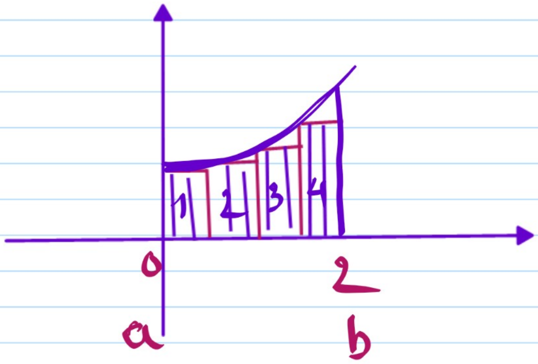
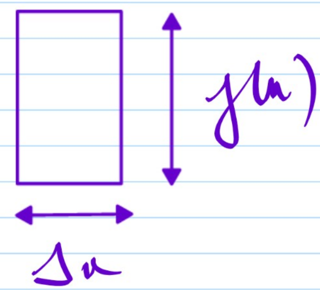


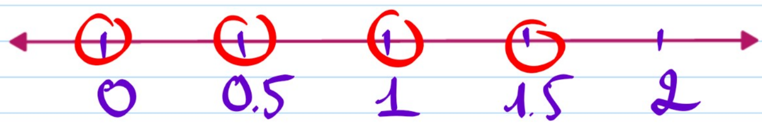


$$A = f(x) \cdot \Delta x$$

$$\sum_{i=1}^{n} \Delta x \cdot f(x) \rightarrow \text{Riemann Sum}$$

$$f(x) = x^2 + 1 \quad , [0; 2]$$

$$\Delta x = \frac{b-a}{n} = \frac{2-0}{4} = \underline{0.5}$$



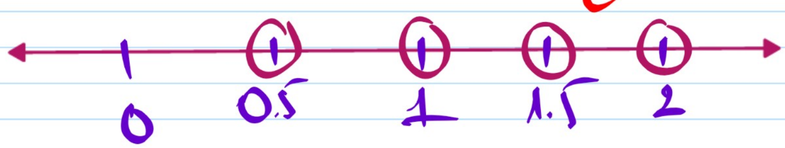
$f(x) = x^2 + 1$

Use Left endpoint | 4 points

$$A = L_n = \sum \Delta x \cdot f(x) = 0.5\left[f(0) + f(0.5) + f(1) + f(1.5)\right]$$

$$= 0.5\left[1 + 1.25 + 2 + 3.25\right]$$

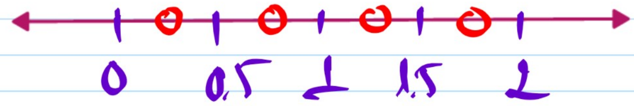
$$= 3.75$$



Right endpoint

$R_n = 0.5\left[f(0.5) + f(1) + f(1.5) + f(2)\right]$

$= 5.75$



Mid endpoint

$M_n = 0.5\left[f(0.25) + f(0.75) + f(1.25) + f(1.75)\right]$

$= 4.625$

$L_n = 8.75$

$R_n = 5.75$

$M_n = 4.625$

$A = 4.6$

$$\frac{L_n + R_n}{2}$$

EX2: $f(x) = x^3$ over $[0;4]$

$\int_0^4 x^3\,dx = 64$

Use $n=4$

Use $n=8$

$\Delta x = 1$

$L_4 = 1 \cdot [f(0) + f(1) + f(2) + f(3)]$

$= 36$

$R_4 = 1 \cdot [f(1) + f(2) + f(3) + f(4)]$

$= 100$

$M_4 = 62$

$\Delta x = \frac{4}{8} = 0.5$

$L_8 = 0.5 [f(0) + f(0.5) + \dots + f(3.5)] = 49$

$R_8 = 81$

$M_8 = 63.5 \approx A = 64$

EX3: Use both Left and Right endpoints approximation to the Area under Curve

$f(x) = x^2$ on the interval $[1, 2]$

Use $n = 4$

With $\underbrace{\text{your Subinterval}}_{n=4}$

$$\Delta x = \frac{2-1}{4} = 0.25$$

$$L_4 = 0.25 \cdot [f(1) + f(1.25) + f(1.5) + f(1.75)]$$
$$= 1.96875$$

$$R_4 = 0.25\,[f(2) + f(1.75) + f(1.5) + f(1.25)]$$
$$= 2.71875$$

The trapezoidal Rule (Third thing)

$$T_n = \frac{1}{2}\Delta x\,[f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(x_n)]$$

Ex: use Trapezoidal Rule, estimate

$\int_0^1 x^2$, $\underbrace{\text{using four subinterval}}_{n=4.}$

$$\Delta x = \frac{1-0}{4} = 0.25$$

$T_4 = \frac{1}{2} \cdot 0.25 \left[f(0) + 2f(0.25) + 2f(0.5) + 2f(0.75) + f(1) \right]$

$\simeq 0{,}22$

The Definite Intergral

$x_i^* \in [x_{i-1}; x_i]$

$x_i = x_0 + i\Delta x$

Riemann Sum : $\sum_{i=1}^{n} f(x_i^*) \cdot \Delta x$

Area under the curve $y = f(x)$, $[a; b]$

$A = \int_a^b f(x)\,dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i^*) \cdot \Delta x$

Ex: Use the definition of definite intergral. to evalue $\int_0^2 x^2\,dx$

$\Delta x = \frac{2-0}{n} = \frac{2}{n}$

$x_i = x_0 + i\Delta x = 2i/n$

$$f(x_i) = \left(\frac{2i}{n}\right)^2 = \frac{4i^2}{n^2}$$

$$\sum_{i=1}^{n} \frac{4i^2}{n^2} \cdot \frac{2}{n} = \frac{8}{n^3} \sum_{i=1}^{n} i^2$$

$$= \frac{8}{n^3} \cdot \frac{n(n+1)(2n+1)}{6}$$

$$\lim \frac{8}{n^3} \frac{n(n+1)(2n+1)}{6} = 8/3$$

Use the definition of the definite integral to evaluate $\int_0^3 (2x-1)\,dx$

$$\Delta x = \frac{3-0}{n} = \frac{3}{n}$$

$$x_i = x_0 + i\Delta x = 3i/n$$

$$f(x_i) = 6i/n - 1$$

$$\sum_{i=1}^{n} \left(\frac{6i}{n} - 1\right) \cdot \frac{3}{n}$$

$$= \sum_{i=1}^{n} \frac{18i}{n^2} - \sum_{i=1}^{n} \frac{3}{n} \rightarrow c \qquad n \cdot \frac{3}{n} = 3$$

$$= \frac{18}{n^2} \sum_{i=1}^{n} i = \frac{18}{n^2} \cdot \frac{n(n+1)}{2}$$

$$A = \lim_{n \to \infty} \left(\frac{18}{n^2} \cdot \frac{n(n+1)}{2} - 3 \right)$$