Antiderivatives

$*\quad F(x) = \int \frac{x^3}{x^4-1}\,dx \qquad \int \frac{1}{x}\,dx = \ln|x|$

$t = x^4 - 1 \Rightarrow \frac{dt}{4} = x^3\,dx$

$F(t) = \int \frac{dt}{4t} = \frac{1}{4}\int \frac{dt}{t} = \frac{1}{4}\ln|t| + C$

$= \frac{1}{4}\ln|x^4-1| + C$

$I = \int \frac{3\cos x}{2+\sin x}\,dx$

$t = 2+\sin x \Rightarrow dt = \cos x\,dx$

$I = \int \frac{3\,dt}{t} = 3\ln|t| + C$

$= 3\ln|2+\sin x| + C$

$I = \int \frac{4x-1}{4x^2-2x+5}\,dx$

$t = 4x^2 - 2x + 5 \Rightarrow dt = (8x-2)\,dx$

$\Rightarrow \frac{dt}{2} = (4x-1)\,dx$

$I = \int \frac{dt}{2t} = \frac{1}{2}\ln|t| + C$

$= \frac{1}{2}\ln|4x^2-2x+5| + C$

$$I = \int \frac{dx}{e^x+1}$$

$$t = e^x + 1 \Rightarrow dt = e^x\,dx \Rightarrow dx = \frac{dt}{e^x}$$

$$\Rightarrow e^x = t-1 \longrightarrow \qquad dx = \frac{dt}{t-1}$$

$$I = \int \frac{dt}{(t-1)t} = \int \left(\frac{1}{t-1} - \frac{1}{t}\right)dt$$

$$= \int \frac{1}{t-1}\,dt - \frac{1}{t}\,dt$$

$$= \ln|t-1| - \ln|t| + C$$

$$= \ln|e^x+1-1| - \ln|e^x+1| + C$$

$$= \ln|e^x| - \ln|e^x+1| + C$$

$$= \ln\frac{e^x}{e^x+1} + C$$

$$I = \int \frac{e^{2x}\,dx}{e^x+1} = \int \frac{e^x \cdot e^x\,dx}{e^x+1}$$

$$t = e^x + 1 \Rightarrow dt = e^x\,dx$$

$$I = \int \frac{(t-1)}{t}\cdot dt = \int \left(1 - \frac{1}{t}\right) dt$$

$$= t - \ln|t| + C$$

$$= e^x + 1 - \ln|e^x+1| + C$$

$$a^m \cdot a^n = a^{m+n} \quad , \quad e^{2x} = e^x \cdot e^x$$

$$I = \int \frac{e^{2x}}{e^x+1}\,dx$$

$$t = e^x + 1 \Rightarrow e^x = t - 1$$

$$\Rightarrow e^{2x} = (t-1)^2$$

$$\cancel{2}\, \underbrace{e^{2x}\,dx} = \cancel{2}\,\underbrace{(t-1)\,dt}$$

$$I = \int \frac{(t-1)\,dt}{t}$$

X.

$$I = \int x\sqrt{x^2+3}\,dx \quad (t = \sqrt{\ })$$

$$t = \sqrt{x^2+3} \Rightarrow t^2 = x^2+3$$

$$\Rightarrow \cancel{2}t\,dt = \cancel{2}x\,dx$$

$$I = \int t \cdot t\,dt = \int t^2\,dt = \frac{t^3}{3} + C$$

$$= \frac{\sqrt{(x^2+3)^3}}{3} + C$$

$$I = \int \frac{x\,dx}{\sqrt{2x^2+3}}$$

$$t = \sqrt{2x^2+3} \Rightarrow t^2 = 2x^2+3$$
$$\Rightarrow 2t\,dt = 4x\,dx$$
$$\Rightarrow x\,dx = \frac{1}{2}t\,dt$$

$$I = \int \frac{\frac{1}{2}t\,dt}{t} = \int \frac{1}{2}dt = \frac{1}{2}t + C$$
$$= \frac{1}{2}\sqrt{2x^2+3} + C$$

$$I = \int \frac{x^3}{\sqrt{2-x^2}}\,dx$$

$$t = \sqrt{2-x^2} \Rightarrow t^2 = 2-x^2$$
$$\Rightarrow 2t\,dt = -2x\,dx$$

$$I = \int \frac{x^2 . x\,dx}{\sqrt{2-x^2}} = \int \frac{(2-t^2)(-t\,dt)}{t}$$
$$= \int (t^2-2)\,dt = \frac{t^3}{3} - 2t + C$$
$$= \frac{\sqrt{(2-x^2)^3}}{3} - 2\sqrt{2-x^2} + C.$$

$$I = \int \frac{dx}{x\ln x}$$

$$(\ln x)' = \frac{1}{x}.$$

$\boxed{t = \ln x} \Longrightarrow dt = \frac{1}{x}dx$

$$I = \int \frac{dt}{t} = \ln|t| + C$$

$$= \ln|\ln x| + C$$

e) $I = \int \frac{(2\ln x + 3)^2}{x} dx$ | $t = \ln x$

$\boxed{t = 2\ln x + 3} \Rightarrow dt = \frac{2}{x}dx$

$$\Rightarrow I = \int t^2 \frac{dt}{2} = \frac{1}{2}\frac{t^3}{3} + C$$

$$= \frac{t^3}{6} + C$$

$$= \frac{(2\ln x + 3)^3}{6} + C$$

f) $I = \int x \cdot e^{x^2+1} dx$ $(t = u)$

$t = x^2 + 1 \Rightarrow \frac{dt}{2} = x\,dx$

$$I = \int e^t \cdot \frac{dt}{2} = \frac{1}{2}e^t + C$$

$$= \frac{1}{2} e^{x^2+1} + C$$

$$\circledast\ I = \int (x-1)\, e^{x^2-2x+3}\, dx$$

$$t = x^2 - 2x + 3 \Rightarrow dt = (2x-2)\, dx$$

$$\Rightarrow \frac{dt}{2} = (x-1)\, dx$$

$$I = \int e^t \frac{dt}{2} = \frac{1}{2} e^t + C$$

$$= \frac{1}{2} e^{x^2-2x+3} + C$$

$$I = \int x(1-x)^{10}\, dx \quad [t = (\ldots)]$$

$$t = 1 - x \Rightarrow dt = -dx$$

$$I = \int (1-t) \cdot t^{10}\, (\ominus dt)$$

$$= \int (t^{11} - t^{10})\, dt$$

$$= \frac{t^{12}}{12} - \frac{t^{11}}{11} + C$$

$$= \frac{(1-x)^{12}}{12} - \frac{(1-x)^{11}}{11} + C$$

$$I = \int \underbrace{\sin^5 x} \cdot \underbrace{\cos x \, dx}$$

$$t = \underbrace{\sin x} \Rightarrow \underbrace{dt} = \underbrace{\cos x \, dx}$$

$$I = \int t^5 \, dt = \frac{t^6}{6} + C$$

$$= \frac{\sin^6 x}{6} + C$$

$$\cos^2 x = 1 - \sin^2 x$$

$$* \; I = \int \sin^2 x \cdot \cos^3 x \, dx$$

$$t = \sin x \Rightarrow dt = \cos x \, dx$$

$$I = \int \sin^2 x \cdot \cos^2 x \cdot \underbrace{\cos x \, dx}$$

$$= \int t^2 (1 - t^2) \, dt$$

$$= \int (t^2 - t^4) \, dt$$

$$= \frac{t^3}{3} - \frac{t^5}{5} + C$$

$$= \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + C$$

$$I = \int \frac{\cot x}{\sin^2 x}\,dx$$

$$t = \cot x \Rightarrow dt = \frac{-1}{\sin^2 x}\,dx$$

$$(\cot x)' = -\frac{1}{\sin^2 x}$$

$$(\tan x)' = \frac{1}{\cos^2 x}$$

$$I = \int t \cdot (-dt)$$

$$= \frac{-t^2}{2} + C = -\frac{\cot^2 x}{2} + C$$

$$I = \int \tan^3 x\,dx \qquad (\cos x \neq 0)$$

$$I = \int \frac{\sin^3 x}{\cos^3 x}\,dx$$

$$t = \cos x \rightarrow dt = \underline{-\sin x\,dx}$$

$$I = \int \frac{\sin^2 x \cdot \sin x\,dx}{\cos^3 x}$$

$$= \int \frac{(1-\cos^2 x)\sin x\,dx}{\cos^3 x}$$

$$= \int \frac{1-t^2}{t^3}(-dt)$$

$$\int t^{-3} = \frac{t^{-2}}{-2}$$

$$= \int \left(\frac{-1}{t^3} + \frac{1}{t} \right) dt \qquad -\frac{1}{2t^2}$$

$$= \frac{1}{2t^2} + \ln|t| + C$$

$$= \frac{1}{2\cos^2 x} + \ln|\cos x| + C$$

$$\underline{I} = \int x \cos x^2 \, dx$$

$$t = x^2 \Rightarrow dt = 2x\,dx$$

$$\underline{I} = \int \cos t \cdot \frac{dt}{2} = \frac{1}{2} \sin t + C$$

$$= \frac{1}{2} \sin x^2 + C$$

$$\underline{I} = \int \frac{1}{\sqrt{x+9} - \sqrt{x}} \, dx$$

$$\underline{I} = \int \frac{\sqrt{x+9} + \sqrt{x}}{9} \, dx$$

$$= \frac{1}{9} \int \left((x+9)^{1/2} + x^{1/2} \right) dx$$

$$= \frac{1}{9}\,\frac{(x+9)^{3/2}}{3/2} + \frac{x^{3/2}}{3/2} + C$$