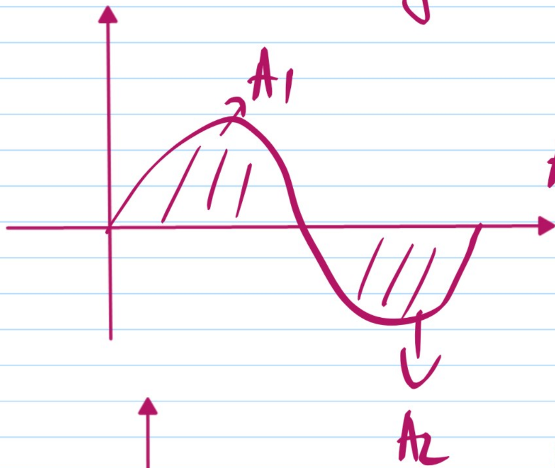
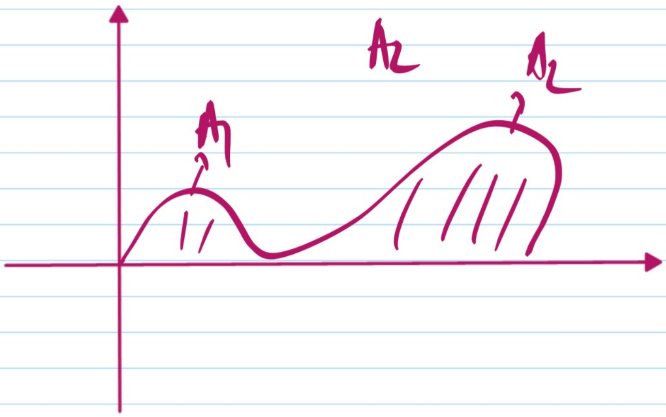
Net Signed area.

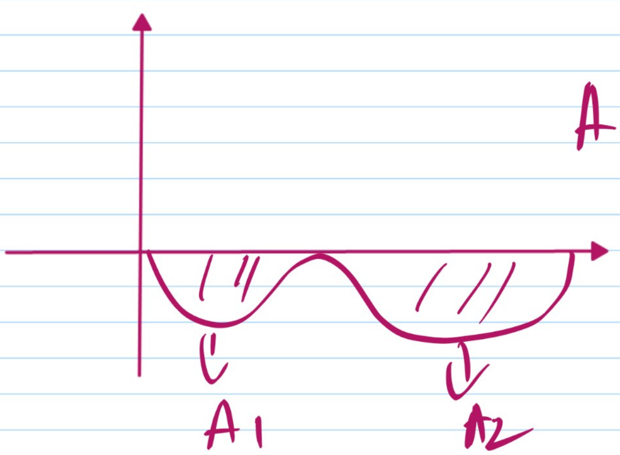


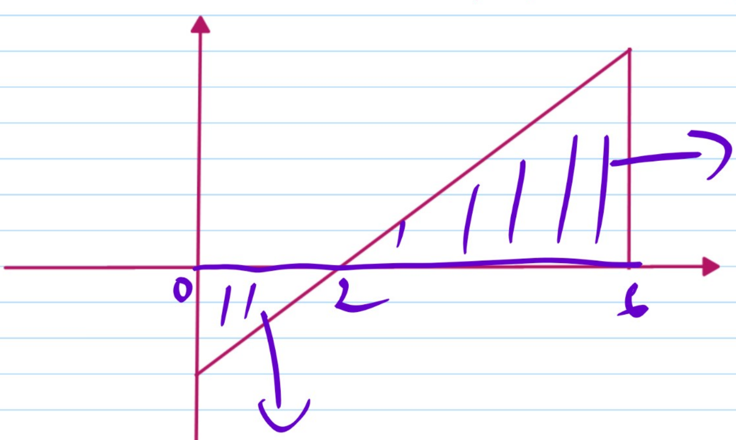
$A = A_1 - A_2$



$A = A_1 + A_2$

$A = -A_1 - A_2$





$\int_2^6 g(x)\,dx = 4$

$\int_0^2 g(x)\,dx = 1$

$\Rightarrow$ Net Signed $= 4 - 1 = 3$

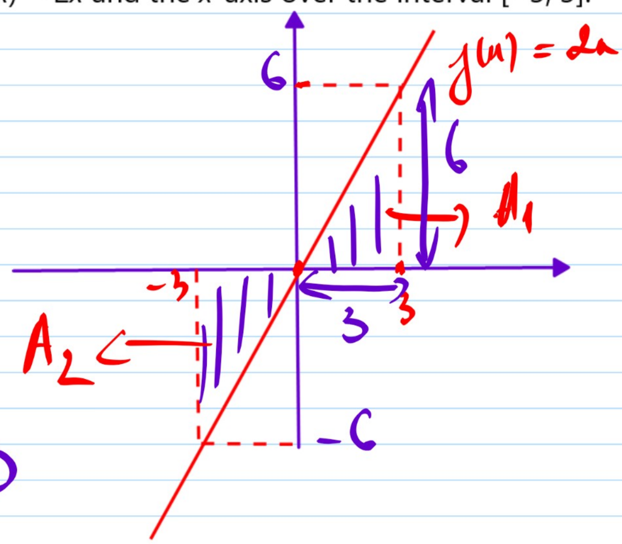
$$\Rightarrow \int_0^6 f(x)\,dx = 3$$

Find the net signed area between the curve of the function $f(x) = 2x$ and the x-axis over the interval [−3, 3].

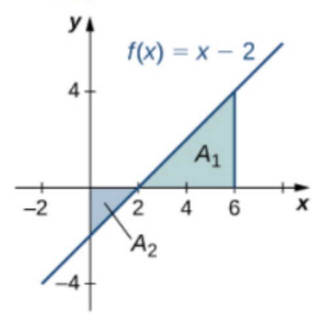
$$A_1 = \frac{1}{2}\cdot 3 \cdot 6 = 9$$

$$A_2 = \frac{1}{2}\cdot 3 \cdot 6 = 9$$

$$\int_{-3}^{3} 2x\,dx = A_1 - A_2 = 0$$



Find the net signed area of $f(x) = x - 2$ over the interval $[0, 6]$, illustrated in the following image.



$$A_1 = \frac{1}{2}\,4\cdot 4 = 8$$

$$A_2 = \frac{1}{2}\,2\cdot 2 = 2$$

$$A = A_1 - A_2 = 6$$

$$\int_0^6 f(x)\,dx = 6$$

Total area

$$\int_a^b |f(x)|\,dx = \underline{A_1 + A_2}$$

Find the total area between $f(x) = x - 2$ and the x-axis over the interval $[\,0, 6\,]$.

$$\int_0^6 (x-2)\,dx = A_1 + A_2$$

$$= 8 + 2 = 10$$

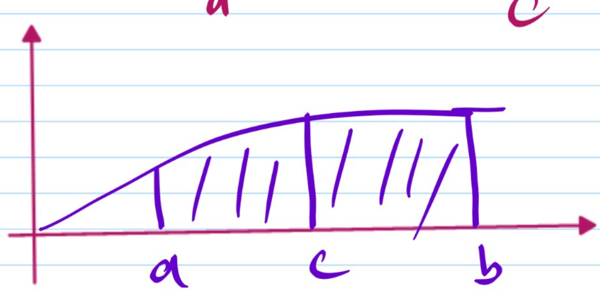
Properties of the Definite integral.

(1) $$\int_a^a f(x)\,dx = 0$$

(2) $$\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

(3) $$\int_a^b c\,f(x)\,dx = c\int_a^b f(x)\,dx$$

(4) $$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$



(5) $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

EX: $\int_0^4 f(x)\,dx = 6$, $\int_0^2 f(x)\,dx = 1$

$\int_0^4 g(x)\,dx = 9$, $\int_2^4 g(x)\,dx = 2$

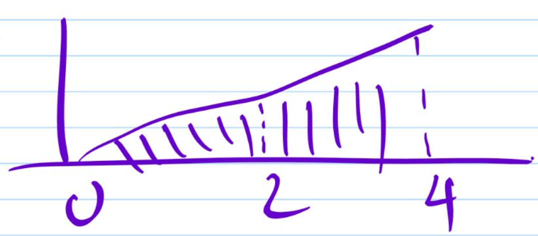
find. $\int_0^4 [f(x)+g(x)]\,dx = 6+9 = 15$

·) $\int_0^4 [f(x)-g(x)]\,dx = 6-9 = -3$

·) $\int_0^2 2f(x)\,dx = 2.1 = 2$

·) $\int_2^4 f(x)\,dx = \int_0^4 f(x)\,dx - \int_0^2 f(x)\,dx = 6-1 = 5$

· $\int_0^2 g(x)\,dx = 9-2 = 7$



EX2: $f(x) = -3x^3+2x+2$, $[-2;1]$

$\int_{-2}^{1} (-3x^3+2x+2)\,dx$

$$= \int_{-2}^{1} -3x^3\,dx + \int_{-2}^{1} 2x\,dx + \int_{-2}^{1} 2\,dx$$

$$= -3\int_{-2}^{1} x^3\,dx + 2\int_{-2}^{1} x\,dx + 2\int_{-2}^{1} dx$$

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$$

1) $$I = \int_0^1 x^3\,dx = \frac{x^4}{4}\Big|_0^1 = \frac{1}{4} - 0 = \frac{1}{4}$$

2) $$I = \int_1^{\ln 2} e^{2x}\,dx = \frac{1}{2}e^{2x}\Big|_1^{\ln 2}$$

$$= \frac{1}{2}\left(e^{2.\ln 2} - e^2\right) = \frac{1}{2}\left(e^{\ln 4} - e^2\right)$$

3) $$I = \int_0^1 \frac{1}{2x-3}\,dx = \frac{1}{2}\ln|2x-3|\Big|_0^1$$

$$= \frac{1}{2}\left(\underbrace{\ln 1}_{0} - \ln 3\right) = -\frac{1}{2}\ln 3$$

4) $$\int_0^1 (3^x + e^x)\,dx = \left(\frac{3^x}{\ln 3} + e^x\right)\Big|_0^1$$

$$= \left(\frac{3}{\ln 3} + e\right) - \left(\frac{1}{\ln 3} + 1\right)$$

$$= \frac{2}{\ln 3} + e - 1$$

5) $\displaystyle\int_0^{\pi/2} \sin 4x \cos 2x \, dx$

$$= \int_0^{\pi/2} 2\sin 2x \cdot \cos^2 2x \, dx$$

$$= \int_0^{\pi/2} 2\sin 2x \left(1 - \sin^2 2x\right) dx$$

$$= \int_0^{\pi/2} \left(2\sin 2x - \underbrace{2\sin^3 2x}\right) dx$$

$$= -\cos 2x \Big|_0^{\pi/2}$$

5) $\displaystyle\int_0^{\pi/2} \sin 4x \cos 2x \, dx = \int_0^{\pi/2} \frac{1}{2}\left[\sin 6x + \sin 2x\right] dx$

$$= \frac{1}{2}\left[-\frac{\cos 6x}{6} - \frac{\cos 2x}{2}\right]\Big|_0^{\pi/2}$$

$$= \frac{1}{2}\left[\left(-\frac{\cos 3\pi}{6} - \frac{\cos \pi}{2}\right) - \left(\frac{-\cos 0}{6} - \frac{\cos 0}{2}\right)\right]$$

$$=$$

Average Value of a function

$f(x)$ be continuous over the interval $[a, b]$

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

Find the average value of $f(x) = x + 1$ over the interval $[0, 5]$

$$f_{ave} = \frac{1}{5-0}\int_0^5 (x+1)\,dx$$

$$= \frac{1}{5}\left(\frac{x^2}{2} + x\right)\Big|_0^5$$

$$= \frac{1}{5}\left(\frac{25}{2} + 5\right) = \frac{7}{2}$$

Find the average value of $f(x) = 6 - 2x$ over the interval [0, 3].

$$f_{ave} = \frac{1}{3}\int_0^3 (6-2x)\,dx =$$

EX 3: Given $\int_0^3 x^2\,dx = 9$, find c such that $f(c)$ equals the average value of $f(x) = x^2$ over $[0; 3]$

$$f(c) = \frac{1}{3-0} \cdot \int_0^3 x^2\,dx = 3$$

$$f(c) = c^2 = 3 \Rightarrow c = \pm\sqrt{3}$$

$$\Rightarrow c = \sqrt{3}$$