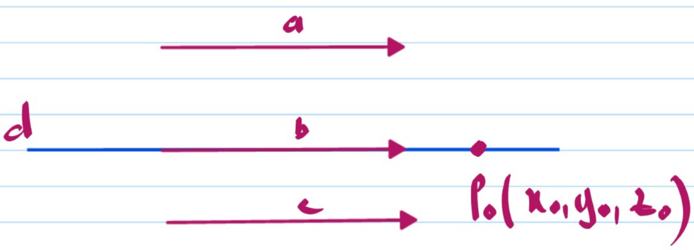
② The distance between P_1 and P_2 is

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

Ex: The distance between $P_1\ (2,-1,3)$ and $P_2\ (1,1,4)$

$$\overrightarrow{P_1P_2} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}, \quad \sqrt{(-1)^2 + 2^2 + 1^2} = \sqrt{6}$$

(2) equation of a line



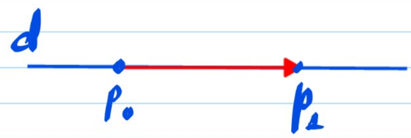
The equation of a line passing through P_0 and parallel $a = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$

$$d = P_0 + at$$

$$\Rightarrow \begin{cases} x = x_0 + at \\ y = y_0 + bt \\ z = z_0 + ct \end{cases} \quad (t \in \mathbb{R})$$

Ex: find the equation of the line through the point $P_0(2,0,1)$
$P_1\ (4,-1,1)$

$$\overrightarrow{P_0P_1} = (2,-1,0)^T$$

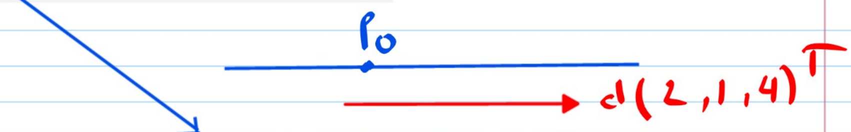


$$d: \begin{cases} x = 2 + 2t \\ y = -t \\ z = 1 \end{cases}$$

Find the equations of the line through $P_0(3,\ -1,\ 2)$ parallel to the line with equations

$$x = -1 + 2t$$
$$y = 1 + t$$
$$z = -3 + 4t$$

$$d \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}$$



$$\Rightarrow \begin{cases} x = 3 + 2t \\ y = 1 + t \\ z = 2 + 4t \end{cases}$$

Exercise 4.1.20 Let P, Q, and R be the vertices of a parallelogram with adjacent sides PQ and PR. In each case, find the other vertex S.

a. $P(3, -1, -1)$, $Q(1, -2, 0)$, $R(1, -1, 2)$

b. $P(2, 0, -1)$, $Q(-2, 4, 1)$, $R(3, -1, 0)$

b. The line passing through $P(3, -1, 4)$ and $Q(1, 0, -1)$.

c. The line passing through $P(3, -1, 4)$ and $Q(3, -1, 5)$.

d. The line parallel to $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and passing through $P(1, 1, 1)$.

e. The line passing through $P(1, 0, -3)$ and parallel to the line with parametric equations $x = -1 + 2t$, $y = 2 - t$, and $z = 3 + 3t$.

f. The line passing through $P(2, -1, 1)$ and parallel to the line with parametric equations $x = 2 - t$, $y = 1$, and $z = t$.

g. The lines through $P(1, 0, 1)$ that meet the line with vector equation $\mathbf{p} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + t \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$ at points at distance 3 from $P_0(1, 2, 0)$.

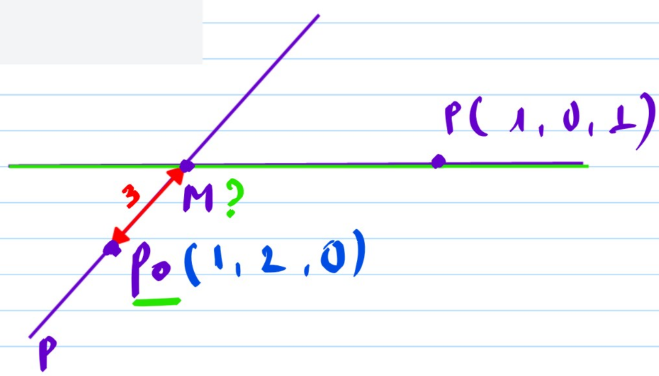
→ intersect ($\cap$: giao)

h. Determine whether the following lines intersect and, if so, find the point of intersection.

$$\begin{aligned} x &= 1 - 3t \qquad & x &= -1 + s \\ y &= 2 + 5t \qquad & y &= 3 - 4s \\ z &= 1 + t \qquad & z &= 1 - s \end{aligned}$$

$$\Leftrightarrow \begin{cases} 1 - 3t = -1 + s \\ 2 + 5t = 3 - 4s \\ 1 + t = 1 - s \end{cases}$$

$$\Leftrightarrow \begin{cases} \\ t = -s \end{cases}$$



$M \in p \Leftrightarrow M(1+2t, 2-t, 2t)$

$$\overrightarrow{MP_0} = \begin{pmatrix} -2t \\ t \\ -2t \end{pmatrix}, \quad \|MP_0\| = 3$$

$$\Rightarrow \sqrt{(-2t)^2 + t^2 + (-2t)^2} = 3$$

$\Rightarrow \quad 9t^2 = 9 \quad \Rightarrow t^2 = 1$

$\Rightarrow t = \pm 1$

Cas : $t = 1 \Rightarrow M(3, 1, 2)$

$$\overrightarrow{MP} = \begin{pmatrix} -2 \\ -1 \\ -1 \end{pmatrix} \Rightarrow \begin{cases} x = 1 - 2t \\ y = -t \\ z = 1 - t \end{cases}$$

(t) $t = -1 \Rightarrow M = (-1, 3, -2)$

$$\overrightarrow{MP} = \begin{pmatrix} 2 \\ -3 \\ 3 \end{pmatrix} \Rightarrow \begin{cases} x = 1 + 2t \\ y = -3t \\ z = 1 + 3t \end{cases}$$