tangent line: $y = f'(x_0)(x - x_0) + y_0$

$f'(x_0)$ ↓ Slope

EX: Find the **Slope** of the tangent line to the Curve:

$$y^3 + x^2 y = 10 \quad \text{at } \underline{(1;2)}$$

$$\frac{d}{dx}(y^3 + x^2 y) = \frac{d}{dx}(10) \qquad (u.v)' = u'v + uv'$$

$$\Leftrightarrow \quad 3y^2 \frac{dy}{dx} + 2xy + x^2 \frac{dy}{dx} = 0$$

$$\Leftrightarrow \quad \frac{dy}{dx} = \frac{-2xy}{3y^2 + x^2}$$

$$\text{Slope} = \left.\frac{-2xy}{3y^2 + x^2}\right|_{\substack{x=1 \\ y=2}}$$

$$\text{Slope} = \frac{-4}{13}$$

$\Rightarrow$ tangent line: $y = \frac{-4}{13}(x - 1) + 2$

$$y = -\frac{4}{13}x - \dots$$

b) $x^4 y - x y^3 = -2$ at $(-1; -1)$

$$\frac{d}{dx}(x^4 y - x y^3) = \frac{d}{dx}(-2)$$

$$\Leftrightarrow \quad 4x^3 y + x^4 \frac{dy}{dx} - y^3 - 3xy^2 \frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = \left.\frac{-4x^3 y + y^3}{x^4 - 3xy^2}\right|_{\substack{x=-1 \\ y=-1}}$$

Slope $= -5/4$

$$y = -\frac{5}{4}(x+1) - 1$$

c) $xy + \sin(x) = 1$ at $(\frac{\pi}{2}; 0)$

$$y + x\frac{dy}{dx} + \cos x = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = \left.\frac{-\cos x - y}{x}\right|_{\substack{x=\pi/2 \\ y=0}}$$

Slope $= 0$

$y = 0$

$y = f'(x_0)(x - x_0) + y_0$

0

d) $xy^2 + \sin(\pi y) - 2x^2 = 10$ at $(2;-3)$

$\Leftrightarrow y^2 + 2xy\dfrac{dy}{dx} + \pi\cos(\pi y)\dfrac{dy}{dx} - 4x = 0$

$$\Leftrightarrow \frac{dy}{dx} = \frac{4x - y^2}{2xy + \pi\cos(\pi y)}\Bigg|_{\substack{x=2\\ y=-3}}$$

$$\Rightarrow \text{Slope} = \frac{1}{\pi + 12} \approx$$

tangent line: $y = \dfrac{1}{\pi+12}(x-2) - 3$

$$y = \frac{1}{\pi+12}x \; \cdots$$

$$\#\; (\sin u)' = u' \cdot \cos u$$

e) $\tan(x.y) = y$ at $(\pi/4 ; 1)$

$$\Rightarrow \frac{1}{\cos^2(xy)} \cdot \left(y + x\frac{dy}{dx}\right) = \frac{dy}{dx} \qquad \left((\tan u)' = \frac{u'}{\cos^2 u} \right)$$

$$\Rightarrow \frac{y}{\cos^2(xy)} + \frac{x}{\cos^2(xy)}\frac{dy}{dx} = \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-y}{\cos^2(xy)} : \left(\frac{x}{\cos^2(xy)} - 1\right) \Bigg|_{\substack{x=\pi/4 \\ y=1}}$$

$$\text{Slope} = \frac{-4}{\pi-2}$$

$$y = \frac{-4}{\pi-2}\left(x - \frac{\pi}{4}\right) + 1$$

$$\left(\cot(u)\right)' = -\frac{1}{\sin^2(u)} \cdot u'$$

$$(\ln x)' = \frac{1}{x} \qquad (\ln u)' = \frac{u'}{u}$$

$$(\log_a x)' = \frac{1}{x \ln a} \qquad (\log_a u)' = \frac{u'}{u \cdot \ln a}$$

$$(a^x)' = a^x \cdot \ln a \qquad (a^u)' = a^u \cdot \ln a \cdot u'$$

$$(\sqrt{x})' = \frac{1}{2\sqrt{x}} \qquad (\sqrt{u})' = \frac{u'}{2\sqrt{u}}$$

$$(e^x)' = e^x \qquad (e^u)' = u' \cdot e^u$$

Ex: Find the Derivative of

a) $y = \ln(x^3+3x-4)$ $\qquad (\ln u)' = \dfrac{u'}{u}$

$$y' = \frac{3x^2+3}{x^3+3x-4}$$

b) $f(x) = \ln\left(\dfrac{x^2 \sin x}{2x+1}\right) =$ $\qquad \left(\dfrac{u}{v}\right)' = \dfrac{u'v - uv'}{v^2}$

$$f'(x) = \frac{\boxed{2x \sin x + x^2 \cos x}\,(2x+1) - x^2 \sin x \, 2}{(2x+1)^2} \cdot \frac{2x+1}{x^2 \sin x}$$

$=$

Find the slope of the tangent line to the graph of

$y = \log_2(3x+1)$ at $x = 1$

$$y' = \frac{3}{(3x+1)\ln 2}\Big|_{x=1}$$

$$\Rightarrow \underline{\text{Slope}} = \frac{3}{4\ln 2} = \boxed{\frac{3}{\ln 16}}$$

$$\underline{y = \frac{3}{\ln 16}(x-1) + \log_2(3\cdot 1+1)}$$

$\ln \boxed{e}$

$\log_{\boxed{10}}$ ✓

*: Find velocity, Acceleration
(vận tốc) (gia tốc)

∘) Velocity: the rate of change (t)

$$V(t) = s'(t)$$

∘) Acceleration: the rate of change of Velocity

$$a(t) = V'(t) = s''(t)$$

EX: The position of a particle is given by the equation: $s(t) = t^3 - 6t^2 + 9t$, where t is measured in seconds and s in meters.

a) What is the Velocity at the time t and after 2s?

$$V(t) = s'(t) = 3t^2 - 12t + 9$$

$$\Rightarrow V(2) = 3.2^2 - 12.2 + 9 = \underline{-3\ m/s.}$$

b) Find the acceleration at time after 4s?

$$a(t) = v'(t) = s''(t) = 6t - 12$$

$$\Rightarrow \quad a(4) = 6.4 - 12 = 12 \ m/s^2$$

EX 2: $s(t) = t^4 - 3t^3 + 20t^2 + 20t$

$v(20) = ?$

$a(0) = ?$

EX 3: $A(t) = 1000\, e^{0,3t}$

t : days

$A(t)$: population

Show that the ratio of the rate of change of the population $A'(t)$ to the population $A(t)$

$\underline{A(t) \text{ is}}$ $\underline{\text{Constant}}$.

$$A'(t) = 1000 \cdot 0,3 \ e^{0,3t}$$

$$= 300 \ e^{0,3t}$$

$$\frac{A'(t)}{A(t)} = \frac{300 \cdot e^{0,3t}}{1000 \cdot e^{0,3t}} = \boxed{0,3}$$

EX: $\boxed{f(x) = \dfrac{x^2-1}{x^2+1}}$ (Substitution Rule / QT thay thế)

Find $f(x) + f\left(\frac{1}{x}\right) = ?$

$$\left(f(x) = \frac{\text{nó}^2 - 1}{\text{nó}^2 + 1}\right) \quad \Bigg| \quad \boxed{f\left(\frac{1}{x}\right) = \frac{\left(\frac{1}{x}\right)^2 - 1}{\left(\frac{1}{x}\right)^2 + 1}} \qquad \frac{\text{nó}^2 - 1}{\text{nó}^2 + 1}$$

$$f(x) + f\left(\frac{1}{x}\right) = \frac{x^2-1}{x^2+1} + \frac{\frac{1}{x^2} - 1}{\frac{1}{x^2} + 1} = \boxed{\frac{x^2-1}{x^2+1} + \frac{1-x^2}{1+x^2}}$$

(tính theo x)

EX: If $f(x) = \dfrac{x}{x-1} = \dfrac{1}{y}$ then $f(y) = ?$

$$\left(f(\text{nó}) = \frac{\text{nó}}{\text{nó} - 1}\right)$$

$$y = \frac{x-1}{x}$$

$$f(y) = f\left(\frac{x-1}{x}\right) = \frac{\frac{x-1}{x}}{\frac{x-1}{x} - 1} = 1 - x$$