Linear Arrival Rate Approximation

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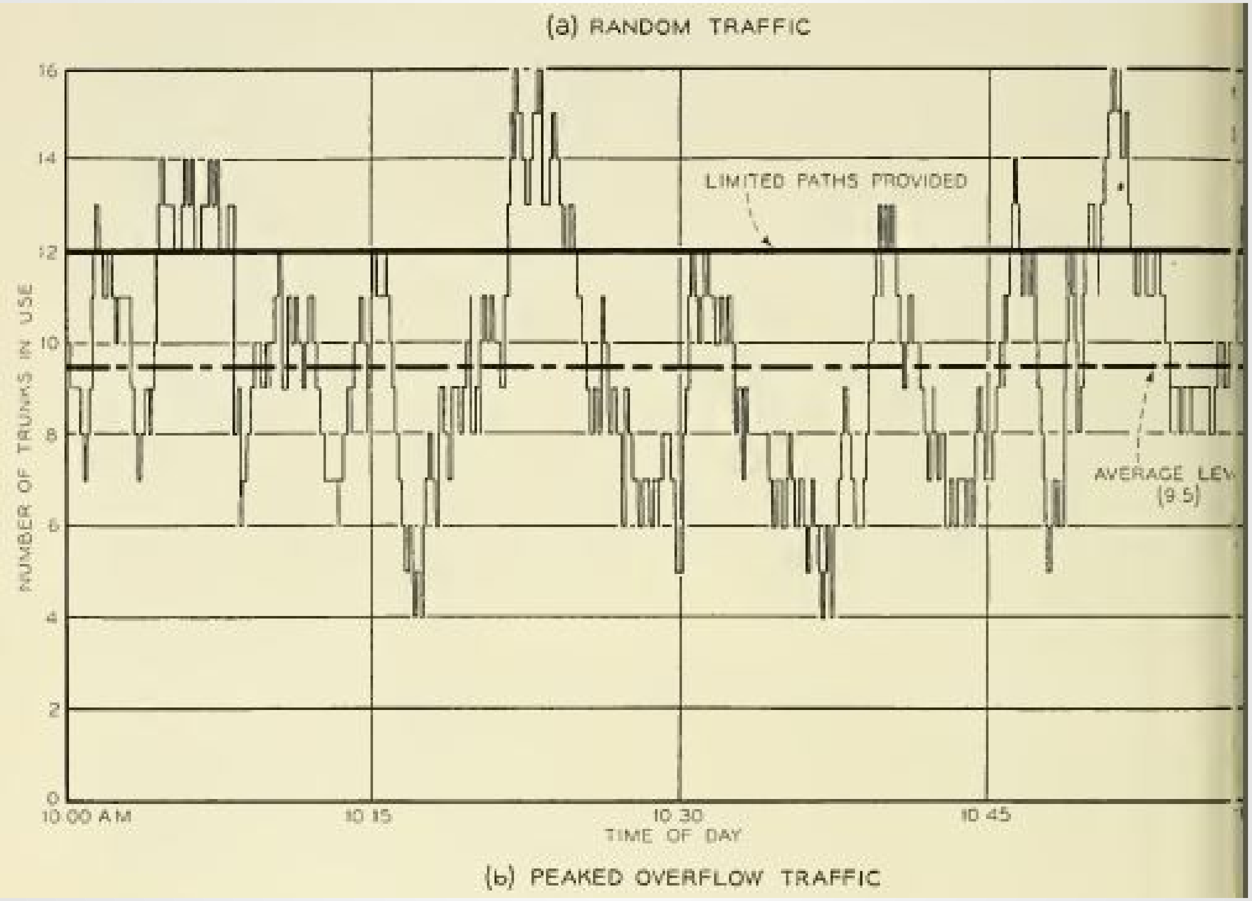
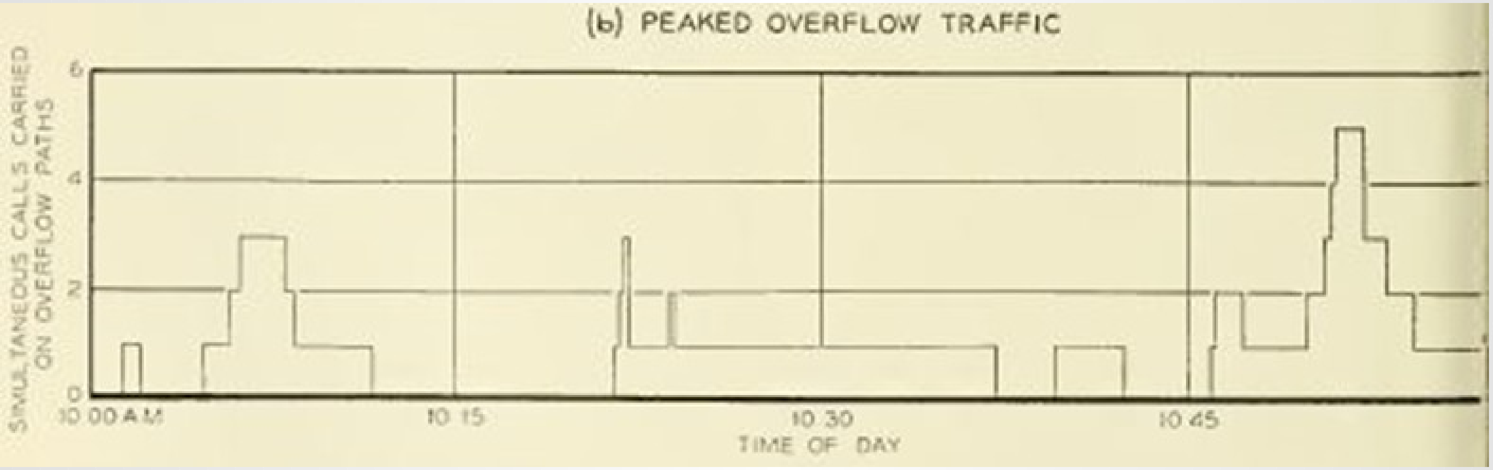
Random Traffic



*From Wilkinson

Overflow Traffic



*From Wilkinson

Question

- “What would be the physical description of a cause system with a variance smaller or larger than the Poisson?”   - Wilkinson

Physical Description

- “If the variance is smaller, there must be forces at work which retard the call arrival rate as the number of calls recently offered exceeds a normal, or average, figure, and which increase the arrival rate when the number recently arrived falls below the normal level. Conversely, the variance will exceed the Poisson’s should the tendencies of the forces be reversed.” -Wilkinson

Linear Arrival Rate System

of calls present

Arbitrary function that modifies call origination rate.
0 < Beta < 1

$$\lambda_k = [\alpha + (k - \alpha)\beta]$$

$$\mu_k = k\mu$$

‘mu’ is set to 1

Base Arrival Rate
alpha >= 0

Birth/Death Process

$$p_k = p_0 \prod_{j=0}^{k-1} \frac{\lambda_j}{\mu_{j+1}}$$

Binomial Coefficient

$$p_k = p_0 \left(\frac{\beta}{\mu}\right)^k \frac{1}{k!} a(a+1)...(a+k-1)$$

where

$$a \equiv \frac{\alpha}{\beta}(1-\beta)$$

Coefficient ‘x’ is

$$\frac{\beta}{\mu}$$

$$(1-x)^{-a} = \sum_{k=0}^{\infty} \frac{a(a+1)\ldots(a+k-1)x^k}{k!}$$

Probability Distribution of LA System

$$p_k = \frac{1}{\left(1-\frac{\beta}{\mu}\right)^{-a}} \left(\frac{\beta}{\mu}\right)^k \frac{a(a+1)\ldots(a+k-1)x^k}{k!}$$

Negative Binomial Distribution (Pascal)

$$p = 1 - \frac{\beta}{\mu}$$

$$n = a$$

$$q = \frac{\beta}{\mu}$$

$$p_k = \binom{n+k-1}{k} p^n q^k$$

$$\mu = 1$$

$$p + q = 1, 0 < p < 1$$

1st and 2nd Moments

$$E(x) = \frac{nq}{p} \Rightarrow \boxed{M = \alpha}$$
$$var(x) = \frac{nq}{p^2} \Rightarrow \boxed{V = \frac{\alpha}{1-\beta}}$$

$$\boxed{Z = \frac{V}{M} = \frac{1}{1-\beta}}$$

*Beta is always between 0 and 1, Z values will always be greater than 1. This translates to a <u>peaked process</u>. (As opposed to a smooth process)

Poisson vs Negative Binomial

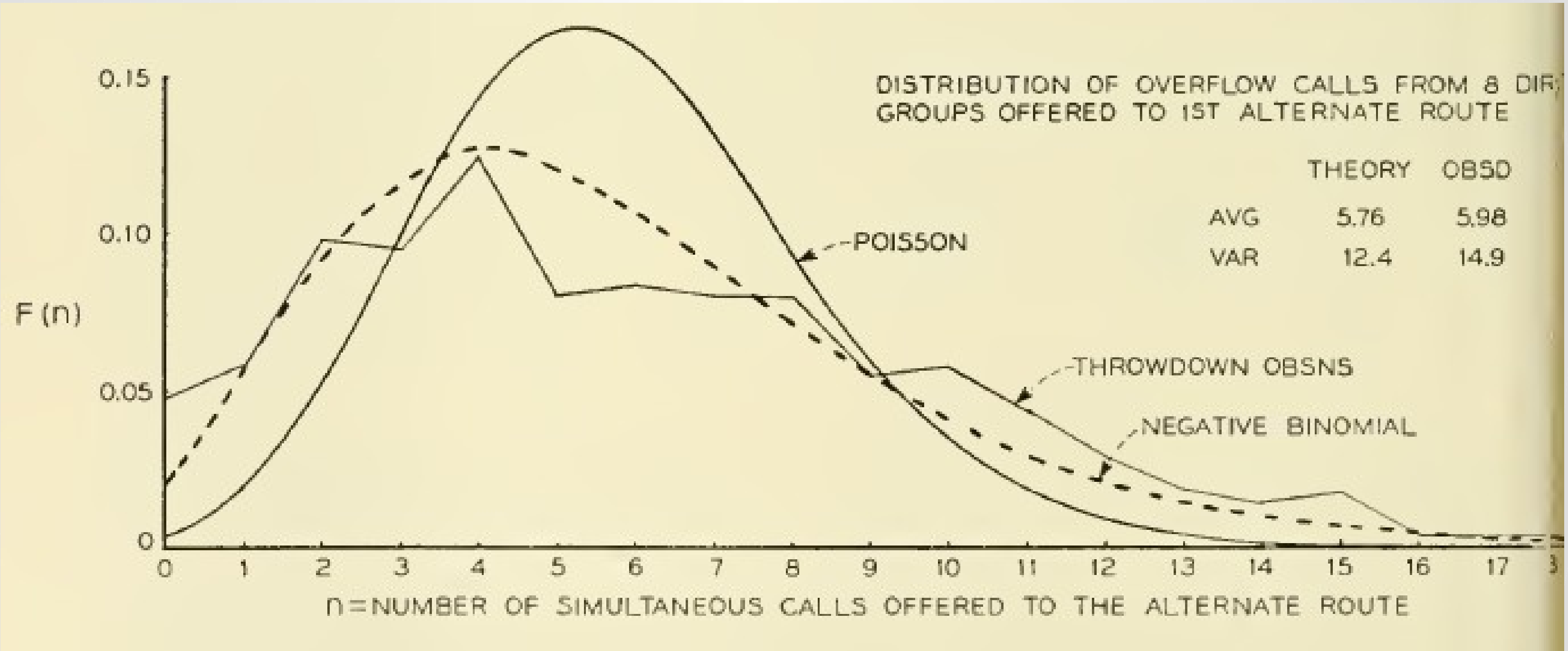


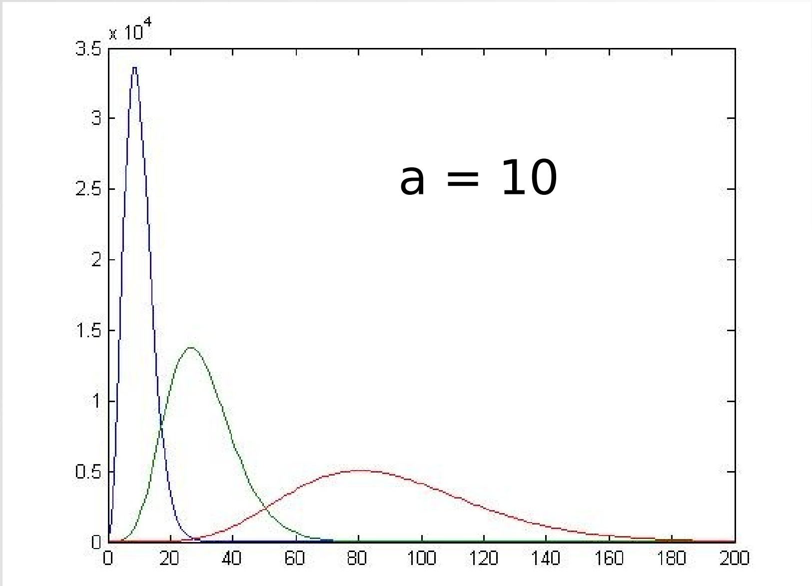
Fig. 21 — Comparison of theoretical and throwdown distributions of simultaneous calls offered to direct groups and to their first alternate route (OST No. 1).

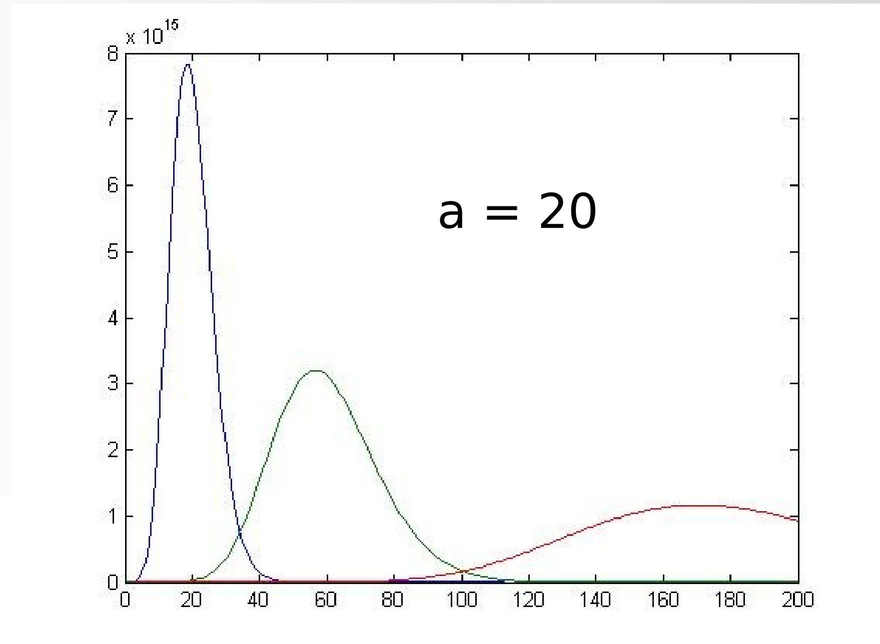
Plotting the Distribution

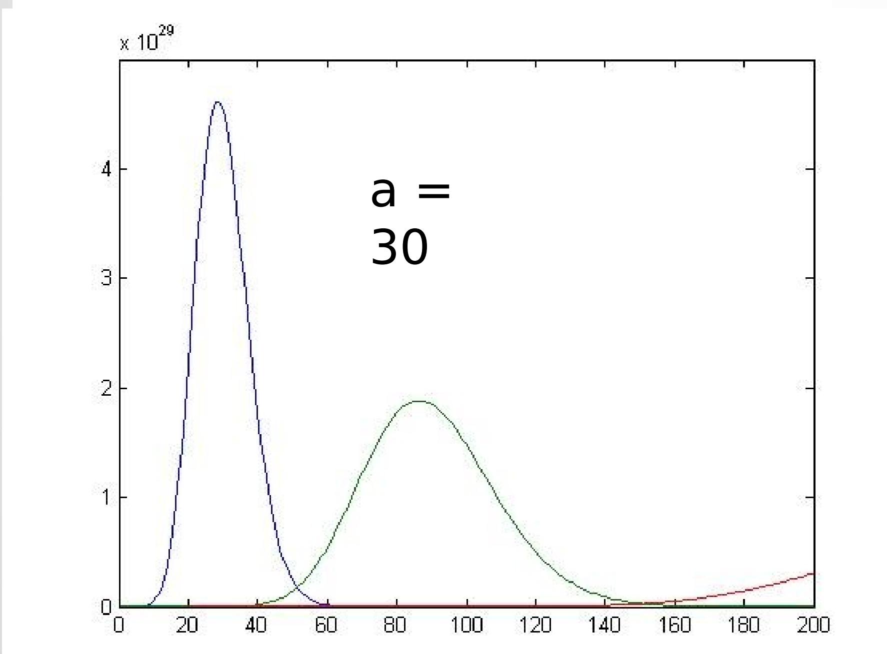
	Beta = 0.5	**Beta = 0.75**	**Beta = 0.9**
a = 10	Alpha = 10	Alpha = 30	Alpha = 90
a = 20	Alpha = 20	Alpha = 60	Alpha = 180
a = 30	Alpha = 30	Alpha = 90	Alpha = 270

$$a \equiv \frac{\alpha}{\beta}(1 - \beta)$$

Plots by value of a





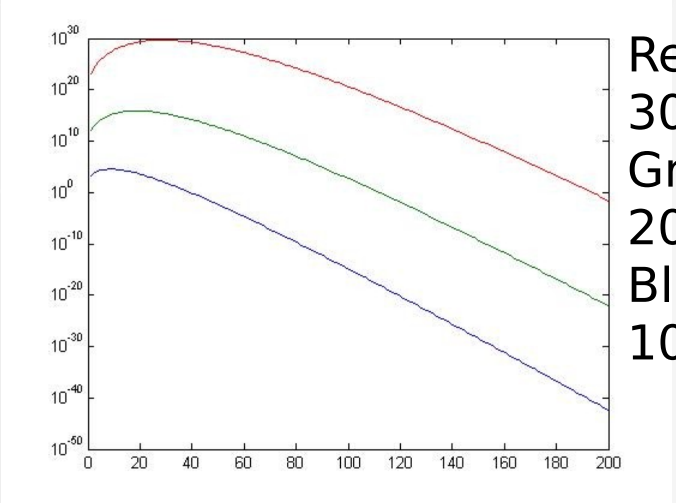


$$\boxed{M = \alpha}$$

$$\boxed{V = \frac{\alpha}{1-\beta}}$$

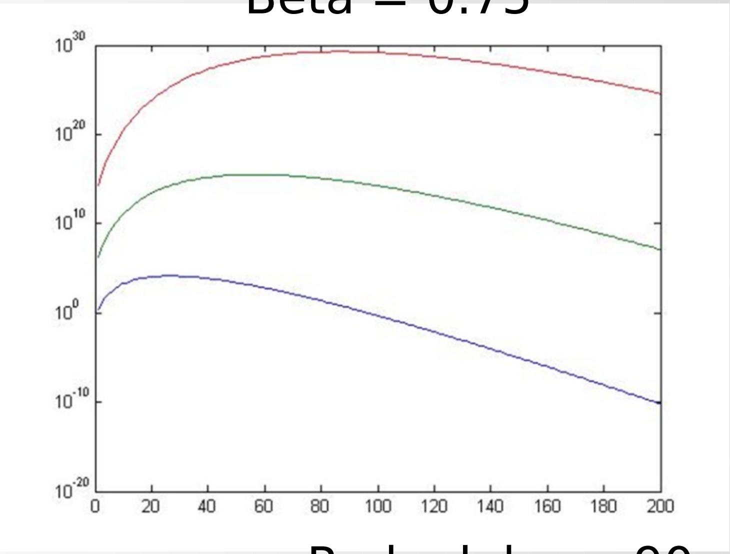
Plots by Beta value

Beta = 0.5



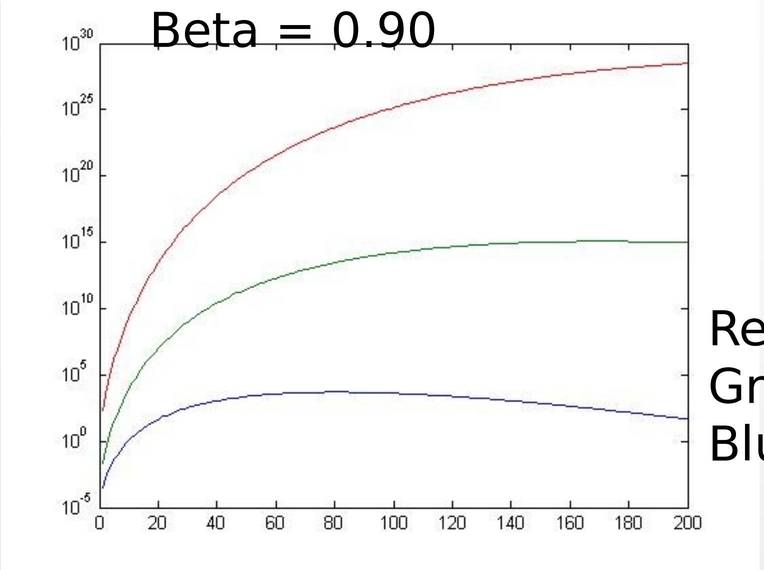
Red: alpha = 30
Green: alpha = 20
Blue: alpha = 10

Beta = 0.75



Red: alpha = 90
Green: alpha = 60
Blue: alpha = 30

Beta = 0.90

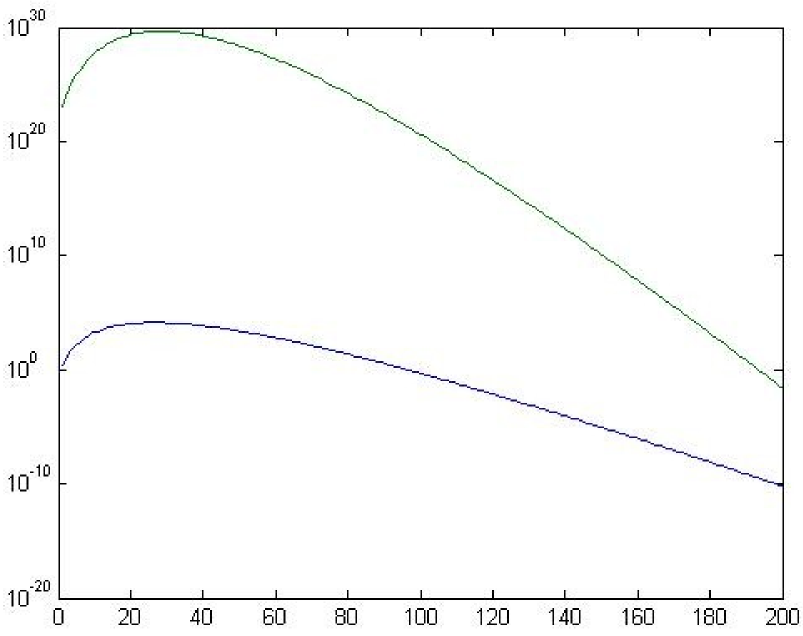


Red: alpha = 270
Green: alpha = 180
Blue: alpha = 90

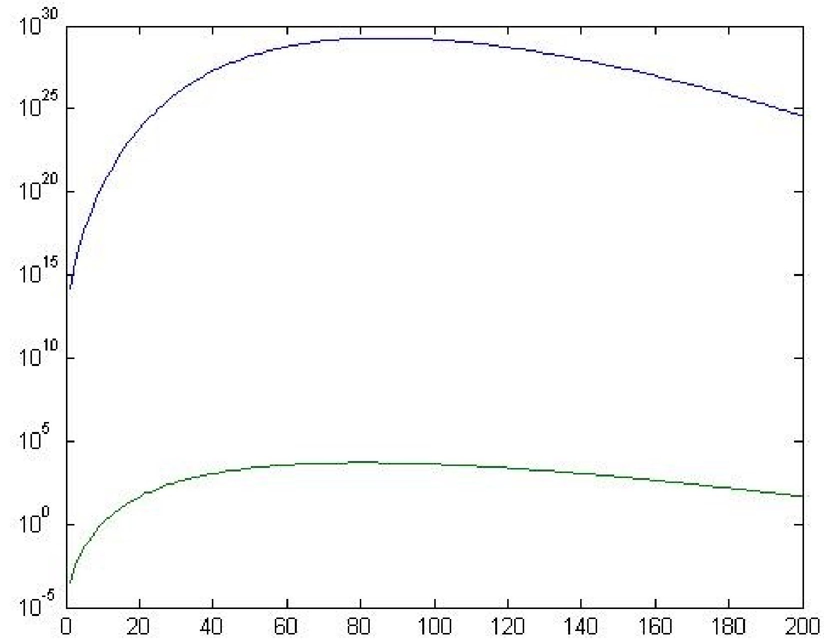
$$\boxed{a \equiv \frac{\alpha}{\beta}(1-\beta)}$$

Plots with same alpha

alpha = 30
Blue: a = 10, beta = 0.75
Green: a = 30, beta = 0.50



alpha = 90
Blue: a = 30, beta = 0.75
Green: a = 10, beta = 0.90

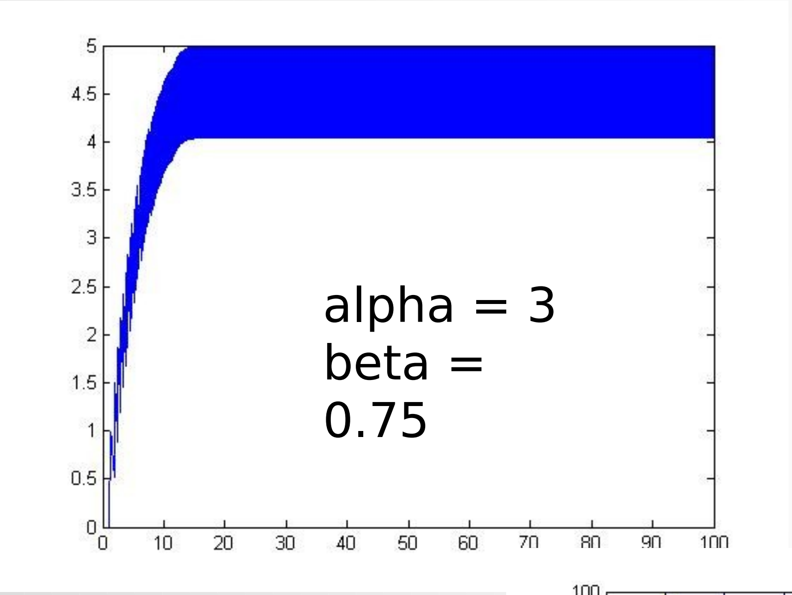


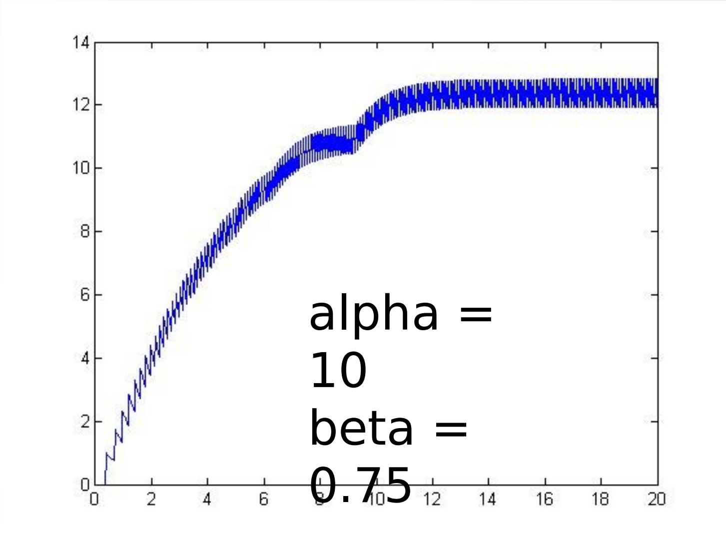
$$a \equiv \frac{\alpha}{\beta}(1-\beta)$$

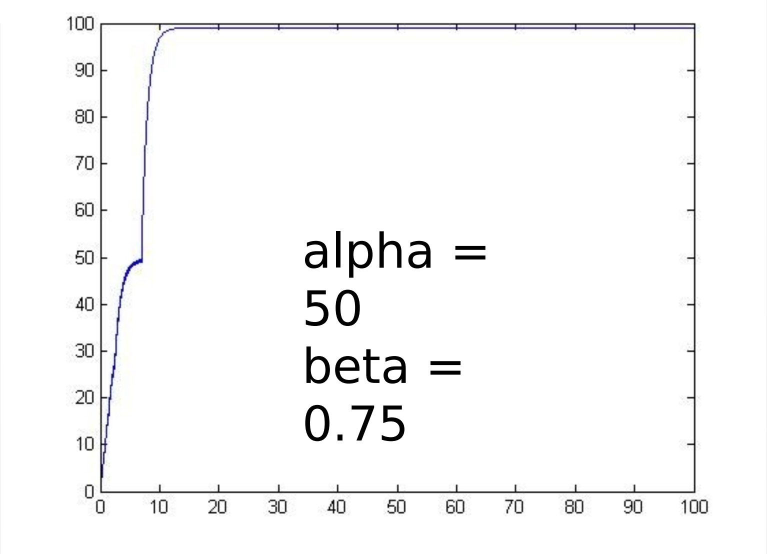
Simulation

	Beta= 0.25	Beta= 0.50	Beta= 0.75	Beta = 0.90	Beta = 0.99
alpha = 3	Mean = 3	Mean = 3.5	Mean = 4.5	Mean = 7.5	Mean = 50
alpha = 10	Mean = 11	Mean = 11	Mean = 12	Mean = 14	Mean = 50
alpha = 50	Mean = 100	Mean = 100	Mean = 100	Mean = 100	Mean = 100

Simulation Plots







$$\boxed{M = \alpha}$$

References

- Andre Girard- Routing and dimensioning in circuit-switched networks
- Wilkinson, R.I., “Theories for toll traffic engineering in the USA,” Bell System Technical Journal, vol. 35, pp 421-514, 1956.

Test Your Comprehension

- Why use Linear Arrival Rates?
- What kind of Distribution does a Linear Arrival Rate system resemble?
- What is the purpose of using Beta when calculating Arrival rate?