Continuous

- $f(x) = \begin{cases} g(x), & x \neq a \\ h(x), & x = a \end{cases}$ is continuous at $x = a$

$$\Leftrightarrow \lim_{x \to a} f(x) = f(a) \qquad \left(\lim_{x \to a} g(x) = h(a) \right)$$

- $f(x) = \begin{cases} g(x), & x \geq a \\ h(x), & x < a \end{cases}$ is continuous at $x = a$

$$\Leftrightarrow \lim_{x \to a^+} f(x) = \lim_{x \to a^-} h(x) = g(a)$$

Ex 1: $g(x) = \begin{cases} \dfrac{6x^2 + x - 2}{2x - 1}, & x \neq \frac{1}{2} \\ 7/2 ; & x = \frac{1}{2} \end{cases}$ ... at $x = \frac{1}{2}$?

Ex 2: $f(x) = \begin{cases} x^2 - e^x, & x < 0 \\ x - 1, & x \geq 0 \end{cases}$ ... at $x = 0$?

Ex 3: Find the point at which the given function is discontinuous

$f(x) = \begin{cases} \dfrac{1}{x - 4}, & x \neq 4 \\ 4, & 4 \end{cases}$

Ex 1: $\displaystyle \lim_{x \to \frac{1}{2}} \left(\frac{6x^2 + x - 2}{2x - 1} \right) = \lim_{x \to \frac{1}{2}} \frac{(2x-1)(3x+2)}{2x - 1} = \frac{7}{2}$

$g\left(\frac{1}{2}\right) = \frac{7}{2} \Rightarrow$ Continuous at $x = \frac{1}{2}$

Ex 2: $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (x-1) = -1$

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (x^2 - e^x) = -1$

$f(0) = -1 \Rightarrow$ continuous at $x = 0$

Ex 3: $\lim_{x \to 4} \left(\frac{1}{x-4}\right) = \nexists$

because $\lim_{x \to 4^-} \frac{1}{x-4} = -\infty$; $\lim_{x \to 4^+} \frac{1}{x-4} = +\infty$

$f(4) = 4 \Rightarrow$ discontinuous at $x = 4$

#) The Chain Rule (Quy tắc chuỗi
(cái ĐH hàm hợp)

1, $h(x) = (f \circ g)(x) = f(g(x))$

$\Rightarrow h'(x) = g'(x) . f'(g(x))$

Ex : Let $h(x) = (f \circ g)(x)$, $g(1) = 5$, $g'(1) = 4$

$f'(5) = 10$. Find $h'(1)$?

$h'(1) = g'(1) . f'(g(1)) = 4 . f'(5) = 4 . 10 = 40$

Ex 2: $h(x) = (f \circ g)(x)$, $g(2) = -3$, $g'(2) = 4$

$f'(3) = 7$. find $h'(2)$?

$$h'(2) = g'(2) \cdot f'(g(2)) = 4 \cdot f'(-3)$$
$$= 4 \cdot 7 = 28$$

(2) $h(x) = (g(x))^n \Rightarrow h'(x) = n\,(g(x))^{n-1} \cdot g'(x)$

Ex: Find the equation of a tangent line to the graph of

a) $h(x) = \dfrac{1}{(3x-5)^2}$ at $x = 2$

b) $f(x) = (x^2-2)^3$ at $x = -2$

a) $h(x) = (3x-5)^{-2}$, $h'(x) = -2\,(3x-5)^{-3} \cdot 3$
$$= \frac{-6}{(3x-5)^3}$$

$h'(2) = -6$

equation tangent line : $y = \underbrace{f'(x_0)}_{\text{slope}}(x - x_0) + y_0$

$\Rightarrow y = -6(x-2) + 1$
$= -6x + 13$

C_2: Cau O $\longrightarrow$

b) $f'(x) = 3\,(x^2-2)^2 \cdot 2x$
$f'(-2) = -48$
$\Rightarrow y = -48\,(x+2) + 8$
$= -48x - 88$

*) Implicit differentiation (Đạo hàm, hàm ẩn)

from : $h(x) = (g(x))^n$

$\Rightarrow h'(x) = n(g(x))^{n-1} \cdot g'(x)$

$$\boxed{\begin{gathered}\frac{d}{dx}(y^n) = n \cdot y^{n-1} \cdot \frac{dy}{dx} \\ \frac{d}{dx}(x^n) = n\, x^{n-1} \frac{dx}{dx} = n\, x^{n-1}\end{gathered}}$$

Ex: $x^2 + y^2 = 25$. Find $\frac{dy}{dx} = ?$

$$\frac{d}{dx}(x^2+y^2) = \frac{d}{dx}(25)$$

$$\Leftrightarrow 2x + 2y\frac{dy}{dx} = 0$$

$$\Leftrightarrow \frac{dy}{dx} = \frac{-x}{y}$$

Ex 2: $10x^2 + 5y^3 = 100$

$$\frac{d}{dx}(10x^2+5y^3) = \frac{d}{dx}(100)$$

$$\Leftrightarrow 100x + 15y^2 \cdot \frac{dy}{dx} = 0$$

$$\Leftrightarrow \frac{dy}{dx} = \frac{20x}{3y}$$

Ex 3: $8x^3 + 20y^4 = 5$

$$\frac{d}{dx}(8x^3 + 20y^4) = \frac{d}{dx}(5)$$

$$\Leftrightarrow \quad 24x^2 + 80y^3\frac{dy}{dx} = 0$$

$$\Leftrightarrow \quad \frac{dy}{dx} = \frac{3x^2}{10y^3}$$

Ex 4: $x^2y + 3xy^2 = 10$. find $\frac{dy}{dx} = ?$

$$\frac{d}{dx}(x^2y + 3xy^2) = \frac{d}{dx}(10)$$

$$2xy + x^2\frac{dy}{dx} + 3y^2 + 6xy\frac{dy}{dx} = 0$$

$$\Leftrightarrow \quad \frac{dy}{dx} = \frac{-2xy - 3y^2}{x^2 + 6xy}$$

Ex 5: $x^3 \sin y + y = 4x + 3$. Find $\frac{dy}{dx} = ?$

$$\frac{d}{dx}(x^3 \sin y + y) = \frac{d}{dx}(4x + 3)$$

$$\Leftrightarrow \quad 3x^2 \sin y + x^3 \cos y \frac{dy}{dx} + \frac{dy}{dx} = 4$$

$$\Leftrightarrow \quad \frac{dy}{dx} = \frac{4 - 3x^2 \sin y}{x^3 \cos y + 1}$$

EXC: Find the equation of the tangent line to the curve

a) $x^2 + y^2 = 25$ at $(3; -4)$

b) $y^3 + x^3 - 3xy = 0$ at $(\frac{3}{2}; \frac{3}{2})$

$$\boxed{\text{Slope} = \frac{dy}{dx}\Big|_{\substack{x_0 = \\ y_0 =}}}$$

a) $$\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx}(25)$$

$$\Leftrightarrow 2x + 2y\frac{dy}{dx} = 0$$

$$\Leftrightarrow \frac{dy}{dx} = \frac{-x}{y} \Rightarrow \text{Slope } \frac{dy}{dx}\Big|_{\substack{x=3 \\ y=-4}} = \frac{3}{4}$$

$\Rightarrow$ tangent line: $y = \frac{3}{4}(x-3) - 4$

$$= \frac{3}{4}x - \frac{25}{4}$$

b) $$\frac{d}{dx}(y^3 + x^3 - 3xy) = 0$$

$$\Leftrightarrow 3y^2\frac{dy}{dx} + 3x^2 - 3y - 3x\frac{dy}{dx} = 0$$

$$\Leftrightarrow \frac{dy}{dx} = \frac{3y - 3x^2}{3y^2 - 3x} = \frac{y - x^2}{y^2 - x}$$

$$\text{Slope: } \frac{dy}{dx}\Big|_{\substack{x = 3/2 \\ y = 3/2}} = -1$$

$\Rightarrow$ tangent line: $y = -1(x - \frac{3}{2}) + \frac{3}{2}$

$$= -x + 3$$

Ex 7: 325 page Volume 1

Ex: 300 → 309

For the following exercises, use implicit differentiation to find $\frac{dy}{dx}$.

300. $x^2 - y^2 = 4$

301. $6x^2 + 3y^2 = 12$

302. $x^2 y = y - 7$

303. $3x^3 + 9xy^2 = 5x^3$

304. $xy - \cos(xy) = 1$

305. $y\sqrt{x+4} = xy + 8$

306. $-xy - 2 = \frac{x}{7}$

307. $y\sin(xy) = y^2 + 2$

308. $(xy)^2 + 3x = y^2$

309. $x^3 y + xy^3 = -8$