Definition : A function F is an antiderivative of the function f if $F'(x) = f(x)$

Indefinite integral of f : $\int f(x)\, dx = F(x) + C$

(tích phân k° xác định)

① $\int [f(x) \pm g(x)]\, dx = \int f(x)\, dx \pm \int g(x)\, dx$

② $\int k \cdot f(x)\, dx = k \int f(x)\, dx$

③ $\int k\, dx = kx + C$

③ $\int x^n\, dx = \dfrac{x^{n+1}}{n+1} + C \qquad (n \neq -1)$

④ $\int \dfrac{1}{x}\, dx = \ln |x| + C$

⑤ $\int e^x\, dx = e^x + C$

⑥ $\int \sin x\, dx = -\cos x + C$

⑦ $\int \cos x\, dx = \sin x + C$

⑧ $\int \dfrac{1}{\cos^2 x}\, dx = \tan x + C$

⑨ $\int \dfrac{1}{\sin^2 x}\, dx = -\cot x + C$

⑩ $\sqrt[n]{a^m} = a^{\frac{m}{n}} \quad , \quad \dfrac{1}{x^n} = x^{-n}$

$\implies$ Antiderivative   Composition

$$y = \int \frac{dy}{du} \cdot du = \int y' \, du$$

EX: $\frac{dy}{du} = \sin u$, $y(0) = 5$. Find $y = ?$

$$y = \int \frac{dy}{du} \cdot du = \int \sin u \cdot du = -\cos u + C.$$

$$y(0) = 5 \Rightarrow -\cos 0 + C = 5$$
$$\Rightarrow C = 6$$
$$\Rightarrow \boxed{y = -\cos u + 6}$$

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EX2: For each of the following functions,
find all antiderivatives.

a) $f(u) = 3u^2$

$$\int 3u^2 = x^3 + C$$

b) $\int (5u^2 - 4u^3 + u + 5) \, du$

c) $\int (u + e^u) \, du$

d) $\int \frac{u^2 + 4\sqrt[3]{u}}{x} \, du$

e) $\int \tan u \cdot \cos u \, du$

EX2: find f ; $f'(t) = 2t - 3\sin t$, $f(0) = 5$

$$y = \int \frac{dt}{dx} \, dt = \int (2t - 3\sin t) \, dt$$

$$= t^2 + 3\cos t + C$$

$$y(0) = 5 \iff 0^2 + 3\cos 0 + C = 5$$

$$\implies C = 2$$

$$\implies y = t^2 + 3\cos t + 2$$

EX 3: Solve the initial-value problem.

$$\frac{dy}{dx} = 3x^{-2} \, , \, y(1) = 2$$

$$y = \int \frac{dy}{dx} \, dx = \int 3x^{-2} \, dx = \frac{3x^{-1}}{-1} + C$$

$$= -3 \cdot \frac{1}{x} + C$$

$$y(1) = 2 \iff -3 \cdot \frac{1}{1} + C = 2$$

$$\implies C = \frac{7}{2}$$

$$\implies y = \frac{-3}{x} + \frac{7}{2}$$

f) $\int \dfrac{14u^3 + 2u + 1}{u^3} \, du$

g) $\int (2\sin x + \sin 2x + \cos 3x)\, dx$

h) $\int \sin x \cdot \cos x \, dx$

Continuous $\longrightarrow$ Đổi biến — Từng phần . . . .