Exponential Smoothing Methods

Time Series Forecasting (Part 2)

Lecture Video Slides

Exponential Smoothing Methods

1. Simple Exponential Smoothing
   - for a series without trend and seasonality.
2. Holt's method
   - for a series with trend but no seasonality.
3. Winters' method (or Holt-Winters' method)
   - for a series with seasonality and possibly trend.

Simple Exponential Smoothing

- Every exponential model has at least one smoothing parameter, between 0 and 1.
- Simple exponential smoothing has a single smoothing constant denoted by α.
- The level of the series at time t (L_t) is an estimate of where the deseasonalised series would be at time t if there is no random error.
- The simple exponential method is defined by the following two equations:

$$L_t = \alpha Y_t + (1 - \alpha) L_{t-1}$$

$$F_{t+k} = L_t$$

  - The *k*-period-ahead forecast, F_{t+k}, of Y_{t+k} made at period *t* is essentially the most recently estimated level, L_t.

Understanding the SES Model

$$L_t = \alpha Y_t + (1 - \alpha) L_{t-1}$$

$$F_{t+k} = L_t$$

- What is the role played by α?

Ans: Smoothing constant (betw 0 and 1) that smooths historical values.

- How many data points are used in calculating L_t?

Ans: All datapoints in the data sample.

- Is L_t used to forecast Y_t?

Ans: No. Y_t is required to compute L_t.

L_t is used to forecast Y_{t+1} onwards.

Holt's method for trend and without seasonality

- When there is a trend in the series, Holt's method deals with it explicitly by including a trend term, T_t, and it's corresponding smoothing constant β.
  - T_t represents an estimate of the *change* in the series from one period to the next.
- The equations for Holt's model are:

$$L_t = \alpha Y_t + (1 - \alpha)(L_{t-1} + T_{t-1})$$

$$T_t = \beta(L_t - L_{t-1}) + (1 - \beta)T_{t-1}$$

$$F_{t+k} = L_t + kT_t$$

Seasonality

- Seasonality is the consistent pattern that repeats each year.
- Three methods for dealing with seasonality:
  - MA Decomposition.
  - Winters' exponential smoothing model.
  - Multiple regression with dummy variables for the seasons.

Winter's method (aka Holt-Winters' method)

- Adds seasonal index for each seasonal effect and a corresponding smoothing constant γ.

$$L_t = \alpha \frac{Y_t}{S_{t-M}} + (1 - \alpha)(L_{t-1} + T_{t-1})$$

$$T_t = \beta(L_t - L_{t-1}) + (1 - \beta)T_{t-1}$$

$$S_t = \gamma \frac{Y_t}{L_t} + (1 - \gamma)S_{t-M}$$

$$F_{t+k} = (L_t + kT_t)S_{t+k-M}$$

- Set gamma = 0 to get Holt's method.
- Set beta = 0 and gamma = 0 to get SES.

Simple Exponential Smoothing

```
35 # Simple Exponential Smoothing ----------------------------------------
36 m.ses <- HoltWinters(flatsales.ts, seasonal = "multiplicative", beta=F, gamma=F)
37
38 m.ses
39 ## Optimal value of alpha = 0.5288797
40
41 plot(m.ses)
42
43 m.ses$fitted
44
45
46
47
48 RMSE.ses <- round(sqrt(m.ses$SSE/nrow(m.ses$fitted)))
49 ## 359
```

```
> m.ses
Holt-Winters exponential smoothing without trend and with
out seasonal component.

Call:
HoltWinters(x = flatsales.ts, beta = F, gamma = F, season
al = "multiplicative")

Smoothing parameters:
 alpha: 0.5288797
 beta : FALSE
 gamma: FALSE

Coefficients:
      [,1]
a 1503.371
```

How to get this?

Ans: Minimize SSE of one period ahead forecast.

Simple Exponential Smoothing

```
35  # Simple Exponential Smoothing ----------------------------------------
36  m.ses <- HoltWinters(flatsales.ts, seasonal = "multiplicative", beta=F, gamma=F)
37
38  m.ses
39  ## Optimal value of alpha = 0.5288797
40
41  plot(m.ses)
42
43  m.ses$fitted
44
45
46
47
48  RMSE.ses <- round(sqrt(m.ses$SSE/nrow(m.ses$fitted)))
49  ## 359
```

$$L_t = \alpha Y_t + (1 - \alpha) L_{t-1}$$

```
> m.ses
Holt-Winters exponential smoothing without trend and with
out seasonal component.

Call:
HoltWinters(x = flatsales.ts, beta = F, gamma = F, season
al = "multiplicative")

Smoothing parameters:
 alpha: 0.5288797
 beta : FALSE
 gamma: FALSE

Coefficients:
      [,1]
a 1503.371
```

What is this?

Ans: The last value of L_t

Simple Exponential Smoothing

```
35 # Simple Exponential Smoothing ----------------------------------------
36 m.ses <- HoltWinters(flatsales.ts, seasonal = "multiplicative", beta=F, gamma=F)
37
38 m.ses
39 ## Optimal value of alpha = 0.5288797
40
41 plot(m.ses)
42
43 m.ses$fitted
44
45 m.ses$alpha*1519 + (1-m.ses$alpha)*1485.8267
46 ## verifying the meaning of coef = 1503.371 is L_{last t}
47
48 RMSE.ses <- round(sqrt(m.ses$SSE/nrow(m.ses$fitted)))
49 ## 359
```

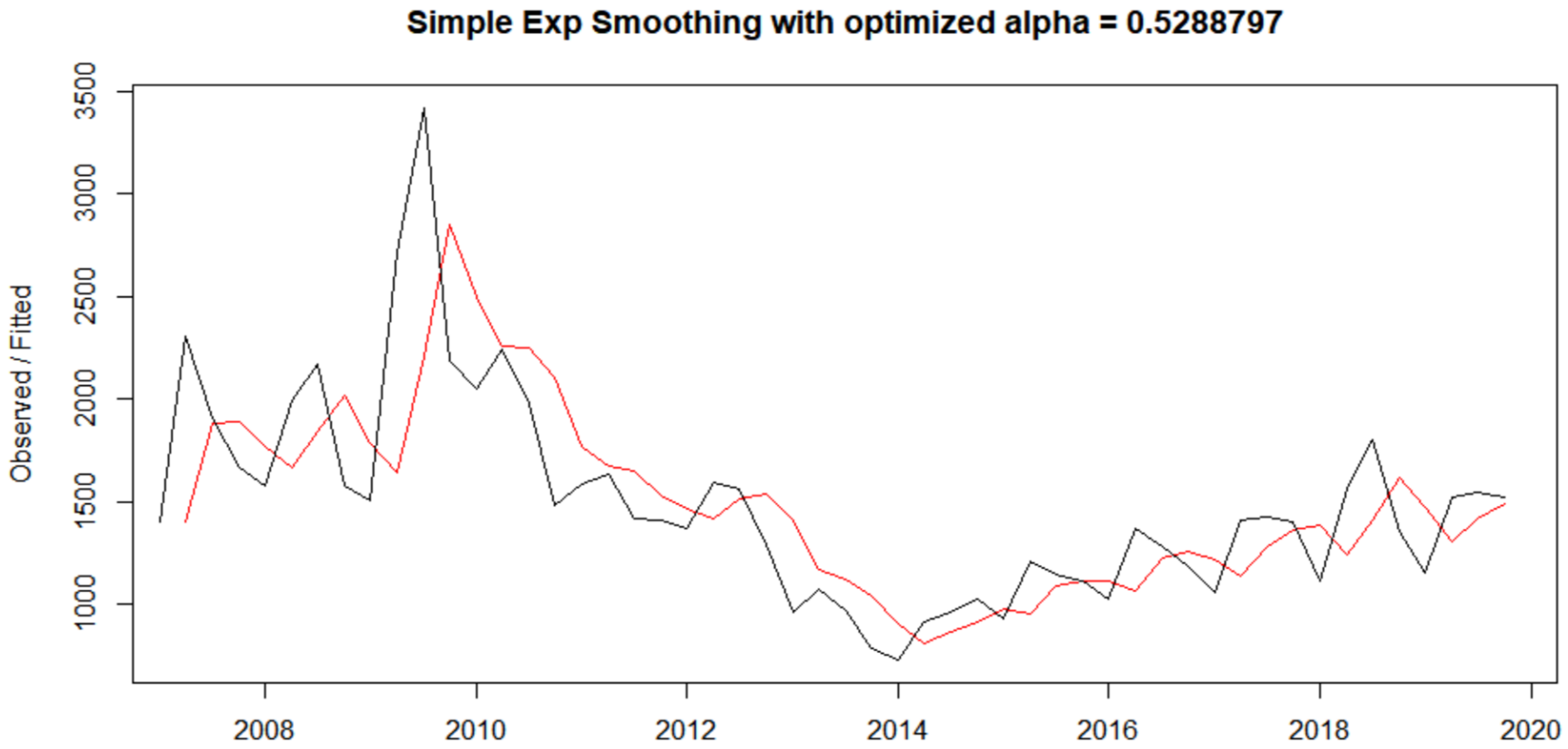
$$L_t = \alpha Y_t + (1 - \alpha) L_{t-1}$$

```
> m.ses
Holt-Winters exponential smoothing without trend and with
out seasonal component.

Call:
HoltWinters(x = flatsales.ts, beta = F, gamma = F, season
al = "multiplicative")

Smoothing parameters:
 alpha: 0.5288797
 beta : FALSE
 gamma: FALSE

Coefficients:
      [,1]
a 1503.371
```

What is this?

Ans: The last value of L_t

```
41  plot(m.ses, main = "Simple Exp Smoothing with optimized alpha = 0.5288797")
```



Q: Which coloured line rep observed vs one period ahead SES forecast?

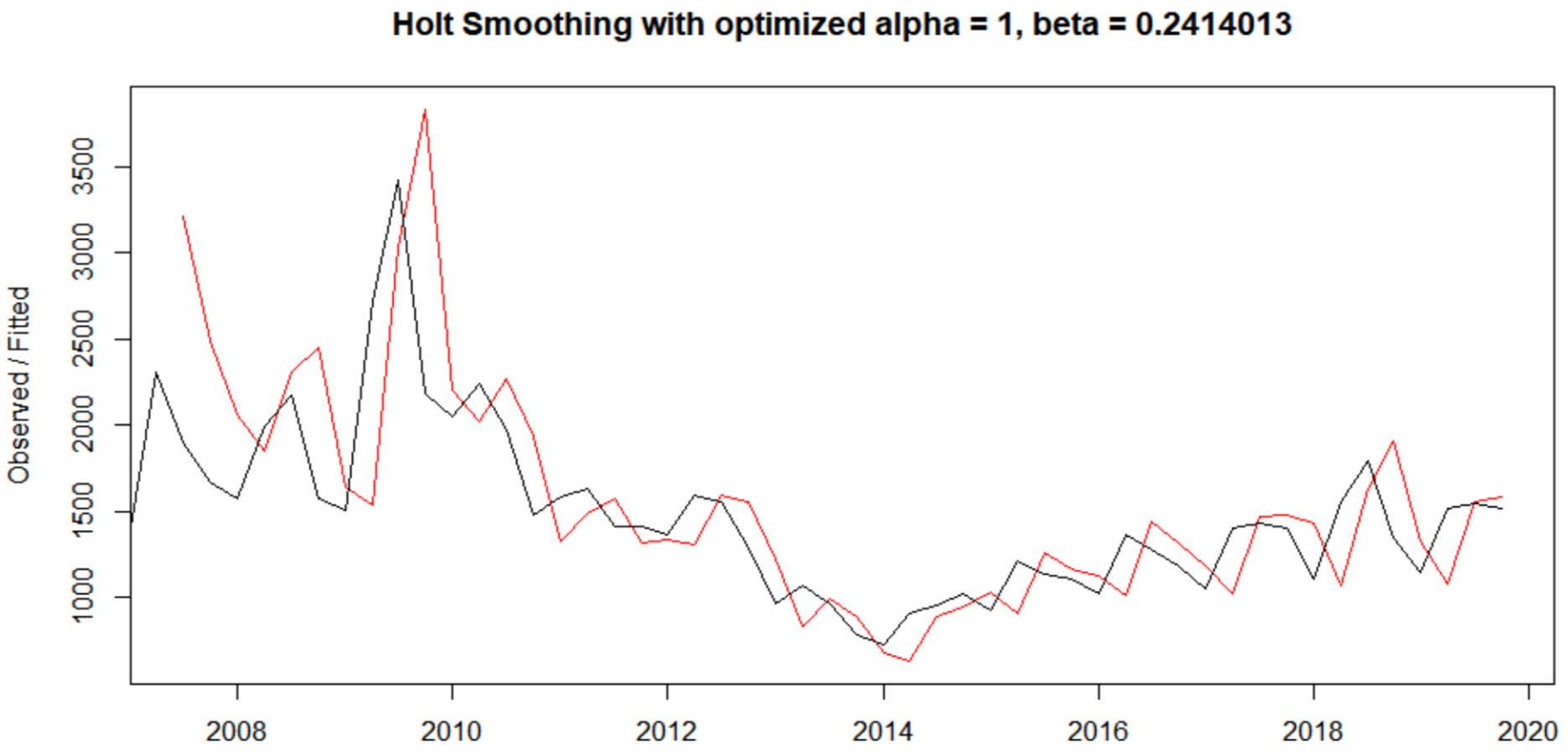
Ans: Black line rep observed data; Red line rep one period ahead forecast.

Holt Smoothing Method

```r
# Holt's method  ------------------------------------------------------------
m.holt <- HoltWinters(flatsales.ts, seasonal = "multiplicative", gamma=F)

m.holt
## Optimal value of alpha = 1, beta = 0.2414013

plot(m.holt, main = "Holt Smoothing with optimized alpha = 1, Beta = 0.2414013")

RMSE.holt <- round(sqrt(m.holt$SSE/nrow(m.holt$fitted)))
```

```
> m.holt
Holt-Winters exponential smoothing with trend and without seasonal component.

Call:
HoltWinters(x = flatsales.ts, gamma = F, seasonal = "multiplicative")

Smoothing parameters:
 alpha: 1
 beta : 0.2414013
 gamma: FALSE

Coefficients:
        [,1]
a 1519.00000
b   21.43562
```

```r
plot(m.holt, main = "Holt Smoothing with optimized alpha = 1, beta = 0.2414013")
```

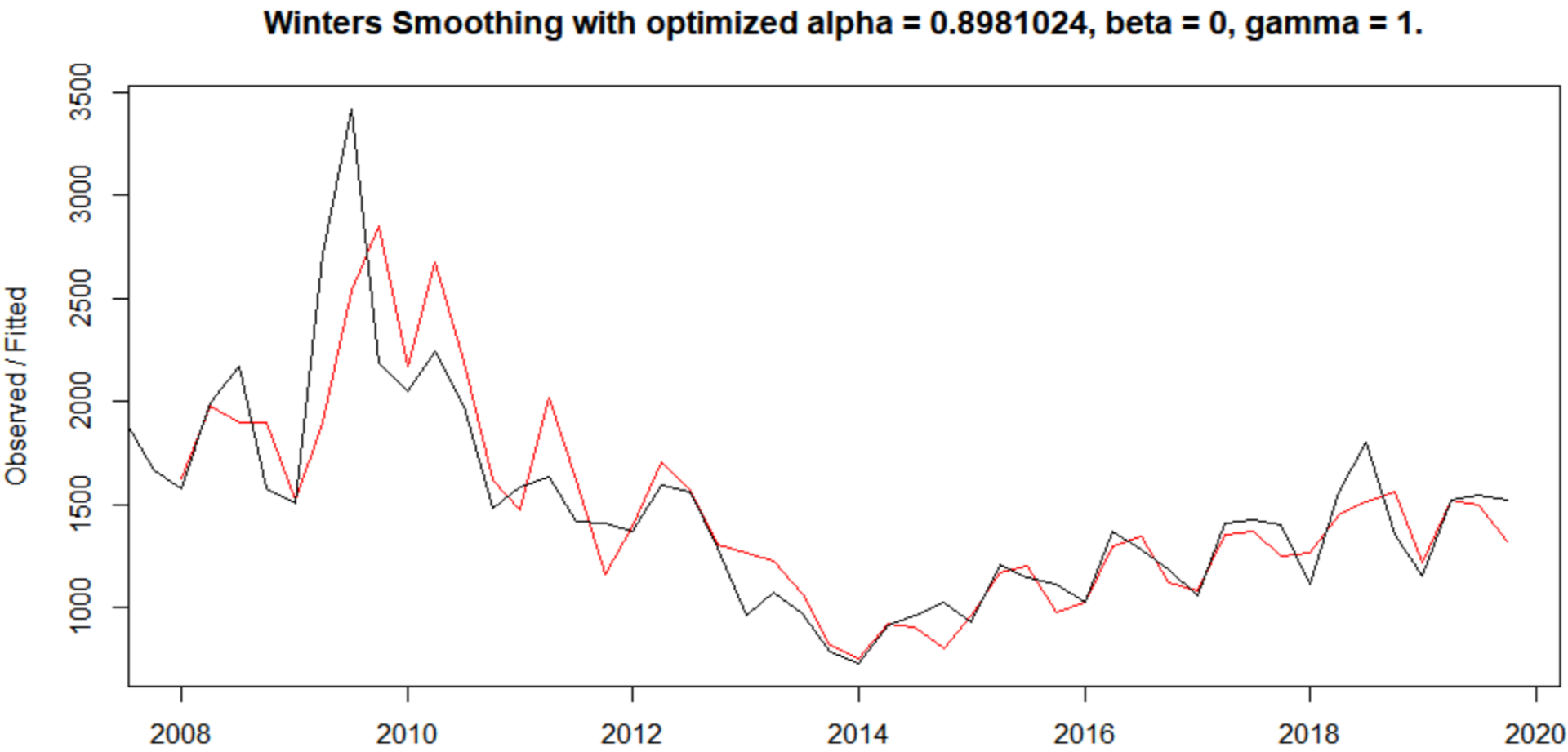


Winters Smoothing Method

```r
# Winter's method ----------------------------------------------------------------
m.winters <- HoltWinters(flatsales.ts, seasonal = "multiplicative")

m.winters
## Optimal value of alpha = 0.8981024, beta = 0, gamma = 1.

plot(m.winters, main = "Winters Smoothing with optimized alpha = 0.8981024, beta = 0, gamma = 1.")

RMSE.winters <- round(sqrt(m.winters$SSE/nrow(m.winters$fitted)))
## Winters method has the lowest RMSE
```

```
> m.winters
Holt-Winters exponential smoothing with trend and multiplicative seasonal
 component.

Call:
HoltWinters(x = flatsales.ts, seasonal = "multiplicative")

Smoothing parameters:
 alpha: 0.8981024
 beta : 0
 gamma: 1

Coefficients:
           [,1]
a  1617.4851043
b    -0.1250000
s1    0.8314238
s2    1.0986646
s3    1.0853405
s4    0.9391122
```

```
68 plot(m.winters, main = "Winters Smoothing with optimized alpha = 0.8981024, beta = 0, gamma = 1.")
```

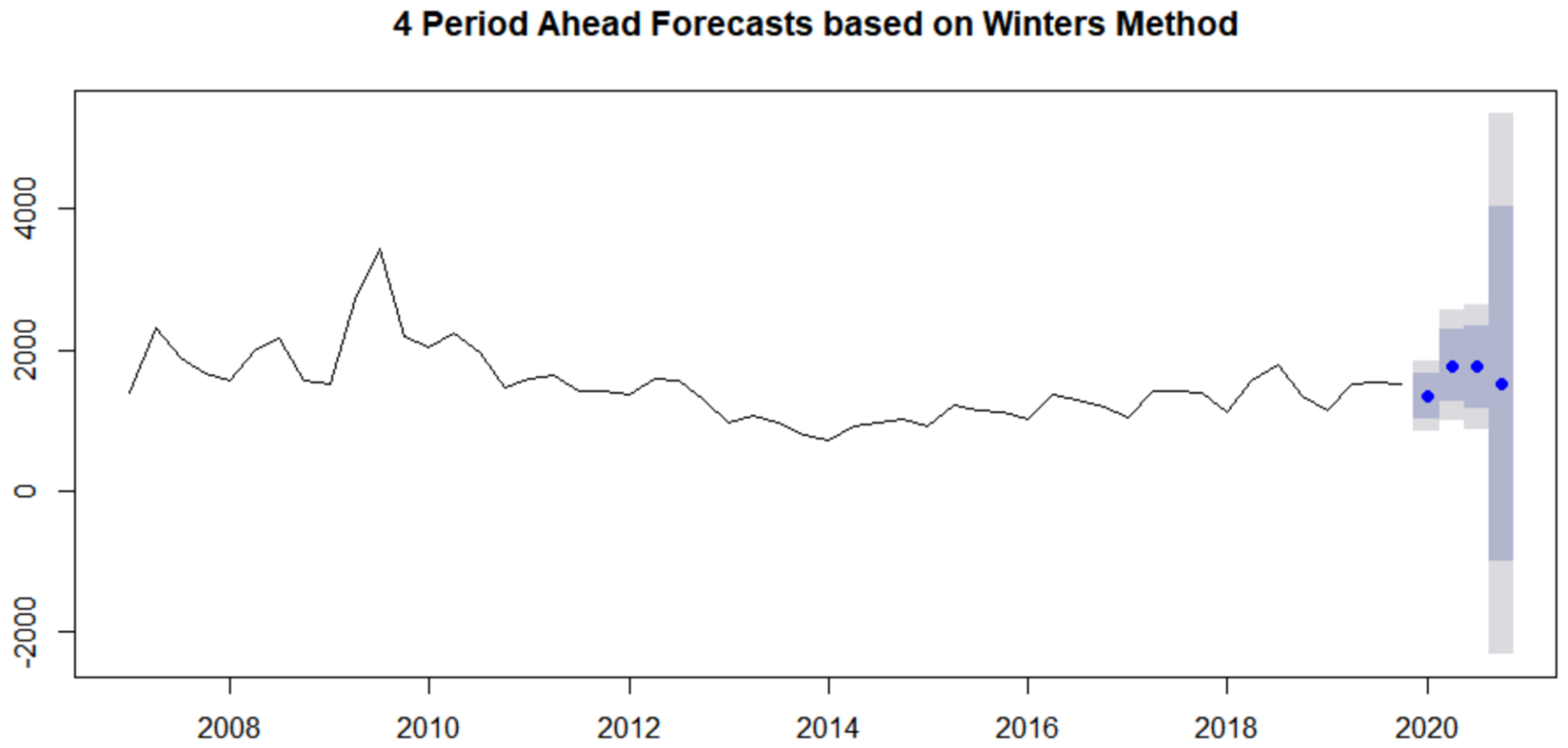


Forecast h periods ahead

```
73  # Forecast 4 periods ahead ------------------------------------
74  m.winters.forecasts <- forecast(m.winters, h = 4)
75
76  m.winters.forecasts
77
78  plot(m.winters.forecasts, main = "4 Period Ahead Forecasts based on Winters Method")
```

```
> m.winters.forecasts
         Point Forecast     Lo 80    Hi 80      Lo 95    Hi 95
2020 Q1       1344.712 1016.6657 1672.758   843.0086 1846.415
2020 Q2       1776.799 1267.6998 2285.898   998.1990 2555.399
2020 Q3       1755.115 1171.8178 2338.412   863.0389 2647.191
2020 Q4       1518.530 -994.0947 4031.156 -2324.1981 5361.259
```

```
78 plot(m.winters.forecasts, main = "4 Period Ahead Forecasts based on Winters Method")
```



Blue Points: Point Forecast, Dark Grey Interval: 80% P.I., Light Grey Interval: 95% P.I.

Summary: Exponential Smoothing

- Uses all the data values
  - Newer data have bigger weights.
- Trend and Seasonal Effects are explicitly estimated.
- Seasonal Effects are computed continuously with time.
- Next: ARIMA
  - Analyze autocorrelations instead of Trend/Seasonal effects.