**QUESTION 1**

**Linear Programming Formulation for Many Looms, many components**

MAX: 300X1 + 320X2 + 340X3 + 360X4 + 380X5 + 400X6 } Maximise profit

**Subject to** : 1X1 + 6X2 + 8X3 + 10X4  + 12X5 + 2X6  <= 1577 } Resource 1

: 6X1 + 1X2 + 5X3 + 4X4 + 2X5 + 3X6 <= 1990 } Resource 2

: 3X1 + 4X2 + 1X3 + 2X4 + 5X5 + 6x6 <= 1321 } Resource 3

: 6X1 + 5X2 + 2X3 +1X4 + 3X5 + 4X6 <= 1920 } Resource 4

: 3X1 + 2X2 + 6X3 + 4X4 + 1X5 + 5X6 <= 1149 } Resource 5

: 2X1 + 5X2 + 6X3 + 3X4 + 4X5 + 1X6 <= 1000 } Resource 6

: 3X1 + 2X2 + 5X3 + 6X4 + 1X5 + 4X6 <= 1231 } Resource 7

: 5X1 + 6X2 + 2X3 + 1X4 + 3X5 + 4X6 <= 2000. } Resource 8

:$B$4 : $G$4 >= 0 } Non- negativity

**Where:**

X1 = # of Loom1 to produce

X2 = # of Loom2 to produce

X3 = # of Loom3 to produce

X4  = # of Loom4 to produce

X5  = # of Loom5 to produce

X6. = # of Loom6 to produce

**Question 1d: Optimal production plan (X1, X2, X3, X4, X5, X6) and the associated profit**

With **reference to the sensitivity report labelled *Q1) Looms Sensitivity Report*** in the excel sheet , it is optimal to produce 269 units of Loom1(X1), 0 unit of Loom2(X2), 1 unit of Loom3(X3), 49 units of Loom4 (X4), 65 units of Loom5(X5) and 15 units of Loom6(X6). The total profit from this production plan (with reference to label Q1)Looms in the excel sheet)

is $129,380.

**Question 1e: Optimal production plan and associated profit when X3 = X4 = X5 = X6 = 0**

With **reference to spreadsheet labelled *Q1e)*** in the excel sheet, the optimal production plan when X3 = X4 = X5 = X6 = 0 is to produce 230 units of Loom1 (X1) , 108 units of Loom2 (X2), 0 unit of Loom3(X3), 0 unit of Loom4 (X4), 0 unit of Loom5(X5) and 0 unit of Loom6(X6). The associated profit from this production plan is $103,560.

**Question 1f: Optimal production plan and associated profit when X4 = X5 = X6 = 0**

With **reference to spreadsheet labelled *Q1f)*** in the excel sheet, the optimal production plan when X4 = X5 = X6 = 0 is to produce 271.90 units of Loom1 (X1) , 40.97 units of Loom2 (X2), 41.90 of Loom3 (X3), 0 unit of Loom4 (X4), 0 unit of Loom5(X5) and 0 unit of Loom6(X6) . The associated profit from this production plan is $108,922.76.

**Question 1g: Optimal production plan and associated profit when X3 = X4 = X5 = X6**

With **reference to spreadsheet labelled *Q1g)*** in the excel sheet, the optimal production plan when X3 = X4 = X5 = X6 is to produce 249.29 units of Loom1 (X1) , 46.29 units of Loom2 (X2), 19.29 of Loom3 (X3), 19.29 unit of Loom4 (X4), 19.29 unit of Loom5(X5) and 19.29 unit of Loom6(X6) . The associated profit from this production plan is $118,140.

**Question 1h: Binding constraints for part 1g**

With **reference to spreadsheet labelled *Q1h)*** in the excel sheet, the constraints which are binding are resource 4, resource 5 and resource 6. This is based on the sensitivity report in the labelled spreadsheet which displays a shadow price higher than zero for resource 4, 5 and 6. This indicates that any changes it the value of those resources would lead to a change in the optimal solution.

**Question 1i: Effect on optimal solution (objective function) from 1d if amount of Resource 1 available is increased by 7**

With **reference to *Q1)Looms*** in the excel sheet, we can see it is shown that total profit is $129,380. Given that in reference to *Q1) Looms Sensitivity Report*, there is an allowable increase of 66.13 with shadow price of $12.57 for resource 1, hence the extra profit from the increase in resource 1 by 7 units would be 7 X $12.57 which will be $88 increase added to the objective function (total profit).

New optimal solution = Initial objective function (total profit) + Unit increase (shadow price of unit)

Hence,

New optimal solution: $129,380 + (7 $12.57) = $129,467.99

**Question 1j: Effect on optimal solution (objective function) from 1d if amount of Resource 8 available increased by 4%**

With **reference to *Q1j)*** in the excel sheet, an increase in 4% of the amount of Resource 8 available will increase the value from 2000 to 2080. However, despite an increase in amount of resource, the optimal solution does not change. This is because the shadow price for resource 8 is zero and is therefore a non-binding constraint. A small change in the RHS (supply) value of a non binding constraint will only typically change the feasible region however it not enough to affect the optimum solution.

**Question 1k.1 answering part d whether solution is degenerate**

A solution is degenerate if the allowable increase or decrease on any constraint is zero. With reference to *Q1) Looms Sensitivity Report* in the excel sheet, we can see that the allowable decrease for constraints Resource 1 used, Resource 4 used, Resource 5 used and Resource 7 used are zero. The allowable increase for the constraints Resource 2 and 3 used is zero. Therefore, the solution for part 1d) is degenerate.

**Question 1l): Effect on** **X1, X2, X3, X4, X5, X6 when profitability of each loom is increased by 2%**

With **reference to Q1l)** in the excel sheet, an increase in profitability by 2% for each loom will lead to unit profit for X1, X2, X3, X4, X5, X6 to be $306, $326.4, $346.8, $367.2 ,$387.6 and $408 respectively. However, despite an increase in profitability, the production quantity remains the same as the original optimum quantity. This is so as the proposed change/ allowable change is below 100%. The resources used is maximised for Resource 1, Resource 2, Resource 3, Resource 4, Resource 5 and Resource 7 as it is equal to resources available for the initial solution, therefore leading it to be the same quantity when the unit profit increase. For Resource 6 and 8 there isn’t an allowable increase when it is at 966 and 1651 resources are used respectively, hence it is already optimum and is similar to the original optimum quantity.

**Question 1m) 9 Extra Resource4 are available at total additional cost of $10. Explanation whether it is worth buying and profit made if so**

With **reference to Q1)Looms Sensitivity Report** in the excel sheet, given that there is an allowable increase of 32.99 and a shadow price of $12.50, the profit for an extra unit of Resource 4 within the allowable increase is $12.50. Given that there is a total additional cost of $10, the unit profit for the additional unit will be $12.50 - $10.00 = $2.50. Hence, the extra profit made from the 9 extra resources will be $2.50 9 = $22.50. It is worth buying as there is an increase in profit of $22.50 from selling extra resource 4.

**Question 1n) Given that Loom 7 is proposed using 1 of each Resources 1,4,5,6 and 8 and with a profitability of $100, is it worth making?**

Given that Loom 7 is proposed using 1 of each Resources 1,4,5,6 and 8 with a profitability of $100, it is worth making it. This is so as with **reference to the Q1)Looms Sensitivity Report** in the excel sheet, the shadow prices for Resources 1,4,5,6 and 8 are $12.57 + $12.50 + $14.49 + 0 + 0 = $39.49. This indicates that there is a profit of $39.49 for an increase in the all those 5 resources. There is an additional of ($100- $39.49) $60.51 profit when producing Loom7 as it is proposed to be sold at $100. Therefore, it is worth making it.

**Question 1o) Given that there is additional constraint as follows: if X5 > 0 then X1 = 0, what are the optimal values of** **X1, X2, X3, X4, X5, X6  and the objective function**

With **reference to Q1o)** in the excel sheet, the optimal values of X1, X2, X3, X4, X5, X6  are 209.30, 90.56, 0, 27.54, 0 and 45.96 respectively with the total profit of $120,069.67. To obtain this solution, a constraint is placed on the condition of the sum of the binary variables in which X5 > 0 and X1 = 0 is <=1. This is so as in the event X5 > 0 then X1 = 0, then the sum of the binary variables will be 1. However, there could potentially be an event where X5 = 0 and X1 = 0 which will make the sum of the binary variables to be 0.

**Question 1p: Given that if X1 > 0 then X2≥ 80, what are the optimal values of** **X1, X2, X3, X4, X5, X6  and the objective function**

With **reference to Q1p)** in the excel sheet, the optimal values of X1, X2, X3, X4, X5, X6  are 216.43, 80.00, 0, 31.03, 8.14 and 41.49 respectively with the total profit of $121,389.27. To obtain this solution, a condition is placed on the condition cell whereby 80 is multiplied to the binary variable of X1. A constraint is placed whereby X2 >= the condition cell. If X1 is more than 0 then the condition cell would be 80 as the binary variable would be 1 , however if X1  is 0 then the condition cell would be 0 as the binary variable would be 0.

**Question 1q: Number of Li produced from different loom is odd number with other looms having Xi = 0 , what are the optimal values and objective function**

With **reference to 1q)** in the excel sheet, the optimal values of X1, X2, X3, X4, X5, X6 are 269, 0, 1, 49, 65 and 15 respectively with the total profit of $129,380.00. To obtain this solution, a constraint is created for the number to be within 0 to 3 and to be multiplied by 2 and added with 1 with a cell titled odd multiplication. A constraint is made whereby the binary sum has to be equal the cell titled odd multiplication to get the condition of odd numbers.

**Question 1r: Number of Li produced from different loom is odd number with other looms having Xi = 0 , what are the optimal values and objective function**

With **reference to 1r)** in the excel sheet, the optimal values of X1, X2, X3, X4, X5, X6 are 269, 0, 1, 49, 65 and 15 respectively with the total profit of $129,380.00. To obtain this solution, a constraint is created for the number to be within 0 to 3 and to be multiplied by 2 and added with a cell titled even multiplication. A constraint is made whereby the binary sum has to be equal the cell titled even multiplication to get the condition of even numbers

**Question 1s: Given that for each Li that is produced , there is a start-up cost of $2000, what is the new objective function and optimal values of X1, X2, X3, X4, X5, X6**

With **reference to Q1s)** in the excel sheet, the optimal values of X1, X2, X3, X4, X5, X6  are 269.40, 0 , 0 , 49.84, 64.99 and 14.70 respectively with the total profit of $121,335.72. The total profit is calculated by multiplying the unit profit and production quantity minus the binary variable in multiplication of fixed cost.

**QUESTION 2 – Trans-shipment**

**Question 2a**

With **reference to Q2a)** in the excel spreadsheet, the total transportation cost of **$1330** is derived from the multiplication of unit shipping with unit cost. Unit shipping per destination is calculated by the solver.

The linear programming formulation for this problem is as follows:

The decision variables defined:

Xij = amount shipped from source i to intermediate nodes j

Xjk = amount shipped from intermediate nodes j to sinks k

Where i = 1 (Aldinga) , 2 (Canberra)

j = 3 (Geelong) , 4 (Kalgoorlie)

k = 5 (Mittagong) , 6 ( Wagga Wagga)

The objective function defined:

Minimize overall shipping costs:

5X13 + 8X14 + 7X23 + 4X24 + 2X35 + 3X36 + 3X45 + 6X46

Constraints Defined:

Amount Out of Aldinga: X13 + X14 = 80

Amount Out of Canberra: X23 + X24 = 90

Amount Through Geelong: X13 + X23 - X35 - X36 = 0

Amount Through Kalgoorie: X14 + X24 - X45 - X46  = 0

Amount Into Mittagong: X35 + X45 = 70

Amount Into Wagga Wagga: X36 + X46  = 100

Non- negativity of variables: Xij ≥ 0, for all i and j.

**Question 2b: modified cost for units from Geelong to Mittagong and Geelong to Wagga Wagga**

With **reference to Q2b)** in the excel spreadsheet, the total transportation cost of **$1390** is derived from the multiplication of total shipping with unit cost. Unit shipping per destination is calculated by the solver.

The linear programming formulation for this problem is as follows:

Xij = amount shipped from source i to intermediate nodes j

Xjk = amount shipped from intermediate nodes j to sinks k

Where i = 1 (Aldinga) , 2 (Canberra)

j = 3 (Geelong) , 4 (Kalgoorlie)

k = 5 (Mittagong) , 6 ( Wagga Wagga)

The objective function defined:

Minimize overall shipping costs:

5X13 + 8X14 + 7X23 + 4X24 + 2X35 + 9X35 + 3X36 + 10X36 + 3X45 + 6X46

Constraints Defined:

Amount Out of Aldinga: X13 + X14 = 80

Amount Out of Canberra: X23 + X24 = 90

Amount Through Geelong: X13 + X23 - X35 - X35 - X36 - X36 = 0

Amount Through Kalgoorie: X14 + X24 - X45 - X46  = 0

Amount Into Mittagong: X35 + X35 + X45 = 70

Amount Into Wagga Wagga: X36 + X36 + X46  = 100

Non- negativity of variables: Xij ≥ 0, for all i and j.

There is an additional condition added in the excel sheet which is the maximum flow of goods from Geelong to Mittagong and Geelong to Wagga Wagga.

**Question 2c: fixed cost charge**

With **reference to Q2c)** in the excel spreadsheet, the total transportation cost of **$2070** is derived from the multiplication of total shipping with unit cost. Unit shipping per destination is calculated by the solver.

The integer linear programming formulation for this problem is as follows:

Xij = amount shipped from source i to intermediate nodes j

Xjk = amount shipped from intermediate nodes j to sinks k

Where i = 1 (Aldinga) , 2 (Canberra)

j = 3 (Geelong) , 4 (Kalgoorlie)

k = 5 (Mittagong) , 6 ( Wagga Wagga)

The objective function defined:

Minimize overall shipping cost:

5X13 + 8X14 + 7X23 + 4X24 + 2X35 + 3X36 + 3X45 + 6X46  - 150Y13 – 150Y14 – 150Y23 – 150Y24 – 150Y35 – 150Y36 – 150Y45 – 150Y46

Constraints defined:

Amount Out of Aldinga: X13 + X14 = 80

Amount Out of Canberra: X23 + X24 = 90

Amount Through Geelong: X13 + X23 - X35 - X35 - X36 - X36 = 0

Amount Through Kalgoorie: X14 + X24 - X45 - X46  = 0

Amount Into Mittagong: X35 + X35 + X45 = 70

Amount Into Wagga Wagga: X36 + X36 + X46  = 100

All Xij , Xjk > 0

Binary Constraints

All Y must be binary

Linking Constraints

X13 – 80Y13 <= 0

X14 – 80Y14 <= 0

X23 – 90Y23 <= 0

X24 – 90Y24 <= 0

X35 – 70Y35 <= 0

X36 – 100Y36 <= 0

X45 – 70Y45 <= 0

X46 – 100Y46 <= 0

**Question 2di)**

With **reference to Q2di)** in the excel spreadsheet, the total transportation cost of **$2230** is derived from the multiplication of total shipping with unit cost. Unit shipping per destination is calculated by the solver.

The integer linear programming formulation for this problem is as follows:

Xij = amount shipped from source i to intermediate nodes j

Xjk = amount shipped from intermediate nodes j to sinks k

Where i = 1 (Aldinga) , 2 (Canberra)

j = 3 (Geelong) , 4 (Kalgoorlie)

k = 5 (Mittagong) , 6 ( Wagga Wagga)

The objective function defined:

Minimize overall shipping cost:

5X13 + 8X14 + 7X23 + 4X24 + 2X35 + 9X35 + 3X36 + 10X36 + 3X45 + 6X46  - 140Y13 – 140Y14 – 140Y23 – 140Y24 – 140Y35– 140Y35 – 140Y36 – 140Y36 – 140Y45 – 140Y46

Constraints defined:

Amount Out of Aldinga: X13 + X14 = 80

Amount Out of Canberra: X23 + X24 = 90

Amount Through Geelong: X13 + X23 - X35 - X35 - X36 - X36 = 0

Amount Through Kalgoorie: X14 + X24 - X45 - X46  = 0

Amount Into Mittagong: X35 + X35 + X45 = 70

Amount Into Wagga Wagga: X36 + X36 + X46  = 100

All Xij , Xjk > 0

Binary Constraints

All Y must be binary

Linking Constraints

X13 – 80Y13 <= 0

X14 – 80Y14 <= 0

X23 – 90Y23 <= 0

X24 – 90Y24 <= 0

X35 – 40Y35 <= 0

X35 – 30Y35 <= 0

X36 – 50Y36 <= 0

X36 – 50Y36 <= 0

X45 – 70Y45 <= 0

X46 – 100Y46 <= 0

**Question 2dii)**

With **reference to Q2dii**) in the excel spreadsheet, the total transportation cost of **$2350** is derived from the multiplication of total shipping with unit cost. Unit shipping per destination is calculated by the solver.

The integer linear programming formulation for this problem is as follows:

Xij = amount shipped from source i to intermediate nodes j

Xjk = amount shipped from intermediate nodes j to sinks k

Where i = 1 (Aldinga) , 2 (Canberra)

j = 3 (Geelong) , 4 (Kalgoorlie)

k = 5 (Mittagong) , 6 ( Wagga Wagga)

The objective function defined:

Minimize overall shipping cost:

5X13 + 8X14 + 7X23 + 4X24 + 2X35 + 9X35 + 3X36 + 10X36 + 3X45 + 6X46  - 160Y13 – 160Y14 – 160Y23 – 160Y24 – 160Y35– 160Y35 – 160Y36 – 160Y36 – 160Y45 – 160Y46

Constraints defined:

Amount Out of Aldinga: X13 + X14 = 80

Amount Out of Canberra: X23 + X24 = 90

Amount Through Geelong: X13 + X23 - X35 - X35 - X36 - X36 = 0

Amount Through Kalgoorie: X14 + X24 - X45 - X46  = 0

Amount Into Mittagong: X35 + X35 + X45 = 70

Amount Into Wagga Wagga: X36 + X36 + X46  = 100

All Xij , Xjk > 0

Binary Constraints

All Y must be binary

Linking Constraints

X13 – 80Y13 <= 0

X14 – 80Y14 <= 0

X23 – 90Y23 <= 0

X24 – 90Y24 <= 0

X35 – 40Y35 <= 0

X35 – 30Y35 <= 0

X36 – 50Y36 <= 0

X36 – 50Y36 <= 0

X45 – 70Y45 <= 0

X46 – 100Y46 <= 0

**Question 2diii)**

With **reference to Q2diii)** in the excel spreadsheet, the total transportation cost of **$2930** is derived from the multiplication of total shipping with unit cost. Unit shipping per destination is calculated by the solver.

The integer linear programming formulation for this problem is as follows:

Xij = amount shipped from source i to intermediate nodes j

Xjk = amount shipped from intermediate nodes j to sinks k

Where i = 1 (Aldinga) , 2 (Canberra)

j = 3 (Geelong) , 4 (Kalgoorlie)

k = 5 (Mittagong) , 6 ( Wagga Wagga)

The objective function defined:

Minimize overall shipping cost:

5X13 + 8X14 + 7X23 + 4X24 + 2X35 + 9X35 + 3X36 + 10X36 + 3X45 + 6X46  - 280Y13 – 280Y14 – 280Y23 – 280Y24 – 280Y35– 280Y35 – 280Y36 – 280Y36 – 280Y45 – 280Y46

Constraints defined:

Amount Out of Aldinga: X13 + X14 = 80

Amount Out of Canberra: X23 + X24 = 90

Amount Through Geelong: X13 + X23 - X35 - X35 - X36 - X36 = 0

Amount Through Kalgoorie: X14 + X24 - X45 - X46  = 0

Amount Into Mittagong: X35 + X35 + X45 = 70

Amount Into Wagga Wagga: X36 + X36 + X46  = 100

All Xij , Xjk > 0

Binary Constraints

All Y must be binary

Linking Constraints

X13 – 80Y13 <= 0

X14 – 80Y14 <= 0

X23 – 90Y23 <= 0

X24 – 90Y24 <= 0

X35 – 40Y35 <= 0

X35 – 30Y35 <= 0

X36 – 50Y36 <= 0

X36 – 50Y36 <= 0

X45 – 70Y45 <= 0

X46 – 100Y46 <= 0

**Question 2e)**

With **reference to Q2e)** in the excel spreadsheet, the total transportation cost of **$1390** is derived from the multiplication of total shipping with unit cost. Unit shipping per destination is calculated by the solver.

The linear programming formulation for this problem is as follows:

The decision variables defined:

Xij = amount shipped from source i to intermediate nodes j

Xjk = amount shipped from intermediate nodes j to sinks k

Where i = 1 (Aldinga) , 2 (Canberra)

j = 3 (Geelong) , 4 (Kalgoorlie)

k = 5 (Mittagong) , 6 ( Wagga Wagga)

The objective function defined:

Minimize overall shipping costs:

5X13 + 8X14 + 7X23 + 4X24 + 2X35 + 3X36 + 3X45 + 6X46

Constraints Defined:

Amount Out of Aldinga: X13 + X14 = 80

Amount Out of Canberra: X23 + X24 = 90

Amount Through Geelong: X13 + X23 - X35 - X36 = 0

Amount Through Kalgoorie: X14 + X24 - X45 - X46  = 0

Amount Into Mittagong: X35 + X45 = 70

Amount Into Wagga Wagga: X36 + X46  = 100

Non- negativity of variables: Xij ≥ 0, for all i and j.

Lower Boundary Constraints

X13 + X45 >= 100

Upper Boundary Constraints

X13 + X45 <= 120

**QUESTION 3- Economic Order Quantity**

**Question 3a: Optimal number of widgets to order at a time**

With **reference to the spreadsheet labelled Q3- EOQ**, the below economic order quantity is tabulated. This is done by substituting values A= 12,000, K = 15, c = 625 and h = 0.4 into the optimal order quantity formula. The calculation below shows the logic of the excel sheet calculation.

Using the formula below to calculate optional number of widgets to order at a time:

Optimal order quantity : Q\* =

A = Annual demand is for 12,000 widgets ( 1000 units/ month 12 months/year)

K = Ordering cost is $15 per order

c = Each widget cost $625 each

h = annual holding cost is 40%

Substituting the values of A,K, c and h into optimal order quantity formula:

Q\* =

Q\* =

Q\* = 37.94 ≈ 37 (approx.)

**Question 3b: Orders placed per annum**

With **reference to the spreadsheet labelled Q3- EOQ**, the below orders placed per annum is tabulated. This is done by substituting values A= 12,000 and Q\* = 37 into the number of orders per year formula. The calculation below shows the logic of the excel sheet calculation.

Using the formula below to calculate orders to be placed per annum:

Number of orders per year =

A = Annual demand is for 12,000 widgets ( 1000 units/ month 12 months/year)

Q\*= Optimal order quantity is 37

Substituting the values of A and Q\* into the number of orders per year formula:

Number of orders per year =

= 324.32 (approx.)

**Question 3c: Optimal number of widgets to order with possibility of back- orders**

With **reference to the spreadsheet labelled Q3- EOQ**, the optimal number of widgets to order with possibility of back-orders is tabulated. This is done by substituting values A= 12,000, k = 15, ch = 250, p = 1000 into the optimal number formula. The calculation below shows the logic of the excel sheet calculation.

Using the formula below to calculate the optimal number of widgets to order with possibility of back-orders:

Q \* =

A = Annual demand is for 12,000 widgets ( 1000 units/ month 12 months/year)

k = Ordering cost is $15 per order.

c = Each widget cost $625 each

h = annual holding cost is 40%

ch = $625 40% = $250 per widget per year

p = Backorder penalty is $250 for 3 months \* 4 months = $1000 per year

Substituting the values of A, k , ch and p into the formula above:

Q\* =

Q\* =

Q\* = 42.42 ≈ 42 (approx.)