Chapter 1: Function and Graphs

Department of Mathematics, FPT University

Chapter 1: Function and Graphs

Objectives

- Four ways to represent a function
- Basis functions and the transformations of functions

1.1

Review of Functions

FUNCTION

- A function f is a rule that assigns to each element x in a set D exactly one element, called f(x), in a set E.
- The set D is called the domain of the function f.

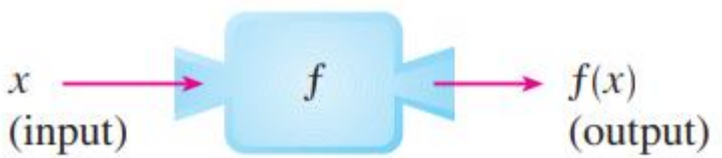


FIGURE 2
Machine diagram for a function f

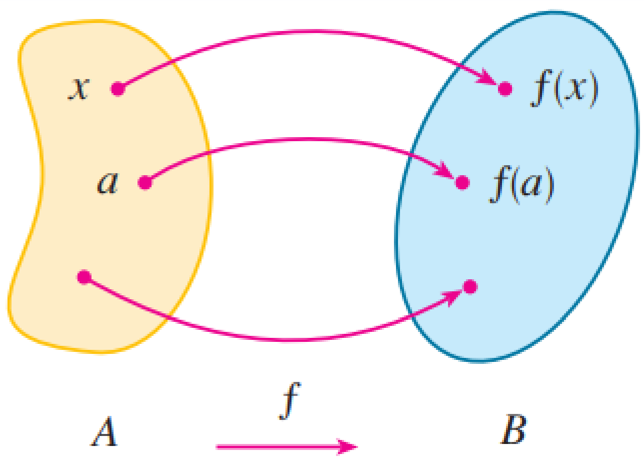
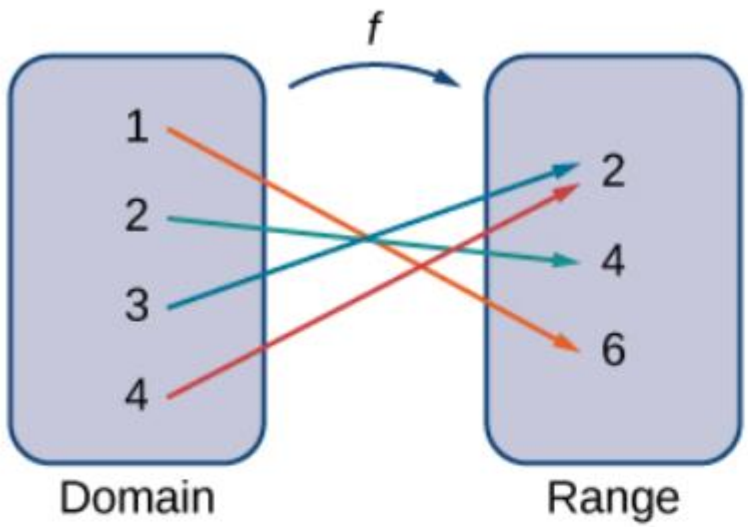


FIGURE 3
Arrow diagram for f

FUNCTION

- The range of f is the set of all possible values of f(x) as x varies throughout the domain.



FUNCTION

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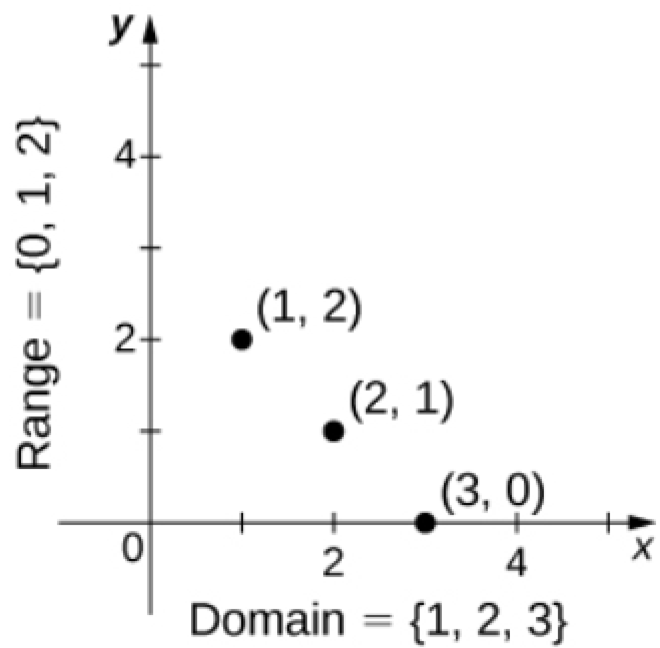
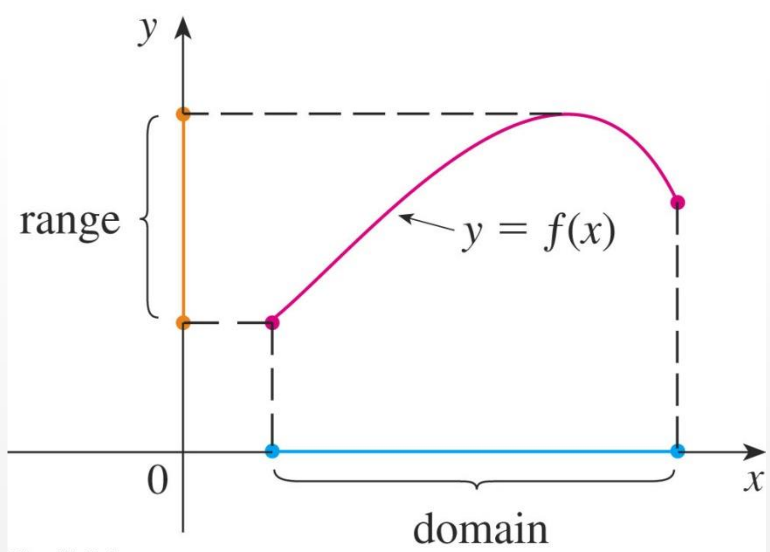


Figure 1.5 Here we see a graph of the function f with domain $\{1, 2, 3\}$ and rule $f(x) = 3 - x$. The graph consists of the points $(x, f(x))$ for all x in the domain.

GRAPH

- The **graph** of f is the **set** *of all points (x, y) in the coordinate plane such that y = f(x) and x is in the domain of f.*

- The graph of *f* also allows us to picture:
  - The **domain of *f*** on the *x*-axis
  - **Its range** on the *y*-axis



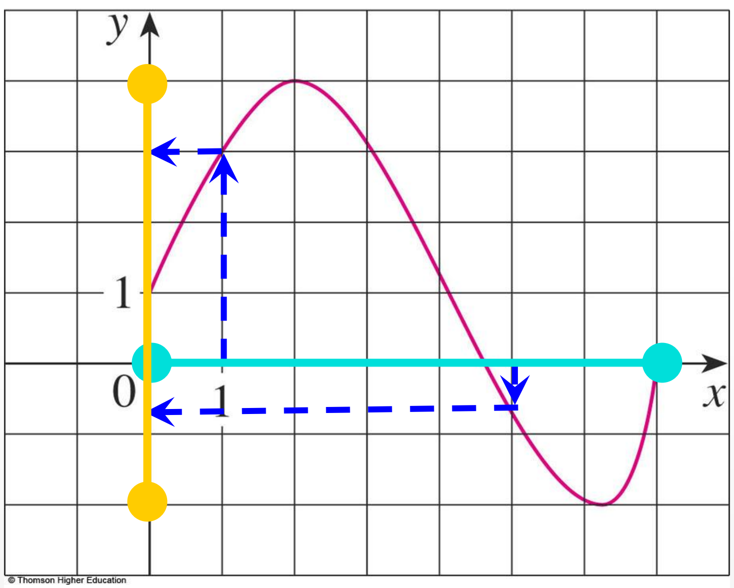
GRAPH

Example 1

The graph of a function *f* is shown.

a. Find the values of *f(1) and f(5).*

b. What is the domain and range of *f*?



f(1) = 3

f(5) = - 0.7

D = [0, 7]

Range(f) = [-2, 4]

DISCUSSION

Find the domain and the range of the following functions:

a) $f(x)=\sqrt{5-2x}$

b) $g(x)=\dfrac{4x-1}{2x+3}$

c) $h(x)=\sqrt{16-x^2}$

d) $q(x)=x^2+4x+7$

REPRESENTATIONS OF FUNCTIONS

There are four possible ways to represent a function:

- Algebraically (by an explicit formula)
- Visually (by a graph)
- Numerically (by a table of values)
- Verbally (by a description in words)

Example 2

The human population of the world P depends on the time t.

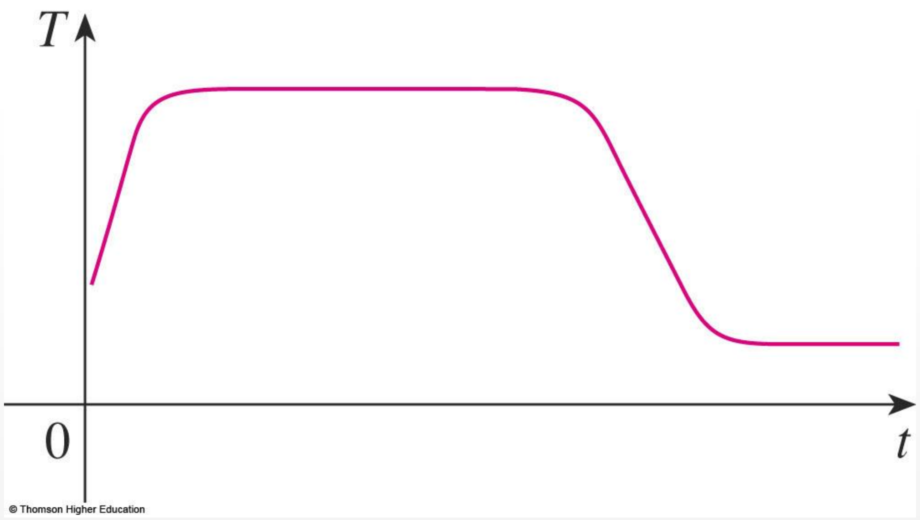
- The table gives estimates of the world population $P(t)$ at time t, for certain years.
- However, for each value of the time t, there is a corresponding value of P, and we say that P is a function of t.

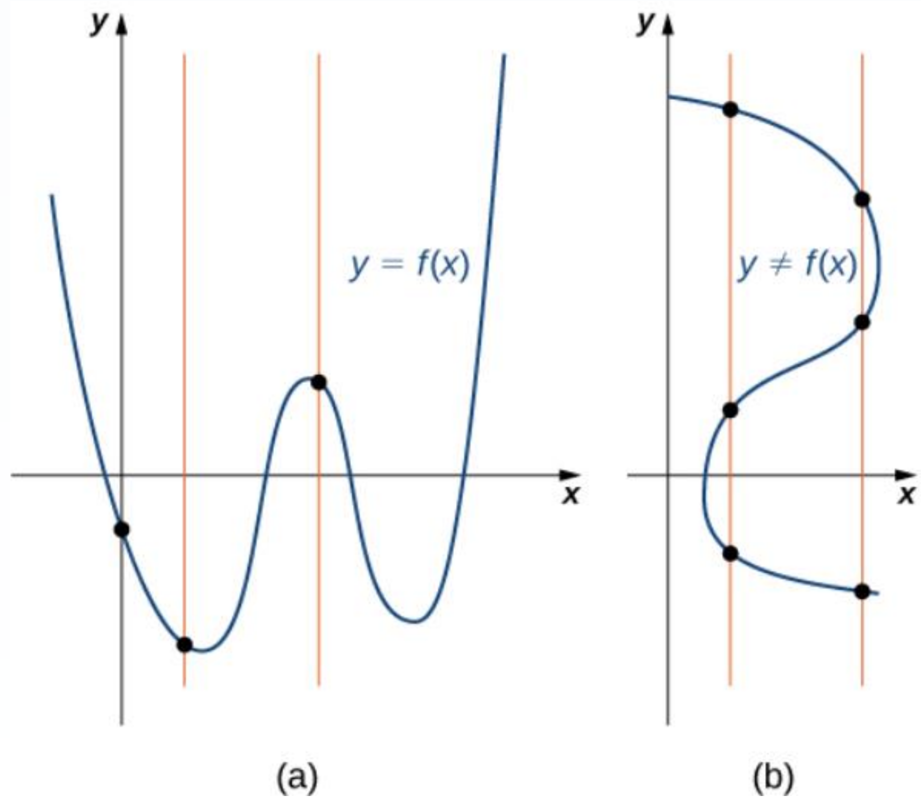
Year	Population (millions)
1900	1650
1910	1750
1920	1860
1930	2070
1940	2300
1950	2560
1960	3040
1970	3710
1980	4450
1990	5280
2000	6080

Example 3

"When you turn on a hot-water faucet, the temperature T of the water depends on how long the water has been running".

Draw a rough graph of T as a function of the time t that has elapsed since the faucet was turned on.



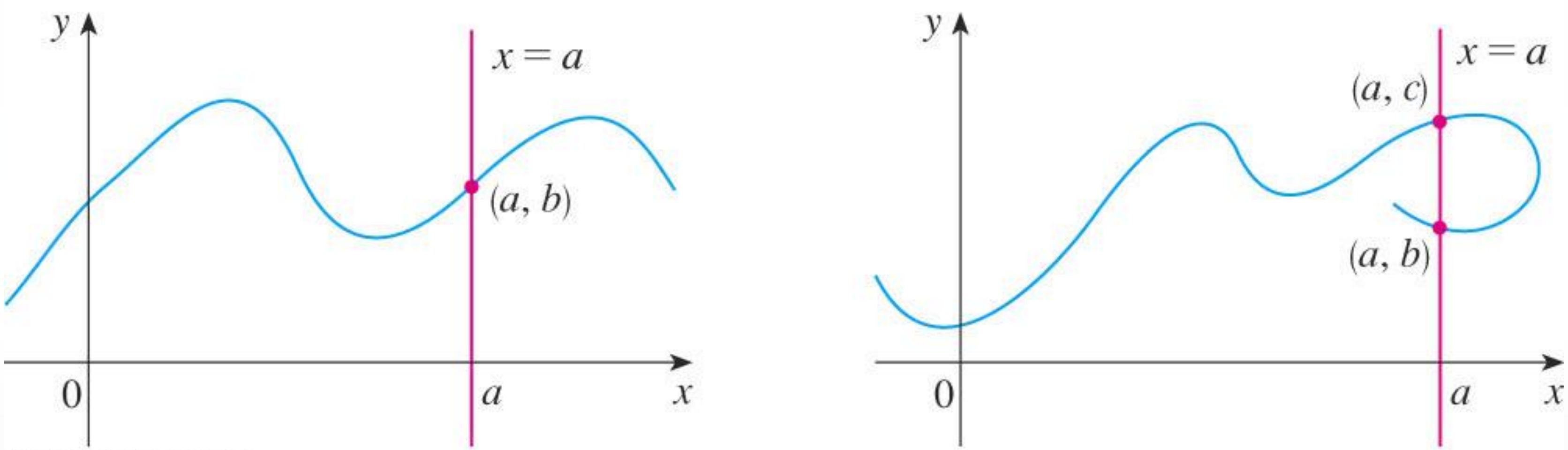


Rule: The vertical line test

A curve in the *xy-plane* is the graph of a function of *x* if and only if **no vertical line** intersects the curve **more than once**.

Rule: The vertical line test

The reason for the truth of the Vertical Line Test can be seen in the figure.



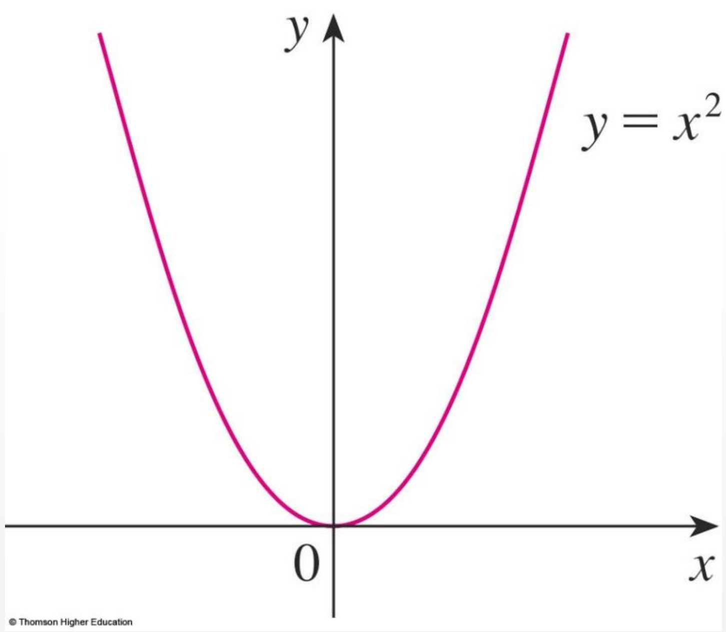
INCREASING AND DECREASING FUNCTIONS

- A function f is called **increasing on an interval I** if:

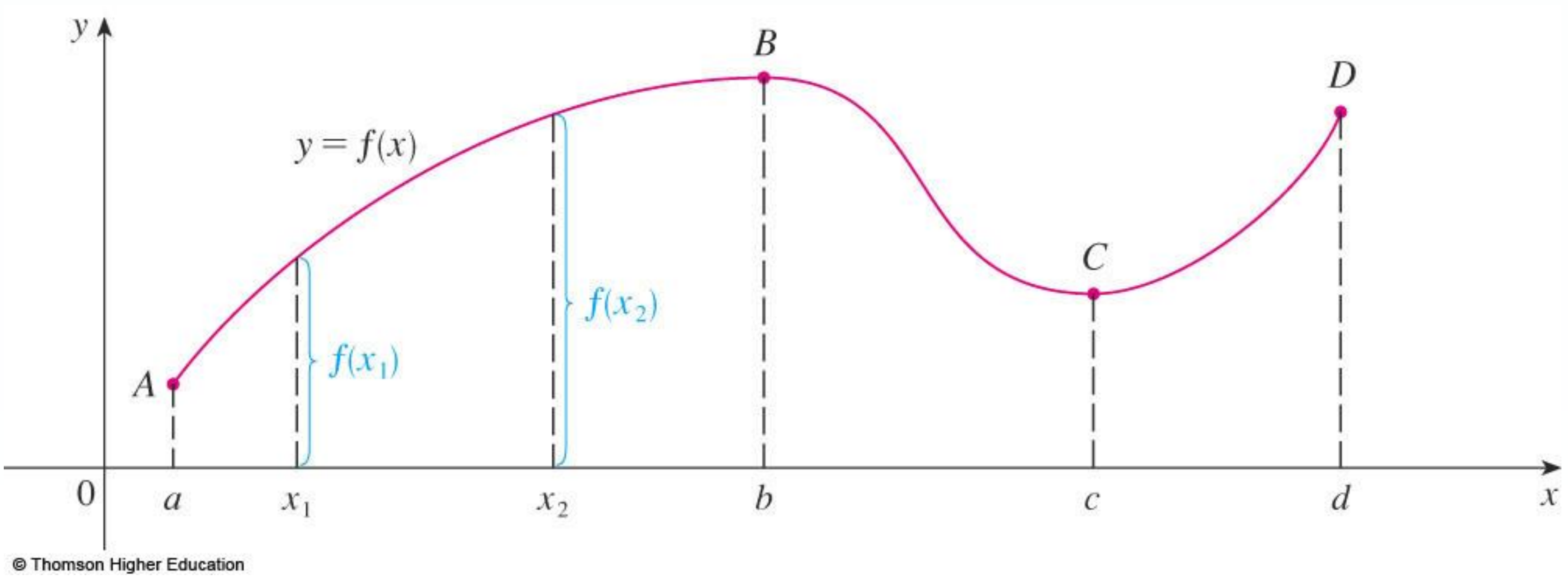
$$f(x_1) < f(x_2) \qquad \text{whenever } x_1 < x_2 \text{ in } I$$

- It is called **decreasing on I** if:

$$f(x_1) > f(x_2) \qquad \text{whenever } x_1 < x_2 \text{ in } I$$



INCREASING AND DECREASING FUNCTIONS



The function f is said to be

- increasing on the interval [a, b],
- decreasing on [b, c],
- and increasing again on [c, d].

SYMMETRY: EVEN FUNCTION

If a function *f* satisfies:

$$f(-x) = f(x), \text{ for all } x \text{ in } D$$

then f is called an **even function.**

- The geometric significance of an even function is that its graph is **symmetric with respect to the *y*-axis**.

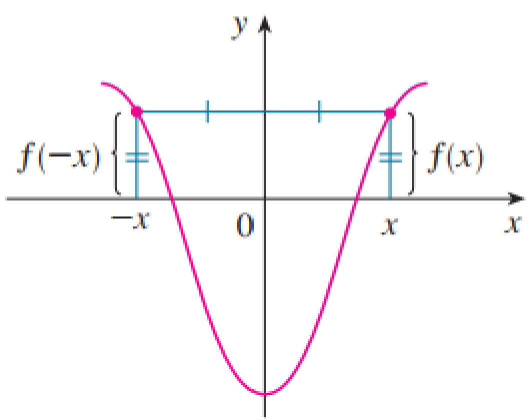
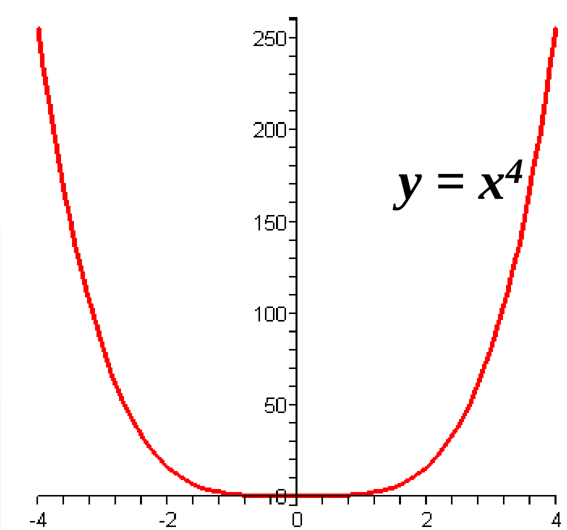


FIGURE 23
An even function



SYMMETRY: ODD FUNCTION

If f satisfies:

$f(-x) = -f(x)$, for all x in D

then f is called an **odd function**.

- The graph of an odd function is **symmetric about the origin**.

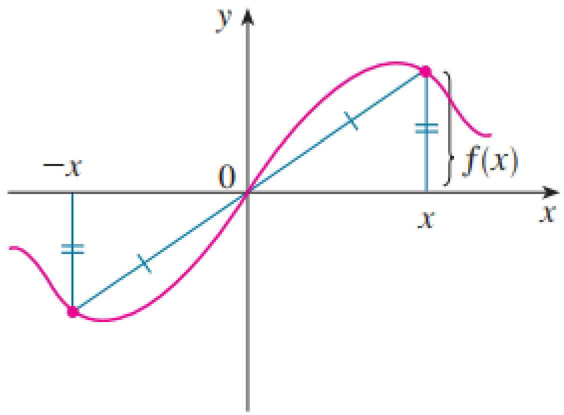


FIGURE 24
An odd function

Example 3

Let f is an **odd function**. If (-3,5) is in the graph of f then which point is also in the graph of f?

a. (3,5) b. (-3,-5) c. (3,-5) d. All of the others

Answer: c

Example 4

Suppose f is an odd function and g is an even function.

What can we say about the function f.g defined by $(f.g)(x)=f(x)g(x)$?

Prove your result.

QUIZ QUESTIONS

1) If f is a function then $f(x+2)=f(x)+f(2)$

a. True

b. False

2) If $f(s)=f(t)$ then $s= t$

a. True

b. False

3) Let f be a function. We can find s and t such that $s=t$ and $f(s)$ is not equal to $f(t)$

a. True

b. False

COMBINATIONS OF FUNCTIONS

- Two functions *f* and *g* can be combined to form new functions:
  - $(f + g)x = f(x) + g(x)$
  - $(f - g)x = f(x) - g(x)$
  - $(fg)(x) = f(x)g(x)$
  - $(f \circ g)(x) = f(g(x))$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

QUIZ QUESTIONS

Let $h(x)=f(g(x))$.

1) If $g(x)=x-1$ and $h(x)=3x+2$ then $f(x)$ is:

a. $3x+3$ b. $3x+4$ c. $3x+1$ d. None of them

2) If $h(x)=3x+2$ and $f(x)=x-1$ then $g(x)$ is:

a. $3x+3$ b. $3x+4$ c. $3x+1$ d. None of them

Answer: 1) d 2) a

QUIZ QUESTIONS

1) If f and g are functions, then *(f o g) = (g o f)*

a. True

b. False

2)

x	1	2	3	4	5	6
$f(x)$	3	2	1	0	1	2
$g(x)$	6	5	2	3	4	6

(f o g)(2) is

a. 5

b. 1

c. 2

d. None of the others

Chapter 1: Function and Graphs

1.2

BASIC CLASSES OF FUNCTIONS

When we say that *y* is a **linear function** of *x*, we mean that the graph of the function is a line.

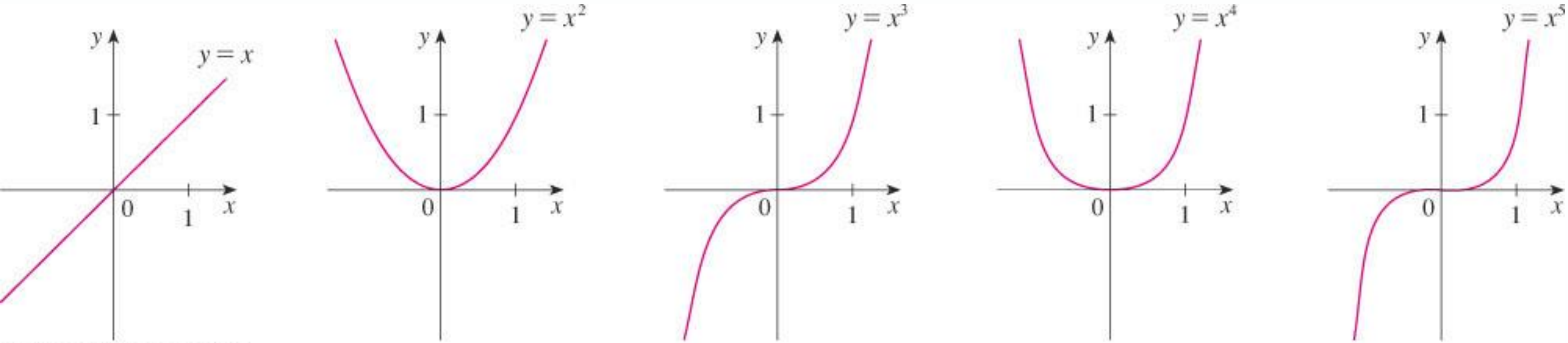
- So, we can use the slope-intercept form of the equation of a line to write a formula for the function as

$$y = f(x) = mx + b$$

where *m* is the slope of the line and *b* is the *y*-intercept.

ALGEBRAIC FUNCTIONS

LINEAR MODELS



A function of the form $f(x) = x^a$, where a is constant, is called

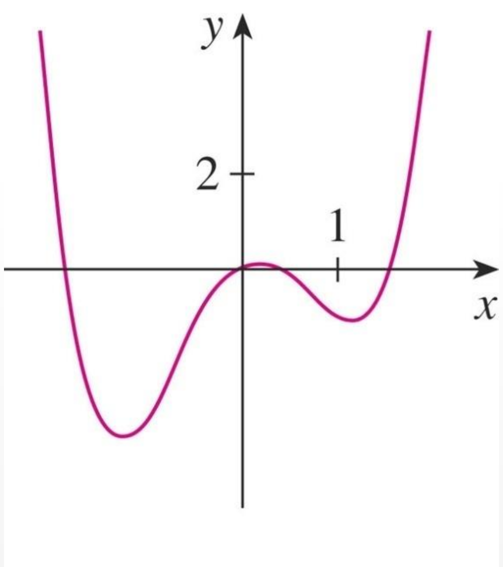
a power function.

POLYNOMIALS

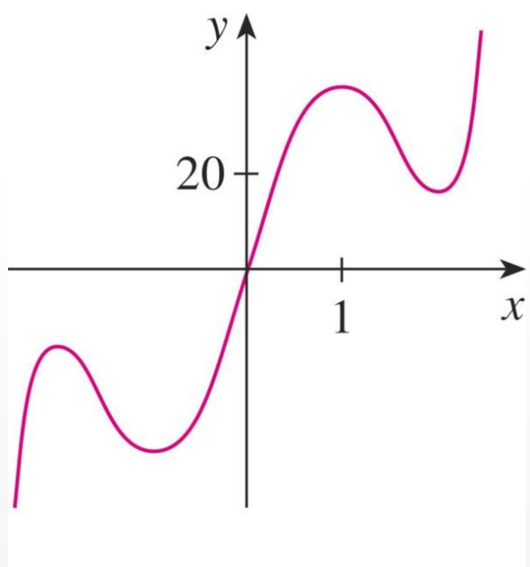
A function P is called a polynomial if

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$$

where n is a nonnegative integer and the numbers $a_0, a_1, a_2, \ldots, a_n$ are constants called the coefficients of the polynomial.



(b) $y = x^4 - 3x^2 + x$



(c) $y = 3x^5 - 25x^3 + 60x$

RATIONAL FUNCTIONS

A rational function f is a ratio of two polynomials

$$f(x) = \frac{P(x)}{Q(x)}$$

where P and Q are polynomials.

- The domain consists of all values of x such that $Q(x) \neq 0$.

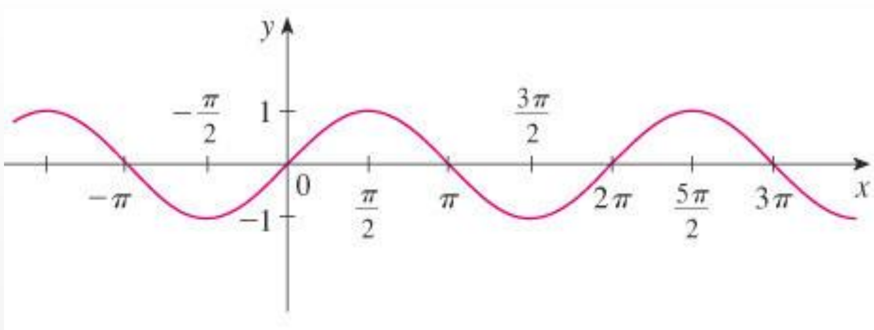
TRANSCENDENTAL FUNCTIONS

TRIGONOMETRIC FUNCTIONS

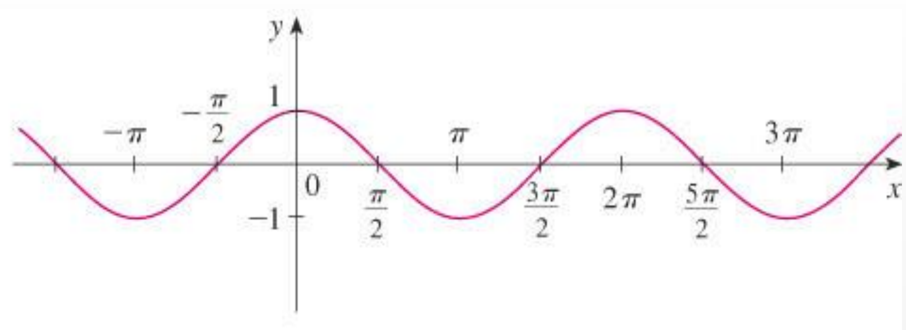
- f(x) = sin x
- g(x) = cos x

$$D = (-\infty, \infty) \qquad Range = [-1,\ 1]$$

$$\sin(x + k2\pi) = \sin x \qquad \cos(x + k2\pi) = \cos x; \quad k \in Z$$



(a) $f(x) = \sin x$



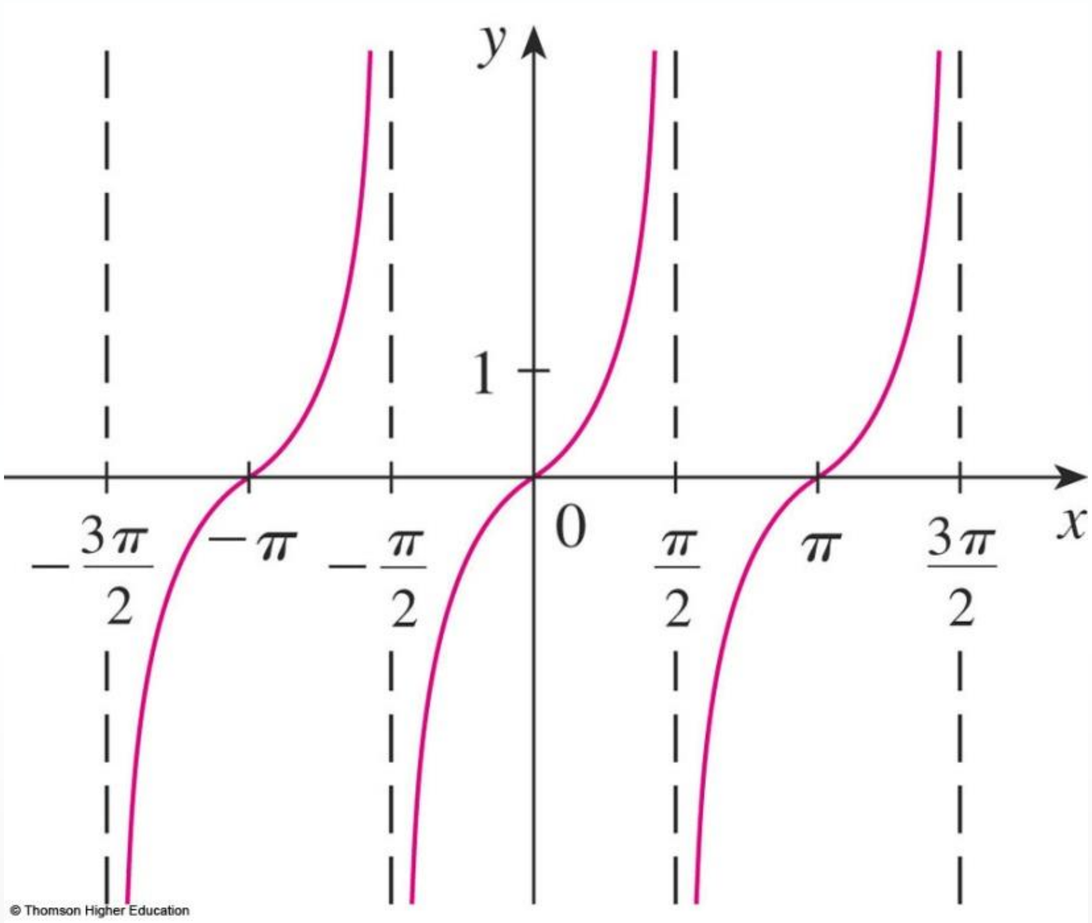
(b) $g(x) = \cos x$

TRANSCENDENTAL FUNCTIONS

TRIGONOMETRIC FUNCTIONS

$$\tan x = \frac{\sin x}{\cos x}$$

- $x \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \cdots$
- $R = (-\infty, \infty)$
- $\tan(x + k\pi) = \tan x; \quad k \in Z$



TRIGONOMETRIC FUNCTIONS

The reciprocals of the sine, cosine, and tangent functions are

$$\cos ecx = \frac{1}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

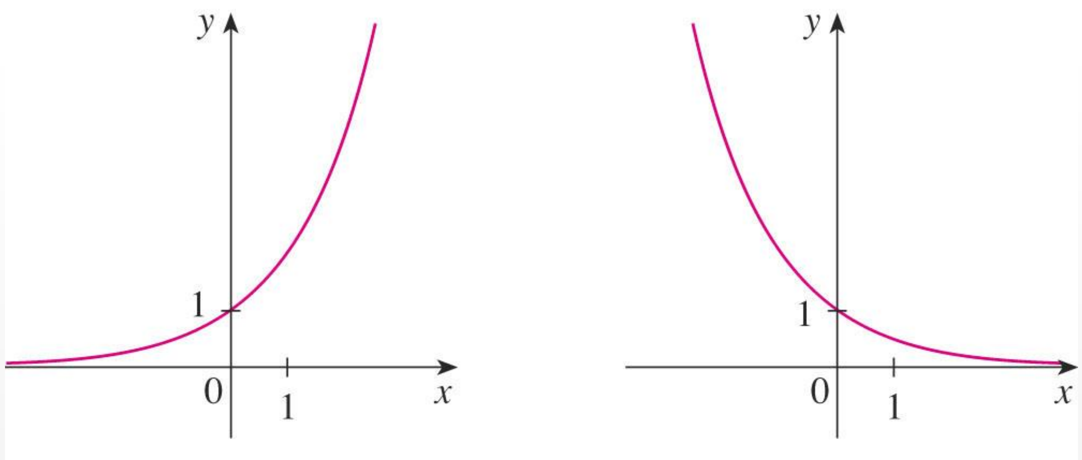
$$\cot anx = \frac{1}{\tan x}$$

TRANSCENDENTAL FUNCTIONS

EXPONENTIAL FUNCTIONS

The **exponential functions** are the functions of the form $f(x) = a^x$, where the base *a* is a positive constant.

- The graphs of $y = 2^x$ and $y = (0.5)^x$ are shown.
- In both cases, the domain is (- ¥ , ¥) and the range is (0, ¥).

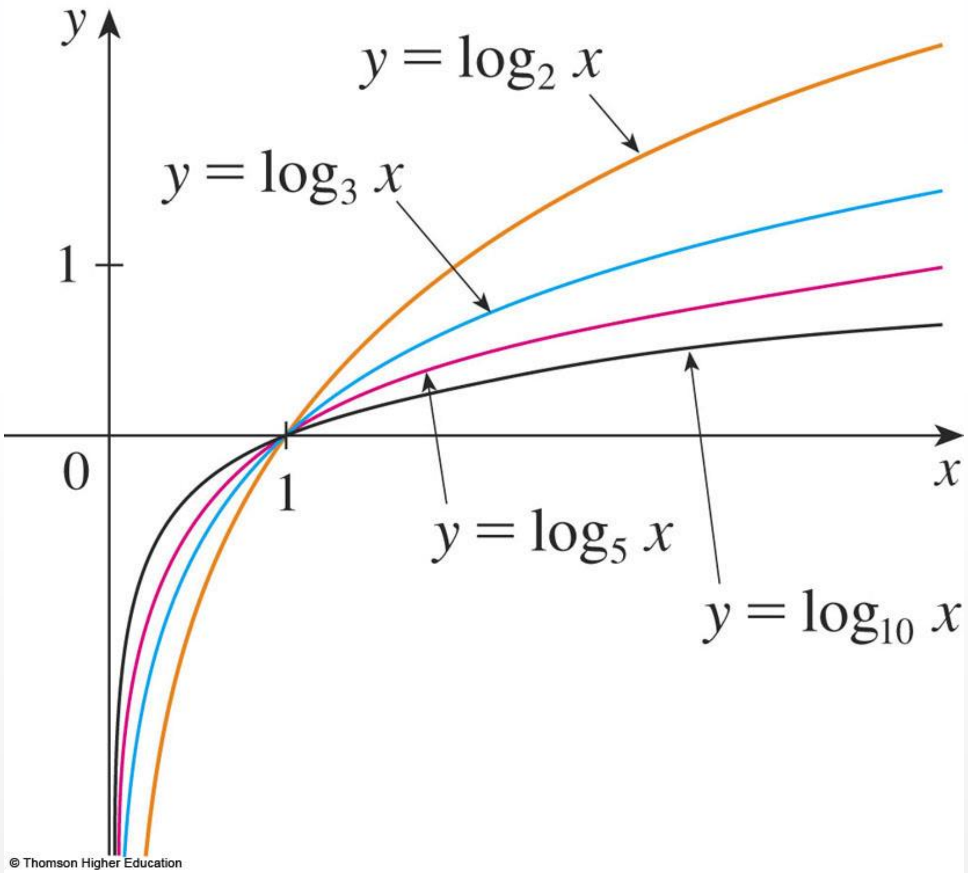


(a) $y = 2^x$ (b) $y = (0.5)^x$

TRANSCENDENTAL FUNCTIONS

LOGARITHMIC FUNCTIONS

The logarithmic functions $f(x) = \log_a x$, where the base *a* is a positive constant, are the inverse functions of the exponential functions.

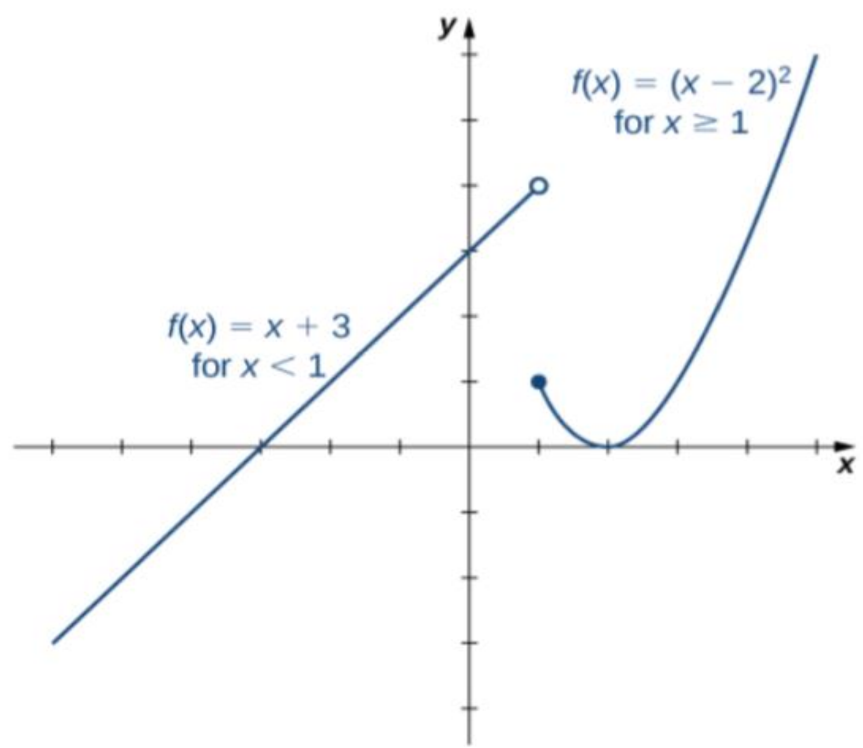


The figure shows the graphs of four logarithmic functions with various bases.

PIECEWISE-DEFINED FUNCTIONS

Example:

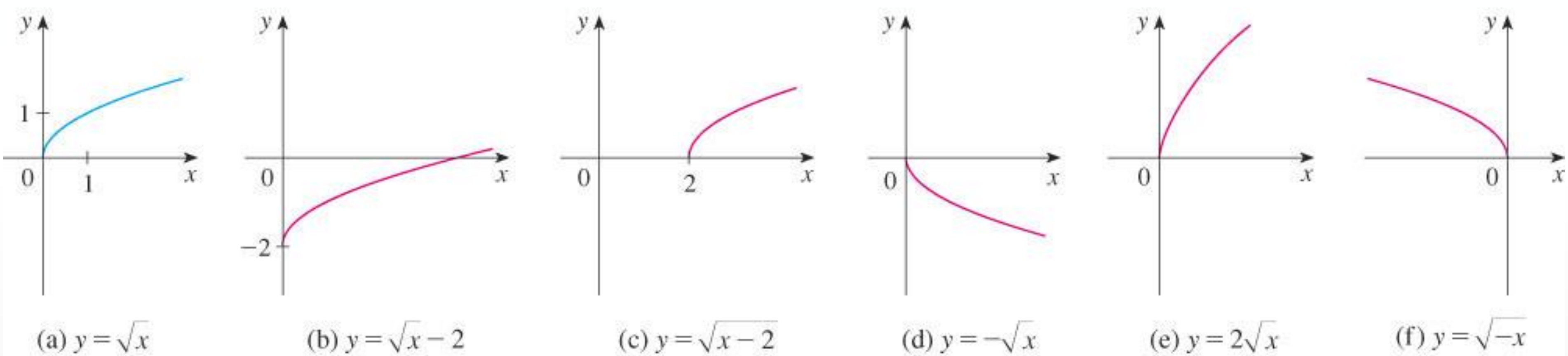
$$f(x)=\begin{cases} x+3, & x<1 \\ (x-2)^2, & x\geq 1 \end{cases}.$$



TRANSFORMATIONS OF FUNCTION

Label the following graph from the graph of the function y=f(x) shown in the part (a)

y=f(x)-2, y=f(x-2), y=-f(x), y=2f(x), y=f(-x)?



SHIFTING

Why don't we consider the case c<0?

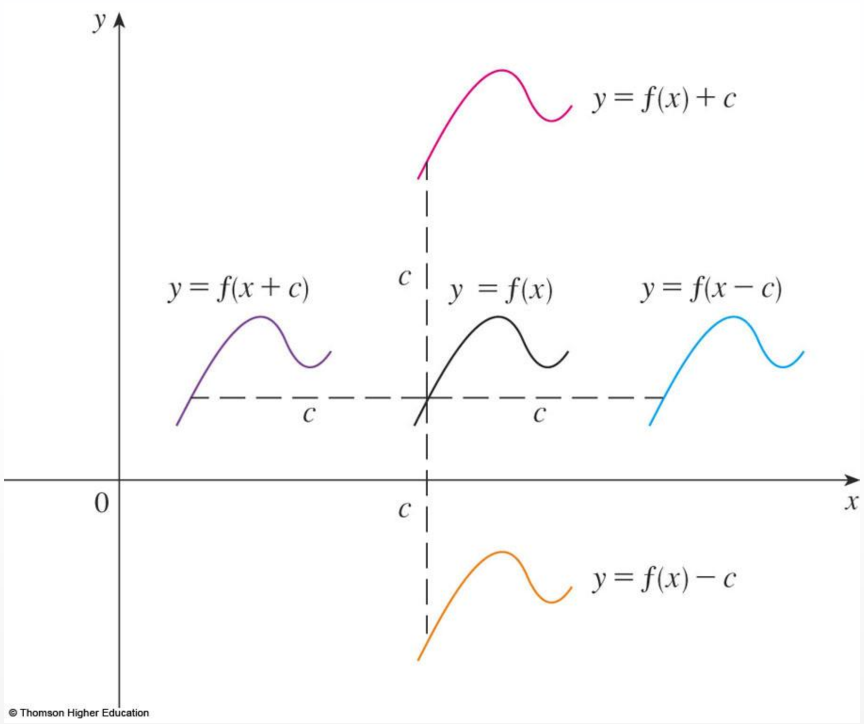
Suppose c > 0.

- *To obtain the graph of y = f(x) + c,*

shift the graph of y = f(x) a distance c units upward.

- *To obtain the graph of y = f(x) - c,*

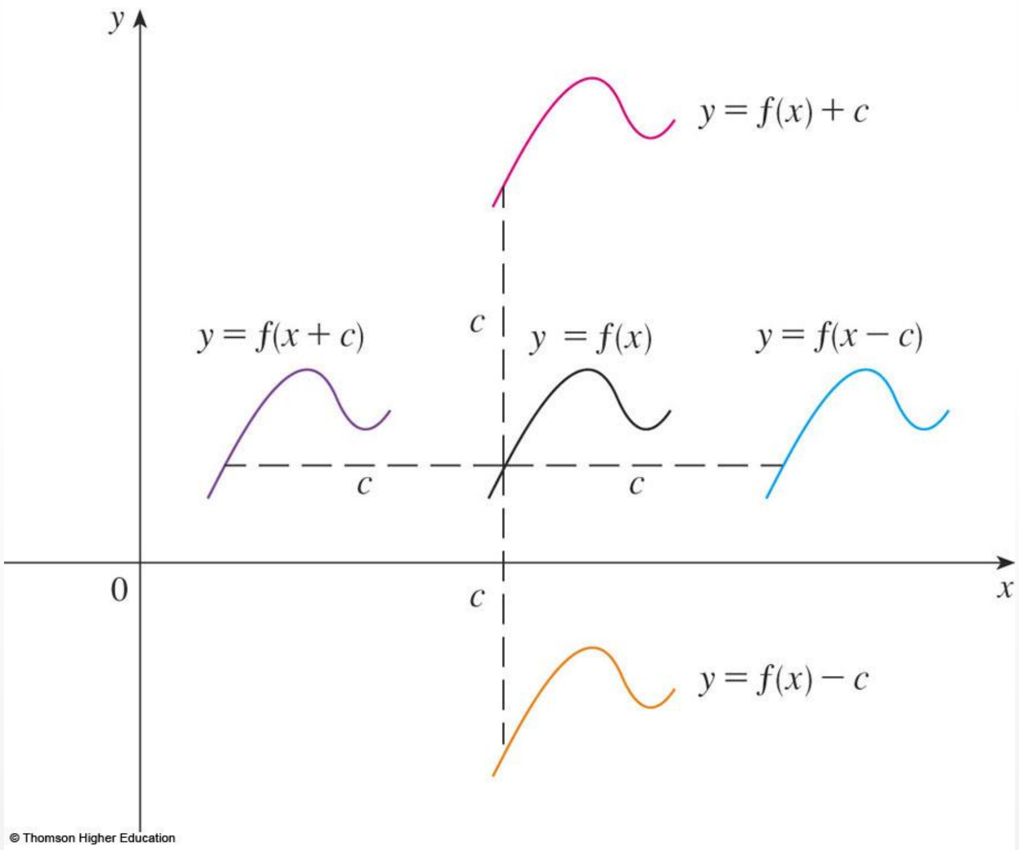
shift the graph of y = f(x) a distance c units downward.



SHIFTING

- *To obtain the graph of $y = f(x - c)$, shift the graph of $y = f(x)$ a distance c units to the right.*

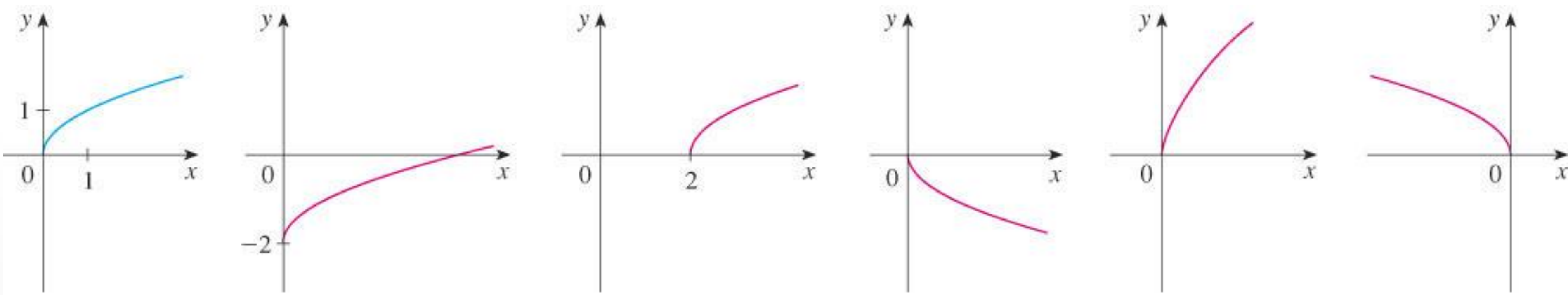
- *To obtain the graph of $y = f(x + c)$, shift the graph of $y = f(x)$ a distance c units to the left.*



NEW FUNCTIONS FROM OLD FUNCTIONS

Label the following graph from the graph of the function y=f(x) shown in the part (a)

y=f(x)-2, y=f(x-2), y=-f(x), y=2f(x), y=f(-x)?



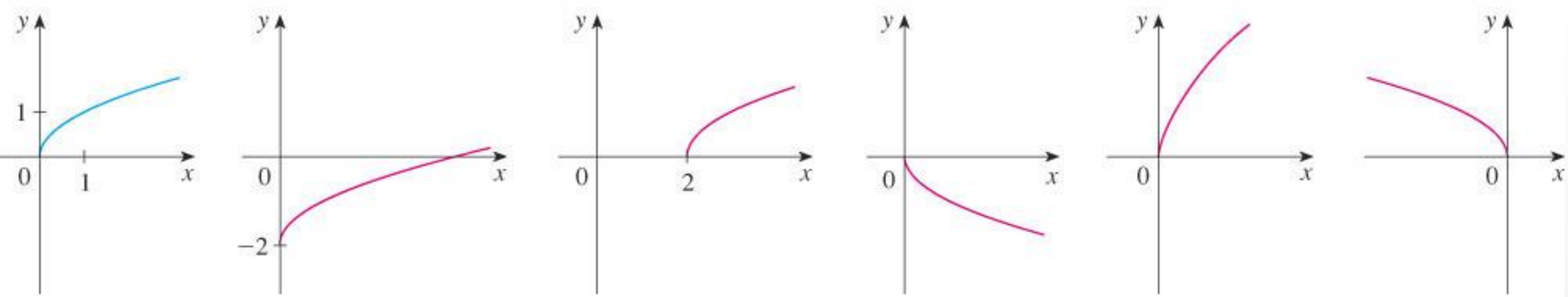
(a) $y=\sqrt{x}$ (b) $y=\sqrt{x}-2$ (c) $y=\sqrt{x-2}$ (d) $y=-\sqrt{x}$ (e) $y=2\sqrt{x}$ (f) $y=\sqrt{-x}$

NEW FUNCTIONS FROM OLD FUNCTIONS

Label the following graph from the graph of the function $y=\sqrt{x}$ shown in the part (a):

y=f(x)-2, y=f(x-2), y=-f(x), y=2f(x), y=f(-x)?

- $y=\sqrt{x}-2$ by shifting 2 units downward.
- $y=\sqrt{x-2}$ by shifting 2 units to the right.



(a) $y=\sqrt{x}$ (b) $y=\sqrt{x}-2$ (c) $y=\sqrt{x-2}$ (d) $y=-\sqrt{x}$ (e) $y=2\sqrt{x}$ (f) $y=\sqrt{-x}$

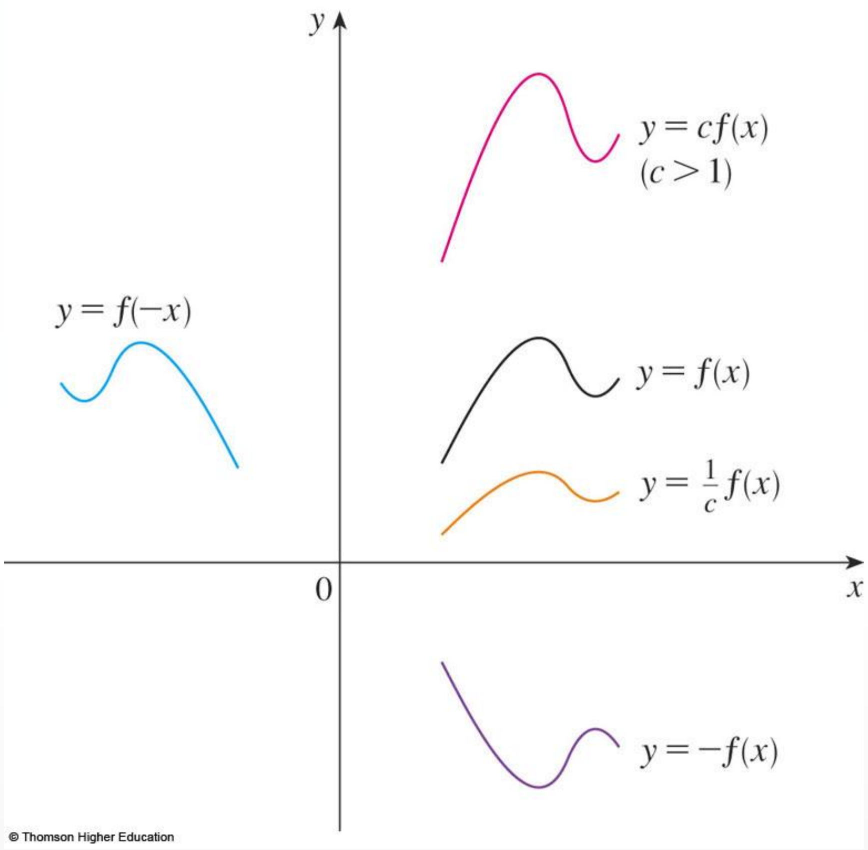
TRANSFORMATIONS

How about the case c<1?

- *Suppose $c > 1$.*
  - *To obtain the graph of $y = cf(x)$,*

    stretch the graph of $y = f(x)$

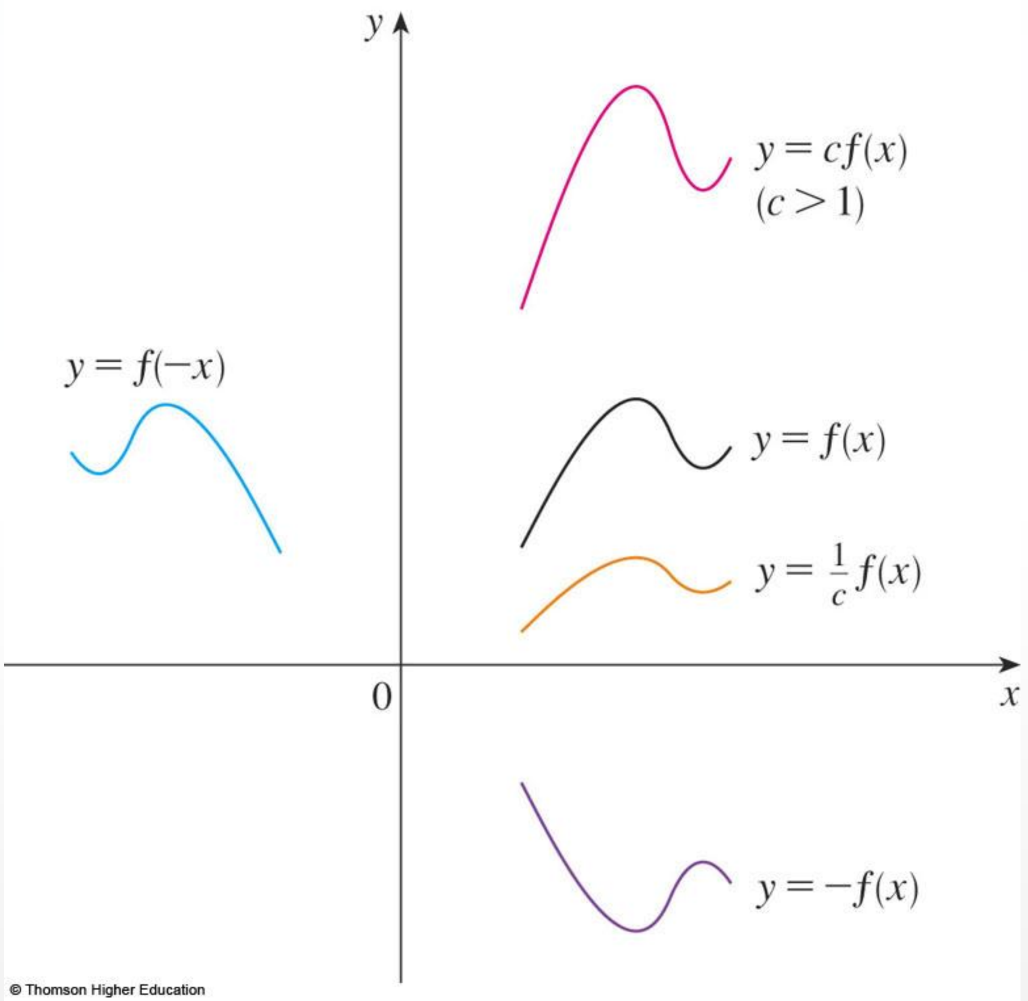
    vertically by a factor of c.
  - *To obtain the graph of $y = (1/c)f(x)$, compress the graph of $y = f(x)$ vertically by a factor of c.*



TRANSFORMATIONS

- *In order to obtain the graph of y = f(cx), compress the graph of y = f(x) horizontally by a factor of c.*

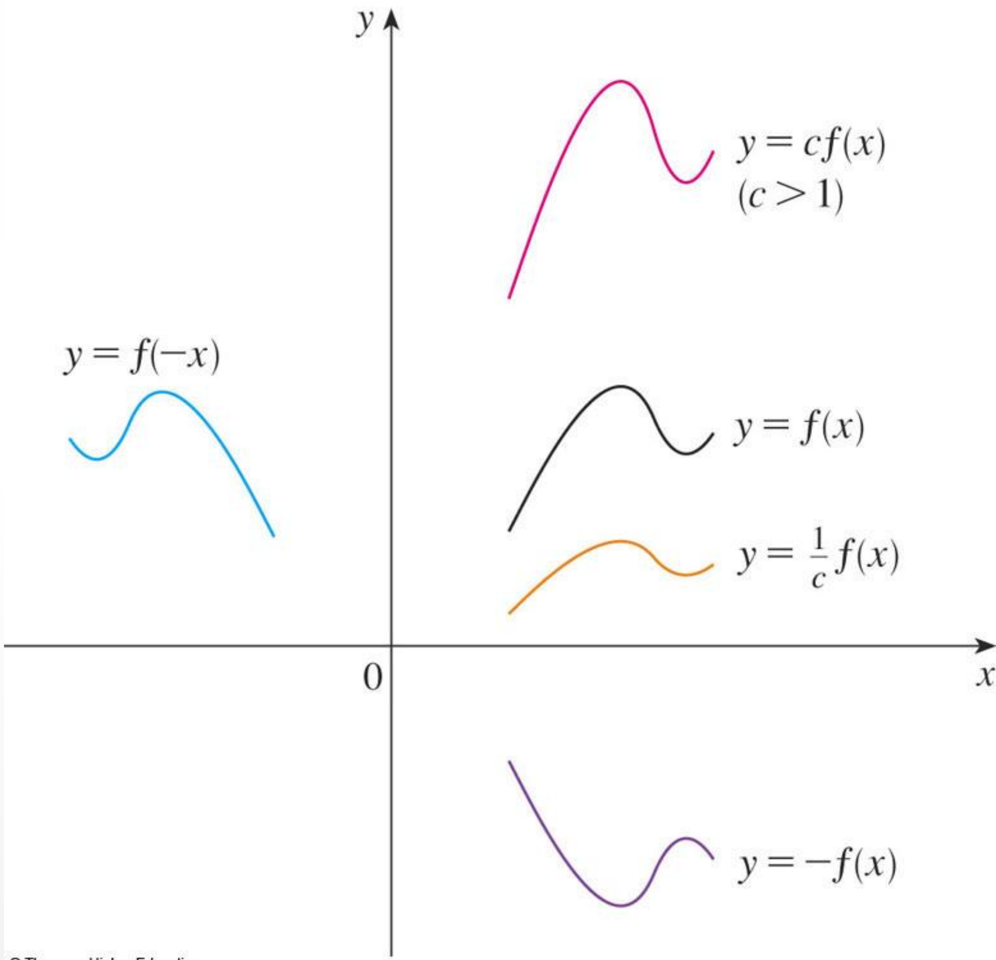
- *To obtain the graph of y = f(x/c), stretch the graph of y = f(x) horizontally by a factor of c.*



TRANSFORMATIONS

- *In order to obtain the graph of y = -f(x), reflect the graph of y = f(x) about the x-axis.*

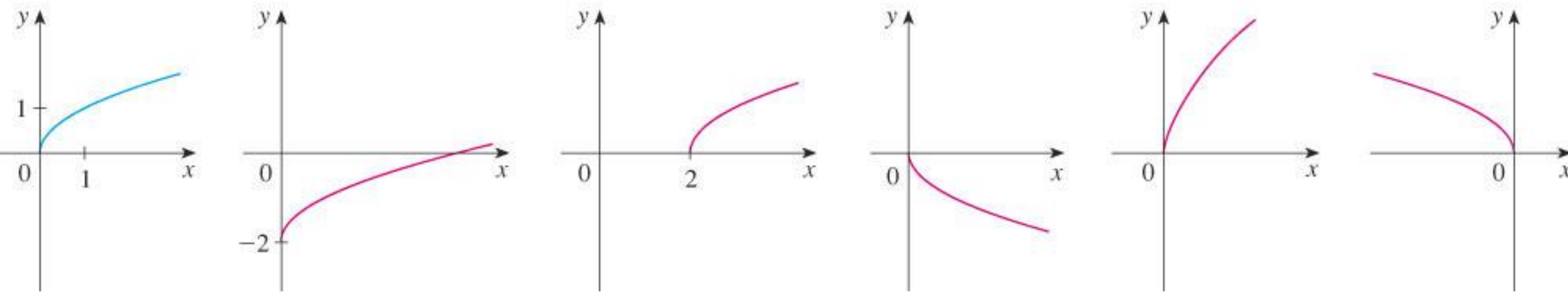
- *To obtain the graph of y = f(-x), reflect the graph of y = f(x) about the y-axis.*



NEW FUNCTIONS FROM OLD FUNCTIONS

Label the following graph from the graph of the function $y = \sqrt{x}$ shown in the part (a):

y=f(x)-2, y=f(x-2), y=-f(x), y=2f(x), y=f(-x)?



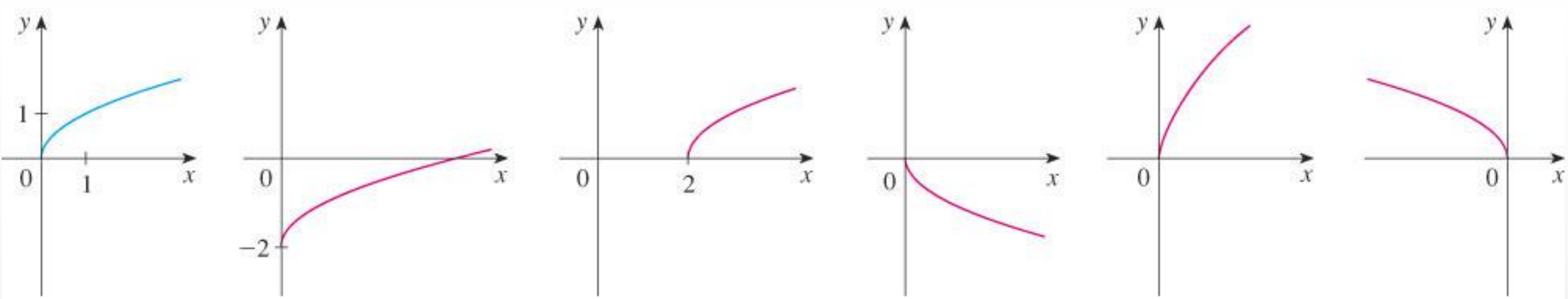
(a) $y=\sqrt{x}$ (b) $y=\sqrt{x}-2$ (c) $y=\sqrt{x-2}$ (d) $y=-\sqrt{x}$ (e) $y=2\sqrt{x}$ (f) $y=\sqrt{-x}$

NEW FUNCTIONS FROM OLD FUNCTIONS

Label the following graph from the graph of the function $y = \sqrt{x}$ shown in the part (a):

y=f(x)-2, y=f(x-2), y=-f(x), y=2f(x), y=f(-x)?

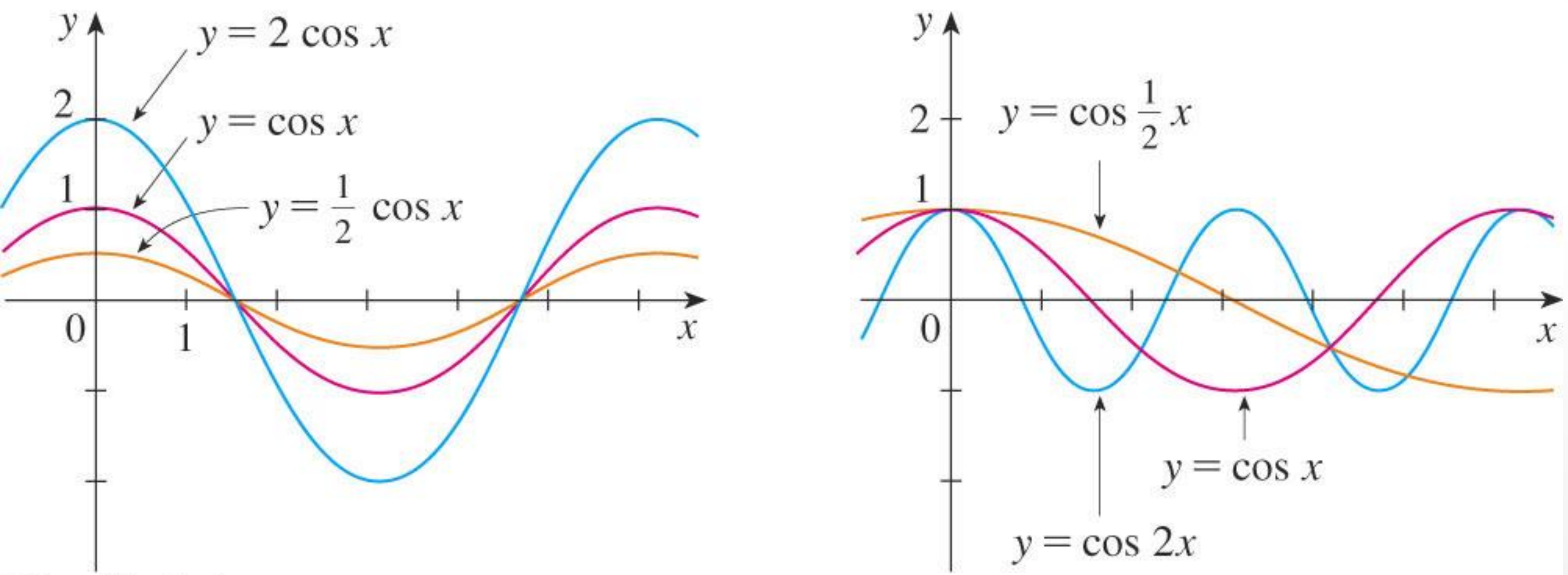
- $y = -\sqrt{x}$ by reflecting about the *x*-axis.
- $y = 2\sqrt{x}$ by stretching vertically by a factor of 2.
- $y = \sqrt{-x}$ by reflecting about the *y*-axis



(a) $y = \sqrt{x}$ (b) $y = \sqrt{x} - 2$ (c) $y = \sqrt{x - 2}$ (d) $y = -\sqrt{x}$ (e) $y = 2\sqrt{x}$ (f) $y = \sqrt{-x}$

TRANSFORMATIONS

- The figure illustrates these stretching
- transformations when applied to the cosine
- function with $c = 2$.



Example 5

Suppose that the graph of f is given.
Describe how the graph of the function $f(x-2)+2$ can be obtained from the graph of f.
Select the correct answer.

a. Shift the graph 2 units to the left and 2 units down.

b. Shift the graph 2 units to the right and 2 units down.

c. Shift the graph 2 units to the right and 2 units up.

d. Shift the graph 2 units to the left and 2 units up.

e. none of these

Answer: c

Thanks