

# Practical Cryptography for Blockchains: Minimizing Trust and Maximizing Security

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## Abstract

In 2008, Satoshi Nakamoto introduced Bitcoin, the first digital currency without a trusted authority whose security is maintained by a decentralized blockchain. Since then, a plethora of decentralized applications have been proposed utilizing blockchains as a public bulletin board. In recent years, it has become clear that this basic functionality is not enough to prevent widespread attacks on both the privacy and security of blockchain users, as evidenced by the blockchain analytics industry and the billions of dollars stolen via cryptocurrency exploits to date.

This work explores the role cryptography has to play in the blockchain ecosystem to both reduce trust and secure user funds. I discuss a new paradigm for rollups called “naysayer proofs” which sits between optimistic and zero-knowledge approaches. Next, I show how to construct efficient, non-interactive, and private on-chain protocols for a large class of elections and auctions. Finally, I introduce a new type of threshold wallet which adds offline components at each custodian to improve security. Furthermore, the construction enables transparency by allowing the wallet owner to request custodians to prove “remembrance” of the secret key material. All three of these works will be deployed in practice or have seen interest from practitioners who I hope will be interested in applying them. Thus, they have real practical impact. I conclude by describing future directions.

## Acknowledgments

## Basis of this Dissertation

- [GGJ<sup>+</sup>24] Sanjam Garg, Noemi Glaeser, Abhishek Jain, Michael Lodder, and Hart Montgomery. Hot-cold threshold wallets with proofs of remembrance, 2024.
- [GSZB23] Noemi Glaeser, István András Seres, Michael Zhu, and Joseph Bonneau. Cicada: A framework for private non-interactive on-chain auctions and voting. Cryptology ePrint Archive, Paper 2023/1473, 2023. <https://eprint.iacr.org/2023/1473>.
- [SGB24] István András Seres, Noemi Glaeser, and Joseph Bonneau. Short paper: Naysayer proofs. In Jeremy Clark and Elaine Shi, editors, *Financial Cryptography and Data Security*, 2024.

## Other Publications by the Author

- [AGRS24] Behzad Abdolmaleki, Noemi Glaeser, Sebastian Ramacher, and Daniel Slamanig. Circuit-succinct universally-composable NIZKs with updatable CRS. In *37th IEEE Computer Security Foundations Symposium*, 2024.
- [GKMR23] Noemi Glaeser, Dimitris Kolonelos, Giulio Malavolta, and Ahmadreza Rahimi. Efficient registration-based encryption. In Weizhi Meng, Christian Damsgaard Jensen, Cas Cremers, and Engin Kirda, editors, *ACM CCS 2023*, pages 1065–1079. ACM Press, November 2023.
- [GMM<sup>+</sup>22] Noemi Glaeser, Matteo Maffei, Giulio Malavolta, Pedro Moreno-Sanchez, Erkan Tairi, and Sri Aravinda Krishnan Thyagarajan. Foundations of coin mixing services. In Heng Yin, Angelos Stavrou, Cas Cremers, and Elaine Shi, editors, *ACM CCS 2022*, pages 1259–1273. ACM Press, November 2022.

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# 1 Introduction

Bitcoin [Nak08] was the first digital currency to successfully implement a fully trustless and decentralized payment system. Underpinning Bitcoin is a *blockchain*, a distributed append-only ledger to record transactions. In Bitcoin, the blockchain’s consistency is enforced via a “proof-of-work” (PoW) consensus mechanism in which participants solve difficult computational puzzles (hash preimages) to append the newest bundle of transactions (a block) to the chain.

Ethereum [But14] introduced programmability via *smart contracts*, special applications which sit on top of the consensus layer and can maintain state and modify it programmatically. This has led to the emergence of a number of *decentralized applications* enabling more diverse functionalities in a trustless manner.

**Notation.** I use  $x \xleftarrow{\$} \mathcal{S}$  to denote sampling an element  $x$  uniformly at random from a set  $\mathcal{S}$ .

# 2 Preliminaries

We use  $[n]$  to denote the range  $\{1, \dots, n\}$ . For other ranges (mostly zero-indexed), we explicitly write the (inclusive) endpoints, e.g.,  $[0, n]$ . Concatenation of vectors  $\mathbf{x}, \mathbf{y}$  is written as  $\mathbf{x} \parallel \mathbf{y}$ . Let  $\lambda$  be the security parameter. We use the uppercase variable  $X$  for the free variable of a polynomial, e.g.,  $f(X)$ . We use a calligraphic font, e.g.,  $\mathcal{S}$  or  $\mathcal{X}$ , to denote sets or domains. When we apply an operation to two sets of equal size  $\ell$  we mean pairwise application, e.g.,  $\mathcal{Z} = \mathcal{X} + \mathcal{Y}$  means  $z_i = x_i + y_i \ \forall i \in [\ell]$ . Sampling an element  $x$  uniformly at random from a set  $\mathcal{X}$  is written as  $x \xleftarrow{\$} \mathcal{X}$ . We use  $:=$  to denote variable assignment,  $y \leftarrow \text{Alg}(x)$  to assign to  $y$  the output of some algorithm  $\text{Alg}$  on input  $x$ , and  $y \xleftarrow{\$} \text{Alg}(x)$  if the algorithm is randomized. When we wish to be explicit about the randomness  $r$  used, we write  $y \leftarrow \text{Alg}(x; r)$ .

For a non-interactive zero-knowledge proof system  $\Pi$ , we write  $\pi \leftarrow \Pi.\text{Prove}(x; w)$  to show that the proving algorithm takes as input an instance  $x$  and witness  $w$  and outputs a proof  $\pi$ . Verification is written as  $\Pi.\text{Vrfy}(x, \pi)$  and outputs a bit  $b$ .

We distinguish the key-pairs used in a signature scheme ( $\text{vk}, \text{sk}$  for “verification” and “signing” key, respectively) from those used in an encryption scheme ( $\text{ek}, \text{dk}$  for “encryption” and “decryption” key, respectively).

## 2.1 Bilinear Pairings

**Definition 1** (bilinear pairing). *A bilinear pairing is a map  $e : \mathbb{G}_1 \times \mathbb{G}_2 \mapsto \mathbb{G}_T$  where  $\mathbb{G}_1, \mathbb{G}_2$ , and  $\mathbb{G}_T$  are cyclic groups of prime order  $p$ . (Often,  $\mathbb{G}_1, \mathbb{G}_2$  are written in additive notation and  $\mathbb{G}_T$  in multiplicative notation.) Let  $g_1, h_1 \in \mathbb{G}_1$  and  $g_2, h_2 \in \mathbb{G}_2$  (generators of their respective groups). The map  $e$  has the following properties:*

**Bilinearity:** For all  $a, b \in \mathbb{Z}_p^*$ , the following hold:

$$\begin{aligned} e(g_1^a, g_2) &= e(g_1, g_2^a) = e(g_1, g_2)^a \\ e(g_1 h_1, g_2) &= e(g_1, g_2) \cdot e(h_1, g_2) \\ e(g_1, g_2 h_2) &= e(g_1, g_2) \cdot e(g_1, h_2) \end{aligned}$$

**Non-degeneracy:**  $e(g_1, g_2) \neq 1$ .

**Computability:** There is an efficient algorithm for computing  $e$ .

Pairings are divided into types based on the (in)equality of the utilized groups  $\mathbb{G}_1, \mathbb{G}_2$  and whether there is an efficiently computable homomorphism between the groups. For our purposes, we will use a type-3 pairing: asymmetric ( $\mathbb{G}_1 \neq \mathbb{G}_2$ ) and with no such efficiently computable homomorphism. In this case, the Decisional Diffie-Hellman (DDH) assumption is believed to hold in both  $\mathbb{G}_1$  and  $\mathbb{G}_2$ ; this is referred to as the *symmetric external Diffie-Hellman (SXDH) assumption*.

## 2.2 Shamir Secret Sharing

Shamir [?] introduced a scheme to share a secret among  $n$  parties such that any  $t$  parties can work together to recover the secret, but with any fewer parties the secret remains information-theoretically hidden.

**Construction 1** (Shamir secret sharing [?]). *Let  $p$  be a prime.*

- $\{s_1, \dots, s_n\} \leftarrow \text{Share}(s, t, n)$ : Given a secret  $s \in \mathbb{Z}_p$  and  $t \leq n \in \mathbb{N}$ , compute a  $t$ -out-of- $n$  sharing of  $s$  by choosing a random degree- $(t-1)$  polynomial  $f(X) \in \mathbb{Z}_p[X]$  such that  $f(0) = s$ . For  $i \in [n]$ , compute  $s_i := (i, f(i))$ .
- $\{s', \perp\} \leftarrow \text{Reconstruct}(S, t, n)$ : Given some set of shares  $S$ , check if  $|S| < t$ . If so, output  $\perp$ . Otherwise, without loss of generality, let  $S' := \{s_1, \dots, s_t\}$  be the first  $t$  entries of  $S$ , where  $s_i := (x_i, y_i)$ . Output the Lagrange interpolation at 0:

$$s' := \sum_{i=1}^t y_i \prod_{j=1, j \neq i}^t \frac{x_j}{x_j - x_i}.$$

The secret sharing scheme is *correct*, since for any secret  $s$  and values  $t \leq n \in \mathbb{N}$ , we have  $\text{Reconstruct}(\text{Share}(s, t, n), t, n) = s$ .

For notational convenience, let  $\text{Share}(s, t, n; r)[i]$  denote the  $i$ th share of  $s$  computed with randomness  $r$ . The reconstruction algorithm can be generalized to interpolate any point  $f(k)$  (not just the secret at  $k = 0$ ) and thereby recover the  $i$ th share:

- $\{s_k, \perp\} \leftarrow \text{Interpolate}(S, k, t, n)$ : If  $|S| < t$ , output  $\perp$ . Otherwise, use the first  $t$  entries  $(x_1, y_1), \dots, (x_t, y_t)$  of  $S$  to interpolate

$$f(k) = \sum_{i=1}^t y_i \prod_{j=1, j \neq i}^t \frac{x_j - k}{x_j - x_i}.$$

Output  $s_k := (k, f(k))$ .

## 2.3 BLS Signatures

**Construction 2** (BLS signature scheme [BLS01]). Let  $\mathbb{G}_1, \mathbb{G}_2$  be elliptic curve groups of order  $p$  generated by  $g_1$  and  $g_2$ , respectively, and  $e : \mathbb{G}_1 \times \mathbb{G}_2 \rightarrow \mathbb{G}_T$  be an efficiently computable asymmetric pairing between them. We also require a hash function  $H : \{0, 1\}^* \rightarrow \mathbb{G}_1$ . The signature scheme works as follows:

- $(\text{sk}, \text{vk}) \leftarrow \text{KGen}(1^\lambda)$ : Given the security parameter  $1^\lambda$ , sample  $x \xleftarrow{\$} \mathbb{Z}_p$  and output the keypair consisting of signing key  $\text{sk} := x$  and verification key  $\text{vk} := g_2^x$ .
- $\sigma \leftarrow \text{Sign}(\text{sk}, m)$ : Given a signing key  $\text{sk} \in \mathbb{Z}_p$  and a message  $m \in \{0, 1\}^*$ , compute a signature  $\sigma := H(m)^{\text{sk}}$ .
- $\{0, 1\} \leftarrow \text{Vrfy}(\text{vk}, m, \sigma)$ : Given a verification key  $\text{vk} \in \mathbb{G}_2$ , message  $m \in \{0, 1\}^*$ , and signature  $\sigma \in \mathbb{G}_1$ , if  $e(\sigma, g_2) = e(H(m), \text{vk})$ , output 1. Else output 0.

The security of BLS relies on the gap co-Diffie Hellman assumption on  $(\mathbb{G}_1, \mathbb{G}_2)$ , i.e., co-DDH being easy but co-CDH being hard on  $\mathbb{G}_1, \mathbb{G}_2$ , as well as the existence of an efficiently computable homomorphism  $\phi : \mathbb{G}_2 \rightarrow \mathbb{G}_1$  (type-2 pairing). Since we require a type-3 pairing for our purposes (i.e., no efficiently computable  $\phi$  exists), we rely on a stronger variant of the co-GDH assumption (see discussion in [BLS01, §3.1] and [SV05, §2.2]).

**Threshold variant** Sharing a BLS signing key  $\text{sk} \in \mathbb{Z}_p$  via Shamir secret sharing leads directly to a robust  $t$ -out-of- $n$  threshold signature [BLS01]. More specifically, each party  $i \in [n]$  receives a  $t$ -out-of- $n$  Shamir secret share  $\text{sk}_i$  of the key. The “partial” public keys  $\text{vk}_i := g_2^{\text{sk}_i}$  are published along with the public key  $\text{vk}$ .

A partial signature is computed in exactly the same way as a regular BLS signature, but under the secret key share:  $\sigma_i := H(m)^{\text{sk}_i}$ . This value is publicly verifiable by checking that  $(g_2, \text{vk}_i, H(m), \sigma_i)$  is a co-Diffie-Hellman tuple (i.e., it is of the form  $(g_2, g_2^a, h, h^a)$  where  $g_2 \in \mathbb{G}_2$  and  $h \in \mathbb{G}_1$ ).

Given  $t$  valid partial signatures on a message  $m \in \{0, 1\}^*$  anyone can recover a regular BLS signature:

- $\sigma \leftarrow \text{Reconstruct}(S := \{(i, \sigma_i)\})$ : Let  $S' \subseteq S$  be the set of valid partial signatures in  $S$ . If  $|S'| < t$ , output  $\perp$ . Otherwise, without loss of generality, assume the first  $t$  valid signatures come from users  $1, \dots, t$  and recover the complete signature as

$$\sigma \leftarrow \prod_{i=1}^t \sigma_i^{\lambda_i}, \text{ where } \lambda_i = \prod_{j=1, j \neq i}^t \frac{j}{j-i} \pmod{p}$$

Notice that the reconstruction simply performs Shamir reconstruction of the signing key shares  $\text{sk}_i$  in the exponent and thus the output will equal  $H(m)^{\text{sk}}$ . Hence, the complete signature is indistinguishable from a regular BLS signature, and verification proceeds exactly as in the regular scheme.

## 2.4 KZG polynomial commitments

We give the asymmetric version of KZG below.

**Construction 3** (KZG polynomial commitments [KZG10]). *Let  $\mathbb{G}_1, \mathbb{G}_2$  be elliptic curve groups of order  $p$  with generators  $g_1, g_2$  and  $e : \mathbb{G}_1 \times \mathbb{G}_2 \mapsto \mathbb{G}_T$  be an elliptic curve pairing. The following is a commitment scheme for polynomials in  $\mathbb{Z}_p[X]$  with degree at most  $d$ .*

- $\text{crs} \leftarrow \text{Setup}(d)$  : Sample  $\tau \xleftarrow{\$} \mathbb{Z}_p$  and output  $\text{crs} := \{g_1, g_1^\tau, g_1^{\tau^2}, \dots, g_1^{\tau^d}, g_2, g_2^\tau\}$ .
- $\text{com}_f \leftarrow \text{Com}(\text{crs}, f(X))$  : Let  $f(X) = a_0 + a_1X + \dots + a_dX^d \in \mathbb{Z}_p[X]$ . Use  $\text{crs}$  to compute and output  $g_1^{f(\tau)} = g_1^{a_0} \cdot (g_1^\tau)^{a_1} \dots (g_1^{\tau^d})^{a_d} = g_1^{a_0 + a_1\tau + \dots + a_d\tau^d} \in \mathbb{G}_1$ .
- $(f(i), \pi_i) \leftarrow \text{Open}(\text{crs}, f(X), i)$  : To open  $f(X)$  at  $i$ , let  $q_i(X) := \frac{f(X) - f(i)}{X - i} \in \mathbb{Z}_p[X]$ <sup>1</sup>. Then compute  $\text{com}_{q_i} \leftarrow \text{Com}(\text{crs}, q_i(X))$  and output  $(f(i), \text{com}_{q_i}) \in \mathbb{Z}_p \times \mathbb{G}_1$ .
- $\{0, 1\} \leftarrow \text{Vrfy}(\text{crs}, \text{com}_f, i, y, \pi_i)$  : To confirm  $y = f(i)$ , it suffices to check that  $q_i(X) = \frac{f(X) - y}{X - i}$  at  $X = \tau$ . This can be done with a single pairing check:  

$$e(\text{com}_f / g_1^y, g_2) \stackrel{?}{=} e(\pi_i, g_2^\tau / g_2^i)$$

The security of the scheme relies on the  $d$ -Strong Diffie Hellman assumption ( $d$ -SDH), which states that given  $\{g_1, g_1^\tau, \dots, g_1^{\tau^d}, g_2, g_2^\tau\}$ , it is difficult to compute  $(c, g_1^{\frac{1}{\tau - c}})$  for any  $c \in \mathbb{Z}_p \setminus \{-\tau\}$ . This assumption is stronger than  $d$ -SDH in  $\mathbb{G}_1$ , which in turn implies DDH in  $\mathbb{G}_1$ .

## 2.5 Pedersen Commitments

Next, we recall Pedersen commitments [Ped92], a commitment scheme which is unconditionally (information-theoretically) hiding and computationally binding (by the discrete logarithm assumption on  $\mathbb{G}$ ). In our construction we will instantiate the scheme over  $\mathbb{G}_1$ , so we let  $\mathbb{G} = \mathbb{G}_1$  in the description below.

**Construction 4** (Pedersen commitment scheme [Ped92]). *Let  $\mathbb{G}_1$  be a group of order  $p$  and  $g_1, h_1$  be generators of  $\mathbb{G}_1$ . The following is a commitment scheme for elements  $x \in \mathbb{Z}_p$ .*

<sup>1</sup>This is a polynomial by Little Bézout's Theorem.



- $(\text{com}, \text{decom}) \leftarrow \text{Com}(x)$  : Sample  $r \xleftarrow{\$} \mathbb{Z}_p$  and return  $\text{com} := g_1^x h_1^r$  and decommitment information  $(x, r)$ .
- $(x, r) \leftarrow \text{Open}(\text{com}, \text{decom})$  : To open  $\text{com}$ , directly output  $\text{decom} = (x, r)$ .
- $\{0, 1\} \leftarrow \text{Vrfy}(\text{com}, x, r)$  : To confirm the opening of  $\text{com}$  to  $x$ , it suffices to check that  $\text{com} = g_1^x h_1^r$ .

A PoK of the committed value can be computed using a Sigma protocol due to Okamoto [Oka93], which can be made non-interactive using the Fiat-Shamir transform [FS87]. We refer to this protocol as  $\Pi_{\text{ped}}$  and present it in Figure 1.

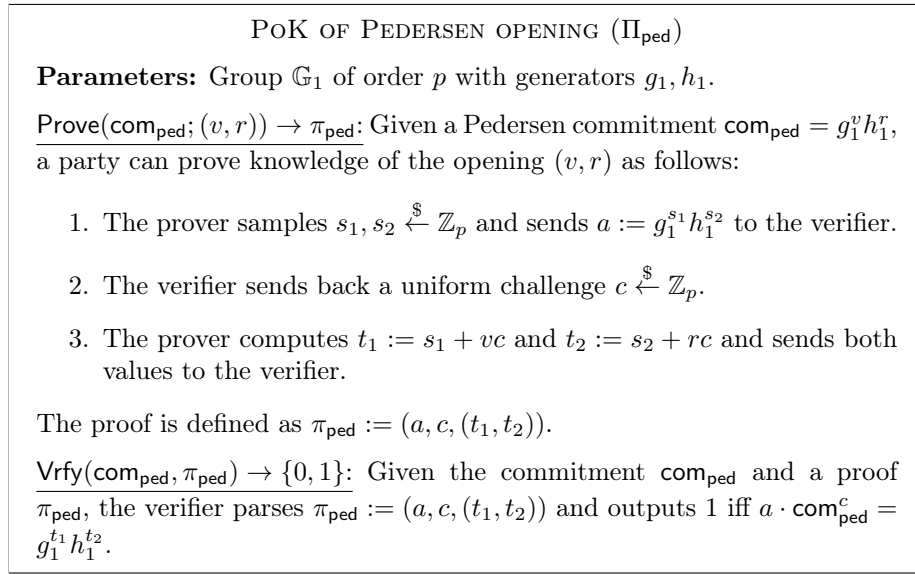


Figure 1: The proof system  $\Pi_{\text{ped}}$  used to prove knowledge of the opening to a Pedersen commitment [Oka93].

## 2.6 Leftover Hash Lemma

We use the presentation of the leftover hash lemma (LHL) [IZ89] from [AMPR19].<sup>2</sup> Let  $(\mathcal{X}, \oplus)$  be a finite group of size  $|\mathcal{X}|$ , and let  $n$  be a positive integer. For any fixed  $2n$ -vector of group elements  $\mathbf{x} = \{x_{j,b}\}_{j \in [n], b \in \{0,1\}} \in \mathcal{X}^{2n}$ , denote by  $\mathcal{S}_{\mathbf{x}}$  the following distribution:

$$\mathcal{S}_{\mathbf{x}} = \left\{ \bigoplus_{j \in [n]} x_{j,r_j} : (r_1, \dots, r_n) \leftarrow \{0, 1\}^n \right\}.$$

<sup>2</sup>We specifically use the improved version from the Journal of Cryptology version of this paper.

Also, let  $\mathcal{U}_{\mathcal{X}}$  denote the uniform distribution over  $\mathcal{X}$ , and let  $\Delta(\mathcal{D}_1, \mathcal{D}_2)$  denote the statistical distance between the distributions  $\mathcal{D}_1$  and  $\mathcal{D}_2$ . We will use the following special case of leftover hash lemma [IZ89]. The proof can be found in the JoC version of [AMPR19].

**Lemma 1.** (Leftover Hash Lemma.) *Let  $(\mathcal{X}, \oplus)$  be a finite group, and let  $\mathcal{S}_{\mathbf{x}}$  and  $\mathcal{U}_{\mathcal{X}}$  be two distributions over  $\mathcal{X}$  as defined above. For any (large enough) positive integer  $n$ , it holds that*

$$\Pr_{\mathbf{x} \leftarrow \mathcal{X}^{2n}} \left[ \Delta(\mathcal{S}_{\mathbf{x}}, \mathcal{U}_{\mathcal{X}}) > \sqrt[4]{\frac{|\mathcal{X}|}{2^n}} \right] \leq \sqrt[4]{\frac{|\mathcal{X}|}{2^n}}.$$

*In particular, for any  $n > \log(|\mathcal{X}|) + \omega(\log(\lambda))$ , if  $\mathbf{x}$  is sampled uniformly then with overwhelming probability the statistical distance between two distributions is negligible.*

## 2.7 Universal Composability (UC) Framework

In the universal composability (UC) framework [Can01], the security requirements of a protocol are defined via an *ideal functionality* which is executed by a trusted party. To prove that a protocol *UC-realizes* a given ideal functionality, we show that the execution of this protocol (in the real or hybrid world) can be *emulated* in the ideal world, where in both worlds there is an additional adversary  $\mathcal{E}$  (the environment) which models arbitrary concurrent protocol executions. Specifically, we show that for any adversary  $\mathcal{A}$  attacking the protocol execution in the real world (by controlling communication channels and corrupting parties involved in the protocol execution), there exists an adversary  $\mathcal{S}$  (the simulator) in the ideal world who can produce a protocol execution which no environment  $\mathcal{E}$  can distinguish from the real-world execution.

Below we describe the UC framework as it is presented in [CLOS02]. All parties are represented as probabilistic interactive Turing machines (ITMs) with input, output, and ingoing/outgoing communication tapes. For simplicity, we assume that all communication is authenticated, so an adversary can only delay but not forge or modify messages from parties involved in the protocol. Therefore, the order of message delivery is also not guaranteed (asynchronous communication). We consider a PPT malicious, adaptive adversary who can corrupt or tamper with parties at any point during the protocol execution.

The execution in both worlds consists of a series of sequential party activations. Only one party can be activated at a time (by writing a message on its input tape). In the real world, the execution of a protocol  $\Pi$  occurs among parties  $P_1, \dots, P_n$  with adversary  $\mathcal{A}$  and environment  $\mathcal{E}$ . In the ideal world, interaction takes place between dummy parties  $\tilde{P}_1, \dots, \tilde{P}_n$  communicating with the ideal functionality  $\mathcal{F}$ , with the adversary (simulator)  $\mathcal{S}$  and environment  $\mathcal{E}$ . Every copy of  $\mathcal{F}$  is identified by a unique session identifier `sid`.

In both the real and ideal worlds, the environment is activated first and activates either the adversary ( $\mathcal{A}$  resp.  $\mathcal{S}$ ) or an uncorrupted (dummy) party by

writing on its input tape. If  $\mathcal{A}$  (resp.  $\mathcal{S}$ ) is activated, it can take an action or return control to  $\mathcal{E}$ . After a (dummy) party (or  $\mathcal{F}$ ) is activated, control returns to  $\mathcal{E}$ . The protocol execution ends when  $\mathcal{E}$  completes an activation without writing on the input tape of another party.

We denote with  $\text{REAL}_{\Pi, \mathcal{A}, \mathcal{E}}(\lambda, x)$  the random variable describing the output of the real-world execution of  $\Pi$  with security parameter  $\lambda$  and input  $x$  in the presence of adversary  $\mathcal{A}$  and environment  $\mathcal{E}$ . We write the corresponding distribution ensemble as  $\{\text{REAL}_{\Pi, \mathcal{A}, \mathcal{E}}(\lambda, x)\}_{\lambda \in \mathbb{N}, x \in \{0,1\}^*}$ . The output of the ideal-world interaction with ideal functionality  $\mathcal{F}$ , adversary (simulator)  $\mathcal{S}$ , and environment  $\mathcal{E}$  is represented by the random variable  $\text{IDEAL}_{\mathcal{F}, \mathcal{S}, \mathcal{E}}(\lambda, x)$  and corresponding distribution ensemble  $\{\text{IDEAL}_{\mathcal{F}, \mathcal{S}, \mathcal{E}}(\lambda, x)\}_{\lambda \in \mathbb{N}, x \in \{0,1\}^*}$ .

The actions each party can take are summarized below:

- Environment  $\mathcal{E}$ : **read** output tapes of the adversary ( $\mathcal{A}$  or  $\mathcal{S}$ ) and any uncorrupted (dummy) parties; then **write** on the input tape of one party (the adversary  $\mathcal{A}$  or  $\mathcal{S}$  or any uncorrupted (dummy) parties).
- Adversary  $\mathcal{A}$ : **read** its own tapes and the outgoing communication tapes of all parties; then **deliver** a pending message to party by writing it on the recipient's ingoing communication tape *or* **corrupt** a party (which becomes inactive: its tapes are given to  $\mathcal{A}$  and  $\mathcal{A}$  controls its actions from this point on, and  $\mathcal{E}$  is notified of the corruption).
- Real-world party  $P_i$ : only follows its code (potentially writing to its output tape or sending messages via its outgoing communication tape).
- Dummy party  $\tilde{P}_i$ : acts only as a simple relay with the ideal functionality  $\mathcal{F}$ , copying inputs from its input tape to its outgoing communication tape (to  $\mathcal{F}$ ) and any messages received on its ingoing communication tape (from  $\mathcal{F}$ ) to its output tape.
- Adversary  $\mathcal{S}$ : **read** its own input tape and the public headers (see below) of the messages on  $\mathcal{F}$ 's and dummy parties' outgoing communication tapes; then **deliver** a message to  $\mathcal{F}$  from a dummy party or vice versa by copying it from the sender's outgoing communication tape to the recipient's incoming communication tape *or* **send** its own message to  $\mathcal{F}$  by writing on the latter's incoming communication tape *or* **corrupt** a dummy party (which becomes inactive: its tapes are given to  $\mathcal{S}$  and  $\mathcal{S}$  controls its actions from this point on, and  $\mathcal{E}$  and  $\mathcal{F}$  are notified of the corruption).
- Ideal functionality  $\mathcal{F}$ : **read** incoming communication tape; then **send** any messages specified by its definition to the dummy parties and/or adversary  $\mathcal{S}$  by writing to its outgoing communication tape.

**Definition 2.** We say a protocol  $\Pi$  UC-realizes an ideal functionality  $\mathcal{F}$  if for any PPT adversary  $\mathcal{A}$ , there exists a simulator  $\mathcal{S}$  such that for any environment  $\mathcal{E}$ , the distribution ensembles  $\{\text{REAL}_{\Pi, \mathcal{A}, \mathcal{E}}(\lambda, x)\}_{\lambda \in \mathbb{N}, x \in \{0,1\}^*}$  and  $\{\text{IDEAL}_{\mathcal{F}, \mathcal{S}, \mathcal{E}}(\lambda, x)\}_{\lambda \in \mathbb{N}, x \in \{0,1\}^*}$  are computationally indistinguishable.

### 3 Naysayer Proofs

### 4 On-chain Voting and Auctions

### 5 Hot-Cold Threshold Wallets

In the cryptocurrency ecosystem, anyone who controls the signing key for some account can take actions on the user’s behalf. This makes signing keys very valuable cryptographic material, since they can allow an attacker to transfer potentially large sums of money out of a user’s account with no recourse. A plethora of solutions has emerged to safeguard these keys in the form of cryptocurrency wallets.

Wallets can be classified into two types: custodial and non-custodial solutions. In a custodial wallet, a client trusts some third party (the custodian) to store the signing key on their behalf and use it to authorize transactions at their request. This clearly requires a strong trust assumption but relieves the client of the need to securely store such a high-value secret. At the other end of the spectrum, non-custodial wallets require clients to store their own signing keys, e.g., on their machine or in a hardware wallet [AGKK19]. This can be a usability issue, since by definition there cannot be a recovery or reset mechanism in case the client loses the key file or hardware wallet, or if the key material is compromised. An increasingly common compromise between these two extremes are threshold wallets [KMOS21, DEF<sup>+</sup>23, BMP22, Eya21]. These are essentially custodial wallets with multiple custodians, where each one is given only a share of the signing key. This maintains the usability advantages of custodial wallets while reducing the trust placed in any single custodian. Unfortunately, the normally highly-interactive threshold signing procedures require custodians to be online during the signing phase, making them vulnerable to attacks aiming to learn their shares [NNN<sup>+</sup>23, Mak23].

**Cold wallets** A common mitigation is to keep only limited funds in the online wallet, with most of a user’s assets kept in a highly secure air-gapped wallet. This can be realized via, e.g., a deterministic wallet [But13, DFL19, ADE<sup>+</sup>20, Hu23, ER22], which enables unlinkable transfers to an offline wallet by specifying how to deterministically derive session keys from a master public-private key pair. Thus, the online wallet can compute and publish the current session public key, allowing anyone to transfer money to the cold wallet (which can derive the corresponding session private key).

This idea is standardized by the BIP32 proposal [Wui12], which also defines how these session keys can be derived in a hierarchical manner to create “child” wallets who can control their own funds. Because the master secret key (and any session/child secret keys) must still be stored in a trusted place, this makes the offline wallet a highly valuable target when it inevitably comes online to transfer funds to the online wallet. Although a recent work [DEF<sup>+</sup>23] shows how to enable threshold child wallets, the issue of a single point of failure still

persists at the root wallet.

**Ensuring availability** Another problem arises when the responsibility to store key material is delegated solely to external custodians who have no explicit incentive to make sure the key stays available: custodians may be lazy and stop storing the key material as a way to save space and money, relying on the assumption that the other custodians will act honestly and store their corresponding shares. Even in the case in which all custodians have honest intentions, a client with a large sum of money in their threshold wallet may still wish to intermittently check that a custodian has access to its key material. To our knowledge, no wallet explicitly implements such a feature, although a similar effect can be achieved by requesting approval of an empty transaction, since this makes it clear that at least  $t$  of the parties still hold their shares (although not necessarily which ones). A more targeted audit could be implemented via generic zero-knowledge proofs requiring a party to prove knowledge of the  $i$ th key share using some time-based challenge, but the practical efficiency of such a proof is unclear.

## 5.1 Our Results

We introduce a new type of wallet which combines offline components with the threshold wallet approach to achieve stronger security guarantees. In our model, each threshold signer consists of two parties: an online *hot* storage and an offline, resource-constrained *cold* storage, e.g., a secure enclave, trusted execution environment, or perhaps something even more sophisticated. Producing a threshold signature on any message should require the involvement of both the hot and cold parties while minimizing the computation and communication on the part of the cold party. Wallet owners should also be able to request proofs from any party, hot or cold, that it retains its share of the secret key. Finally, it should be possible to periodically refresh the shares of the online parties to thwart an adversary who slowly and incrementally corrupts parties in the system. We describe each of our contributions in more detail below.

**Stronger threat model** In a hot-cold threshold wallet, an attacker must corrupt *both* the cold and hot components of a signer to obtain its key share. This requirement to corrupt a threshold  $t$  of hot-cold *pairs* is different from a generic  $2t$ -out-of- $2n$  threshold wallet, since the attacker cannot forge a signature by corrupting any arbitrary set of  $2t$  parties: in our model, corrupting, e.g.,  $2t$  hot parties should be of no use in forging a signature (in fact, even corrupting all  $n$  hot parties should be useless). Indeed, our threat model is instead comparable to a Boolean signing policy which requires at least  $t$  pairs of hot *and* cold parties to contribute, which is much more difficult to attack since the cold components are almost always offline.

**Threshold BLS construction** We show how to construct such a hot-cold threshold signing protocol for the BLS signature scheme [BLS01, Bol03]. Although ECDSA [GGN16, DKLS18, LN18, GG18, AHS20, GKSŠ20, DJN<sup>+</sup>20, CGG<sup>+</sup>20, CCL<sup>+</sup>20, CCL<sup>+</sup>21, Pet21, ANO<sup>+</sup>22] and Schnorr [KG20, BCK<sup>+</sup>22, BLM22] are the most popular threshold signature schemes in the literature, their more complicated structure makes a threshold signing procedure with offline parties difficult. BLS signatures have the advantage of being simple to understand and deploy. This has led to their widespread use in production systems, including in Ethereum’s consensus protocol [Edg23, §2.9.1], Filecoin [Fil], transactions on the Chia Network [Chi24], and a BLS smart contract wallet [Pri22]. With the event of account abstraction on Ethereum [Eth24], users will be able to specify alternative signature schemes to verify their transactions, further easing the adoption of BLS-based wallets. The homomorphic structure of BLS allows us to give a simple non-interactive hot-cold threshold signature protocol. Our construction also has the added benefit that the cold parties can generate their secret key shares independently, without interacting with the hot parties or with each other.

**Proofs of remembrance** We show how the cold and hot parties in our protocol can produce “proofs of remembrance”, i.e., zero-knowledge proofs that they still possess their key material. In order to support proactive refreshes (see below), our proof systems must accommodate periodic updates to individual key shares while guaranteeing that the underlying secret key is still available. We show how to meet both needs simultaneously, allowing a client to intermittently and independently audit its (hot or cold) custodians. This allows it to ensure that its key material has not been overwritten or forgotten, and its funds are still accessible.

**Proactive refresh** Many threshold wallets support proactive refresh [KMOS21], meaning secret key shares are regularly updated. This forces an attacker to compromise at least  $t$  parties within a single epoch in order to mount a successful attack: otherwise, any partial key material which has been obtained is made obsolete by the refresh operation. Our protocol allows proactive refresh of the hot key shares while still letting hot parties prove knowledge of those shares and their consistency with the (unchanging) verification key.

**Efficient open-source implementation** We implemented our protocol and show that it is practically efficient for essentially all reasonable settings of  $t, n$ . Producing a signature takes less than 1ms and computing proofs of remembrance is on the order of milliseconds. Our implementation is publicly available in Hyperledger Labs<sup>3</sup> and is Apache 2.0 licensed.

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<sup>3</sup><https://github.com/hyperledger-labs/agora-key-share-proofs>

## 5.2 Technical overview

We will use the superscripts *hot* and *cold*, respectively, to denote the hot and cold components of some value, and a subscript  $i$  to denote the value corresponds to the  $i$ th party. For example, the  $i$ th party’s cold signature is written as  $\sigma_i^{\text{cold}}$ . We use the words “user” and “client” interchangeably to refer to the wallet owner.

A starting point for constructing a threshold signature with our desired access structure would be to first compute a  $t$ -out-of- $n$  sharing  $\text{sk}_1, \dots, \text{sk}_n$  of the signing key  $\text{sk}$ , then give each hot-cold pair a 2-out-of-2 (additive) sharing  $\text{sk}_i^{\text{hot}}, \text{sk}_i^{\text{cold}}$  of one of the threshold shares. The signing protocol has to be designed carefully, since we need to ensure that any communication from the cold party to the hot is “bound” to the message  $m$  being signed. Otherwise, the hot storage could replay the communication and produce a signature on some other message  $m' \neq m$  without the cold storage’s cooperation, violating our threat model.

Our scheme takes advantage of the natural homomorphism of BLS signatures to use the additive sharing idea. At a high level, the cold and hot parties will receive shares  $r$  and  $\text{sk}_i + r$ , respectively, of the threshold signing key share. Then the cold storage can send the hot storage a partial signature  $\sigma_i^{\text{cold}} := H(m)^r$  which is “bound” to the message  $m$ , and the hot storage computes its own partial signature  $\sigma_i^{\text{hot}} := H(m)^{\text{sk}_i + r}$ . The threshold BLS signature  $\sigma_i$  can be computed by the hot storage as  $\sigma_i^{\text{hot}} / \sigma_i^{\text{cold}} = H(m)^{\text{sk}_i}$ . This scheme also enables proactive refresh of the hot shares basically “for free” due to the malleability of the additive sharing: the wallet owner can simply send every hot storage a  $t$ -out-of- $n$  sharing of zero, which the hot storage adds to its current share.

Our idea for realizing this additive sharing is to allow each cold party to sample an encryption key-pair  $(\text{ek}_i, \text{dk}_i)$  and let the cold share  $r$  be a deterministic function of  $\text{ek}_i$  and the client signing key  $\text{sk}$ . In particular, we want to set  $r := \mathcal{H}(\text{ek}_i^{\text{sk}})$  for a carefully-designed efficient hash function  $\mathcal{H}$ . Put another way, this means the hot key share will be an encryption of  $\text{sk}_i$  under the cold party’s public key  $\text{ek}_i$ . This allows the hot key material to be computed by a key generation protocol which outputs the verification key  $\text{vk} = g^{\text{sk}}$  and each hot party’s key share  $\text{sk}_i + \mathcal{H}(\text{ek}_i^{\text{sk}})$  without interacting with the cold parties. The crucial piece is that each cold encryption key-pair will be instantiated in the same group and using the same generator as the signing key-pair, so  $\text{ek}_i^{\text{sk}} = (g^{\text{dk}_i})^{\text{sk}} = \text{vk}^{\text{dk}_i}$ . Thus, given the verification key, each cold storage can now independently recover its signing key share  $r$  as  $\mathcal{H}(\text{vk}^{\text{dk}_i})$ . This means that a partial signature is effectively computed by decrypting the hot key share “in the exponent” of the partial BLS signature.

To prove remembrance, each type of party uses its own proof system. For cold parties, a standard proof of knowledge of discrete logarithm (for  $\text{dk}_i$  corresponding to  $\text{ek}_i$ ) suffices. For the hot parties, the relation is more complicated since the shares are no longer simply points on a degree- $t$  polynomial. We give a proof system based on KZG commitments which leverages their multiplicative homomorphism to enable compatibility with hot share refreshes.

### 5.3 Related Work

Blokh et al. [BMP22] describe another threshold signing protocol which combines online (hot) and offline (cold) parties. Their protocol is tailored to ECDSA signatures and uses  $n$  hot parties and single cold party. These  $(n + 1)$  parties are independent (i.e., the cold party is not paired with any hot party, as in our setting); thus, their security guarantee is still a classic  $t$ -out-of- $n$  threshold guarantee, unlike our protocol, which is secure up to corruption of  $t$  *pairs*. Furthermore, in their protocol, hot parties engage in an MPC to generate a pre-signature, which is then finalized and output by the cold party. This is in contrast to our protocol, where the cold parties send messages to their corresponding hot parties, and the hot parties output the final signature (shares).

### 5.4 Model

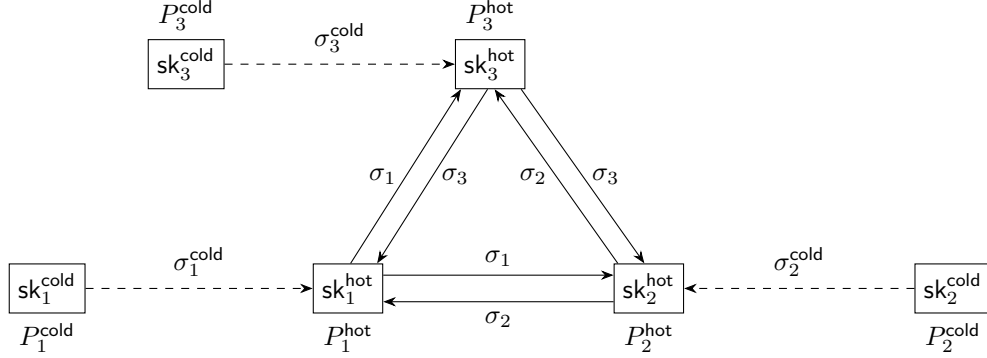


Figure 2: A hot-cold threshold wallet with  $n = 3$ . Given a message  $m$ , each  $P_i^{\text{cold}}$  uses its cold share  $\text{sk}_i^{\text{cold}}$  to compute a cold partial signature  $\sigma_i^{\text{cold}}$  on  $m$  and sends it to its hot storage.  $P_i^{\text{hot}}$  uses  $m$  and its hot share  $\text{sk}_i^{\text{hot}}$  to compute  $\sigma_i^{\text{hot}}$ , which it combines with  $\sigma_i^{\text{cold}}$  to get  $\sigma_i$ . The hot parties broadcast their partial signatures to each other to reconstruct the full signature  $\sigma$  on  $m$ . The dashed lines between each cold and hot storage represent the authenticated channel between them which is only active at signing time. The hot parties are always online and connected, represented by the solid lines.

Let  $I_1, \dots, I_n$  be parties (each representing an institution) who will store shares of some user's signing key  $\text{sk}$ . Each institution  $I_i$  controls two parties: a hot storage ( $P_i^{\text{hot}}$ ) and a cold storage ( $P_i^{\text{cold}}$ ). Thus we represent an institution by the tuple  $I_i = (P_i^{\text{hot}}, P_i^{\text{cold}})$ . As in the standard threshold wallet setting, the hot parties are connected to each other via authenticated (but not private) channels and we also assume the parties can send broadcast messages (e.g. by posting to a blockchain). In contrast, each cold storage is connected by an authenticated channel only to its corresponding hot storage. The channel is only active during the signing phase, further reducing the cold party's attack



surface. (For example, the cold storage could be a read-only USB device which is plugged into a PC (the hot storage) only briefly to produce a signature.) An illustration of this model is given in Figure 2 for  $n = 3$ .

We assume  $P_i^{\text{hot}}$  has more storage space and computational power, while  $P_i^{\text{cold}}$  has limited storage (in particular, we want the space complexity to be independent of the number of users in the system). Therefore, our protocol aims to minimize the computation done by the cold storage.

## 5.5 Malleable Encryption for eXponents (MEX)

As explained above, the core idea of our construction is for each hot party to store an encryption of the institution's signing key share  $\text{sk}_i$ , with the corresponding decryption key kept on the cold storage. That is, each institution will generate an encryption key pair  $(\text{ek}_i, \text{dk}_i)$  and store the hot key share  $\text{sk}_i^{\text{hot}} := \text{Enc}(\text{ek}_i, \text{sk}_i)$  in the hot storage and  $\text{sk}_i^{\text{cold}} := \text{dk}_i$  in the cold storage. We want to enable threshold signing of a message  $m$  by allowing the hot and cold parties to derive signature shares  $\sigma_i^{\text{hot}}, \sigma_i^{\text{cold}}$  for  $m$  from their secret material  $\text{sk}_i^{\text{hot}}, \text{sk}_i^{\text{cold}}$  (the secret key and ciphertext, respectively). Together, the signature shares can be used to recover a partial signature  $\sigma_i$  of  $m$  under  $\text{sk}_i$ .

In this section, we describe an encryption scheme based on ElGamal [ELG84] whose malleability allows decryption of the hot key share “in the exponent” of a partial BLS signature. Recall that the ElGamal ciphertext for a message  $m \in \mathbb{G}$  is computed as  $(g^r, \text{ek}^r \cdot m) \in \mathbb{G}^2$  where  $\mathbb{G}$  is a prime order group. In our case, however,  $m = \text{sk}_i$  will be a scalar element of  $\mathbb{Z}_p$  (an “exponent”). Adapting the encryption scheme to scalar messages, we can compute the ciphertext as  $(g_2^r, \mathcal{H}(\text{ek}^r) + m)$ , where  $\mathcal{H}$  is some function which maps  $\text{ek}^{\text{sk}}$  to  $\mathbb{Z}_p$  in a way that masks  $m$ . We will see below that this can be achieved by defining our encryption key  $\text{ek}$  to be a tuple  $(\text{ek}_1, \text{ek}_2) \in \mathbb{G}^2$  and letting  $\mathcal{H} : \mathbb{G}^2 \rightarrow \mathbb{Z}_p$ . Now we can mask  $m$  with the output of  $\mathcal{H}$ , and a cold party can undo this masking (in the exponent of the BLS signature) using  $g^r$  and  $\text{dk}_1, \text{dk}_2$ . We call the resulting scheme Malleable Encryption for eXponents (MEX).

Like the original ElGamal encryption and as the name suggests, MEX is malleable (in this case, additively rather than multiplicatively), which allows “shifting” of the message  $m$  in a ciphertext by an additive factor (via the Shift algorithm). We will use this property to enable proactive refresh of the hot (encrypted) key shares.

**Construction 5 (MEX).** *Let  $\mathbb{G}$  be a DLog-hard group of order  $p$  with generator  $g$ . Let  $\mathcal{H} : \mathbb{G} \rightarrow \mathbb{Z}_p$  be a hash function.*

- $(\text{ek}, \text{dk}) \leftarrow \text{KGen}(1^\lambda)$ : *Sample  $\text{dk} \xleftarrow{\$} \mathbb{Z}_p$ . Set  $\text{ek} := g^{\text{dk}}$  and output  $(\text{ek}, \text{dk})$ .*
- $ct \leftarrow \text{Enc}(\text{ek}, m; r)$ : *Given an encryption key  $\text{ek} \in \mathbb{G}$  and a message  $m \in \mathbb{Z}_p$ , use randomness  $r \in \mathbb{Z}_p$  to compute the ciphertext  $ct := (g^r, m + \mathcal{H}(\text{ek}^r))$ .*
- $m' \leftarrow \text{Dec}(\text{dk}, ct)$ : *Given a secret key  $\text{dk} \in \mathbb{Z}_p$  and a ciphertext  $ct \in \mathbb{G} \times \mathbb{Z}_p$ , parse  $ct$  as  $(ct_0, ct_1)$  and return  $ct_1 - \mathcal{H}(ct_0^{\text{dk}})$ .*

- $ct' \leftarrow \text{Shift}(ct, \delta)$ : Given a ciphertext  $ct \in \mathbb{G} \times \mathbb{Z}_p$  and a shift  $\delta \in \mathbb{Z}_p$ , parse  $ct$  as  $(ct_0, ct_1)$  and output the shifted ciphertext  $ct' := (ct_0, ct_1 + \delta)$ .

### 5.5.1 Additive Secret Sharing from MEX

The Enc algorithm in Construction 5 takes an explicit randomness input  $r$ . In the context of our wallet construction, encryption will take place at the same time as signing key generation, and instead of sampling fresh randomness  $r \in \mathbb{Z}_p$  for each hot party's ciphertext (hot key share), we will sample a single value  $r$ . The signing key-pair is set to  $(vk, sk) := (g^r, r)$ .<sup>4</sup> The result is that each party  $P_i^{\text{hot}}$ 's ciphertext uses the mask  $\mathcal{H}(\text{ek}_i^{\text{sk}}) = \mathcal{H}(vk^{\text{dk}_i})$ , which allows the corresponding cold party  $P_i^{\text{cold}}$  to decrypt using only a client's verification key  $vk$  and without receiving or storing any additional per-client randomness.

Because the input to  $\mathcal{H}$  is no longer uniformly random, we now need  $\mathcal{H}$  to be a hash function where the Leftover Hash Lemma (see Section 2.6) holds on random inputs. In order for the output of  $\mathcal{H}$  to have sufficient entropy to mask  $m$ , this requires two group elements as input. Therefore, in our wallet construction we will use  $\text{ek} := (g^{\text{dk}_1}, g^{\text{dk}_2})$  and  $ct := m + \mathcal{H}(\text{ek}_1^{\text{sk}}, \text{ek}_2^{\text{sk}})$ .

Although any function  $\mathcal{H}$  which meets the above requirements suffices, it is desirable to have a very efficient construction since  $\mathcal{H}$  will have to be computed by the cold party at signing time. One such  $\mathcal{H}$  is a random subset sum. In more detail,  $\mathcal{H}$  first represents its inputs  $x_1, x_2 \in \mathbb{G}$  as  $\ell$ -bit vectors  $\mathbf{x}_1, \mathbf{x}_2 \in \{0, 1\}^\ell$ , where  $\ell = \log p$ . Let  $\mathbf{r}_1, \mathbf{r}_2 \in \mathbb{Z}_p^{2\ell}$  be (public) uniform vectors with  $\mathbf{r}_k = (\mathbf{r}_{k,0}, \mathbf{r}_{k,1})$  for  $k = 1, 2$ . We will use bracket notation to index into the vector, i.e.,  $\mathbf{r}_{1,0}[i]$  is the  $i$ th element of  $\mathbf{r}_{1,0}$ . Let  $\mathcal{H}(x_1, x_2) := \mathcal{H}'(\mathbf{r}_1, \mathbf{x}_1) + \mathcal{H}'(\mathbf{r}_2, \mathbf{x}_2)$ , where  $\mathcal{H}' : \mathbb{Z}_p^{2\ell} \times \{0, 1\}^\ell \rightarrow \mathbb{Z}_p$  is the subset sum function

$$\mathcal{H}'(\mathbf{r} := (\mathbf{r}_0, \mathbf{r}_1), \mathbf{x}) := \sum_{b_i \in \mathbf{x}} \mathbf{r}_{b_i}[i]$$

When we want to be specific about the randomness used in  $\mathcal{H}$ , we write  $\mathcal{H}(x_1, x_2; \mathbf{k})$  for  $\mathbf{k} \in \mathbb{Z}_p^{4\ell}$ . By Lemma 1 in Section 2.6, the output of  $\mathcal{H}$  is statistically indistinguishable from uniform.

## 5.6 Proofs of Remembrance

We will use non-replayable zero-knowledge proofs of knowledge (ZKPoKs) for two languages. We refer the reader to [Tha23] for the definition of a ZKPoK. The PoKs for both our hot and cold parties are Sigma protocols, made non-interactive via the Fiat-Shamir transform [FS87]. Non-replayability is enforced by including some unpredictable timestamp (e.g., current block number of some blockchain) in the payload of the Fiat-Shamir hash.

To prove knowledge of the cold share, i.e., its decryption key, each cold storages will use a proof of knowledge of discrete logarithm  $\Pi_{\text{DL}}$  for the language

<sup>4</sup>This means we can leave out the redundant first element, so each ciphertext will consist of a single group element.

$\{y \in \mathbb{G} : \exists w \in \mathbb{Z}_p \text{ s.t. } y = g^w\}$ . This can be instantiated with the classic Schnorr protocol [Sch90].

The proof of knowledge for the hot storages is more complicated. At setup time, each hot storage must receive, along with its encrypted share, a proof of knowledge for the share's well-formedness. This should prove that the hot share equals  $\mathcal{H}(\text{ek}_{i,1}^x, \text{ek}_{i,2}^x) + x_i$  for some secret value  $x$  such that  $g^x = \text{vk}$  and  $x_i = \text{Share}(x, t, n)$ , and that the same value of  $x$  is used for every party. Our core idea is to use a KZG commitment to the polynomial used in the Shamir sharing of  $x$  and distribute an evaluation proof to each hot storage. The additive homomorphism of the commitments allows the public commitment, as well as each party's evaluation proof, to be updated when the shares are refreshed. We begin with a strawman example where the hot storage shares are unencrypted and then show how to adapt the construction to encrypted shares.

### 5.6.1 PoK of (Unencrypted) Key Share

Let  $\text{crs}$  be a degree- $(t-1)$  KZG CRS. At time  $T = 0$ , the client  $C$  picks a random degree- $(t-1)$  polynomial  $f_0(X) \in \mathbb{Z}_p[X]$  (its evaluation at 0 will be the signing key  $\text{sk}$ ) and publishes  $\text{com}_0 \leftarrow \text{KZG.Com}(\text{crs}, f_0(X))$ . Each hot storage  $P_i^{\text{hot}}$  receives an opening  $f_0(i)$  (i.e., the key share  $\text{sk}_i$ ) and evaluation proof  $\pi_{0,i}$ .

Whenever it wants the servers to refresh their shares and thus transition from epoch  $T-1$  to  $T$ ,  $C$  will commit to (as  $\text{ucom}_T$ ) a new random degree- $t$  polynomial  $z_T(X)$  such that  $z_T(0) = 0$ , publish an evaluation proof  $\zeta_{T,0}$  at  $X = 0$ , and send each  $P_i^{\text{hot}}$  its opening  $z_T(i)$  and the corresponding evaluation proof  $\zeta_{T,i}$ . Everyone can check the correctness of the update by verifying  $\zeta_{T,0}$  with respect to  $\text{ucom}_T$ . If the check passes, they can compute the commitment to the new polynomial  $f_T(X)$  homomorphically from the commitments to  $f_{T-1}(X)$  and  $z_T(X)$ , namely as  $\text{com}_T = \text{com}_{T-1} \cdot \text{ucom}_T$ . Then each party verifies its own update proof  $\zeta_{T,i}$  before updating its previous key share  $f_{T-1}(i)$  and proof  $\pi_{T-1,i}$ , respectively, to  $f_T(i) := f_{T-1}(i) + z_T(i)$  and  $\pi_{T,i} := \pi_{T-1,i} \cdot \zeta_{T,i}$ . By the homomorphic nature of the KZG commitment scheme,  $P_i^{\text{hot}}$  now has an evaluation of  $(f_{T-1} + z_T)(X) = f_T(X)$  at  $i$  and a corresponding evaluation proof  $\pi_{T,i}$ .

There is a problem with this scheme:  $P_i^{\text{hot}}$  needs to reveal  $f_T(i)$  in order for anyone to verify  $\pi_{T,i}$ ...but  $f_T(i)$  is its current key share! The solution is for  $P_i^{\text{hot}}$  to prove it knows  $f_T(i)$  in *zero-knowledge* via a blinded KZG evaluation proof (a modified version of the protocol in [ZBK<sup>+</sup>22, §6.1]). It commits to the key share  $f_T(i)$  using a Pedersen commitment  $\text{com}_{\text{ped}} := g_1^{f_T(i)} h_1^r$  and computes  $\pi_{\text{ped}} \leftarrow \Pi_{\text{ped}}.\text{Prove}(\text{com}_{\text{ped}}; (f_T(i), r))$ . It also samples  $s \xleftarrow{\$} \mathbb{Z}_p$  and computes a blinded version of the evaluation proof as  $\bar{\pi}_{T,i} := \pi_{T,i} h_1^s$ . The final ZKPoK is defined as  $(\text{com}_{\text{ped}}, \pi_{\text{ped}}, \bar{\pi}_{T,i}, g^{s_{T,i}(\tau)})$ , where  $s_{T,i}(X) := -r - s(X - i)$ . The client accepts if and only if

$$e(\text{com}_T / \boxed{\text{com}_{\text{ped}}}, g_2) \stackrel{?}{=} e(\boxed{\bar{\pi}_{T,i}}, g_2^\tau / g_2^i) \cdot \boxed{e(h_1, g_2^{s_{T,i}(\tau)})}$$

(where the boxed parts are changes to the original KZG verification check due to the blinding).

### 5.6.2 PoK of Encrypted Key Share

Next, we modify the previous construction to accommodate an *encrypted* initial key share  $\tilde{x}_i := \mathcal{H}(\text{ek}_{i,1}^x, \text{ek}_{i,2}^x) + x_i$ , where  $x = f_0(0)$  and  $x_i = f_0(i)$ . After having computed every party's plaintext share  $x_i$  (as above), the client  $C$  will compute each encrypted key share  $\tilde{x}_i$  and interpolate the degree- $(n-1)$  polynomial  $\tilde{f}_0(X)$  where  $\tilde{f}_0(i) = \tilde{x}_i$  for  $i \in [n]$ . For now, we assume the  $\tilde{x}_i$  and  $\tilde{f}_0(X)$  are computed correctly (we will return to this assumption in Section 5.7). Thus, we don't prove the correctness of the  $\tilde{x}_i$  values themselves, only that they are equal to  $\tilde{f}_0(i)$  where  $\tilde{f}_0(X)$  is fixed for all parties.

$C$  commits to  $\tilde{f}_0(X)$  publicly and, in a similar fashion as before, sends each hot storage  $P_i^{\text{hot}}$  the opening  $\tilde{f}_0(i) =: \tilde{x}_i$  and the corresponding evaluation proof  $\pi_{0,i}$ . Now the hot storage can prove remembrance of its current share  $\tilde{f}_T(i)$  in zero-knowledge in the same way as before, namely by committing to the share and blinding the evaluation proof  $\pi_{T,i}$  as described in the previous section.

Share refreshes in epoch  $T$  also proceed as before with a commitment  $\text{ucom}_T$  to a polynomial  $z_T(X)$  such that  $z_T(0) = 0$  (confirmed via a public evaluation proof  $\zeta_{T,0}$  at  $X = 0$ ). Parties receive their update value  $z_T(i)$  and corresponding evaluation proof  $\zeta_{T,i}$ , and can update their (now encrypted) share homomorphically just like before. (This still works because our encryption scheme allows additive shifts of the plaintext via addition to the ciphertext.) The only difference is that, because the encrypted shares now lie on a degree- $(n-1)$  polynomial instead of a degree- $(t-1)$  polynomial, the KZG CRS must accommodate polynomials up to degree  $n-1$ . (In practice, because different clients in the system may chose different values of  $n$ , the KZG CRS will actually have some degree  $d$  which is as large as the maximum allowed value of  $n-1$ .) As with the original polynomial  $\tilde{f}_0(X)$ , we assume  $z_T(X)$  is chosen honestly by  $C$ . In Section 5.8.1, we show how to avoid trusting  $C$  in the refresh stage.

The final hot proof of remembrance is summarized as  $\Pi_{\text{EKS}}$  in Figure 3.

## 5.7 Hot-Cold Threshold BLS

In this section, we show how to use the MEX from Section 5.5 and the proofs of remembrance from Section 5.6 to construct a hot-cold threshold wallet for BLS signatures (recalled in Section 2.3) with proofs of remembrance and proactive refresh of the hot shares.

Let  $\mathbb{G}_1, \mathbb{G}_2$  be elliptic curve groups of order  $p$  generated by  $g_1$  and  $g_2$ , respectively, and  $e : \mathbb{G}_1 \times \mathbb{G}_2 \rightarrow \mathbb{G}_T$  is an efficiently computable asymmetric (type-3) pairing between them. Since  $\text{vk} \in \mathbb{G}_2$ , we will also instantiate the MEX over  $\mathbb{G}_2$ . Let  $H : \{0,1\}^* \rightarrow \mathbb{G}_1$  and  $\mathcal{H} : \mathbb{G}_2^2 \rightarrow \mathbb{Z}_p$  be hash functions as defined in Section 2.3 and Section 5.5, respectively.

### HOT STORAGE PROOFS OF ENCRYPTED KEY SHARE ( $\Pi_{\text{EKS}}$ )

**Parameters:** Generators  $g_1, h_1 \in \mathbb{G}_1$  and  $g_2 \in \mathbb{G}_2$ ; a degree- $d$  KZG common reference string  $\text{crs} = \{g_1, g_1^\tau, \dots, g_1^{\tau^d}, g_2, g_2^\tau\}$ .

$\text{Prove}((\text{crs}, \text{com}_T, i); (\tilde{x}_i, \pi_{T,i})) \rightarrow \pi_i^{\text{hot}}$ : Given  $\text{crs}$ , a KZG commitment  $\text{com}_T$  to the current polynomial  $\tilde{f}_T(X)$ , and its index  $i$ , a hot storage uses its key share  $\tilde{x}_i = \tilde{f}_T(i)$  and corresponding opening proof  $\pi_{T,i}$  to compute a ZKPoK of  $\tilde{x}_i$  as follows:

1. Sample  $r \xleftarrow{\$} \mathbb{Z}_p$ , let  $\text{com}_{\text{ped}} := g_1^{\tilde{x}_i} h_1^r$ , and compute  $\pi_{\text{ped}} \leftarrow \Pi_{\text{ped}}.\text{Prove}(\text{com}_{\text{ped}}; (\tilde{x}_i, r))$  (see Section 2.5).
2. Sample  $s \xleftarrow{\$} \mathbb{Z}_p$  and let  $s_i(X) := -r - s(X - i)$ . Compute  $\bar{\pi}_{T,i} := \pi_{T,i} h_1^s$  and  $S := g_2^{s_i(\tau)}$  using  $\text{crs}$ .
3. Output  $\pi_i^{\text{hot}} := (\text{com}_{\text{ped}}, \pi_{\text{ped}}, \bar{\pi}_{T,i}, S)$ .

$\text{Vrfy}((\text{crs}, \text{com}_T, i), \pi_i^{\text{hot}}) \rightarrow \{0, 1\}$ : Given  $\text{crs}$ , a KZG commitment  $\text{com}_T$ , and a party index  $i$ , verify the hot storage proof  $\pi_i^{\text{hot}} = (\text{com}_{\text{ped}}, \pi_{\text{ped}}, \bar{\pi}_{T,i}, S)$  by outputting 1 iff the following hold:

$$\begin{aligned} \Pi_{\text{ped}}.\text{Vrfy}(\text{com}_{\text{ped}}, \pi_{\text{ped}}) &= 1 \\ e(\text{com}_T / \text{com}_{\text{ped}}, g_2) &= e(\bar{\pi}_{T,i}, g_2^\tau / g_2^i) \cdot e(h_1, S). \end{aligned}$$

Figure 3: The proof system  $\Pi_{\text{EKS}}$  used by each  $P_i^{\text{hot}}$  to show possession of a valid encrypted key share with respect to the current KZG commitment.

**Subprotocols** We assume the existence of the following ideal functionalities, which we will use as building blocks for our protocol:

**(Encrypted) secret share generation  $\mathcal{F}_{\text{SS}}$ :** Presented in Figure 4, this functionality is executed between a client  $C$  and a set of institutional entities. The functionality allows the client to choose which institutional entities it wants to use. We require that the client provide the public keys of the institutional servers that it wants to engage. A direct way to implement the  $\mathcal{F}_{\text{SS}}$  functionality would be for (trusted)  $C$  to execute the steps of  $\mathcal{F}_{\text{SS}}$  on input  $\text{SSSetup}$  locally and send  $\tilde{x}_i$  to  $P_i^{\text{hot}}$  for each  $i$  via a point-to-point channel. While this suffices for a number of applications, we model it as an abstract ideal functionality as we will be interested in settings where security is desired even if  $C$  (i) does not have access to a source of true randomness, or (ii) is (or, may in future be) corrupted. In Section 5.8.1, we show that one can use an additional proof system  $\Pi_{\text{Ref}}$  and a public bulletin-board with limited programmability to avoid trusting  $C$  beyond the key generation phase. We leave a fully decentralized key generation

**Public parameters:** Groups  $\mathbb{G}_1, \mathbb{G}_2$  of prime order  $p$  with generators  $g_1, g_2$ , respectively; a degree- $d$  KZG common reference string  $\text{crs}$ .

- On input  $(\text{sid}, \text{SSSetup}, C, (t, \mathcal{P}, \{\text{ek}_i\}_{i \in [n]}))$ , where  $\mathcal{P} = \{P_1, \dots, P_n\}$  is a set of parties, for  $i \in [n]$ ,  $\text{ek}_i \in \mathbb{G}_2$  is an encryption key, and  $t \leq |\mathcal{P}|$ , it proceeds as follows:
  1. Sample  $x \xleftarrow{\$} \mathbb{Z}_p \setminus \{0\}$ . Let  $y := g_2^x$ .
  2. Generate  $t$ -out-of- $n$  Shamir Shares of  $x$  as  $x_1, \dots, x_n \in \mathbb{Z}_p$ . Let  $y_i := g_2^{x_i} \forall i \in [n]$ .
  3. Interpolate the degree- $n$  polynomial  $\tilde{f}$  such that  $\tilde{f}(i) = \mathcal{H}(\text{ek}_i^x) + x_i \forall i \in [n]$ . Compute  $\text{com} \leftarrow \text{KZG.Com}(\text{crs}, \tilde{f})$ .
  4. Delete any entries  $(C, *, *, *, *, *) \in D$ . Add  $(C, \mathcal{P}, t, y, \text{com}, \text{time} := 1)$  to  $D$ .
  5. For each  $i \in [n]$ , compute  $(\tilde{x}_i, \pi_i) \leftarrow \text{KZG.Open}(\text{crs}, \tilde{f}, i)$  and output  $(\text{sid}, \text{SecretShare}, P_i, (C, i, \tilde{x}_i, \pi_i))$ .
  6. Finally, output  $(\text{sid}, \text{SSSetupDone}, C, (y, \{y_i\}_{i \in [n]}))$ .
- On input  $(\text{sid}, \text{ZeroSetup}, C, (t, \mathcal{P}))$ , where  $\mathcal{P} = \{P_1, \dots, P_n\}$  is a set of parties and  $t \leq |\mathcal{P}|$ , it proceeds as follows:
  1. Generate  $t$ -out-of- $n$  Shamir Shares of 0 as  $x_1, \dots, x_n \in \mathbb{Z}_p$ ; let  $f$  be the polynomial used.
  2. Compute  $\text{com}_0 \leftarrow \text{KZG.Com}(\text{crs}, f)$ .
  3. Retrieve  $(C, \mathcal{P}, t, y, \text{com}, \text{time}) \in D$  for the maximum value of  $\text{time}$ . Add  $(C, \mathcal{P}, t, y, \text{com} \cdot \text{com}_0, \text{time}++)$  to  $D$ .
  4. For each  $i \in [n]$ , if  $C$  is corrupt ask  $\mathcal{A}$  for a bit  $b_i^*$ . If  $b_i^* = 1$ , compute  $(\delta_i, \pi_i) \leftarrow \text{KZG.Open}(\text{crs}, f, i)$ . Otherwise set  $(\delta_i, \pi_i) := (\perp, \perp)$ .
  5. If  $P_i$  is corrupt, ask  $\mathcal{A}$  for values  $(\delta'_i, \pi'_i)$  and set  $(\delta_i, \pi_i) = (\delta'_i, \pi'_i)$ . Output  $(\text{sid}, \text{ZeroShare}, P_i, (C, \delta_i, \pi_i))$ .
  6. Finally, output  $(\text{sid}, \text{ZeroSetupDone}, C, (1))$ .
- On input  $(\text{sid}, \text{AuxRecover}, P_i, (C))$  for some client  $C$ , retrieve  $(C, *, *, y, \text{com}, \text{time}) \in D$  for the maximum value of  $\text{time}$  and output  $(\text{sid}, \text{AuxInfo}, P_i, (C, y, \text{com}))$ .

Figure 4: The encrypted secret sharing functionality  $\mathcal{F}_{\text{SS}}$ .

- On input  $(\text{sid}, \text{PKSetup}, P)$ , it proceeds as follows:
  1. Sample  $\text{dk} \xleftarrow{\$} \mathbb{Z}_p$  and set  $\text{ek} := g_2^{\text{dk}}$ .
  2. Delete any existing entries  $(P, *, *, *, *) \in L$  and add  $(P, \text{ek}, \text{dk}, \text{time} := 1, \text{unleaked} := 1)$  to  $L$ .
  3. Output  $(\text{sid}, \text{PKSetupResult}, P, (\text{ek}, \text{dk}))$ .
- On input  $(\text{sid}, \text{PKRecover}, P, (Q))$ , retrieve  $(Q, \text{ek}, *, *, *) \in L$  and output  $(\text{sid}, \text{PKRecoverResult}, P, (Q, \text{ek}))$ .

Figure 5: The public key functionality  $\mathcal{F}_{\text{PK}}$ .

protocol to future work.

**Public key infrastructure  $\mathcal{F}_{\text{PK}}$ :** Finally, we assume a functionality  $\mathcal{F}_{\text{PK}}$  (Figure 5) which allows a party (institutional cold servers in our case) to obtain a secret (decryption) key  $\text{dk}$  sampled from  $\mathbb{Z}_p$  while its public (encryption) key  $\text{ek} := g_2^{\text{dk}}$  is made public, and can be retrieved reliably by any client. Again this functionality can be implemented by a party executing it locally. We abstract it out to separate the implementation details of this functionality from our modeling.

**Construction** Our protocol for BLS signatures is given in Figures 6 and 7. Each cold storage is set up separately and independently of any client using the **ColdRegister** protocol. When a client registers, it specifies a set of  $n$  institutions  $\mathcal{I}$  and a threshold  $t \leq n$  of them required for signing. The **ClientRegister** protocol is an interactive protocol between  $C$  and the  $n$  hot parties specified by  $\mathcal{I}$  which outputs a verification key  $\text{vk}$  to the client and an encrypted secret key share  $\tilde{x}_i$  to each hot party. Each hot party also receives a proof  $\pi_i$  which will allow it to prove that  $\tilde{x}_i$  was the output of **ClientRegister**.

To sign a message  $m$  on behalf of  $C$ , each institution (consisting of a hot and cold party) separately produces a partial signature on  $m$ . Upon receiving a signature request (which in most implementations would be passed to the cold party via the hot party), each component can honor the request by producing a partial signature  $\sigma_i^{\text{hot}}$  or  $\sigma_i^{\text{cold}}$ , respectively, which are combined by the hot party into  $\sigma_i$ .

Proving remembrance of each party’s key material is done via **CProof** and **HProof**, respectively. We write that  $P_i^{\text{cold}}$  sends its proof directly to  $C$ , which in practice can be achieved by passing the message via  $P_i^{\text{hot}}$  assuming eventual delivery. Finally, the hot key shares can be proactively refreshed via an interactive protocol **ShareRefresh** between  $C$  and the hot parties, which is similar to **ClientRegister** and outputs some update information  $\delta_i$  and a proof  $\zeta_i$  of its correctness to each hot party.

**Parameters:** Groups  $\mathbb{G}_1, \mathbb{G}_2$  of prime order  $p$  with generators  $g_1, g_2$ , respectively; a degree- $d$  KZG common reference string  $\text{crs}$ ; hash functions  $H : \{0, 1\}^* \rightarrow \mathbb{G}_1$  and  $\mathcal{H} : \mathbb{G}_2^2 \rightarrow \mathbb{Z}_p$  as defined in 2.3 and 5.5, resp.

#### SETUP

Registering a cold storage  $P_i^{\text{cold}}$ : On input  $(\text{sid}, \text{ColdRegister}, P_i^{\text{cold}})$ ,  $P_i^{\text{cold}}$  calls  $\mathcal{F}_{\text{PK}}$  on input  $(\text{sid}, \text{PKSetup}, P_i^{\text{cold}})$  and receives response  $(\text{sid}, \text{PKSetupResult}, P_i^{\text{cold}}, (\text{ek}_i, \text{dk}_i))$ . It stores  $\text{ek}_i, \text{dk}_i$  and outputs  $(\text{sid}, \text{ColdRegistered}, P_i^{\text{cold}}, \text{ek}_i)$ .

Registering a client  $C$ : On input  $(\text{sid}, \text{ClientRegister}, C, (t, \mathcal{I}))$ , where  $\mathcal{I} = \{(P_i^{\text{hot}}, P_i^{\text{cold}})\}_{i \in [n]}$  is a set of institutional entities and  $t \leq n$  a signing threshold:

1.  $C$  calls  $\mathcal{F}_{\text{PK}}$  on input  $(\text{sid}, \text{PKRecover}, C, (P_i^{\text{cold}}))$  for all  $i \in [n]$ . It waits until it receives a response  $(\text{sid}, \text{PKRecoverResult}, C, (P_i^{\text{cold}}, \text{ek}_i))$  for all  $i \in [n]$ .
2.  $C$  calls  $\mathcal{F}_{\text{SS}}$  on input  $(\text{sid}, \text{SSSetup}, C, (t, \{P_i^{\text{hot}}\}_{i \in [n]}, \{\text{ek}_i\}_{i \in [n]}))$ . The latter outputs  $(\text{sid}, \text{SecretShare}, P_i^{\text{hot}}, (C, i, \tilde{x}_i, \pi_i))$  to  $P_i^{\text{hot}} \forall i \in [n]$ . It also outputs  $(\text{sid}, \text{SSSetupDone}, C, (\text{vk}, \{\text{vk}_i\}_{i \in [n]}))$  to  $C$ .
3. Each  $P_i^{\text{hot}}$  stores the tuple  $(C, \tilde{x}_i, \pi_i)$  in a list  $L_i$ . Then it outputs  $(\text{sid}, \text{ClientRegistered}, P_i^{\text{hot}}, (C, b_i = 1))$ .
4. Meanwhile,  $C$  stores the parameters and the values it received from  $\mathcal{F}_{\text{SS}}$  in the tuple  $D = (\text{vk}, \{\text{vk}_i\}_{i \in [n]}, t, \mathcal{I})$ . Finally, it outputs  $(\text{sid}, \text{ClientRegistered}, C, \text{vk}, \{\text{vk}_i\}_{i \in [n]})$ .

#### SIGNING

Threshold signing: On input a signature request  $(\text{sid}, \text{TSign}, P_i^{\text{hot}}, (C, m))$  from a client  $C$ ,

1. If  $P_i^{\text{hot}}$  decides to honor the request, it calls  $\mathcal{F}_{\text{SS}}$  on  $(\text{sid}, \text{AuxRecover}, P_i^{\text{hot}}, (C))$  to get  $\text{vk}$  and sends  $(\text{vk}, m)$  to  $P_i^{\text{cold}}$ . Otherwise, it aborts.
2. If  $P_i^{\text{cold}}$  decides to honor the request, it sends  $\sigma_i^{\text{cold}} := H(m)^r$  to  $P_i^{\text{hot}}$ , where  $r := \mathcal{H}(\text{vk}^{\text{dk}_{i,1}}, \text{vk}^{\text{dk}_{i,2}})$ .
3. Meanwhile,  $P_i^{\text{hot}}$  retrieves  $(C, \tilde{x}_i, *) \in L_i$  and computes  $\sigma_i^{\text{hot}} := H(m)^{\tilde{x}_i}$ .
4. Once it receives  $\sigma_i^{\text{cold}}$ ,  $P_i^{\text{hot}}$  outputs  $(\text{sid}, \text{TSignResult}, P_i^{\text{hot}}, (C, m, \sigma_i := \sigma_i^{\text{hot}} / \sigma_i^{\text{cold}}))$ .

Figure 6: Our BLS hot-cold threshold wallet protocol (setup and threshold signing).



**Parameters:** Groups  $\mathbb{G}_1, \mathbb{G}_2$  of prime order  $p$  with generators  $g_1, g_2$ , respectively; a degree- $d$  KZG common reference string  $\text{crs}$ .

#### PROOFS OF REMEMBRANCE

**Cold proof:** A client  $C$  can verify that an institutional cold storage  $P_i^{\text{cold}}$  still retains its key material (namely  $\text{dk}_i$ ). On input  $(\text{sid}, \text{CProof}, C, (P_i^{\text{cold}}))$ ,  $C$  sends a designated proof request message to  $P_i^{\text{cold}}$ . Then:

1.  $P_i^{\text{cold}}$  retrieves its stored  $\text{ek}_i = (\text{ek}_{i,1}, \text{ek}_{i,2})$ ,  $\text{dk}_i = (\text{dk}_{i,1}, \text{dk}_{i,2})$  and computes a non-replayable proof  $\pi_{i,k}^{\text{cold}} \leftarrow \Pi_{\text{DL}}.\text{Prove}(\text{ek}_{i,k}; \text{dk}_{i,k})$  for  $k = 1, 2$ . It sends  $\pi_i^{\text{cold}} := (\pi_{i,1}^{\text{cold}}, \pi_{i,2}^{\text{cold}})$  to  $C$ .
2. To verify,  $C$  calls  $\mathcal{F}_{\text{PK}}$  on input  $(\text{sid}, \text{PKRecover}, C, (P_i^{\text{cold}}))$  and receives  $(\text{sid}, \text{PKRecoverResult}, C, (P_i^{\text{cold}}, \text{ek}_i))$ . It parses  $\text{ek}_i = (\text{ek}_{i,1}, \text{ek}_{i,2})$  and computes  $b_k \leftarrow \Pi_{\text{DL}}.\text{Vrfy}(\text{ek}_{i,k}, \pi_{i,k}^{\text{cold}})$  for  $k = 1, 2$ . Finally it outputs  $(\text{sid}, \text{CProofResult}, C, (P_i^{\text{cold}}, b_1 \wedge b_2))$ .

**Hot proof:** A client  $C$  can also verify that an institutional hot storage  $P_i^{\text{hot}}$  still retains its key material (namely  $\tilde{x}_i$ ). On input  $(\text{sid}, \text{HProof}, C, (P_i^{\text{hot}}))$ ,  $C$  sends a designated proof request message to  $P_i^{\text{hot}}$ . Then:

1.  $P_i^{\text{hot}}$  calls  $\mathcal{F}_{\text{SS}}$  on input  $(\text{sid}, \text{AuxRecover}, P_i^{\text{hot}}, (C))$  and receives  $(\text{sid}, \text{AuxInfo}, P_i^{\text{hot}}, (C, \text{vk}, \text{com}))$ . Then it uses  $\text{com}$  and the stored tuple  $(C, \tilde{x}_i, \pi_i) \in L_i$  to compute  $\pi_i^{\text{hot}} \leftarrow \Pi_{\text{EKS}}.\text{Prove}((\text{crs}, \text{com}, i); (\tilde{x}_i, \pi_i))$ , which it sends to  $C$ .
2. To verify,  $C$  calls  $\mathcal{F}_{\text{SS}}$  on input  $(\text{sid}, \text{AuxRecover}, C, (C))$  and receives  $(\text{sid}, \text{AuxInfo}, C, (C, *, \text{com}))$ . Then it lets  $b \leftarrow \Pi_{\text{EKS}}.\text{Vrfy}((\text{crs}, \text{com}, i), \pi_i^{\text{hot}})$  and outputs  $(\text{sid}, \text{HProofResult}, C, (P_i^{\text{hot}}, b))$ .

#### PROACTIVE REFRESH

**Hot share refresh:** A client  $C$  can trigger a refresh of its hot key shares. On input  $(\text{sid}, \text{ShareRefresh}, C)$ ,  $C$  retrieves its stored wallet parameters  $D = (\text{vk}, *, t, \mathcal{I})$  where  $\mathcal{I} = \{(P_i^{\text{hot}}, P_i^{\text{cold}})\}_{i \in [n]}$ . Then:

1.  $C$  calls  $\mathcal{F}_{\text{SS}}$  on input  $(\text{sid}, \text{ZeroSetup}, C, (t, \{P_i^{\text{hot}}\}_{i \in [n]}))$ .
2.  $\mathcal{F}_{\text{SS}}$  outputs  $(\text{sid}, \text{ZeroShare}, P_i^{\text{hot}}, (C, \delta_i, \zeta_i))$  to  $P_i^{\text{hot}}$  for all  $i \in [n]$ . It also outputs  $(\text{sid}, \text{ZeroSetupDone}, C, (b))$  to  $C$ .
3. Each  $P_i^{\text{hot}}$  checks if  $(\delta_i, \zeta_i) = (\perp, \perp)$ . If so, it sets  $b_i = 0$ ; otherwise, it sets  $b_i = 1$  and updates  $(C, \tilde{x}_i, \pi_i) \in L_i$  to  $(C, \tilde{x}_i + \delta_i, \pi_i \cdot \zeta_i)$ . Then it outputs  $(\text{sid}, \text{ShareRefreshResult}, P_i^{\text{hot}}, (C, b_i))$ .
4. Meanwhile,  $C$  outputs  $(\text{sid}, \text{ShareRefreshResult}, C, (b))$ .

Figure 7: Our BLS hot-cold threshold wallet protocol (proofs of remembrance and proactive refresh).

**Correctness** Let  $\mathcal{I}$  be the set of  $n$  institutions  $C$  registers with using threshold  $t$ . For  $i \in [n]$ , let  $\text{ek}_i$  be the output of `ColdRegister` for  $P_i^{\text{cold}}$ ,  $\text{vk}$  be the output of `ClientRegister`,  $\text{com}_T$  be the polynomial commitment after  $T$  executions of `ShareRefresh`, and  $\pi_i^{\text{hot}}$  (resp.  $\pi_i^{\text{cold}}$ ) be the output of `HProof` for  $P_i^{\text{hot}}$  and  $C$  (resp. `CProof`,  $P_i^{\text{cold}}$ , and  $C$ ). For correctness, we require that for all  $T$ , if there exists some set  $S = \{i : i \in [n]\}$  with  $|S| \geq t$  such that  $\Pi_{\text{EKS}}.\text{Vrfy}(\text{crs}, \text{com}_T, \pi_i^{\text{hot}}) = 1$  and  $\Pi_{\text{DL}}.\text{Vrfy}((\text{ek}_i, g_2), \pi_i^{\text{cold}}) = 1$  for all  $i \in S$ , then  $\text{BLS}.\text{Vrfy}(\text{vk}, m, \text{BLS}.\text{Reconstruct}(\{\sigma_i\}_{i \in S})) = 1$  with all but negligible probability, where  $\sigma_i$  is the output of `TSign` for  $P_i^{\text{hot}}, C, m$ .

Recall that `TSign` outputs computes  $\sigma_i$  as  $\sigma_i^{\text{hot}}/\sigma_i^{\text{cold}}$ , which by construction equals  $H(m)^{\text{sk}_i + \mathcal{H}(\text{ek}_i^{\text{sk}})} \cdot H(m)^{-\mathcal{H}(\text{vk}^{\text{dk}_i})} = H(m)^{\text{sk}_i} = \text{BLS}.\text{TSign}(\text{sk}_i, m)$ . Thus correctness follows by construction, the soundness of  $\Pi_{\text{DL}}, \Pi_{\text{EKS}}$ , and the correctness of threshold BLS.

**Efficiency and compactness of cold storage** We wish to point out that our construction optimizes storage-, computation-, and communication-efficiency for the cold storage. Each  $P_i^{\text{cold}}$  only stores a single decryption key  $\text{dk}_i \in \mathbb{G}_2^2$ , regardless of the number of clients registered with its institution. To produce a cold partial signature, it computes two  $\mathbb{G}_2$  exponentiations, one addition in  $\mathbb{Z}_p$ , and a single evaluation of  $\mathcal{H}$ , which is highly efficient since it is a simple subset sum (see Section 5.5). Computing a proof of remembrance requires 2  $\mathbb{G}_2$  exponentiations, 1 hash function evaluation (for Fiat-Shamir), and 2 additions and multiplications each in  $\mathbb{Z}_p$ . Finally, in terms of communication, the cold storage only needs to send a single  $\mathbb{G}_1$  element per signing operation. A cold proof of remembrance consists of 2  $\mathbb{G}_2$  and 2  $\mathbb{Z}_p$  elements.

### 5.7.1 Security Analysis

We prove our scheme secure in the universal composability framework<sup>5</sup>, which we summarize in Section 2.7.

Messages to and from the ideal functionalities consist of two pieces: a *header* and the *contents*. The header normally consists of a session identifier `sid`, a description of the action the functionality should take/has taken, and the sender/recipient. In this paper, we will use the convention of putting the message contents in parentheses so it is clear where the (public) header ends and the (private) contents begin, e.g.,  $(\text{sid}, \text{Action}, P_{\text{sender}}, (\text{contents}))$  or  $(\text{sid}, \text{ActionDone}, P_{\text{recipient}}, \text{publicvars}, (\text{contents}))$ .

**Ideal Functionality** We now present our hot-cold threshold BLS functionality  $\mathcal{F}_{\text{HC}}$ , whose interfaces can only be called by *either* an institutional hot or cold storage (specified by the superscripts *hot* and *cold*) or a client. For readability, we split  $\mathcal{F}_{\text{HC}}$  into three figures. The first, Figure 8, describes how parties

<sup>5</sup>Specifically, we use the version presented in [CLOS02].

<sup>6</sup>We assume share refreshes are sequential and delivery to all  $P_i$  precedes any new refreshes.

**Public parameters:** Groups  $\mathbb{G}_1, \mathbb{G}_2$  of prime order  $p$  with generators  $g_1, g_2$ , respectively; a degree- $d$  KZG common reference string  $\text{crs}$ ; a hash function  $H : \{0, 1\}^* \rightarrow \mathbb{G}_1$ ; an extractor  $\mathcal{H} : \mathbb{G}_2^2 \rightarrow \mathbb{Z}_p$ .

**Internal variables:**

$L_{\text{cold}} = \{(P^{\text{cold}}, \text{ek}, \text{dk}, \text{leaked}, \text{tampered}, \text{corrupted}, \text{allowc})\}$ , a table with the state of each cold storage's key material.

$D = \{(C, \mathcal{I}, t, y, \text{com})\}$ , a table of registered clients and their metadata.

$L_{\text{hot}} = \{(P^{\text{hot}}, C_j, \tilde{x}, \pi, \text{time}, \text{leaked}, \text{tampered})\}$ , which keeps track of the key material of every hot storage for each of its clients, and whether it has been leaked or tampered with.

$S_{\text{hot}} = \{P^{\text{hot}}, \text{corrupted}, \text{allowc}\}$ , which keeps track of whether each hot party has been corrupted.

### Registration

- On input  $(\text{sid}, \text{ColdRegister}, P_i^{\text{cold}})$ , it proceeds as follows:
  1. Sample  $\text{dk}_i \xleftarrow{\$} \mathbb{Z}_p$  and set  $\text{ek}_i := g_2^{\text{dk}_i}$ .
  2. Delete any existing entries for  $P_i^{\text{cold}}$  in  $L_{\text{cold}}$  and add  $(P_i^{\text{cold}}, \text{ek}_i, \text{dk}_i, \text{leaked} := 0, \text{tampered} := 0, \text{corrupted} := 0, \text{allowc} := 1)$  to  $L_{\text{cold}}$ .
  3. Output  $(\text{sid}, \text{ColdRegistered}, P_i^{\text{cold}}, \text{ek}_i)$ .
- On input  $(\text{sid}, \text{ClientRegister}, C, (t, \mathcal{I}))$ , where  $\mathcal{I} = \{(P_i^{\text{hot}}, P_i^{\text{cold}})\}_{i \in [n]}$  is a set of institutional entities and  $t \leq n$  a signing threshold, it proceeds as follows:
  1. For each  $i \in [n]$ , retrieve  $(P_i^{\text{cold}}, \text{ek}_i, *, *, *, *, *)$  from  $L_{\text{cold}}$ . If a public key for some party is unavailable then output  $y := \perp$ .
  2. Otherwise, sample  $x \xleftarrow{\$} \mathbb{Z}_p \setminus \{0\}$ . Let  $y := g_2^x$ .
  3. Generate  $t$ -out-of- $n$  Shamir Shares of  $x$  as  $x_1, \dots, x_n \in \mathbb{Z}_p$ . Let  $y_i := g_2^{x_i} \forall i \in [n]$ .
  4. Interpolate the degree- $n$  polynomial  $\tilde{f}$  such that  $\tilde{f}(i) = \mathcal{H}(\text{ek}_i^x) + x_i \forall i \in [n]$ . Compute  $\text{com} \leftarrow \text{KZG.Com}(\text{crs}, \tilde{f})$ .
  5. Delete any existing entries  $(C, *, *, *, *, *) \in D$  and add  $(C, \mathcal{I}, t, y, \text{com})$  to  $D$ . Send  $(\text{sid}, \text{ClientRegistered}, C, y, \{y_i\}_{i \in [n]})$  to  $C$ .
  6. For each  $i \in [n]$ :
    - (a) Compute  $(\tilde{x}_i, \pi_i) \leftarrow \text{KZG.Open}(\text{crs}, \tilde{f}, i)$ .
    - (b) Output  $(\text{sid}, \text{ClientRegistered}, P_i^{\text{hot}}, (C, b_i = 1))$ .
    - (c) Once  $\mathcal{A}$  has delivered/allowed delivery of the message, delete any existing entries  $(P_i^{\text{hot}}, C, *, *, *, *, *) \in L_{\text{hot}}$  and add  $(P_i^{\text{hot}}, C, \tilde{x}_i, \pi_i, \text{time} := 0, \text{leaked} := 0, \text{tampered} := 0)$  to  $L_{\text{hot}}$ . Delete any existing entries  $(P_i^{\text{hot}}, \text{corrupted}, \text{allowc}) \in S_{\text{hot}}$  and add  $(P_i^{\text{hot}}, \text{corrupted} := 0, \text{allowc} := 1)$ .

Figure 8: The BLS hot-cold threshold wallet functionality  $\mathcal{F}_{\text{HC}}$  (registration).

## Signing

- On input  $(\text{sid}, \text{TSign}, P_i^{\text{hot}}, (C, m))$ , retrieve  $(C, \mathcal{I}, *, \text{vk}, *) \in D$  and  $(P_i^{\text{hot}}, P_i^{\text{cold}}) \in \mathcal{I}$ . Then:
  1. Get  $(P_i^{\text{cold}}, *, \text{dk}_i, *, \text{tampered}, \text{corrupt}_{\text{cold}}, \text{allowc}) \in L_{\text{cold}}$ . If  $\text{allowc} = 1$ , set it to 0.
    - If  $\text{corrupt}_{\text{cold}} = 1$ , send  $(\text{sid}, \text{CSignRequest}, \mathcal{A}, (P_i^{\text{cold}}, C, m))$  to  $\mathcal{A}$ , wait for response  $b$ , and set  $b_{\text{cold}} := (b \wedge \neg \text{tampered})$ . Otherwise ( $\text{corrupt}_{\text{cold}} = 0$ ), set  $b_{\text{cold}} := \neg \text{tampered}$ .
  2. Retrieve  $(P_i^{\text{hot}}, C, \tilde{x}_i, *, \text{time}, *, \text{tampered}, \text{corrupt}_{\text{hot}}, *) \in L_{\text{hot}}$  for the maximum time.
    - If  $\text{corrupt}_{\text{hot}} = 1$ , send  $(\text{sid}, \text{HSignRequest}, \mathcal{A}, (P_i^{\text{hot}}, C, m))$  to  $\mathcal{A}$ , wait for response  $b$ , and set  $b_{\text{hot}} := (b \wedge \neg \text{tampered})$ . Otherwise, set  $b_{\text{hot}} := \neg \text{tampered}$ .
  3. If  $b_{\text{cold}} \wedge b_{\text{hot}}$ , compute  $\sigma_i^{\text{cold}} := H(m)^{\mathcal{H}(\text{vk}^{\text{dk}_i})}$  and  $\sigma_i^{\text{hot}} := H(m)^{\tilde{x}_i}$ . Let  $\sigma_i := \sigma_i^{\text{hot}} / \sigma_i^{\text{cold}}$ . Output  $(\text{sid}, \text{TSignResult}, P_i^{\text{hot}}, (C, m, \sigma_i))$ .
    - If  $(\text{corrupt}_{\text{cold}} \wedge \neg \text{corrupt}_{\text{hot}})$ , also send  $\sigma_i^{\text{hot}}$  to  $\mathcal{A}$ . If  $(\neg \text{corrupt}_{\text{cold}} \wedge \text{corrupt}_{\text{hot}})$ , send  $\sigma_i^{\text{cold}}$  to  $\mathcal{A}$ .
    - Additionally, for every party  $P_j^{\text{hot}}$  such that  $(P_j^{\text{hot}}, *) \in \mathcal{I}$ , retrieve  $(P_j^{\text{hot}}, *, \text{allowc}) \in S_{\text{hot}}$  and set  $\text{allowc}$  to 0.
  4. If instead  $\neg(b_{\text{cold}} \wedge b_{\text{hot}})$ , output  $(\text{sid}, \text{TSignResult}, P_i^{\text{hot}}, (C, m, \perp))$ .
    - If  $(\text{corrupt}_{\text{cold}} \wedge \neg \text{corrupt}_{\text{hot}})$  or  $(\neg \text{corrupt}_{\text{cold}} \wedge \text{corrupt}_{\text{hot}})$ , also sample  $r \xleftarrow{\$} \mathbb{Z}_p$  and send  $H(m)^r$  to  $\mathcal{A}$ .

## Proofs of Remembrance

- On input  $(\text{sid}, \text{CProof}, P_i^{\text{cold}}, (C))$ , retrieve  $(P_i^{\text{cold}}, \text{ek}_i, \text{dk}_i, *, *, \text{corrupted}, *) \in L_{\text{cold}}$ . If  $\text{corrupted} = 1$ , send  $(\text{sid}, \text{CProofRequest}, \mathcal{A}, (P_i^{\text{cold}}))$  to the adversary, who will send back a bit  $b^*$  to represent whether the query should be responded to honestly. Set  $b := b^* \wedge (\text{ek}_i = g_2^{\text{dk}_i})$  and output  $(\text{sid}, \text{CProofResult}, C, (P_i^{\text{cold}}, b))$ .
- On input  $(\text{sid}, \text{HProof}, C, (P_i^{\text{hot}}))$ , retrieve  $(P_i^{\text{hot}}, C, \tilde{x}_i, \pi_i, *, *, *, \text{corrupted}, *) \in L_{\text{hot}}$  and  $(C, *, *, *, \text{com}) \in D$ . If  $\text{corrupted} = 1$ , send  $(\text{sid}, \text{HProofRequest}, \mathcal{A}, (P_i^{\text{hot}}))$  to the adversary, who will send back a bit  $b^*$ . Set  $b := b^* \wedge \text{KZG.Vrfy}(\text{crs}, \text{com}, i, \tilde{x}_i, \pi_i)$  and output  $(\text{sid}, \text{HProofResult}, C, (P_i^{\text{hot}}, b))$ .

## Proactive Refresh

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- On input  $(\text{sid}, \text{ShareRefresh}, C)$ , it proceeds as follows:
  1. Output  $\perp$  if a prior share refresh from  $C$  is still pending.<sup>6</sup>
  2. Retrieve  $(C, \mathcal{I}, t, y, \text{com}, \text{time}) \in D$  for the maximum value of  $\text{time}$ .
  3. Generate  $t$ -out-of- $n$  Shamir shares of 0 as  $x_1, \dots, x_n \in \mathbb{Z}_p$  using polynomial  $f$

register in the system. The signing, share refresh, and proof of remembrance interfaces of  $\mathcal{F}_{\text{HC}}$  are described in Figure 9.

The most complicated part of the functionality is the signing interface, which provides partial BLS signatures  $\sigma_i$  on requested messages. For uncorrupted institutions  $I_i$  (that is, neither  $P_i^{\text{cold}}$  and  $P_i^{\text{hot}}$  are corrupt),  $\sigma_i$  is computed honestly. If the hot party has been corrupted but the cold remains honest, the functionality asks the adversary whether to use the correct value for the hot signature; if so, it computes  $\sigma_i$  correctly and also sends the cold signature to the adversary. Otherwise, it outputs  $\sigma_i = \perp$  and sends  $H(m)^r$  for a uniform  $r$  to  $\mathcal{A}$  (as the “corresponding” cold signature). (The idea is that in this case, the hot signature is incorrect so  $\sigma_i$  will not verify, and the cold signature should be “useless” without it: it should look random to the adversary.) If the hot party is honest and the cold is corrupt, the functionality behaves in the same way but with the hot and cold roles reversed. Finally, if both parties in a pair are corrupt, the adversary will get no information about the hot or cold partial signatures from the functionality, which only outputs either the correct  $\sigma_i$  or  $\perp$ , depending on whether the adversary says to compute the hot and cold partial signatures correctly.

We will argue that this implements threshold BLS signatures, so the security of a protocol implementing  $\mathcal{F}_{\text{HC}}$  follows from security of threshold BLS.

**Adversarial interference** Figure 10 gives the leak and tamper interfaces of  $\mathcal{F}_{\text{HC}}$ , which an adversary can use to interfere with the information stored by the hot and cold storages in the system. We also give an explicit corruption interface, which only allows non-adaptive corruptions of the cold and hot parties (in the latter case, on a per-epoch basis). The interfaces also capture the fact that the client  $C$  is out of scope for corruption.

**Theorem 1** (security). *Our hot-cold threshold wallet construction (??) UC-realizes  $\mathcal{F}_{\text{HC}}$  in the  $\mathcal{F}_{\text{SS}}, \mathcal{F}_{\text{PK}}$ -hybrid model.*

*Proof.* Let  $\mathcal{A}$  be the adversary interacting with the parties (namely,  $P_1^{\text{hot}}, P_1^{\text{cold}}, \dots, P_n^{\text{hot}}, P_n^{\text{cold}}, C$ ) running the protocol presented in ??. We will construct a simulator  $\mathcal{S}$  running in the ideal world against  $\mathcal{F}_{\text{HC}}$  so that no environment  $\mathcal{E}$  can distinguish an execution of the ideal-world interaction from the real protocol.  $\mathcal{S}$  will interact with  $\mathcal{F}_{\text{HC}}$ ,  $\mathcal{E}$ , and invoke a copy of  $\mathcal{A}$  to run a simulated interaction of the protocol (we call this simulated interaction between  $\mathcal{A}$ ,  $\mathcal{E}$ , and the parties the *internal interaction* to distinguish it from the *external interaction* between  $\mathcal{S}$ ,  $\mathcal{E}$ , and  $\mathcal{F}_{\text{HC}}$ ).

Each protocol/algorithm begins with a party receiving some input. In the ideal world, this input is received by some dummy party, who immediately copies it to its outgoing communication tape where  $\mathcal{S}$  can read it (public header only) and potentially deliver it to the ideal functionality  $\mathcal{F}_{\text{HC}}$ . (Recall that for simplicity we assume authenticated communication, so  $\mathcal{S}$  cannot modify the messages it delivers to/from  $\mathcal{F}_{\text{HC}}$ .) To complete the protocol,  $\mathcal{S}$  will deliver the response of  $\mathcal{F}_{\text{HC}}$  to the dummy party, who copies it to its output tape (which is visible to  $\mathcal{E}$ ).

### Leak

- On input  $(\text{sid}, \text{Leak}, \mathcal{A}, (P_i^{\text{hot}}))$ , for every entry  $(P_i^{\text{hot}}, C_j, \tilde{x}_i, \pi_i, \text{time}, \text{leaked}, *) \in L_{\text{hot}}$ , set **leaked** to 1 and send  $(\text{sid}, \text{LeakResult}, \mathcal{A}, ((P_i^{\text{hot}}, C_j, \tilde{x}_i, \pi_i, \text{time})))$  to  $\mathcal{A}$ .
- On input  $(\text{sid}, \text{Leak}, \mathcal{A}, (P_i^{\text{cold}}))$ , retrieve the entry  $(P_i^{\text{cold}}, \text{ek}_i, \text{dk}_i, \text{leaked}, *, *, *) \in L_{\text{cold}}$ . Set **leaked** to 1 and send  $(\text{sid}, \text{LeakResult}, \mathcal{A}, ((P_i^{\text{cold}}, \text{dk}_i)))$  to  $\mathcal{A}$ .

### Tamper

- On input  $(\text{sid}, \text{Tamper}, \mathcal{A}, (P_i^{\text{hot}}, C, f, g))$ , where  $f, g$  are functions:
  1. Retrieve  $(P_i^{\text{hot}}, C, \tilde{x}_i, \pi_i, \text{time}, \text{leaked}, \text{tampered}) \in L_{\text{hot}}$  for the maximum value of **time**. Let  $\tilde{x}'_i := f(\tilde{x}_i)$  and  $\pi'_i := g(\pi_i)$ .
  2. If  $\tilde{x}'_i, \pi'_i \neq \perp$ , let  $b := 1$  and update the entry to  $(C, P_i^{\text{hot}}, \tilde{x}'_i, \pi'_i, \text{time}, \text{leaked}, \text{tampered} = 1)$ . Otherwise let  $b := 0$ .
  3. Send  $(\text{sid}, \text{TamperDone}, \mathcal{A}, (C, P_i^{\text{hot}}, b))$  to  $\mathcal{A}$ .
- On input  $(\text{sid}, \text{Tamper}, \mathcal{A}, (P_i^{\text{cold}}, f))$ , where  $f$  is a function:
  1. Retrieve  $(P_i^{\text{cold}}, \text{ek}_i, \text{dk}_i, \text{leaked}, \text{tampered}, \text{corrupt}, \text{allowc}) \in L_{\text{cold}}$ . Let  $\text{dk}'_i := f(\text{dk}_i)$ .
  2. If  $\text{dk}'_i \neq \perp$ , let  $b := 1$  and update the entry to  $(P_i^{\text{cold}}, \text{ek}_i, \text{dk}'_i, \text{leaked}, \text{tampered} = 1, \text{corrupt}, \text{allowc})$ . Otherwise let  $b := 0$ .
  3. Send  $(\text{sid}, \text{TamperDone}, \mathcal{A}, (P_i^{\text{cold}}, b))$  to  $\mathcal{A}$ .

### Corrupt

- On input  $(\text{sid}, \text{Corrupt}, \mathcal{A}, (P_i^{\text{hot}}))$ , retrieve  $(P_i^{\text{hot}}, \text{corrupted}, \text{allowc}) \in S_{\text{hot}}$  and check that **allowc** = 1. If so, set **corrupted** to 1. ( $\mathcal{A}$  receives  $P_i^{\text{hot}}$ 's tapes and  $\mathcal{E}$  is notified.)
- On input  $(\text{sid}, \text{Corrupt}, \mathcal{A}, (P_i^{\text{cold}}))$ , retrieve  $(P_i^{\text{cold}}, *, *, *, *, \text{corrupted}, \text{allowc}) \in L_{\text{cold}}$  and check if **allowc** = 1. If so, set **corrupted** to 1. ( $\mathcal{A}$  receives  $P_i^{\text{cold}}$ 's tapes and  $\mathcal{E}$  is notified.)

Figure 10: The BLS hot-cold threshold wallet functionality  $\mathcal{F}_{\text{HC}}$  (adversarial interfaces).

**Message delivery**  $\mathcal{S}$  waits to deliver any messages until  $\mathcal{A}$  delivers the corresponding message in the internal interaction.

**Corruptions** Whenever  $\mathcal{A}$  corrupts a party  $P_i^{\text{cold}}$  or  $P_i^{\text{hot}}$  in the internal execution,  $\mathcal{S}$  corrupts the corresponding dummy party (via the **Corrupt** interface).

**Leak and Tamper** Whenever  $\mathcal{A}$  leaks or tampers with the inputs of a party in the internal execution,  $\mathcal{S}$  uses the corresponding interface of  $\mathcal{F}_{\text{HC}}$  to learn/change the same information (it can use any functions  $f, g$  for tampering).

**Cold Registration**  $\mathcal{S}$  will deliver the message  $(\text{sid}, \text{ColdRegister}, P_i^{\text{cold}})$  from  $\tilde{P}_i^{\text{cold}}$  to  $\mathcal{F}_{\text{HC}}$  once  $\mathcal{A}$  delivers  $(\text{sid}, \text{PKSetup}, P_i^{\text{cold}})$  to  $\mathcal{F}_{\text{PK}}$  in the internal interaction.

- If  $\tilde{P}_i^{\text{cold}}$  is corrupt,  $\mathcal{S}$  records  $(P_i^{\text{cold}}, \text{ek}_i^*, \text{corrupt} = 1)$  in a local database  $L'_{\text{cold}}$ , where  $\text{ek}_i^*$  is the value sent by  $\mathcal{A}$  on (the corrupted)  $\tilde{P}_i^{\text{cold}}$ 's behalf.
- Otherwise, if  $\tilde{P}_i^{\text{cold}}$  is honest, it stores  $(P_i^{\text{cold}}, \text{ek}_i, \text{corrupt} = 0) \in L'_{\text{cold}}$ , where  $\text{ek}_i$  is the value from  $\mathcal{F}_{\text{HC}}$ 's response.

It delivers the response to  $\tilde{P}_i^{\text{cold}}$  once  $\mathcal{A}$  delivers the result of  $\mathcal{F}_{\text{PK}}$  in the internal interaction.

**Client Registration and Share Refreshes** Recall that  $C$  is out of scope for corruptions. Thus, in any case  $\mathcal{S}$  will deliver the message  $(\text{sid}, \text{ClientRegister}, C, (t, \mathcal{I}))$  on  $\tilde{C}$ 's outgoing communication tape to  $\mathcal{F}_{\text{HC}}$  once  $\mathcal{A}$  delivers  $(\text{sid}, \text{SSSetup}, C, (t, \{P_i^{\text{hot}}\}_{i \in [n]}, *))$  from  $C$  to  $\mathcal{F}_{\text{SS}}$  in the internal interaction.  $\mathcal{F}_{\text{HC}}$  will send  $(\text{sid}, \text{ClientRegistered}, C, (\text{vk}))$  to  $\tilde{C}$  and  $(\text{sid}, \text{ClientRegistered}, P_i^{\text{hot}}, (C, b_i))$  to  $\tilde{P}_i^{\text{hot}}$ .  $\mathcal{S}$  delivers these messages to  $\tilde{C}$  and each *honest* party  $\tilde{P}_i^{\text{hot}}$ , respectively, once the corresponding response by  $\mathcal{F}_{\text{SS}}$  is delivered to them in the internal interaction. For any corrupt  $\tilde{P}_i^{\text{hot}}$ ,  $\mathcal{S}$  instead outputs on its behalf the same bit  $b_i$  as  $\mathcal{A}$  does in the internal interaction. The simulation for share refreshes works the same way.

**Signing** We will use the fact that partial BLS signatures  $\sigma_i$  can be verified with respect to the partial verification key  $\text{vk}_i$ . Let  $\text{PVerfy}(\text{vk}_i, m, \sigma_i)$  be the partial verification algorithm, which in the case of BLS consists of checking that  $(g_2, \text{vk}_i, H(m), \sigma_i)$  is a co-Diffie-Hellman tuple (see ??). On input  $(\text{sid}, \text{TSign}, P_i^{\text{hot}}, (C, m))$ , the simulator first delivers the message to  $\mathcal{F}_{\text{HC}}$  once  $\mathcal{A}$  delivers the corresponding request from  $C$  to  $P_i^{\text{hot}}$  in the internal interaction. Then it retrieves the identity of the corresponding  $P_i^{\text{cold}}$  and behaves as follows:

- If  $P_i^{\text{hot}}, P_i^{\text{cold}}$  are both honest,  $\mathcal{S}$  immediately delivers  $\mathcal{F}_{\text{HC}}$ 's output  $(\text{sid}, \text{TSignResult}, P_i^{\text{hot}}, (C, m, \sigma_i))$ .

- If  $P_i^{\text{hot}}$  is honest and  $P_i^{\text{cold}}$  is corrupt,  $\mathcal{S}$  will receive a **CSignRequest** from  $\mathcal{F}_{\text{HC}}$  to which it must respond with a bit  $b$ . It looks at the values  $\sigma_i^{\text{hot}}$  and  $\sigma_i^{\text{cold}*}$  in the internal interaction (the latter is output by  $\mathcal{A}$  on behalf of the corrupt  $P_i^{\text{cold}}$ ) and responds with  $b := \text{PVrfy}(\text{vk}_i, m, \sigma_i^{\text{hot}}/\sigma_i^{\text{cold}*})$ , where  $\text{vk}_i, m$  are the  $i$ th partial verification key and message requested in the *internal* execution (all known to  $\mathcal{S}$ ). As before, it delivers  $\mathcal{F}_{\text{HC}}$ 's output to  $\tilde{C}$  immediately.
- If  $P_i^{\text{hot}}$  is corrupt and  $P_i^{\text{cold}}$  is honest,  $\mathcal{S}$  will receive an **HSignRequest** from  $\mathcal{F}_{\text{HC}}$  to which it must again respond with a bit  $b$ . Similarly, it now looks at the values  $\sigma_i^{\text{hot}*}$  (output by  $\mathcal{A}$ ) and  $\sigma_i^{\text{cold}}$  in the internal execution and responds with  $b := \text{PVrfy}(\text{vk}_i, m, \sigma_i^{\text{hot}*}/\sigma_i^{\text{cold}})$ . Again it immediately delivers  $\mathcal{F}_{\text{HC}}$ 's output to  $\tilde{C}$ .
- Finally, if both  $P_i^{\text{hot}}$  and  $P_i^{\text{cold}}$  are corrupt,  $\mathcal{S}$  checks if  $\text{PVrfy}(\text{vk}_i, m, \sigma_i^{\text{hot}*}/\sigma_i^{\text{cold}*}) = 1$  in the internal execution. If so, it responds 1 to both **CSignRequest** and **HSignRequest**; otherwise (w.l.o.g.) it sends 0 to both. Then it delivers  $\mathcal{F}_{\text{HC}}$ 's output to  $\tilde{C}$ .

**Proofs of Remembrance** On input  $(\text{sid}, \text{CProof}, C, (P_i^{\text{cold}}))$ , the simulator delivers the message to  $\mathcal{F}_{\text{HC}}$  once  $\mathcal{A}$  delivers the corresponding request from  $C$  to  $P_i^{\text{cold}}$  in the internal interaction.

- If  $\tilde{P}_i^{\text{cold}}$  is honest,  $\mathcal{S}$  delivers  $(\text{sid}, \text{CProofResult}, C, (P_i^{\text{cold}}, b))$  to  $\tilde{C}$  once  $\mathcal{A}$  delivers the output of  $\mathcal{F}_{\text{PK}}$  to  $C$  in the internal execution.
- On the other hand, if  $\tilde{P}_i^{\text{cold}}$  is corrupted,  $\mathcal{S}$  will receive a message from  $\mathcal{F}_{\text{HC}}$  requesting a bit  $b^*$ . It retrieves the party's  $\text{ek}_i$  from  $L'_{\text{cold}}$  and the proof  $\pi_i^{\text{cold}}$  computed by  $\mathcal{A}$  in the internal execution and sends back  $b^* := \Pi_{\text{DL}}.\text{Vrfy}((\text{ek}_i, g_2), \pi_i^{\text{cold}})$ . Finally it delivers  $\mathcal{F}_{\text{HC}}$ 's output  $(\text{sid}, \text{CProofResult}, C, (P_i^{\text{cold}}, b))$  to  $\tilde{C}$  once  $\mathcal{A}$  delivers the message from  $\mathcal{F}_{\text{PK}}$  to  $C$  in the internal execution.

On input  $(\text{sid}, \text{HProof}, P_i^{\text{hot}}, (C))$ ,  $\mathcal{S}$  acts in the same way as **CProof**, except in the corrupted case it sets  $b^* := \Pi_{\text{EKS}}.\text{Vrfy}(\text{com}, \pi_i^{\text{hot}})$ , where both  $\text{com}, \pi_i^{\text{cold}}$  are the party's values in the internal execution.

It is straightforward to see that the simulated registration and share refreshes are identical to a real-world execution. The simulation of the signing protocol is computationally indistinguishable from the real-world execution by the correctness and security of threshold BLS signatures, which guarantees that  $\mathcal{S}$  sends  $b = 1$  iff  $\sigma_i^{\text{cold}*} = H(m)^{\mathcal{H}(\text{vk}^{\text{dk}_i})}$ , resp.  $\sigma_i^{\text{hot}*} = H(m)^{\tilde{x}_i}$ . Similarly, the indistinguishability of simulating proofs of remembrance follows from the correctness and soundness of  $\Pi_{\text{DL}}$  and  $\Pi_{\text{EKS}}$ .  $\square$

Finally, notice that the signing interface of  $\mathcal{F}_{\text{HC}}$  is identical to the functionality offered by threshold BLS, except that  $\mathcal{A}$  additionally learns either  $\sigma_i^{\text{hot}}$  or  $\sigma_i^{\text{cold}}$  (in step 3) or a uniform value  $H(m)^r$  (in step 4). Clearly the uniform value is independent of any private information can be simulated perfectly as



a uniform  $\mathbb{G}_2$  element. As for step 3, the simulator can again send a uniform  $\mathbb{G}_2$  element, now in place of  $\sigma_i^{\text{hot}}$  (alternatively,  $\sigma_i^{\text{cold}}$ ), and the simulation is statistically indistinguishable by Lemma 1.

## 5.7.2 Implementation and Evaluation

We implemented our construction in Rust using the BLS12-381 elliptic curve.<sup>7</sup> Each element in  $\mathbb{G}_1$  is 48 bytes in compressed form,  $\mathbb{G}_2$  is 96 bytes, and  $\mathbb{Z}_p$  is 32 bytes. Thus the sizes of  $\text{vk}$  and  $\text{ek}_i$  are both 96 bytes. The cold and hot shares  $\text{dk}_i$  and  $\tilde{x}_i$ , as well as the share refresh information  $\delta_i$ , are all 32 bytes. The (partial) signatures  $\sigma_i^{\text{hot}}$ ,  $\sigma_i^{\text{cold}}$ , and  $\sigma_i$ , as well as the commitment  $\text{com}_T$ , are 48 bytes.

We report the runtimes of each of our algorithms for small  $(t, n) = (3, 5)$ , medium  $(5, 20)$ , and large  $(67, 100)$  parameter settings. The times for cold registration and proofs, threshold signing, and processing hot share refreshes (which includes running  $\Pi_{\text{Ref.HVrfy}}$  to check  $\zeta_i$ ) are all independent of  $(t, n)$  and are shown in Table 1. Hot share generation ( $\text{ClientRegister}$ ), producing hot share refreshes (in our implementation, by  $C$  running the code of  $\mathcal{F}_{\text{SS}}$ ), **Noemi: does this include  $\Pi_{\text{Ref.UCVrfy}}$ ? i.e., checking that the update is a sharing of zero (verify KZG opening at zero) and that it's degree- $t$  (check  $\text{dcom}$ ). So, it's the bullet of  $\text{ClientUpdate}$  in  $\text{spec.tex}$  that starts with “The system should check that...”** and hot proofs depend on the specific values of  $(t, n)$  and are shown in Table 2. The benchmarks are an average over 1000 runs using a machine with 8-core AMD Ryzen 9 5900HX with 64GB RAM and cache: L1 512KB, L2 4MB, L3 16 MB at 5GHz. The proof sizes are given in Table 3.

	Time
ColdRegister	360 $\mu\text{s}$
TSign	890 $\mu\text{s}$
ShareRefresh ( $P_i^{\text{hot}}$ )	5ms
Cold Prove ( $\Pi_{\text{DL.Prove}}$ )	370 $\mu\text{s}$
Cold Verify ( $\Pi_{\text{DL.Vrfy}}$ )	560 $\mu\text{s}$

Table 1: Benchmarks of our algorithms regardless of threshold

## 5.8 Extensions

### 5.8.1 Trustless proactive refresh using a bulletin-board

Recall from Section 5.6.2 that the correctness of our proactive refresh protocols relies on the update polynomial  $z_T(X)$  being of degree  $t - 1 \leq d$  and having  $z_T(0) = 0$ . We can use a folklore technique [CHM<sup>+</sup>20, §2.5] to enforce the degree requirement: the client publishes an additional public “degree commitment”

<sup>7</sup><https://github.com/hyperledger-labs/agora-key-share-proofs/>

$(t, n) =$	Time (ms)		
	(3, 5)	(5, 20)	(67, 100)
ClientRegister	10	40	170
ShareRefresh ( $C$ )	11	41	172
Hot Prove ( $\Pi_{\text{EKS}}.\text{Prove}$ )	10	40	170
Hot Verify ( $\Pi_{\text{EKS}}.\text{Vrfy}$ )	14	43	4

Table 2: Benchmarks of our algorithms for each setting of  $(t, n)$ . Times for CProof and HProof are not included since they are identical to the corresponding rows of Table 3. The ShareRefresh times for  $C$  and  $P_i^{\text{hot}}$  include the UCVrfy resp. HVrfy times of the refresh proofs in Table 3.

	$ \pi $ (B)
Cold proof ( $\Pi_{\text{DL}}$ )	256
Hot proof ( $\Pi_{\text{EKS}}$ )	304
Refresh proof ( $\zeta_i$ )	48

Table 3: Sizes for our various proof systems regardless of threshold

$\text{dcom}_T := g_1^{\tau^{d-t+1} \cdot z_T(\tau)}$ , which is verified by checking

$$e(\text{dcom}_T, g_2) = e(\text{ucom}_T, g_2^{\tau^{d-t+1}}),$$

where  $d$  is the degree of the KZG CRS. This ensures that the polynomial  $z_T(X)$  committed to by  $\text{ucom}_T$  is of degree at most  $t-1$ . To enforce the evaluation at zero,  $C$  can simply provide a KZG opening proof for  $\text{ucom}_T$  at  $X=0$ .

Additionally, the hot parties should also be sure that the client is using the *same* polynomial  $z_T(X)$  for all of them when computing their share updates  $\delta_i$ . This is easily done, since each  $P_i^{\text{hot}}$  is already provided with an evaluation proof  $\zeta_i$  for  $\delta_i$ . In the case of a trusted client, these were assumed to be computed correctly so  $P_i^{\text{hot}}$  only used them to blindly update the its key share and opening proof  $(\tilde{x}_i, \pi_i)$ . In the case of an untrusted client, parties will instead first check their correctness by ensuring  $\text{KZG.Vrfy}(\text{crs}, \text{ucom}_T, i, \delta_i, \zeta_i) = 1$ , and only update their key share and opening proof if verification passes.

Figure 11 gives the proof system  $\Pi_{\text{Ref}}$  used to prove correctness of share refreshes, with verification split between verifying the well-formedness of the update polynomial  $z_T(X)$  committed to by  $\text{ucom}_T$  (via UCVrfy) and of each hot party's refresh information  $\delta_i$  (via HVrfy).

Given  $\Pi_{\text{Ref}}$ , it is fairly simple to realize  $\mathcal{F}_{\text{SS}}$  in a manner that avoids trusting  $C$  except for the setup phase. To do this, we assume a public bulletin board functionality  $\mathcal{F}_{\text{BB}}$  with limited programmability (Figure 12). This functionality will store the most up-to-date commitment  $\text{com}$  to the hot shares  $\tilde{x}_1, \dots, \tilde{x}_n$ . Whenever the shares are refreshed, it will also check the correctness of the update to  $\text{com}$  (namely  $\text{ucom}$ ) before making the new commitment available.

Specifically, instead of only running the steps of  $\mathcal{F}_{SS}$  locally,  $C$  will compute some additional values using  $\Pi_{Ref}$  to let  $\mathcal{F}_{BB}$  and each  $P_i^{hot}$  check the correctness of the share and update commitments. The proof for the update commitment  $ucom$  will be checked by  $\mathcal{F}_{BB}$  before changing the stored commitment. Additionally,  $C$  will let  $\mathcal{F}_{BB}$  store and distribute  $ucom$  instead. We describe the full protocol in Figure 13, where changes with respect to locally running the ideal functionality  $\mathcal{F}_{SS}$  are shown in blue.

**Theorem 2.** *The encrypted secret sharing protocol in Figure 13 UC-realizes  $\mathcal{F}_{SS}$  in the  $\mathcal{F}_{BB}$ -hybrid model.*

*Proof.* **todo:**

□

## 6 Conclusion

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SHARE REFRESH PROOFS ( $\Pi_{\text{Ref}}$ )

**Parameters:** Degree- $d$  KZG common reference string  $\text{crs} = \{g_1, g_1^\tau, \dots, g_1^{\tau^d}, g_2, g_2^\tau\}$ .

Prove $((\text{crs}, \text{ucom}_T, t-1, \{\delta_i\}_{i \in [n]}; z_T(X)) \rightarrow (\{\zeta_{T,i}\}_{i \in [n]}, \pi_z))$ : Given  $\text{crs}$ , a KZG commitment  $\text{ucom}_T$  to update polynomial  $z_T(X)$ , the latter's degree  $t-1$ , and each party's share refresh information  $\delta_i$ , use  $z_T(X)$  to compute the following:

1. For each  $i \in [n]$ , prove that  $\delta_i = z_T(i)$  by computing  $(\delta_i, \zeta_{T,i}) \leftarrow \text{KZG.Open}(\text{crs}, z_T(X), i)$ .
2. Prove that  $z_T(0) = 0$  and  $z_T(X)$  has degree  $t-1 \leq d$  by computing  $(0, \zeta_{T,0}) \leftarrow \text{KZG.Open}(\text{crs}, z_T(X), 0)$  and  $\text{dcom}_T := g_1^{\tau^{d-t+1} \cdot z_t(\tau)}$  using  $\text{crs}$ . Let  $\pi_{\text{ucom}} := (\zeta_{T,0}, \text{dcom}_T)$ .
3. Output  $(\{\zeta_{T,i}\}_{i \in [n]}, \pi_{\text{ucom}})$ .

HVrfy $((\text{crs}, \text{ucom}_T, i, \delta_i), \zeta_{T,i}) \rightarrow \{0, 1\}$ : Given  $\text{crs}$ , a KZG commitment  $\text{ucom}_T$ , party index  $i$ , and share refresh information  $\delta_i$ , output  $\text{KZG.Vrfy}(\text{crs}, \text{ucom}_T, i, \delta_i, \zeta_{T,i})$ .

UCVrfy $((\text{crs}, \text{ucom}_T, t-1), \pi_{\text{ucom}}) \rightarrow \{0, 1\}$ : Given  $\text{crs}$ , a KZG commitment  $\text{ucom}_T$ , and its supposed degree  $t-1$ , verify the proof  $\pi_{\text{ucom}} = (\zeta_{T,0}, \text{dcom}_T)$  by outputting 1 iff the following hold:

$$\begin{aligned} \text{KZG.Vrfy}(\text{crs}, \text{ucom}_T, 0, 0, \zeta_{T,0}) &= 1 \\ e(\text{dcom}_T, g_2) &= e(\text{ucom}_T, g_2^{d-t+1}) \end{aligned}$$

Figure 11: The proof system  $\Pi_{\text{Ref}}$  used to prove correctness of the every hot party's share refresh information  $\delta_i$  and the commitment update  $\text{ucom}_t$ . Each hot party verifies its own update information using  $\text{HVrfy}$ , and the correctness of  $\text{ucom}$  is verified separately via  $\text{UCVrfy}$ .



**Public parameters:** Groups  $\mathbb{G}_1, \mathbb{G}_2$  of order  $p$  with generators  $g_1, g_2$ , respectively.

- On input  $(\text{sid}, \text{Setup}, C, (\text{vk}, t, \text{com}))$ , delete any existing entries  $(C, *, *, *, *) \in D$  and  $(C, *, *) \in U$ . Add  $(C, \text{vk}, t, \text{com}, \text{time} := 0)$  to  $D$  and output  $(\text{sid}, \text{SetupResult}, C, (1))$ .
- On input  $(\text{sid}, \text{ComUpdate}, C, (\text{ucom}, \pi_{\text{ucom}}))$ :
  1. Retrieve  $(C, \text{vk}, t, \text{com}, \text{time}) \in D$  for the maximum value of **time**.
  2. If  $\Pi_{\text{Ref}}.\text{UCVrfy}((\text{crs}, \text{ucom}, t), \pi_{\text{ucom}}) = 1$ , set  $b := 1$  and add  $(C, \text{vk}, t, \text{com} \cdot \text{ucom}, \text{time}++)$  to  $D$  and  $(C, \text{ucom}, \text{time}++)$  to  $U$ . Otherwise set  $b := 0$ .
  3. Output  $(\text{sid}, \text{ComUpdateResult}, C, (b))$ .
- On input  $(\text{sid}, \text{ClientInfoRecover}, P, (C))$ , retrieve  $(C, \text{vk}, *, \text{com}, \text{time}) \in D$  for the maximum value of **time** and output  $(\text{sid}, \text{ClientInfo}, P, (C, \text{vk}, \text{com}))$ .
- On input  $(\text{sid}, \text{UComRecover}, P, (C))$ , retrieve  $(C, \text{ucom}, \text{time}) \in U$  for the maximum value of **time** and output  $(\text{sid}, \text{UCom}, P, (C, \text{ucom}))$ .

Figure 12: **The Programmable Bulletin-Board Functionality  $\mathcal{F}_{\text{BB}}$**

### ENCRYPTED SECRET SHARING PROTOCOL

**Public parameters:** Groups  $\mathbb{G}_1, \mathbb{G}_2$  of prime order  $p$  with generators  $g_1, g_2$ , respectively; a degree- $d$  KZG common reference string  $\text{crs}$ .

Setup: On input  $(\text{sid}, \text{SSSetup}, C, (t, \mathcal{P}, \{\text{ek}_i\}_{i \in [n]}))$ , where  $t, n \in \mathbb{N}$  s.t.  $n = |\mathcal{P}|$  and  $t \leq n \leq d$ ,  $\mathcal{P} = \{P_1 \dots P_n\}$  a set of parties, and  $\{\text{ek}_i\}_{i \in [n]}$  a set of public (encryption) keys ( $\forall i \in [n], \text{ek}_i \in \mathbb{G}_2$ ),  $C$  proceeds as follows:

1. Sample  $x \xleftarrow{\$} \mathbb{Z}_p \setminus \{0\}$ . Let  $y := g_2^x$ .
2. Generate  $t$ -out-of- $n$  Shamir Shares of  $x$  as  $x_1, \dots, x_n \in \mathbb{Z}_p$ . Let  $y_i := g_2^{x_i} \forall i \in [n]$ .
3. Interpolate the degree- $n$  polynomial  $\tilde{f}$  such that  $\tilde{f}(i) = \mathcal{H}(\text{ek}_i^x) + x_i \forall i \in [n]$ . Compute  $\text{com} \leftarrow \text{KZG.Com}(\text{crs}, \tilde{f})$  and send  $(\text{sid}, \text{Setup}, C, (y, t, n, \text{com}))$  to  $\mathcal{F}_{\text{BB}}$ .
4. Delete any existing entries  $(C, *, *) \in D$  and add  $(C, \mathcal{P}, t)$  to  $D$ .
5. For each  $i \in [n]$ , compute  $(\tilde{x}_i, \pi_i) \leftarrow \text{KZG.Open}(\text{crs}, \tilde{f}, i)$  and output  $(\text{sid}, \text{SecretShare}, P_i, (C, i, \tilde{x}_i, \pi_i))$ .
6. Finally, output  $(\text{sid}, \text{SSSetupDone}, C, (y, \{y_i\}_{i \in [n]}))$ .

Generating share refreshes: On input  $(\text{sid}, \text{ZeroSetup}, C, (t, \mathcal{P}))$ , where  $t, n \in \mathbb{N}$  s.t.  $n = |\mathcal{P}|$  and  $t \leq n \leq d$ , and  $\mathcal{P} = \{P_1 \dots P_n\}$  a set of parties,  $C$  will proceed as follows:

1. Generate  $t$ -out-of- $n$  Shamir Shares of 0 as  $x_1, \dots, x_n \in \mathbb{Z}_p$ ; let  $f$  be the polynomial used.
2. Compute  $\text{com}_0 \leftarrow \text{KZG.Com}(\text{crs}, f)$  and  $(\{\zeta_i\}_{i \in [n]}, \pi_z) \leftarrow \Pi_{\text{Ref}}.\text{Prove}((\text{crs}, \text{com}_0, t - 1, \{x_i\}_{i \in [n]}); f(X))$ . Send  $(\text{sid}, \text{ComUpdate}, C, (\text{com}_0, \pi_z))$  to  $\mathcal{F}_{\text{BB}}$ , which returns  $(\text{sid}, \text{ComUpdateResult}, C, (b))$ .
3. Send  $(x_i, \zeta_i)$  to  $P_i$  for all  $i \in [n]$ , then output  $(\text{sid}, \text{ZeroSetupDone}, C, (b))$ .
4. Each party  $P_i$  for  $i \in [n]$  will send  $(\text{sid}, \text{UComRecover}, P_i, (C))$  to  $\mathcal{F}_{\text{BB}}$  and receive  $(\text{sid}, \text{UCom}, P_i, (C, \text{ucom}))$  in return. Check that  $\Pi_{\text{Ref}}.\text{HVerfy}((\text{crs}, \text{ucom}, i, x_i), \zeta_i) = 1$ . If not, set  $x_i, \zeta_i = \perp$ . Output  $(\text{sid}, \text{ZeroShare}, P_i, (C, x_i, \zeta_i))$ .

Providing auxiliary information: On input  $(\text{sid}, \text{AuxRecover}, P, (C))$  for some client  $C$ ,  $P$  sends  $(\text{sid}, \text{ClientInfoRecover}, C)$  to  $\mathcal{F}_{\text{BB}}$  to get  $(\text{vk}, \text{com})$ . It outputs  $(\text{sid}, \text{AuxInfo}, P, (C, \text{vk}, \text{com}))$ .

Figure 13: Protocol realizing  $\mathcal{F}_{\text{SS}}$  in the  $\mathcal{F}_{\text{BB}}$ -hybrid model