Matrix Multiplication

$\downarrow$

nhân

$\Rightarrow A_{m\times n}\cdot B_{n\times p}=C_{m\times p}$

$A_{3\times 4}\cdot B_{4\times 2}=C_{3\times 2}$

EX: $A_{3\times \underline{4}}$, $B_{\underline{m}\times n}$. Find $m=?$ $\exists A.B$

$\Rightarrow \boxed{m=4}$

$$A_{m\times k}\cdot B_{k\times n}=C_{m\times n}=(C_{ij})_{m\times n}$$

$$C_{ij}=(\text{row } i \text{ of } A)\cdot(\text{column } j \text{ of } B)$$

$$A=\begin{pmatrix}1 & 2\\ -1 & 1\\ 3 & 0\end{pmatrix}_{3\times 2},\quad B=\begin{pmatrix}1 & 0 & 3\\ 4 & -1 & 6\end{pmatrix}_{2\times 3}$$

$$C=A.B=\begin{pmatrix}9 & -2 & 15\\ 3 & -1 & 3\\ 3 & 0 & 9\end{pmatrix}_{3\times 3}$$

EX: $$A=\begin{pmatrix}1 & -1 & 0\\ 2 & 3 & 1\\ 1 & -1 & 2\end{pmatrix}_{3\times 3},\quad B=\begin{pmatrix}3 & -1\\ 1 & 0\\ 1 & -1\end{pmatrix}_{3\times 2}$$

$$C=A.B=\begin{pmatrix}2 & -1\\ 10 & -3\\ 4 & -3\end{pmatrix}_{3\times 2}$$

The inverse of a matrix

A , A^{-1} : is call the inverse of Matrix A

$$\boxed{A \cdot A^{-1} = A^{-1} \cdot A = I_n}$$

$$I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} , \quad I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

\# Iverse of 2×2 Matrix.

$$\boxed{A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \longrightarrow A^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}}$$

$$A = \begin{pmatrix} 2 & -3 \\ 1 & -4 \end{pmatrix} \longrightarrow A^{-1} = \frac{1}{-5} \begin{pmatrix} -4 & 3 \\ -1 & 2 \end{pmatrix}$$

$$B = \begin{pmatrix} 7 & 2 \\ 17 & 5 \end{pmatrix} \longrightarrow B^{-1} = \frac{1}{1} \begin{pmatrix} 5 & -2 \\ -17 & 7 \end{pmatrix}$$

$$B \cdot B^{-1} = \begin{pmatrix} 7 & 2 \\ 17 & 5 \end{pmatrix} \cdot \begin{pmatrix} 5 & -2 \\ -17 & 7 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I_2$$

Ex: $A^2 \cdot B = ?$ $A_{2\times 2}$, $B_{2\times 5}$ $\Rightarrow A^2 B_{2\times 5}$

$d^2_{2\times 2}$

1) The inverse of nxn matrices (3×3)

$$[A \mid I_n] \xrightarrow{\text{gauss}} [I_n \mid A^{-1}]$$

Ex:

$$A = \begin{pmatrix} 1 & -1 & 2 \\ 0 & -1 & 3 \\ 0 & 2 & -5 \end{pmatrix} \qquad A^{-1} = ?$$

$$\rightarrow \left(\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & -1 & 3 & 0 & 1 & 0 \\ 0 & 2 & -5 & 0 & 0 & 1 \end{array}\right) \xrightarrow{-r_2} \left(\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -3 & 0 & -1 & 0 \\ 0 & 2 & -5 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{r_3 - 2r_2} \left(\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -3 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 2 & 1 \end{array}\right) \xrightarrow[r_2 + 3r_3]{r_1 - 2r_3} \left(\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & -4 & -2 \\ 0 & 1 & 0 & 0 & 5 & 3 \\ 0 & 0 & 1 & 0 & 2 & 1 \end{array}\right)$$

$$\xrightarrow{r_1 + r_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 5 & 3 \\ 0 & 0 & 1 & 0 & 2 & 1 \end{array}\right) \rightarrow A^{-1} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 5 & 3 \\ 0 & 2 & 1 \end{pmatrix}$$

$$\underbrace{\qquad}_{I_3} \mid A^{-1}$$

$$B = \begin{pmatrix} 1 & 2 & -1 \\ -2 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix} \quad \text{find } B^{-1} ?$$

$$\to \left(\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 & 1 \end{array}\right) \xrightarrow[r_3 - r_1]{r_2 + 2r_1} \left(\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 4 & -1 & 2 & 1 & 0 \\ 0 & -3 & 1 & -1 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{4r_3 + 3r_2} \left(\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 4 & -1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 & 3 & 4 \end{array}\right) \xrightarrow{r_2 + r_3} \left(\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 4 & 0 & 4 & 4 & 4 \\ 0 & 0 & 1 & 2 & 3 & 4 \end{array}\right)$$

$$\to \left(\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 & 3 & 4 \end{array}\right) \xrightarrow{r_1 + r_3} \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 3 & 3 & 4 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 & 3 & 4 \end{array}\right)$$

$$\xrightarrow{r_1 - 2r_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 & 3 & 4 \end{array}\right) \to B^{-1} = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 1 & 1 \\ 2 & 3 & 4 \end{pmatrix}$$

$$I_3 \mid B^{-1}$$

Matrix and linear transformation
(phép biến đổi ttính)

- Linear transformation:

Df: $T : \mathbb{R}^n \to \mathbb{R}^m$

$$\boxed{\Leftrightarrow \begin{cases} T(x+y) = T(x)+T(y) \\ T(ax) = aT(x) \end{cases}}, \quad x, y \in \mathbb{R}^n, \quad a \in \mathbb{R}.$$

& $y \in \mathbb{R}^n$: linear combination of $x_1, x_2, \dots, x_n$
([illegible])

$$y = a_1x_1 + a_2x_2 + \dots + a_nx_n$$

$$\boxed{\begin{aligned} \Rightarrow T(y) &= T(a_1x_1 + a_2x_2 + \dots + a_nx_n) \\ &= a_1T(x_1) + a_2T(x_2) + \dots + a_nT(x_n) \end{aligned}}$$

Ex: linear transformation? let $T: \mathbb{R}^2 - \mathbb{R}^2$

A. $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+2y \\ 2\,5 \end{pmatrix}$ B. $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1\,5 \\ y \end{pmatrix}$

C. $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+y \\ 3y \end{pmatrix}$ D. $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \cdot y \\ 0 \end{pmatrix} 5$

E. $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+y \\ 0 \end{pmatrix}$ F. $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x/y \;5 \\ x+2y \end{pmatrix}$

$$\boxed{T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax \pm by \\ cx \pm dy \end{pmatrix}}$$

$(cx + 0y)$

Ex: $T: \mathbb{R}^2 \to \mathbb{R}^2$. $T\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$, $T\begin{pmatrix} 3 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$

a) find $T\left[2\begin{pmatrix} 1 \\ -1 \end{pmatrix} + 3\begin{pmatrix} 3 \\ 0 \end{pmatrix}\right] = T\left(2\begin{pmatrix} 1 \\ -1 \end{pmatrix}\right) + T\left(3\begin{pmatrix} 3 \\ 0 \end{pmatrix}\right)$

$T\,a(x) = aT(x)$

b) find $T\begin{pmatrix}5\\-2\end{pmatrix} = ?$

a) $= 2T\begin{pmatrix}1\\-1\end{pmatrix} + 3T\begin{pmatrix}3\\0\end{pmatrix} = 2\begin{pmatrix}-2\\3\end{pmatrix} + 3\begin{pmatrix}4\\5\end{pmatrix}$

$= \begin{pmatrix}-4\\6\end{pmatrix} + \begin{pmatrix}12\\15\end{pmatrix} = \begin{pmatrix}8\\21\end{pmatrix}$

b) $\begin{pmatrix}5\\-2\end{pmatrix} = a\begin{pmatrix}1\\-1\end{pmatrix} + b\begin{pmatrix}3\\0\end{pmatrix}$

$\Leftrightarrow \begin{cases} 5 = a + 3b \\ -2 = -a + 0b \end{cases} \Leftrightarrow \begin{cases} b = 1 \\ a = 2 \end{cases}$

$\Rightarrow T\begin{pmatrix}5\\-2\end{pmatrix} = T\left[2\begin{pmatrix}1\\-1\end{pmatrix} + 1\begin{pmatrix}3\\0\end{pmatrix}\right]$

$= 2T\begin{pmatrix}1\\-1\end{pmatrix} + T\begin{pmatrix}3\\0\end{pmatrix}$

$= 2\begin{pmatrix}-2\\3\end{pmatrix} + \begin{pmatrix}4\\5\end{pmatrix} = \begin{pmatrix}-4\\6\end{pmatrix} + \begin{pmatrix}4\\5\end{pmatrix}$

$= \begin{pmatrix}0\\11\end{pmatrix}$

EX: $T: \mathbb{R}^2 \to \mathbb{R}^2$. $T\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}2\\-3\end{pmatrix}$, $T\begin{pmatrix}1\\-2\end{pmatrix} = \begin{pmatrix}5\\1\end{pmatrix}$

find $T\begin{pmatrix}4\\3\end{pmatrix} = ?$

$\begin{pmatrix}4\\3\end{pmatrix} = a\begin{pmatrix}1\\1\end{pmatrix} + b\begin{pmatrix}1\\-2\end{pmatrix}$

$\Leftrightarrow \begin{cases} 4 = a + b \\ 3 = a - 2b \end{cases} \Leftrightarrow \begin{cases} a = 11/3 \\ b = 1/3 \end{cases}$

$$T\begin{pmatrix}4\\3\end{pmatrix} = T\left[\frac{11}{3}\begin{pmatrix}1\\1\end{pmatrix} + \frac{1}{3}\begin{pmatrix}1\\-2\end{pmatrix}\right]$$

$$= \frac{11}{3}T\begin{pmatrix}1\\1\end{pmatrix} + \frac{1}{3}T\begin{pmatrix}1\\-2\end{pmatrix}$$

$$= \frac{11}{3}\begin{pmatrix}2\\-3\end{pmatrix} + \frac{1}{3}\begin{pmatrix}5\\1\end{pmatrix}$$

$$= \frac{1}{3}\begin{pmatrix}27\\-32\end{pmatrix}$$