CHAPTER

THREE

REACTOR HEAT GENERATION

I INTRODUCTION

Determination of the heat-generation distribution throughout the nuclear reactor is achieved via a neutronic analysis of the reactor. Accurate knowledge of the heat source is a prerequisite for analysis of the temperature field, which in turn is required for definition of the nuclear and physical properties of the fuel, coolant, and structural materials. Therefore coupling the neutronic and thermal analyses of a nuclear core is required for accurate prediction of its steady state as well as its transient conditions. For simplicity, the neutronic and thermal analyses during thermal design evaluations may not be coupled, in which case the level and distribution of the heat-generation rate are assumed fixed, and thermal analysis is carried out to predict the temperature field in the reactor core.

It should be noted that the operational power of the core is limited by thermal, not nuclear, considerations. That is, in practice, the allowable core power is limited by the rate at which heat can be transported from fuel to coolant without reaching, either at steady state or during specified transient conditions, excessively high temperatures, which would cause degradation of the fuel, structures, or both. The design limits are discussed in Chapter 2.

II ENERGY RELEASE AND DEPOSITION

A Forms of Released Energy

The energy released in a reactor is produced by exothermic nuclear reactions in which part of the nuclear mass is transformed to energy. Most of the energy is released when nuclei of heavy atoms split as they absorb neutrons. The splitting of these nuclei is called *fission*. A small fraction of the reactor energy comes from nonfission neutron capture in the fuel, moderator, coolant, and structural materials.

The fission energy, which is roughly 200 MeV (or 3.2×10^{-11} J) per fission, appears as kinetic and decay energy of the fission fragments, kinetic energy of the newborn neutrons, and energy of emitted γ-rays and neutrinos. Many of the fission fragments are radioactive, undergoing β-decay accompanied by neutrons. The β-emission makes certain isotopes unstable, with resultant delayed neutron and γ-ray emission.

Immediately upon capture, the neutron-binding energy, which ranges from 2.2 MeV in hydrogen to 6 to 8 MeV in heavy materials, is released in the form of γ-rays. Many capture products are unstable and undergo decay by emitting β-particles, neutrons, and γ-rays. An approximate accounting of the forms of the released energy is given in Figure 3-1, and the energy distribution among these forms is outlined in Table 3-1.

The neutrons produced in the fission process have a relatively high kinetic energy and are therefore called *fast neutrons.* Most of these prompt neutrons have energies between 1 and 2 MeV, although some may have energies up to 10 MeV. The potential for a neutron to cause fission is improved if its energy is reduced to levels comparable to the surroundings by a slowing process, called *neutron moderation*. The slow neutrons, referred to as *thermal neutrons,* have energies in the range of 0.01 to 0.10 eV. The best moderating materials are those of low atomic masses. Hence moderators such as carbon, hydrogen, and deuterium have been used in power reactors that rely mostly on fission from slow (thermal) neutrons.

Table 3-1 Approximate distribution of energy release and deposition in thermal reactors

Type	Process	Percent of total released energy	Principal position of energy deposition
Fission			
I: instantaneous energy	Kinetic energy of fission fragments	80.5	Fuel material
	Kinetic energy of newly born fast neutrons	2.5	Moderator
	γ Energy released at time of fission	2.5	Fuel and structures
II: delayed energy	Kinetic energy of delayed neutrons	0.02	Moderator
	β-Decay energy of fission products	3.0	Fuel materials
	Neutrinos associated with β decay	5.0	Nonrecoverable
	γ-Decay energy of fission products	3.0	Fuel and structures
Neutron Capture			
III: instantaneous and delayed energy	Nonfission reactions due to excess neutrons plus β- and γ-decay energy of (n, γ) products	3.5	Fuel and structures
Total		100	

Source: Adapted from El-Wakil [2].

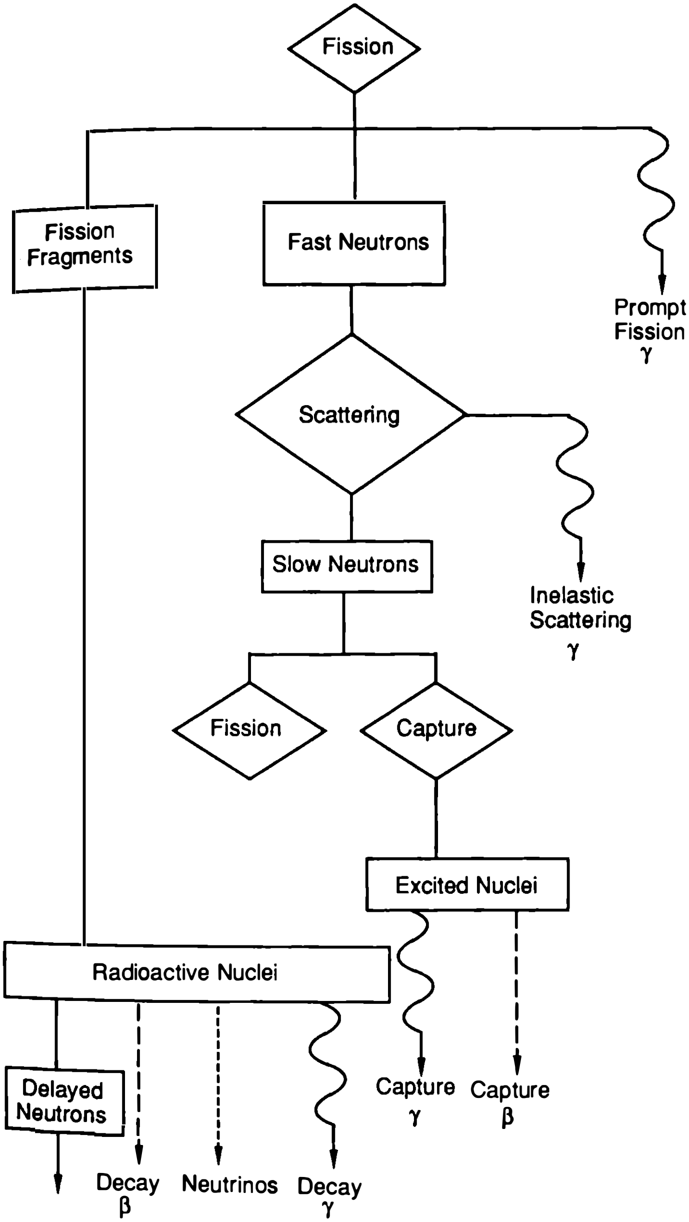

Figure 3-1 Forms of energy release in a reactor.

Splitting a fissile atom produces two smaller atoms and two or more neutrons. Uranium 235 is the only fissile material naturally present in extractable amounts, at roughly 0.7% of all uranium. Other fissile materials are generated by neutron capture in so-called fertile atoms. Thus fissile plutonium 239 and plutonium 241 are produced by neutron absorption in atoms of uranium 238 and plutonium 240, respectively. The only other practical fissile material is uranium 233, which is produced by neutron capture in thorium 232.

The energy release upon fission (E_f) is slightly dependent on the different fissile materials. Lamarsh [5] suggested the following relation for energy release due to fission by a thermal neutron:

$$E_f\,(^{233}\mathrm{U}) = 0.98\,E_f\,(^{235}\mathrm{U})$$
$$E_f\,(^{239}\mathrm{Pu}) = 1.04\,E_f\,(^{235}\mathrm{U})$$

In a typical LWR only about one-half of the neutrons are absorbed in fissile isotopes; the other one-half are captured by the fertile isotopes, control, and structural materials. The LWR fresh fuel is composed of uranium-based fuel, i.e., UO_2, slightly enriched in uranium 235. The plutonium produced in the LWR core during operation also participates in the energy-release process and may contribute up to 50% of the fission energy release.

B Energy Deposition

The fission products moving through the core material lose their energy through interaction with the surrounding matter. The energy loss rate depends mainly on the penetrating ability of the fission products. The neutrino energy is unrecoverable, as it does not interact with the surrounding materials. The fission fragments that carry most of the fission energy have a short range (<0.25 mm). The γ-rays' energy is released in structural as well as fuel materials. A considerable amount of the kinetic energy of neutrons is released in the moderator and the structure.

Let us consider the heat-generation rate per unit volume at any position, $q'''(\vec{r})$. It should be recognized that $q'''(\vec{r})$ is due to products of reaction events at all neighboring positions, as these reaction products pass through position $\vec{r}$. Let the volumetric heat-generation rate at position $\vec{r}$ due to reaction products of type i and energy E be $q_i'''(\vec{r},E)$. To obtain the total volumetric heat-generation rate at position $\vec{r}$ we must sum over all particle (and photon) types and energy spectrum:

$$q'''(\vec{r}) = \sum_i \int_o^\infty q_i'''(\vec{r},E)dE \tag{3-1}$$

Thus to calculate exactly the heat generation at a particular point of the reactor is difficult. However, the heat generation can be well approximated in the various parts of the reactor by established reactor physics analysis methods.

Information on the typical distribution of energy deposition in fuel and nonfuel materials in an LWR is given in Table 3-1. The amount of energy produced within the various parts of the reactor depends on reactor materials and geometry and hence on the reactor type. Roughly speaking, 87% of the total energy released per fission in an LWR is recovered in the fuel, 3% in the moderator, and 5% in the structure; 5% is unrecoverable (neutrino energy). Accounting for capture energy, the fraction of recoverable energy, i.e., core power, deposited in the fuel becomes about 95%.

III HEAT-GENERATION PARAMETERS

A Heat Generation and Neutron Flux in Thermal Reactors

The heat-generation rate in the fuel, $q'''(\vec{r})$, is typically computed by assuming that the energy released by a fission reaction is recovered at the position of the fission event, except for the fraction carried away by neutrinos and the fraction deposited in nonfuel materials.* In other words, the spatial distribution of energy deposition from the fission fragments, γ's and β's, is assumed to follow the spatial distribution of the fission reaction rate $RR_f(\vec{r})$. Therefore the heat-generation rate $q'''(\vec{r},E)$ also becomes proportional to $RR_f(\vec{r})$.

We define the energy per fission reaction of isotope j, which is deposited in the fuel, as χ_f^j. For a typical thermal reactor, χ_f^{25} is about 190 MeV per fission.

Evaluation of the heat source distribution requires the knowledge of fission reaction rates, RR_f^j, which when summed over all atom types yield the total fission reaction rate, $RR_f(\vec{r})$:

$$RR_f(\vec{r}) = \sum_j RR_f^j(\vec{r}) \tag{3-2}$$

Let $\sigma_a^j(E)$ be the microscopic absorption cross section of isotope j, which is the equivalent projected area of an atom for an absorption reaction. The cross section $\sigma_a^j(E)$ is proportional to the probability that one atom of type j absorbs an incident neutron of energy E. Because the absorption of a neutron by an atom can lead to either a fission process or a nonfission (or capture) process, the absorption cross section is the sum of the fission cross section σ_f and the capture cross section σ_c. The units of the microscopic cross section are normally given by square centimeters or barns (1 barn $= 10^{-24}$ cm^2). Typical values of the absorption and fission cross sections of thermal neutrons are given in Table 3-2.

The macroscopic fission cross section is the sum of all microscopic cross sections for a fission reaction due to all atoms of type j within a unit volume interacting with an incident neutron per unit time. The macroscopic cross section is defined by:

$$\Sigma_f^j(\vec{r},E) \equiv N^j(\vec{r})\sigma_f^j(E) \tag{3-3}$$

where N^j is the atomic density of isotope j. The atomic density (N^j) can be obtained from the mass density (ρ_j) using the relation:

$$N^j = A_v\, \rho_j/M_j \tag{3-4}$$

where A_v = Avogadro's number for molecules in 1 gram mole (0.60225) $\times\ 10^{24}$ molecules/gram mole); M_j = molecular mass of the isotope j.

The fission rate of isotope j at position $\vec{r}$ due to the neutron flux $\phi(\vec{r},E)$ within the interval of neutron energy of E to $E + dE$ is obtained from:

*In addition to the loss of the neutrino energy and the energy deposited outside the fuel, the magnitude of the energy released per reaction, which appears as heat in the fuel, should theoretically be reduced by conversion of a small part to potential energy of various metallurgical defects. It is, however, a negligible effect.

Table 3-2 Thermal (0.0253 eV) neutron cross sections

	Cross section (barns)	
Material	Fission σ_f	Absorption σ_a
Uranium 233	531	579
Uranium 235	582	681
Uranium 238	—	2.70[a]
Uranium, natural	4.2	7.6
Plutonium 239	743	1012
Boron	—	759
Cadmium	—	2450
Carbon	—	0.0034
Deuterium	—	0.0005
Helium	—	<0.007
Hydrogen	—	0.33
Iron	—	2.55
Oxygen	—	0.00027
Sodium	—	0.53
Zirconium	—	0.19

[a]The effective absorption cross section of ^{238}U in a typical LWR is substantially higher owing to the larger cross section at epithermal energies.

$$RR_f^j(\vec{r},E)dE = \Sigma_f^j(\vec{r},E)\ \phi(\vec{r},E)dE$$

The fission reaction rate for isotope j due to neutrons of all energies is:

$$RR_f^j(\vec{r}) = \int_0^\infty RR_f^j(\vec{r},E)\ dE \tag{3-5}$$

The heat-generation rate per unit volume at $\vec{r}$ due to isotope j is:

$$q_j'''(\vec{r}) = \int_0^\infty \chi_f^j\, RR_f^j\,(\vec{r},E)dE \tag{3-6}$$

Summing over all isotopes yields the total volumetric heat-generation rate:

$$q'''(\vec{r}) = \sum_j \int_0^\infty \chi_f^j\, RR_f^j(\vec{r},E)dE$$

or

$$q'''(\vec{r}) = \sum_j \int_0^\infty \chi_f^j\, \Sigma_f^j\,(\vec{r},E)\ \phi(\vec{r},E)dE \tag{3-7}$$

In practice, the energy range is subdivided into a few intervals or groups. A multi-energy group model is then used to calculate the neutron fluxes; thus:

$$q'''(\vec{r}) = \sum_j \sum_{k=1}^{K} \chi_f^j\, \Sigma_{fk}^j\,(\vec{r})\phi_k(\vec{r}) \tag{3-8}$$

where K = number of energy groups; Σ_{fk} and ϕ_k = equivalent macroscopic fission cross section and neutron flux, respectively, for the energy group k.

If we use a one energy group approximation, which gives good results for homogeneous thermal reactors at locations far from the reactor core boundaries, we have:

$$q'''(\vec{r}) = \sum_j \chi_f^j \, \Sigma_{f1}^j(\vec{r}) \, \phi_1(\vec{r}) \tag{3-9}$$

If we also assume uniform fuel material composition, then:

$$q'''(\vec{r}) = \sum_j \chi_f^j \, \Sigma_{f1}^j \phi_1(\vec{r}) \tag{3-10}$$

where Σ_{f1}^j is independent of position $\vec{r}$.

Assuming that $\chi_f^j = \chi_f$ for all fissile material, the volumetric heat-generation rate may be given by:

$$q'''(\vec{r}) = \chi_f \, \Sigma_{f1} \phi_1(\vec{r}) \tag{3-11}$$

where

$$\Sigma_{f1} = \sum_j \Sigma_{f1}^j \tag{3-12}$$

It should be noted that the thermal neutron flux in a typical LWR is only 15% of the total neutron flux. However, the fission cross section of ^{235}U and ^{239}Pu is so large at thermal energies that thermal fissions constitute 85 to 90% of all fissions.

Example 3-1 Determination of the neutron flux at a given power in a thermal reactor

PROBLEM A large PWR designed to produce heat at a rate of 3083 MW has 193 fuel assemblies each loaded with 517.4 kg of UO_2. If the average isotopic content of the fuel is 2.78 weight percent ^{235}U, what is the average thermal neutron flux in the reactor?

Assume uniform fuel composition, and χ_f = 190 MeV/fission (3.04×10^{-11} J/fission). Also assume that 95% of the reactor energy, i.e., of the recoverable fission energy is from the heat generated in the fuel. The effective thermal fission cross section of ^{235}U (σ_f^{25}) for this reactor is 350 barns. Note that this effective value of σ_f^{25} is smaller than that appearing in Table 3-2, as it is the average value of $\sigma_f^{25}(E)$ over the energy spectrum of the neutron flux, including the epithermal neutrons.

SOLUTION Use Eq. 3-11 to find $\phi_1(\vec{r})$.

$$\phi_1(\vec{r}) = \frac{q'''(\vec{r})}{\chi_f \, \Sigma_f} \tag{3-13a}$$

but because $\Sigma_f = \Sigma_f^{25}$ in this reactor, Σ_f^{25} is obtained from Eq. 3-3, yielding:

$$\phi_1(\vec{r}) = \frac{q'''(\vec{r})}{\chi_f \sigma_f^{25} N^{25}} \tag{3-13b}$$

Consequently, the core-average value of the neutron flux in a reactor with uniform density of ^{235}U is obtained from the average heat-generation rate by:

$$\langle \phi_1 \rangle = \frac{\langle q''' \rangle}{\chi_f \sigma_f^{25} N^{25}} \tag{3-13c}$$

Multiplying both the numerator and denominator by the UO_2 volume (V_{UO_2}) we get:

$$\langle \phi_1 \rangle = \frac{\langle q''' \rangle V_{UO_2}}{\chi_f \sigma_f^{25} N^{25} V_{UO_2}} = \frac{\dot{Q}}{\chi_f \sigma_f^{25} N^{25} V_{UO_2}} \tag{3-13d}$$

To use the above equation only N^{25} needs to be calculated. All other values are given. The value of N^{25} can be obtained from the uranium atomic density if the ^{235}U atomic fraction (a) is known:

$$N^{25} = aN^{U} \tag{3-14}$$

The uranium atomic density is equal to the molecular density of UO_2, as each molecule contains one uranium atom ($N^U = N^{UO_2}$). Thus from Eqs. 3-4 and 3-14:

$$N^{25} = a \frac{A_v \rho_{UO_2}}{M_{UO_2}} \tag{3-15a}$$

Multiplying each side of Eq. 3-15a by V_{UO_2} we get:

$$N^{25} V_{UO_2} = aN^{UO_2} V_{UO_2} = \frac{a(A_v) m_{UO_2}}{M_{UO_2}} \tag{3-15b}$$

Now

$$m_{UO_2} = 193 \text{ (assemblies)} \times 517.4 \left[\frac{\text{kg UO}_2}{\text{assembly}}\right] \times \frac{1000 \text{ g}}{\text{kg}} = 9.9858 \times 10^7 \text{ g}$$

The molecular mass of UO_2 is calculated from:

^{235}U M_{25} = 235.0439 [g/mole]
^{238}U M_{28} = 238.0508
Oxygen M_O = 15.9994

$$M_{UO_2} = M_U + 2M_O \tag{3-16a}$$

$$M_U = a\, M_{25} + (1 - a) M_{28} \tag{3-16b}$$

$$\therefore M_{UO_2} = a\, M_{25} + (1 - a) M_{28} + 2M_O \tag{3-16c}$$

To obtain the value of a, we use the known ^{235}U weight fraction, or enrichment (r):

$$r = \frac{aM_{25}}{M_U} = \frac{aM_{25}}{aM_{25} + (1 - a)M_{28}} \tag{3-17}$$

Equation 3-17 can be solved for a

$$a = \frac{r}{r + \dfrac{M_{25}}{M_{28}}(1 - r)} = \frac{0.0278}{0.0278 + \dfrac{235.0439}{238.0508}(0.9722)} = 0.028146 \qquad (3\text{-}18)$$

From Eq. 3-16c:

$$\begin{aligned} M_{UO_2} &= 0.028146(235.0439) + (0.971854)(238.0508) + 2(15.9994) \\ &= 237.9657 + 2(15.9994) = 269.9645 \end{aligned}$$

Then from Eq. 3-15b:

$$\begin{aligned} N^{25}V_{UO_2} &= \frac{0.028146\left(0.60225 \times 10^{24} \dfrac{\text{atoms}}{\text{g} \cdot \text{mole}}\right)(9.9858 \times 10^7 \text{ g})}{269.9645 \text{ g/mole}} \\ &= 6.27 \times 10^{27} \text{ atoms U}^{25} \end{aligned}$$

Now using Eq. 3-13d, the value for the average flux is calculated:

$$\langle \phi_l \rangle = \frac{0.95\ (3083\ \text{MW})\left(10^6 \dfrac{\text{W}}{\text{MW}}\right)}{\left[3.04 \times 10^{-11} \dfrac{\text{J}}{\text{fission}}\right]\left[350 \times 10^{-24} \dfrac{\text{cm}^2}{\text{atom-neutron}}\right][6.270 \times 10^{27}\ \text{atom}]}$$

Answer: $\langle \phi_l \rangle = 4.38 \times 10^{13}$ neutron/cm$^2 \cdot$ s

B Relation Between Heat Flux, Volumetric Heat Generation, and Core Power

1 Single pin parameters. Three thermal parameters, introduced in Chapter 2, are related to the volumetric heat generation in the fuel: (1) the fuel pin power or rate of heat generation ($\dot{q}$); (2) the heat flux ($\vec{q}''$), normal to any heat transfer surface of interest that encloses the fuel (e.g., the heat flux may be defined at the inner and outer surfaces of the cladding or the surface of the fuel itself); (3) the power rating per unit length (linear heat-generation rate) of the pin (q').

At steady state the three quantities are related by Eqs. 2-1, 2-2, and 2-3, which can be combined and applied to the nth fuel pin to yield:

$$\dot{q}_n = \iiint_{V_{fn}} q'''(\vec{r})dV = \iint_{S_n} \vec{q}'' \cdot \vec{n}\, dS = \int_L q'\, dz \qquad (3\text{-}19)$$

where V_{fn} = volume of the energy-generating region of a fuel element; $\vec{n}$ = outward unity vector normal to the surface S_n surrounding V_{fn}; L = length of the active fuel element.

It is also useful to define the mean heat flux through the surface of our interest:

$$\{q''\}_n = \frac{1}{S_n} \iint_{S_n} \vec{q}''(\vec{r}) \cdot \vec{n}\, dS = \frac{\dot{q}_n}{S_n} \tag{3-20}$$

where $\{\ \}$ = a surface averaged quantity.

The mean linear power rating of the fuel element is obtained from:

$$q'_n = \frac{1}{L} \int_L q' dz = \frac{\dot{q}_n}{L} \tag{3-21}$$

Let us apply the relations of Eq. 3-19 to a practical case. For a cylindrical fuel rod of pellet radius R_{fo}, outer clad radius R_{co}, and length L, the total rod power is related to the volumetric heat-generation rate by:

$$\dot{q}_n = \int_{L/2}^{L/2} \int_0^{R_{fo}} \int_0^{2\pi} q'''(r,\theta,z) r d\theta dr dz \tag{3-22a}$$

The pin power can be related to the heat flux at the cladding outer surface (q''_{co}) by:

$$\dot{q}_n = \int_{-L/2}^{L/2} \int_0^{2\pi} q''_{co}(\theta,z) R_{co} d\theta dz \tag{3-22b}$$

Here we have neglected axial heat transfer through the ends of the rod and heat generation in the cladding and gap. Finally, the rod power can be related to the linear power by:

$$\dot{q}_n = \int_{-L/2}^{L/2} q'(z) dz = q'_n L \tag{3-22c}$$

The mean heat flux through the outer surface of the clad is according to Eq. 3-20:

$$\{q''_{co}\}_n = \frac{1}{2\pi R_{co} L} \int_{-L/2}^{L/2} \int_0^{2\pi} q''_{co}(\theta,z)\ R_{co} d\theta dz = \frac{\dot{q}_n}{2\pi R_{co} L} \tag{3-23}$$

It should be recognized that the linear power at any axial position is equal to the heat flux integrated over the perimeter:

$$q'(z) = \int_0^{2\pi} q''_{co}(\theta,z)\ R_{co} d\theta \tag{3-24a}$$

The linear power can also be related to the volumetric heat generation by:

$$q'(z) = \int_0^{R_{fo}} \int_0^{2\pi} q'''(r,\theta,z)\ r d\theta dr \tag{3-24b}$$

Then, for any fuel rod:

$$\dot{q}_n = L\ q'_n = L\ 2\pi R_{co} \{q''_{co}\}_n = L\ \pi R_{fo}^2 \langle q''' \rangle_n \tag{3-25}$$

where the average volumetric generation rate in the pin is:

$$\langle q''' \rangle_{\mathrm{n}} = \frac{\dot{q}_{\mathrm{n}}}{\pi R_{\mathrm{fo}}^2 L} = \frac{\dot{q}_{\mathrm{n}}}{V_{\mathrm{fn}}} \tag{3-26}$$

2 Core power and fuel pin parameters. Consider a core consisting of N fuel pins. The overall power generation in the core is then:

$$\dot{Q} = \sum_{n=1}^{N} \dot{q}_{\mathrm{n}} + \dot{Q}_{\mathrm{nonfuel}} \tag{3-27}$$

Defining γ as the fraction of power generated in the fuel:

$$\dot{Q} = \frac{1}{\gamma} \sum_{n=1}^{N} \dot{q}_{\mathrm{n}} \tag{3-28}$$

From Eqs. 3-25 and 3-28, for N fuel pins of identical dimensions:

$$\dot{Q} = \frac{1}{\gamma} \sum_{n=1}^{N} \dot{q}_{\mathrm{n}} = \frac{1}{\gamma} \sum_{n=1}^{N} L\, q'_{\mathrm{n}} = \frac{1}{\gamma} \sum_{n=1}^{N} L\, 2\pi R_{\mathrm{co}} \{q''_{\mathrm{co}}\}_{\mathrm{n}} = \frac{1}{\gamma} \sum_{n=1}^{N} L \pi R_{\mathrm{fo}}^2 \langle q''' \rangle_{\mathrm{n}} \tag{3-29}$$

We can define core-wide thermal parameters for an average pin as:

$$\frac{\dot{Q}}{N} = \frac{1}{\gamma} \langle \dot{q} \rangle = \frac{L}{\gamma} \langle q' \rangle = \frac{L}{\gamma} 2\pi R_{\mathrm{co}} \langle q''_{\mathrm{co}} \rangle = \frac{L}{\gamma} \pi R_{\mathrm{fo}}^2 \langle q''' \rangle \tag{3-30}$$

When all the energy release is assumed to occur in the fuel, Eq. 3-30 becomes identical to Eq. 2-5.

The core-average volumetric heat generation rate in the fuel is given by:

$$\langle q''' \rangle = \frac{\gamma \dot{Q}}{V_{\mathrm{fuel}}} = \frac{\gamma \dot{Q}}{N V_{\mathrm{fn}}} \tag{3-31}$$

The core-wide average fuel volumetric heat-generation rate $\langle q''' \rangle$ should not be confused with the core power density Q''', defined in Chapter 2 as:

$$Q''' = \dot{Q} / V_{\mathrm{core}} \tag{3-32}$$

which takes into account the volume of all the core constituents: fuel, moderator, and structures.

Example 3-2 Heat transfer parameters in various power reactors

PROBLEM For the set of reactor parameters given below, calculate for each reactor type:

1. Equivalent core diameter and core length
2. Average core power density Q''' (MW/m^3)
3. Core-wide average linear heat-generation rate of a fuel rod, $\langle q' \rangle$ (kW/m)

4. Core-wide average heat flux at the interface between the rod and the coolant $\langle q''_{co} \rangle$ (MW/m²)

Quantity	PWR	BWR	PHWR[a] (CANDU)	LMFBR[b]	HTGR[c]
Core power level (MWt)	3800	3579	2140	780	3000
% of power deposited in fuel rods	96	96	95	98	100
Fuel assemblies/core	241	732	12 × 380 = 4560	198	8 × 493 = 3944
Assembly lateral spacing (mm)	207 (square pitch)	152 (square pitch)	280 (square pitch)	144 (across hexagonal flats)	361 (across hexagonal flats)
Fuel rods/assembly	236	62	37	217	72
Fuel rod length (mm)	3810	3760	480	914	787
Fuel rod diameter (mm)	9.7	12.5	13.1	5.8	21.8

[a]CANDU, 12 fuel assemblies are stacked end to end at 380 locations. CANDU fuel rods are oriented horizontally.

[b]Blanket assemblies are excluded from the LMFBR calculation.

[c]HTGR, eight fuel assemblies are stacked end to end at 493 locations. Fuel rod dimensions given actually refer to coolant holes.

SOLUTION Only the *PWR* case is considered in detail here; the results for the other reactors are summarized.

1. Equivalent core diameter and length calculation

$$\text{Fuel assembly area} = (0.207 \text{ m})^2 = 0.043 \text{ m}^2$$
$$\text{Core area} = (0.043 \text{ m}^2)(241 \text{ fuel assemblies}) = 10.36 \text{ m}^2$$
$$\text{Equivalent circular diameter: } \frac{\pi D^2}{4} = 10.36 \text{ m}^2$$
$$\therefore D = 3.64 \text{ m}$$
$$\text{Core length } (L) = 3.81 \text{ m}$$
$$\text{Total core volume} = 3.81 \, \frac{\pi(3.64)^2}{4} = 39.65 \text{ m}^3$$

2. Average power density in the core from Eq. 3-32:

$$Q''' = \frac{\dot{Q}}{\pi R^2 L} = \frac{\dot{Q}}{V_{\text{core}}} = \frac{3800 \text{ MW}}{39.65 \text{ m}^3} = 95.85 \text{ MW/m}^3$$

3. Average linear heat generation rate in a fuel rod can be obtained from Eq. 3-30 as:

$$\langle q' \rangle = \frac{\gamma \dot{Q}}{NL} = \frac{0.96 \text{ (3800 MW)}}{(236 \text{ rods/assembly})(241 \text{ assemblies})(3.81 \text{ m/rod})} = 16.8 \text{ kW/m}$$

4. Average heat flux at the interface between a rod and the coolant: From Eq. 3-30 we can obtain $\langle q''_{co} \rangle$ as:

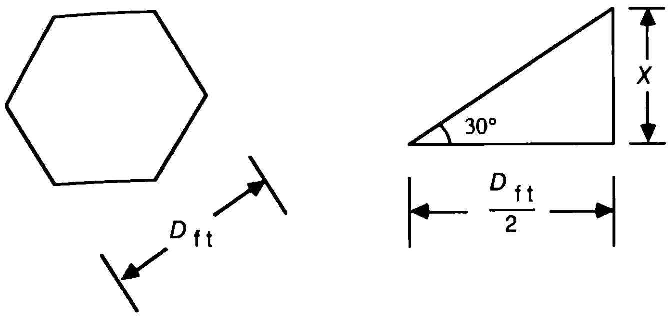

Figure 3-2 Hexagonal assembly.

$$\langle q''_{co} \rangle = \frac{\gamma \dot{Q}}{NL2\pi R_{co}} = \frac{\gamma \dot{Q}}{NL\pi D_{co}} = \frac{\langle q' \rangle}{\pi D_{co}}$$

$$= \frac{(16.8 \text{ kW/m})(10^{-3} \text{ MW/kW})}{\pi(0.0097 \text{ m})} = 0.552 \text{ MW/m}^2$$

Calculations for the other reactor types are left as an exercise for the reader. The solutions are given below.

Quantity	BWR	CANDU	LMFBR	HTGR
Equivalent core diameter (m)	4.64	6.16	2.13	8.42
Core length (m)	3.76	5.76	0.914	6.3
Core power density (MW/m^3)	56.3	12.5	239.5	8.55
Average linear heat generation of a fuel rod (kW/m)	20.1	25.1	19.9	13.4
Average heat flux at the interface of fuel rod and coolant (MW/m^2)	0.512	0.61	1.09	0.20

Note: Area for hexagon-shaped assembly (Fig. 3-2) is given by:

$$\text{Area hexagon} = \frac{12}{2}(D_{ft}/2)(D_{ft}/2)(\tan 30°) = \frac{\sqrt{3}}{2} D_{ft}^2$$

where D_{ft} = distance across hexagonal flats.

IV POWER PROFILES IN REACTOR CORES

We shall consider simple cases of reactor cores to form an appreciation of the overall power distribution in various geometries. The simplest core is one in which the fuel is homogeneously mixed with the moderator and uniformly distributed within the core volume. Consideration of such a core is provided here as a means to establish the general tendency of neutron flux behavior. In a neutronically heterogeneous power reactor the fuel material is dispersed in lumps within the moderator (see Chapter 1).

In practice, different strategies may be sought for the fissile material distribution in the core. For example, to burn the fuel uniformly in the core, uniform heat generation is desired. Therefore various enrichment zones may be introduced, with the highest enrichment located at the low neutron flux region near the core periphery.

A Homogeneous Unreflected Core

In the case of a homogeneous unreflected core, the whole core can be considered as one fuel element. Using the one energy group scheme, it is clear from Eq. 3-11 that $q'''(\vec{r})$ is proportional to $\phi_1(\vec{r})$.

Solving the appropriate one-group neutron diffusion equation, simple analytical expressions for the neutron flux, and hence the volumetric heat-generation rate, have been obtained for simple geometries. The general distribution is given by*:

$$q'''(\vec{r}) = q'''_{\max} F(\vec{r}) \tag{3-34}$$

where $q'''_{\max}$ is the heat generation at the center of the homogeneous core. Expressions for $F(\vec{r})$ are given in Table 3-3. Thus for a cylindrical core:

$$q'''(r,z) = q'''_{\max} J_o \left(2.4048 \frac{r}{R_e}\right) \cos\left(\frac{\pi z}{L_e}\right) \tag{3-35}$$

where r and z are measured from the center of the core.

The shape of q''' as a function of r is shown in Figure 3-3. It is seen that the neutron flux becomes zero at a small distance δR from the actual core boundary. The

Table 3-3 Distribution of heat generation in a homogeneous unreflected core

Geometry	Coordinate	$q'''(\vec{r})/q'''_{\max}$ or $F(\vec{r})$	$q'''_{\max}/\langle q'''\rangle$ (ignoring extrapolation lengths)
Infinite slab	x	$\cos \dfrac{\pi x}{L_e}$	$\dfrac{\pi}{2}$
Rectangular parallelepiped	x,y,z	$\cos\left(\dfrac{\pi x}{L_{xe}}\right) \cos\left(\dfrac{\pi y}{L_{ye}}\right) \cos\left(\dfrac{\pi z}{L_{ze}}\right)$	$\dfrac{\pi^3}{8}$
Sphere	r	$\dfrac{\sin\left(\dfrac{\pi r}{R_e}\right)}{\pi r/R_e}$	$\dfrac{\pi^2}{3}$
Finite cylinder	r,z	$J_o\left(2.405 \dfrac{r}{R_e}\right) \cos\left(\dfrac{\pi z}{L_e}\right)$	$2.32\left(\dfrac{\pi}{2}\right)$

$L_e = L + 2\delta L$; $R_e = R + \delta R$; L_e, R_e = extrapolated dimensions; L, R = fuel physical dimensions. Source: Rust [8].

*Note that in the homogeneous reactor $\langle q'''\rangle$ and Q''' are identical.

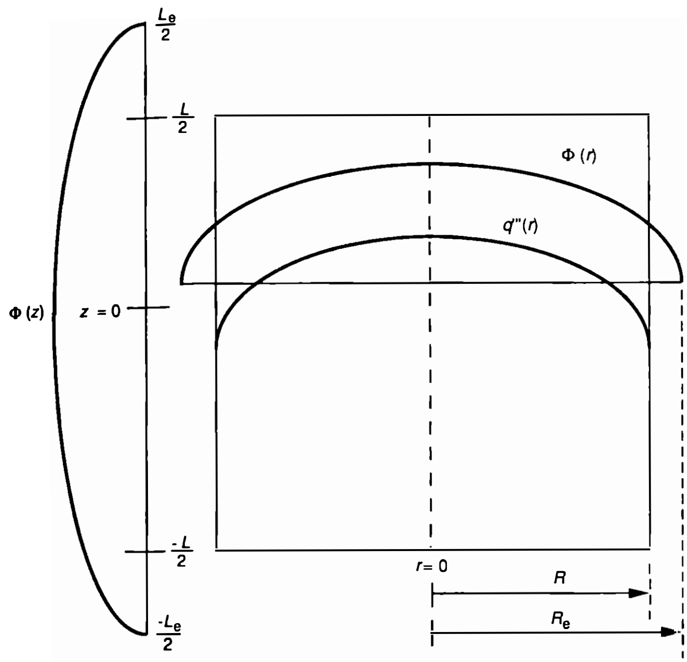

Figure 3-3 Neutron flux and heat-generation rate profiles in a homogeneous cylindrical reactor.

distances δR and δL are called the *extrapolation lengths* and are usually small relative to L and R, respectively.

The overall core heat generation rate is given by:

$$\dot{Q} = q'''_{\max} \iiint_{V_{\text{core}}} F(\vec{r})dV \tag{3-36}$$

In real reactors, the higher burnup of fuel at locations of high neutron fluxes leads to flattening of radial and axial power profiles.

B Homogeneous Core with Reflector

For a homogeneous core with a reflector it is also possible to use a one-group scheme inside the reactor. For the region near the boundary between the core and the reflector, a two-group approximation is usually required. An analytical expression for q''' in this case is more difficult. The radial shape of the thermal neutron flux is shown in Figure 3-4.

C Heterogeneous Core

In the case of a heterogeneous thermal reactor, heat is produced mainly in the fuel elements, and the thermal neutron flux is generated in the moderator.

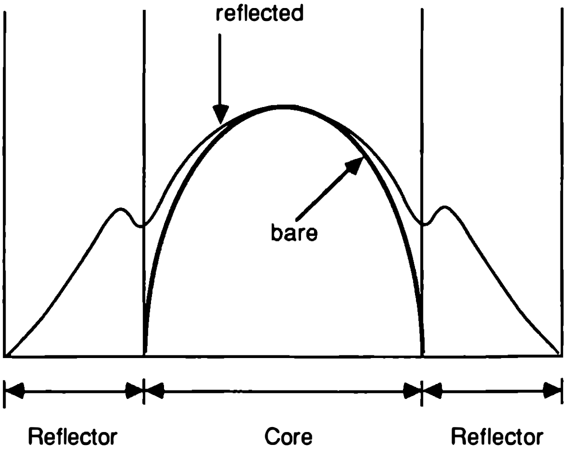

Figure 3-4 Effect of neutron reflector on the thermal neutron flux radial distribution.

In most power reactors, there are large numbers of rods. Thus with little error we can approximate the profile of the heat-generation rate in a fresh core with uniform enrichment by the previous expressions for a homogeneous core, provided that now $q'''(r)$ is understood to represent the heat-generation rate in a fuel rod that is a distance r from the center of the reactor core. However, in practice, a fresh core may have zones of variable enrichment, which negates the possibility of using the homogeneous core expressions. With fuel burnup, various amounts of plutonium and fission products are introduced. A typical power distribution in a PWR mid-burnup core is shown in Figure 3-5. It is clear that no analytic expression can easily describe the spatial distribution.

D Effect of Control Rods

The control rods depress the neutron flux radially and axially. Thus the radial power profile is also depressed near the rods (Fig. 3-6). In many reactors, some control material is uniformly mixed with the fuel in several fuel pins so that it is burned up as the fuel burns, thereby readily compensating for loss of fuel fissile content. In addition, soluble poison is routinely used for burnup cycle control in PWRs.

Example 3-3 Local pin power for a given core power

PROBLEM For a heavy-water-moderated reactor with uniform distribution of enriched UO_2 fuel in a cylindrical reactor core, calculate the power generated in a fuel rod located half-way between the centerline and the outer boundary. The important parameters for the core are as follows.

1. Core diameter (R) $=$ 8 ft (2.44 m)
2. Core height (L) $=$ 20 ft (6.10 m)
3. Fuel pellet outside diameter $=$ 0.6 in. (1.524 cm)
4. Maximum thermal-neutron flux $= 10^{13}$ neutrons/cm$^2 \cdot$ s

Center-line	A	B	C	D	E	F	G	H
A	7FG 27841 35997 0.72	9AH 10506 20929 0.94	8GB 22820 31824 0.81	9FG 8021 18967 1.03	9FF 12007 22721 1.03	8AH 20927 29456 0.82	8BG 23452 32601 0.93	10AH 0 10505 1.12
B	9HA 10506 20928 0.94	9BG 12827 23454 0.97	9CH 9884 20535 0.98	8DH 19247 28621 0.86	9DH 8086 19249 1.08	8EG 21118 30125 0.88	10BG 0 12826 1.36	10BH 0 10847 1.16
C	8GB 22820 31820 0.81	9HC 9884 20530 0.98	9GB 12827 22827 0.91	8FG 18968 27840 0.81	9EG 10514 21117 1.02	8CH 20532 29348 0.86	8DG 23409 32506 0.92	10CH 0 9883 1.03
D	9FG 8021 18956 1.02	8HD 19236 28594 0.86	8GF 19382 28183 0.80	8FF 22722 31183 0.79	9BH 10847 21660 1.07	9DG 12829 23410 1.07	10DG 0 12830 1.37*	10DH 0 8086 0.83
E	9FF 12007 22714 1.03	9HD 8086 19238 1.08	9GE 10514 21098 1.02	9HB 10847 21656 1.07	9GF 8021 19382 1.15	8BH 21656 31071 0.95	10EG 0 10516 1.10	
F	8HA 20926 29454 0.82	8GE 21099 30105 0.88	8HC 20527 29340 0.86	9GD 12829 23408 1.07	8HB 21651 31067 0.95	10FF 0 12010 1.25	10FG 0 8022 0.82	
G	8BG 23452 32601 0.93	10GB 0 12825 1.36	8GD 23407 32504 0.92	10GD 0 12829 1.37*	10GE 0 10516 1.10	10GF 0 8022 0.82		
H	10HA 0 10505 1.12	10HB 0 10847 1.16	10HC 0 9883 1.03	10HD 0 8086 0.83				

*Maximum Relative Power

Fuel Lots 7, 8, 9, 10 Initially 3.20 w/o U-235

Cycle Average Burnup = 10,081 MWd/MT
Cycle Thermal Energy = 896.8 GWd

Key

1AA	Assembly Number
0	BOC Burnup, MWd/MT
17302	EOC Burnup, MWd/MT
1.04	BOC Relative Power (Assembly/Average)

Figure 3-5 Typical PWR assembly power and burnup distribution, assuming fresh fuel is introduced at the outer core locations. *(From Benedict et al.* [1].*)*

Assume that the extrapolated dimensions can be approximated by the physical dimensions and that enrichment and average moderator temperature are such that:

$$q'''(\text{Btu/hr ft}^3) = 6.99 \times 10^{-7}\ \phi\ (\text{neutrons/cm}^2 \cdot \text{s}) \text{ at every position}$$
$$q'''(\text{W/m}^3) = 7.27 \times 10^{-6}\ \phi\ (\text{neutrons/cm}^2 \cdot \text{s})$$

SOLUTION This reactor can be approximated as a homogeneous unreflected core in the form of a finite cylinder. From Table 3-3:

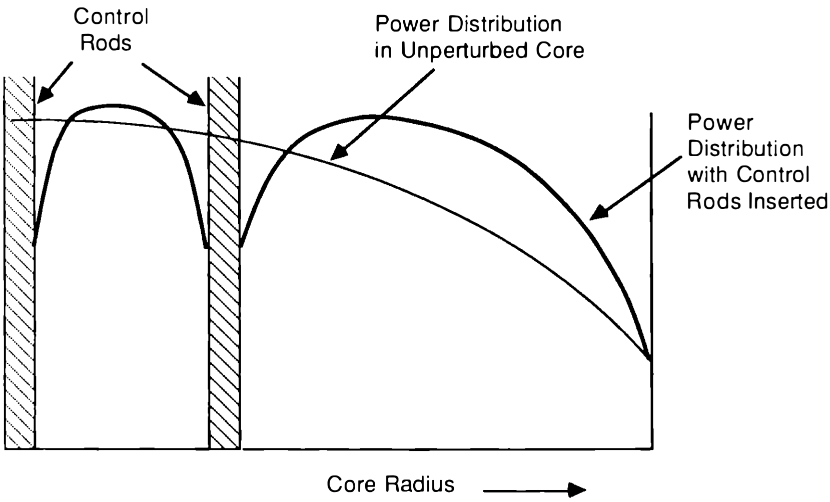

Figure 3-6 Radial power profile in a cylindrical reactor with inserted control rods.

$$q'''(r,z) = q'''_{\max} J_o \left(2.4048 \frac{r}{R_e}\right) \cos\left(\frac{\pi z}{L_e}\right) \tag{3-37}$$

The value of $q'''_{\max}$ is readily established as

$$q'''_{\max} = 6.99 \times 10^{-7} \phi_{\max} = 6.99 \times 10^{6} \text{ Btu/hr ft}^3 \ (72.68 \text{ MW/m}^3)$$

for $r/R_e = 0.5$.

$$q'''(r,z) = q'''_{\max} J_o(1.202) \cos\left(\frac{\pi z}{L_e}\right)$$

For the heat generated from a single fuel rod, Eq. 3-22a yields:

$$\dot{q}_n = \int_{-L/2}^{L/2} q'''_n (r,z) A \, dz$$

where $A = \dfrac{\pi D_{fo}^2}{4}$. Taking Le as L

$$\dot{q}_n = \int_{-L/2}^{L/2} q'''_{\max} A J_o(1.202) \cos\left(\frac{\pi z}{L}\right) dz$$

$$= q'''_{\max} A J_o(1.202) \frac{L}{\pi} \sin \frac{\pi z}{L}\Bigg|_{-L/2}^{L/2}$$

$$= q'''_{\max} A J_o(1.202) \frac{2L}{\pi}$$

The value of $J_o(1.202) = 0.6719$ [10]. Hence

$$\dot{q}_n = (6.99 \times 10^6) \frac{\pi\left(\frac{0.6}{12}\right)^2}{4} (0.6719) \frac{2}{\pi} (20)$$

Answer: $\dot{q}_n = 1.17 \times 10^5$ Btu/hr (34.5 kW)

V HEAT GENERATION WITHIN A FUEL PIN

A Fuel Pins of Thermal Reactors

Consider a cylindrical fuel pin inside the reactor. The heat-generation rate at a particular point of the rod, $q'''(r, \theta, z)$ depends on the position of the rod in the reactor and the concentration of the various fissionable materials at this point.

In thermal reactors with uniform fuel enrichment, the profile of the heat-generation rate follows approximately the thermal neutron flux. For single-phase cooled reactors, in many cases the axial profile of q''' can be approximated by a cosine function, i.e.,

$$q'''(z) \sim \cos\left(\frac{\pi z}{L_e}\right) \tag{3-38}$$

As mentioned before, for fresh fuel this formula gives adequate results for illustrative purposes.

Radially within the fuel element, q''' is expected to be reduced at the center because of thermal neutron flux depression. This depression is often neglected for small-diameter low-absorbing fuel rods but should not be ignored for thick, highly absorbing rods.

B Fuel Pins of Fast Reactors

The dependence of q''' on z is similar to that of thermal reactors. However, the shape of q''' as a function of r within a rod is different because in fast reactors energetic neutrons, even fission neutrons, contribute to the fission reaction directly without being slowed. The radial profile of q''' tends to be flatter than in thermal and epithermal systems. Fortunately, in the real cases the diameter of the fuel rod is relatively small and the mean free path of the neutrons relatively large. Therefore the assumption that q''' is independent of r and θ is a good approximation.

VI HEAT GENERATION WITHIN THE MODERATOR

The energy deposition in the moderator mainly comes from (1) neutron slowing by scattering due to collisions with the nuclei of the moderator material; and (2) γ-ray absorption. The dominant mechanism of heat production is neutron slowing due to elastic scattering. Neutrons also lose energy through inelastic collisions as a result of excitation of the target nuclei. However, for light nuclei, moderation by inelastic scattering is less important than by elastic scattering. With heavy nuclei, such as those involved in structures, the inelastic scattering is the principal mechanism for neutron moderation.

With elastic scattering the energy lost from the neutrons appears as kinetic energy of the struck nucleus. With inelastic scattering the energy lost by the neutrons appears as γ-rays.

Let $\Sigma_{s,e\ell}(\vec{r},E)$ be the macroscopic elastic scattering cross section of neutrons of energy E at position $\vec{r}$, and let $\phi(\vec{r},E)$ be the neutron flux at this position and energy. Hence the elastic scattering reaction rate $RR_{s,e\ell}(\vec{r},E)dE$ within the energy interval E to $E + dE$ is given by:

$$RR_{s,e\ell}(\vec{r},E)dE = \Sigma_{s,e\ell}(\vec{r},E)\ \phi(\vec{r},E)dE \tag{3-39}$$

If $\Delta E(E)$ is the mean energy loss per collision at neutron energy E, the heat generation due to $\phi(r,\vec{E})$ is:

$$q'''_{e\ell}(\vec{r},E)dE = \Delta E(E)\Sigma_{s,e\ell}(\vec{r},E)\ \phi(\vec{r},E)dE \tag{3-40}$$

Define the mean logarithmic energy decrement per collision (ξ) by

$$\xi \equiv \overline{\ell n\ \frac{E}{E'}} \tag{3-41}$$

where E' = neutron energy after one collision (i.e., $\Delta E = E - E'$). By expanding the logarithm in terms of ΔE, we get:

$$\ell n\ \frac{E}{E'} \simeq \frac{\Delta E}{E} + \frac{1}{2}\left(\frac{\Delta E}{E}\right)^2 + \frac{1}{3}\left(\frac{\Delta E}{E}\right)^3 + \ldots\ldots. \tag{3-42}$$

Assuming $\frac{\Delta E}{E}$ is sufficiently small, we can approximate:

$$\overline{\ell n\ \frac{E}{E'}} \simeq \overline{\frac{\Delta E}{E}} \tag{3-43}$$

Then Eq. 3-40 becomes:

$$q'''_{e\ell}(\vec{r},E)dE = \xi E\ \Sigma_{s,e\ell}(\vec{r},E)\ \phi(\vec{r},E)dE \tag{3-44}$$

Usually the moderator can be considered a homogeneous material, so that Σ_s is independent of $\vec{r}$ within the moderator. Then:

$$q'''_{e\ell}(\vec{r},E)dE = \xi E\ \Sigma_{s,e\ell}(E)\ \phi(\vec{r},E)dE \tag{3-45}$$

The heat-generation rate from neutron elastic scatterings at all energies is:

$$q'''_{e\ell}(\vec{r}) = \int_{E_c}^{\infty} \xi E\ \Sigma_{s,e\ell}(E)\ \phi(\vec{r},E)dE \tag{3-46}$$

where E_c = an energy level under which the energy loss by neutrons is negligible (e.g., E_c = 0.1 eV).

In order to assess the conditions for the validity of Eq. 3-43, consider isotropic elastic scattering in the center of mass system. Let j be a particular isotope. Then [3]:

$$\overline{\Delta E} = \frac{E - \alpha_j E}{2} \tag{3-47}$$

where

$$\alpha_j = \left(\frac{A^j - 1}{A^j + 1}\right)^2 \tag{3-48}$$

and A^j is the atomic mass number. Then

$$\frac{\overline{\Delta E}}{E} = \frac{1 - \alpha_j}{2} \tag{3-49}$$

We see that for all possible values of A^j, $0 \leq \alpha_j < 1$, and $\frac{\overline{\Delta E}}{E}$ ranges from 0 to 0.5. Thus because elastic scattering is close to being isotropic, $\frac{\overline{\Delta E}}{E}$ is small; therefore it can be concluded that Eq. 3-43 is a good approximation to Eq. 3-42.

VII HEAT GENERATION IN THE STRUCTURE

The main sources of heat generation in the structure are (1) γ-ray absorption; (2) elastic scattering of neutrons; and (3) inelastic scattering of neutrons.

A γ-Ray Absorption

The photon "population" at the particular point $\vec{r}$ of the structure is mainly due to: (1) γ-rays born somewhere in the fuel, which arrive without scattering at position $\vec{r}$ within the structure; (2) γ-rays born within the structural materials, which arrive unscattered at $\vec{r}$; and (3) scattered photons (Compton effect).

Consider the quantity $N_\gamma(\vec{r},E)dE$, which is the photon density at a particular position r within the structure having energy between E and $E + dE$. The energy flux is defined as:

$$I_\gamma(\vec{r},E)dE \equiv EN_\gamma(\vec{r},E)dE \tag{3-50}$$

The absorption rate is described by application of the linear energy absorption coefficient $\mu_a(E)$ as follows:

$$q'''_\gamma(\vec{r},E)dE = \mu_a(E)\, I_\gamma(\vec{r},E)dE \tag{3-51}$$

where $q'''_\gamma(\vec{r},E)$ = the absorbed energy density per unit time from the γ-ray energy flux within the interval E to $E + dE$; $\mu_a(E)$ = a function of the material, as can be found in Table 3-4. The total heat-generation rate then is:

Table 3-4 Linear γ-ray attenuation and absorption coefficients

γ-Ray energy (MeV)	Coefficient (m^{-1}) Water	Iron	Lead	Concrete
0.5				
μ	9.66	65.1	164	20.4
μ_a	3.30	23.1	92.4	7.0
1.0				
μ	7.06	46.8	77.6	14.9
μ_a	3.11	20.5	37.5	6.5
1.5				
μ	5.74	38.1	58.1	12.1
μ_a	2.85	19.0	28.5	6.0
2.0				
μ	4.93	33.3	51.8	10.5
μ_a	2.64	18.2	27.3	5.6
3.0				
μ	3.96	28.4	47.7	8.53
μ_a	2.33	17.6	28.4	5.08
5.0				
μ	3.01	24.6	48.3	6.74
μ_a	1.98	17.8	32.8	4.56
10.0				
μ	2.19	23.1	55.4	5.38
μ_a	1.65	19.7	41.9	4.16

Source: Templin [11]

$$q_\gamma'''(\vec{r}) = \int_0^{E_\infty} q_\gamma'''(\vec{r},E)dE = \int_0^{E_\infty} \mu_a(E)I_\gamma(\vec{r},E)dE \tag{3-52}$$

where E_∞ is selected at a sufficiently high value.

$I_\gamma(\vec{r},E)$ can be found by solving the appropriate transport equation. However, practical calculation of γ-ray attenuation is often greatly simplified by the use of so-called buildup factors and the uncollided γ-ray flux. With this procedure the simplified transport equation is first solved neglecting the scattering process to yield the uncollided flux $I_\gamma^*(\vec{r},E)$. For example, for a plane geometry the energy of the uncollided γ-ray flux is obtained from:

$$I_\gamma^* = I_\gamma^\circ e^{-\mu(E)x} \tag{3-53}$$

where $\mu(E)$ = linear attenuation coefficient for photons at energy E due to absorption and scattering (Table 3-4); I_γ° = unattenuated γ-ray flux. If $I_\gamma(\vec{r},E')$ is the real energy flux at a point $\vec{r}$ resulting from I_γ°, the buildup factor (B) is defined as:

$$B(\vec{r},\mu,E) = \frac{\int_0^{E_\infty} \mu_a(E')I_\gamma(\vec{r},E')dE'}{\mu_a(E)I_\gamma^*(\vec{r},E)} \tag{3-54}$$

Note that by definition $B(\vec{r},E)$ is greater than unity and depends primarily on the boundary conditions, and the energy level of the uncollided photons (through the linear attenuation coefficient and the scattered photon source distribution). The values for the cases of interest are tabulated elsewhere [4, 6]. Utilizing this definition of B, Eq. 3-51 becomes:

$$q'''_{\gamma}(\vec{r},E)\,dE = B\mu_a(E)\,I^*_{\gamma}(\vec{r},E)dE \tag{3-55}$$

The following are mathematical expressions for simplified cases for the evaluation of Eq. 3-55 when $I^*_{\gamma}(\vec{r},E)$ can be analytically evaluated.

1. Point isotropic source emitting S photons of energy E_o per second. In this case:

$$q'''_{\gamma}(r) = S\,B\,\mu_a(E_o)\,E_o\,\frac{e^{-\mu r}}{4\pi r^2} \tag{3-56}$$

   where r is the distance from the point source.
2. Infinite plane source emitting S photons with energy E_o per unit time per unit surface in the positive direction of the x-axis.

$$q'''_{\gamma}(x) = S\,B\,\mu_a(E_o)\,E_o e^{-\mu x} \tag{3-57}$$

3. Plane isotropic source emitting S photons of energy E_o per unit source area, per second in all directions.

$$q'''_{\gamma}(x) = S\,B\mu_a(E_o)\,\frac{E_o}{2}\int_{\mu x}^{\infty}\frac{e^{-t}}{t}\,dt \tag{3-58}$$

B Neutron Slowing

In the structure, the neutrons slow by (1) elastic scattering and (2) inelastic scattering. For elastic scattering we can use the approximation of Eq. 3-46:

$$q'''_{e\ell}(\vec{r}) = \int_{E_c}^{\infty}\xi\,E\Sigma_{s,e\ell}(E)\,\phi(\vec{r},E)dE$$

For inelastic scattering, the approach is more complicated owing to the generation of γ-rays. Taking into consideration that (1) inelastic scattering heating is not large compared with γ-heating, and (2) the γ-rays due to inelastic scattering are of moderate energies and therefore are absorbed in relatively short distances [3], the heat can be assumed to be released at the point of the inelastic scattering event. Then $q'''_{i\ell}(\vec{r})$ is given by an expression similar to that of Eq. 3-46:

$$q'''_{i\ell}(\vec{r}) = \int_{E_c}^{\infty} Ef(E)\Sigma_{s,i\ell}(E)\phi(\vec{r},E)dE \tag{3-59}$$

where $f(E)$ = the fraction of the neutron energy E lost in the collision. The parameter f is a function of E and the material composition.

Finally, the heat generation within the structure is:

$$q'''(\vec{r}) = q'''_{\gamma}(\vec{r}) + q'''_{e\ell}(\vec{r}) + q'''_{i\ell}(\vec{r}) \tag{3-60}$$

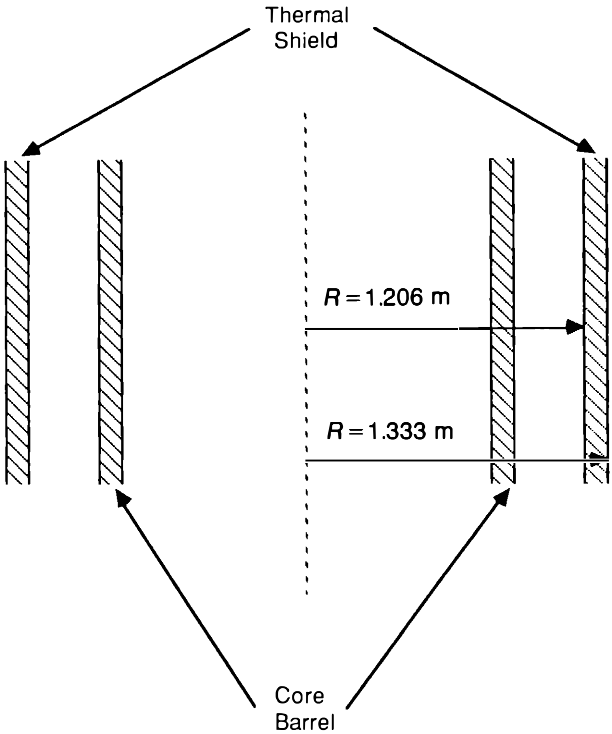

Figure 3-7 Thermal shield.

Example 3-4 Power deposition in a thermal shield

PROBLEM In a PWR the core is surrounded by a thermal shield (Fig. 3-7) to protect the pressure vessel from γ-ray heating and neutron-induced radiation damage. For an iron thermal shield with the radiation values given below, calculate the volumetric heat generation rate in the shield at its outermost position.

Assume that the core of the reactor is equivalent to an infinite plane source and that the shield can be treated as a slab owing to the small thickness-to-radius ratio.

Note that at energies well above the inelastic scattering threshold the total cross section is due to scattering. For steel, the inelastic scattering rate may be assumed to be equal to the elastic scattering rate:

$$\Sigma_{s,e\ell} \approx \Sigma_{s,i\ell} \tag{3-61}$$

The total neutron cross section can be approximated by the sum of both scatterings:

$$\Sigma_T \approx \Sigma_{s,e\ell} + \Sigma_{s,i\ell} \tag{3-62}$$

Use the following values for the radiation flux parameters.

* γ Radiation	* Neutron Radiation
$E_o = 2$ MeV	$\phi_{fast} = 10^{14}$ neutrons/cm^2 · s
$S = 10^{14}\ \gamma$/cm^2 s	Effective neutron energy = 0.6 MeV
$B_{Fe} = 4.212$	Neutron total cross section = σ_T (0.1 < E < 15) = 3 barns
	f(2 MeV) = 0.1

SOLUTION Equation 3-57 may be used to find $q'''_\gamma(x)$.

$$q'''_\gamma(x) = SB\mu_a(E_o)E_o e^{-\mu x}$$

where x is measured from the inner wall:

$$x = 1.333 \text{ m} - 1.206 \text{ m} = 0.127 \text{ m} = 12.7 \text{ cm}$$

From Table 3-4, μ_a for iron at $E_o = 2.0$ MeV is 0.182 cm^{-1} and μ for iron at $E_o = 2.0$ MeV is 0.333 cm^{-1}.

$$\therefore q'''_\gamma = (10^{14})(4.212)(0.182)(2)e^{-0.333(12.7)} = 2.23 \times 10^{12} \text{ MeV/cm}^3 \cdot \text{s}$$

To solve for the elastic and inelastic scattering of the neutrons, Eqs. 3-46 and 3-59 must be solved:

$$q'''_{e\ell}(\vec{r}) = \int_{E_c}^{\infty} \xi\, E\Sigma_{s,e\ell}(E)\, \phi(r,E)dE \tag{3-46}$$

$$q'''_{i\ell}(\vec{r}) = \int_{E_c}^{\infty} Ef(E)\Sigma_{s,i\ell}(E)\phi(\vec{r},E)dE \tag{3-59}$$

Using a one-group approximation for the integral and defining ϕ_{fast} as the fast neutron flux, the above equations reduce to:

$$q'''_{e\ell}(r) = \bar{\xi}\bar{E}\,\Sigma_{s,e\ell}\phi_{fast} \tag{3-63}$$

$$q'''_{i\ell}(r) = f(\bar{E})\bar{E}\,\Sigma_{s,i\ell}\phi_{fast} \tag{3-64}$$

where $\bar{E}$ is the effective energy of the flux.

Now, from Eqs. 3-42 and 3-49:

$$\xi = \overline{\ln \frac{E}{E'}} = \frac{\overline{\Delta E}}{E} = \frac{1}{2}(1 - \alpha)$$

where $\alpha = \left(\dfrac{A - 1}{A + 1}\right)^2$ (Eq. 3-48).

For iron, $A = 55.85$ so that $\alpha = 0.9309$ and:

$$\xi = \frac{1}{2}\left(1 - \left(\frac{54.85}{56.85}\right)^2\right) = 0.0346$$

The total removal cross section can be obtained from:

$$\Sigma_T = \frac{\rho_{Fe} A_v}{M_{Fe}} \sigma_T = \frac{7.87 \text{ g/cm}^3 (0.6022 \times 10^{24} \text{ atom/mole})}{55.85 \text{ g/mole}} (3 \times 10^{-24} \text{ cm}^2)$$
$$= 0.254 \text{ cm}^{-1}$$

Therefore, with the assumption that $\Sigma_{s,el} = \Sigma_{s,i\ell}$, each would equal one-half of Σ_T:

$$\Sigma_{s,i\ell} = \Sigma_{s,e\ell} \simeq 0.5\, \Sigma_T = 0.127 \text{ cm}^{-1}$$

Now the heat-generation rate due to neutron scattering from this monoenergetic neutron flux can be calculated:

$$\begin{aligned} q'''_{e\ell} &= \bar{\xi}\, \bar{E}\, \Sigma_{s,e\ell}\, \phi_{fast} \\ &= (0.0346)(0.6 \text{ MeV})(0.127 \text{ 1/cm})(10^{14} \text{ neutron/cm}^2 \cdot \text{s}) \\ &= 0.26 \times 10^{12} \text{ MeV/cm}^3 \cdot \text{s} \end{aligned} \tag{3-63}$$

$$\begin{aligned} q'''_{i\ell} &= f(\bar{E})\bar{E}\, \Sigma_{s,i\ell}\, \phi_{fast} \\ &= (0.1)(0.6 \text{ MeV})(0.127 \text{ 1/cm})(10^{14} \text{ neutron/cm}^2 \cdot \text{s}) \\ &= 0.76 \times 10^{12} \text{ MeV/cm}^3 \cdot \text{s} \end{aligned} \tag{3-64}$$

$$\begin{aligned} \therefore q''' &= q'''_{\gamma} + q'''_{e\ell} + q'''_{i\ell} \\ &= 2.23 \times 10^{12} + 0.26 \times 10^{12} + 0.76 \times 10^{12} \\ &= 3.25 \times 10^{12} \text{ MeV/cm}^3 \cdot \text{s} = 0.52 \text{ W/cm}^3 \end{aligned}$$

Note that the heat deposition due to γ-rays is the principal source of heat generation in the structure. In fact, because of the high buildup factor (B) for iron, the calculated heat generation due to γ-rays may exceed the incident flux of photons. A more refined transport calculation in which the photon energy and scattering properties are accounted for in detail is needed if exact prediction of the heat-generation level is desired.

VIII SHUTDOWN HEAT GENERATION

It is important to evaluate the heat generated in a reactor after shutdown for determining cooling requirements under normal conditions and accident consequences following abnormal events. Reactor shutdown heat generation is the sum of heat produced from the following: (1) fissions from delayed neutron or photoneutron emissions; and (2) decay of fission products, fertile materials, and other activation products from neutron capture. These two sources initially contribute equal amounts to the shutdown heat generation. However, within minutes from shutdown fissions from delayed neutron emission are reduced to a negligible amount.

A Fission Heat After Shutdown

The heat generated from fissions by delayed neutrons is obtained by solving the neutron kinetic equations after a large negative insertion of reactivity. Assuming a single group of delayed neutrons, the time-dependent neutron flux can be given by [5]:

$$\phi(t) = \phi_o \left[\frac{\beta}{\beta - \rho} e^{-\gamma_1 t} - \frac{\rho}{\beta - \rho} e^{-\frac{(\beta - \rho)t}{\ell}} \right] \tag{3-65}$$

where ϕ_o = steady-state neutron flux prior to shutdown; β = total delayed neutron fraction; ρ = step reactivity change; ℓ = prompt neutron lifetime; γ_1 = decay constant for longest-lived delayed neutron precursor; t = time after initiation of the transient.

Substituting typical values for a ^{235}U-fueled, water-moderated reactor of $\gamma_1 = 0.0124\ \text{s}^{-1}$, $\beta = 0.006$, and $\ell = 10^{-4}$ s into Eq. 3-65 for a reactivity insertion of $\rho = -0.09$, the fractional power, which is proportional to the flux, is given by:

$$\frac{\dot{Q}}{\dot{Q}_o} = 0.0625\ e^{-0.0124t} + 0.9375\ e^{-960t} \tag{3-66}$$

where t is in seconds.

The second term in Eq. 3-66 becomes negligible in less than 0.01 second. Consequently, the reactor power decreases exponentially over a period of approximately 80 seconds, which is about the half-life of the longest-lived delayed neutron precursor.

B Heat from Fission Product Decay

The major source of shutdown heat generation is fission product decay. Simple, empirical formulas for the rate of energy release due to β and γ emissions from decaying fission products are given by [3]:

$$\begin{aligned} \beta \text{ energy release rate} &= 1.40\ t'^{-1.2}\ \text{MeV/fission} \cdot \text{s} \\ \gamma \text{ energy release rate} &= 1.26\ t'^{-1.2}\ \text{MeV/fission} \cdot \text{s} \end{aligned} \tag{3-67}$$

where t' = time after the occurrence of fission in seconds.

The equations above are accurate within a factor of 2 for 10 s < t' < 100 days. Integrating the above equations over the reactor operation time yields the rate of decay energy released from fission products after a reactor has shut down.

Assuming 200 MeV are released for each fission, 3.1×10^{10} fissions per second would be needed to produce 1 watt of operating power. Thus a fission rate of $3.1 \times 10^{10}\ q_o'''$ fissions/cm^3 · s is needed to produce q_o''' W/cm^3. The decay heat at a time τ seconds after reactor startup due to fissions occurring during the time interval between τ' and $\tau' + d\tau'$ is given by the following (see Figure 3-8 for time relations).

$$dP_\beta = 1.40(\tau - \tau')^{-1.2}\ (3.1 \times 10^{10}) q_o''' d\tau'\ \text{MeV/cm}^3 \cdot \text{s} \tag{3-68a}$$

$$dP_\gamma = 1.26(\tau - \tau')^{-1.2}\ (3.1 \times 10^{10}) q_o''' d\tau'\ \text{MeV/cm}^3 \cdot \text{s} \tag{3-68b}$$

For a reactor operating at a constant power level over the period τ_s, we integrate Eqs. 3-68a and 3-68b to get the decay heat from all fissions:

$$P_\beta = 2.18 \times 10^{11} q_o'''[(\tau - \tau_s)^{-0.2} - \tau^{-0.2}]\ \text{MeV/cm}^3 \cdot \text{s} \tag{3-69a}$$

$$P_\gamma = 1.95 \times 10^{11} q_o'''[(\tau - \tau_s)^{-0.2} - \tau^{-0.2}]\ \text{MeV/cm}^3 \cdot \text{s} \tag{3-69b}$$

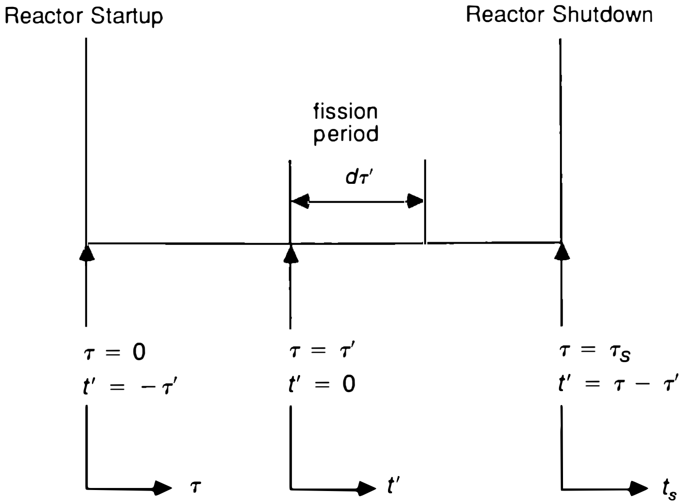

Figure 3-8 Time intervals. τ = time after reactor startup; $t' = \tau - \tau'$ = time after fission; $t_s = \tau - \tau_s$ = time after shutdown.

The decay heat level may be expressed as a fraction of the constant operating power level (P_o)—which is associated with the steady-state volumetric heat-generation rate (q_o''')—by multiplying P_γ and P_β in Eqs. 3-69a and 3-69b by 1.602×10^{-13} to convert the units to W/cm^3—and rearranging to obtain:

$$\frac{P_\beta}{P_o} = 0.035\ [(\tau - \tau_s)^{-0.2} - \tau^{-0.2}] \tag{3-70a}$$

$$\frac{P_\gamma}{P_o} = 0.031\ [(\tau - \tau_s)^{-0.2} - \tau^{-0.2}] \tag{3-70b}$$

The total fission power decay heat rate (P) is then given by:

$$\frac{P}{P_o} = 0.066[(\tau - \tau_s)^{-0.2} - \tau^{-0.2}] \tag{3-70c}$$

This equation may also be written as

$$\frac{P}{P_o} = 0.066[t_s^{-0.2} - (t_s + \tau_s)^{-0.2}]$$

Although all the energy from the β-particles is deposited in the fuel material, depending on the reactor configuration, only a fraction of the γ energy is deposited in the fuel material. The rest is deposited within the structural materials of the core and the surrounding supporting structures.

The decay heat rate predicted by Eq. 3-70c is plotted in Figure 3-9 as a function of time after reactor shutdown for various times of reactor operation. Note that for

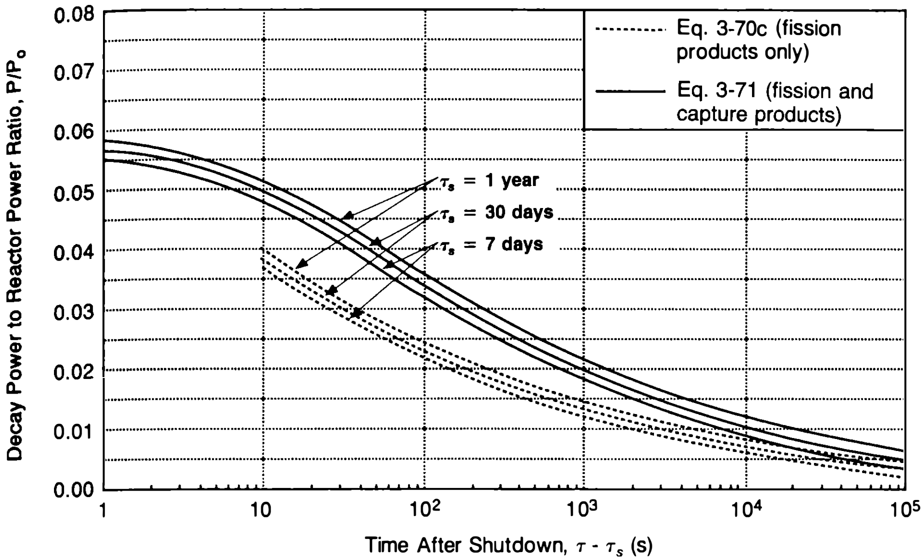

Figure 3-9 Decay heat rate from empirical relations as a function of shutdown time.

operating times (τ_s) of a few days or longer, the initial decay power is independent of reactor operating time. However, the reactor operating time is relatively important for determining the long-term decay heat.

For comparison, another equation that was later experimentally obtained from a 1-in. diameter uranium rod is also plotted in Figure 3-9. The resulting equation is given by [3]:

$$\frac{P}{P_o} = 0.1[(\tau - \tau_s + 10)^{-0.2} - (\tau + 10)^{-0.2} + 0.87\,(\tau + 2 \times 10^7)^{-0.2} - 0.87\,(\tau - \tau_s + 2 \times 10^7)^{-0.2}] \tag{3-71}$$

This equation, as may be observed in Figure 3-9, predict higher decay powers due to, among other factors, the inclusion of the decay heat of the actinides ^{239}U and ^{289}Np along with decay of ^{235}U fission products. The effect of neutron capture in fission products is to increase the decay heat on the order of a few percent, depending on the level of burnup and the operating time.

C ANS Standard Decay Power

In 1961 the data from several experiments were combined to provide a more accurate method for predicting fission product decay heat power [9]. The results (Figure 3-10) were adopted in 1971 by the American Nuclear Society (ANS) as the basis for a draft standard (ANS-5.1/N18.6) for reactor shutdown cooling requirements. The curve, which spans a time range from 1 second to 10^9 seconds, refers to reactors initially

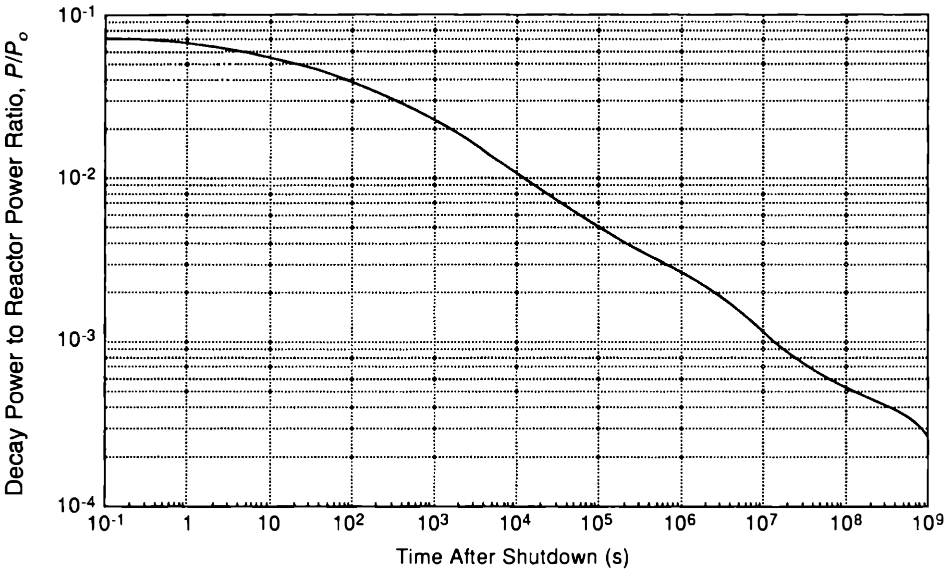

Figure 3-10 Fission-product decay heat power as a function of time after shutdown. *(From ANS-5.1/N18.6.)*

fueled with uranium and operated at a constant power (P_o) for an infinite* period before being instantaneously shut down. The value of P/P_o for reactors operated for a finite period (τ_s) may be obtained from Figure 3-10 by subtracting the value of P/P_o at the time $\tau_s + t_s$ from the value of P/P_o at the time t_s, where t_s is the cooling time after shutdown.

Some important observations may be derived from Figure 3-10. Consider the values of P/P_o for reactor operating times of 20 days, 200 days, and infinity and for various cooling times. These values are given in Table 3-5. For all three reactor operating times, the fractional decay power at 1 hour is approximately 1.3%. For cooling times less than 1 day the ratio P/P_o is independent of τ_s for $\tau_s > 20$ days. This situation is due to the rapid decay of fission products of short half-lives, which reach their saturation values during short periods of operation. The decay heat at longer cooling times is due to fission products with longer half-lives. Because the amount of these products present at shutdown is dependent on the reactor operation time (τ_s) the decay heat after cooling times longer than 1 day is also dependent on the operating time. The uncertainty associated with the 1971 proposed ANS standard was given as

$$
\begin{array}{rl}
t_s < 10^3 \text{ seconds} & +20\%,\ -40\% \\
10^3 < t_s < 10^7 \text{ seconds} & +10\%,\ -20\% \\
t_s > 10^7 \text{ seconds} & +25\%,\ -50\%
\end{array}
$$

*An infinite period is considered to occur when all fission products have reached saturation levels.

Table 3-5 Decay heat power after shutdown

	Fraction of thermal operating power (P/P_0) at various times after shutdown (cooling period)		
Operating time	1 Hour	1 Day	100 Days
20 Days	0.013	2.5×10^{-3}	3×10^{-4}
200 Days	0.013	4.1×10^{-3}	5×10^{-4}
Infinite	0.013	5.1×10^{-3}	12×10^{-4}

Investigation of the inaccuracies in the 1971 ANS proposed standard due to the assumptions that (1) decay heats from different fission products are equal and (2) neutron capture effects are negligible led to a new standard that was developed in 1979 and reaffirmed in 1985. The revised ANS standard [8], which consists of equations based on summation calculations, explicitly accounts for decay heat from ^{235}U, ^{238}U, and ^{239}Pu fission products. Neutron capture in fission products is included through a correction factor multiplier. The new standard is also capable of accounting for changes in fissile nuclides with fuel life. Accuracy within the first 10^4 seconds after shutdown was emphasized in the new standard's development for accident consequence evaluation.

The old ANS standard is compared with the data on decay heat associated with one fission from ^{238}U, ^{235}U, and ^{239}Pu atoms in Figure 3-11. As shown, the ^{238}U data are higher for the first 20 seconds, whereas the ^{239}Pu and ^{235}U data are lower. For decay times less than 10^3 seconds, the new standard predicts decay powers lower than those of the old standard, whereas for decay times of more than 10^4 seconds the revised standard may give higher decay powers. This increase is due to the inclusion of neutron capture effects.

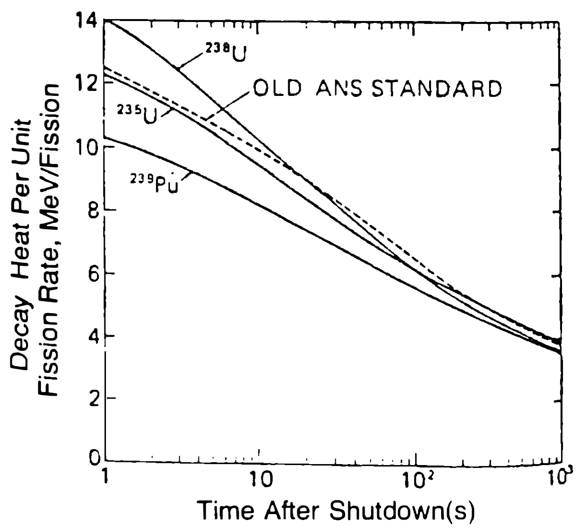

Figure 3-11 Comparison of the ANS 1971 revised standard with the ANS 1978 standard for the essentially infinite irradiation time of 10^{13} seconds [8]. Decay power can be obtained by multiplying the above shown values by the fission rate during operation.

Table 3-6 Comparison of the 1971 and 1979 ANS decay heat standards
A. Reactor operating history

Interval	Duration (days)	Power	Fractional power during interval			
			^{235}U	^{239}Pu	^{238}U	^{241}Pu
1	300	Full	0.8	0.13	0.06	0.01
2	60	Zero				
3	300	Full	0.6	0.29	0.07	0.04
4	60	Zero				
5	300	Full	0.40	0.42	0.08	0.10

B. Decay heat power based on above history

Time after shutdown (s)	1979 Revised standard[a]			Decay power ratio (Revised/old)	
	Capture multiplier	Uncertainty 1σ (%)	$\frac{P}{P_o}$	Nominal[b]	Upper bound[c]
1	1.005	5.1	0.0577	0.925	0.860
10	1.005	3.5	0.0448	0.890	0.808
10^2	1.005	3.1	0.0295	0.909	0.802
10^3	1.006	3.1	0.0178	1.02	0.879

Source: Schrock [8].
[a]Based on $E_f = 200$ MeV/fission.
[b]1979 Revised standard nominal from column 4. Old standard nominal from standard curve.
[c]New standard: $(1 + 2\sigma) \times$ nominal; old standard: $1.2 \times$ nominal.

A more meaningful comparison can be made by choosing a specific reactor application. Such a comparison is presented in Table 3-6, where the new standard upper bound was chosen as $1 + 2\sigma$ times the nominal results, where σ is the standard deviation as prescribed by the ANS standard. This comparison shows that for typical end-of-life core composition, the lower-decay heat from ^{239}Pu plays an important role in reducing the decay heat power predicted by the 1979 standard and that the capture effect is small for a short time after shutdown. Calculated uncertainties in the 1979 standard are much lower than in the older standard, giving upper bound decay power results that are lower by 12 to 19%. A similar comparison made for ''younger'' fuel would show somewhat smaller differences because the change in the ^{239}Pu role is the major effect. Because of the sharp increase in the capture effect for $t_s > 10^4$ seconds, the revised standard may give higher decay power than did the old standard for long time after shutdown.

REFERENCES

1. Benedict, M., Pigford, T. H., and Levi, H. W. *Nuclear Chemical Engineering.* New York: McGraw-Hill, 1981.
2. El-Wakil, M. M. *Nuclear Heat Transport.* Scranton, PA: International Textbook Company, 1971.

3. Glasstone, S., and Sesonske, A. *Nuclear Reactor Engineering*. New York: Van Nostrand Reinhold, 1967 (2nd ed.), 1981 (3rd ed.).
4. Goldstein, H. *Fundamental Aspects of Reactor Shielding*. Reading, MA: Addison Wesley, 1959.
5. Lamarsh, J. *Nuclear Reactor Theory*. Reading, MA: Addison Wesley, 1966.
6. Leipunskii, O., Noroshicov, B. V., and Sakharov, V. N. *The Propagation of Gamma Quanta in Matter*. Oxford: Pergamon Press, 1965.
7. Rust, J. H. *Nuclear Power Plant Engineering*. Buchanan, GA: Haralson Publishing, 1979.
8. Schrock, V. E. A revised ANS standard for decay heat from fission products. *Nucl. Technol.* 46:323, 1979; and ANSI/ANS-5.1-1979: *Decay Heat Power in Light Water Reactors*. Hinsdale, IL: American Nuclear Society, 1979.
9. Shure, K. *Fission Product Decay Energy*. USAEC report WAPD-BT-24, 1961, Pittsburgh, PA, pp. 1–17, 1961.
10. Spiegel, U. S. *Mathematical Handbook*, Outline Series. New York: McGraw-Hill: 1968, p. 111.
11. Templin, L. T. (ed.). *Reactor Physics Constants* (2nd ed.). Argonne, IL: ANL-5800. 1963.

PROBLEMS

Problem 3-1 Thermal design parameters for a cylindrical fuel pin (section III)

Consider the PWR reactor of Example 3-1.

1. Evaluate the average thermal neutron flux if the enrichment of the fuel is 3.25%.
2. Evaluate the average power density of the fuel in MW/m^3. Assume the fuel density is 90% of theoretical density.
3. Calculate the average linear power of the fuel, assuming there are 207 fuel rods per assembly. Assume the fuel rod length to be 3810 mm.
4. Calculate the average heat flux at the cladding outer radius, when the cladding diameter is 10 mm.

Answers:

1. $\langle\phi\rangle = 3.75 \times 10^{13}$ neutrons/cm$^2 \cdot$ s
2. $\langle q''' \rangle = 289.6$ MW/m^3
3. $\langle q' \rangle = 19.24$ kW/m
4. $\langle q'' \rangle = 612.7$ kW/m^2

Problem 3-2 Power profile in a homogeneous reactor (section IV)

Consider an ideal core with the following characteristics: The ^{235}U enrichment is uniform throughout the core, and the flux distribution is characteristic of an unreflected, uniformly fueled cylindrical reactor, with extrapolation distances δz and δR of 10 cm. How closely do these assumptions allow prediction of the following characteristics of a PWR?

1. Ratio of peak to average power density and heat flux?
2. Maximum heat flux?
3. Maximum linear heat generation rate of the fuel rod?
4. Peak-to-average enthalpy rise ratio, assuming equal coolant mass flow rates in every fuel assembly?
5. Temperature of water leaving the central fuel assembly?

Calculate the heat flux on the basis of the area formed by the cladding outside diameter and the active fuel length. Use as input only the following values.

Total power = 3411 MWt (Table 1-2)
Equivalent core diameter = 3.37 m (Table 2-3)
Active length = 3.66 m (Table 2-3)

Fraction of energy released in fuel = 0.974
Total number of rods = 50,352 (Tables 2-2, 2-3)
Rod outside diameter = 9.5 mm (Table 1-3)
Total flow rate = 17.4×10^3 kg/s (Table 1-2)
Inlet temperature = 286°C (Table 1-2)
Core average pressure = 15.5 MPa (Table 1-2)

Answers:

1. $\phi_o/\bar{\phi} = 3.11$
2. $q''_{max} = 1.88$ MW/m^2
3. $q'_{max} = 56.1$ kW/m
4. $(\Delta h)_{max}/\Delta\bar{h} = 2.08$
5. $(T_{out})_{max} = 344.9$°C

Problem 3-3 Power generation in a thermal shield (section VII)

Consider the heat-generation rate in the PWR core thermal shield discussed in Example 3-4.

1. Calculate the total power generation in the thermal shield if it is 4.0 m high.
2. How would this total power change if the thickness of the shield is increased from 12.7 cm to 15 cm?

Assume uniform axial power profile.

Answers:

1. $\dot{Q} = 23.85$ MW
2. $\dot{Q} = 24.37$ MW

Problem 3-4 Decay heat energy (section VIII)

Using Eq. 3-70c, evaluate the energy generated in a 3000 MWth LWR after the reactor shuts down. The reactor operated for 1 year at the equivalent of 75% of total power.

1. Consider the following time periods after shutdown:
   a. 1 Hour
   b. 1 Day
   c. 1 Month
2. How would your answers be different if you had used Eq. 3-71 (i.e., would higher or lower values be calculated)?

Answers:

1a. 0.113 TJ: 1TJ = 10^{12} J
1b. 1.24 TJ
1c. 13.1 TJ
2. Higher

Problem 3-5: Effect of continuous refueling on decay heat (section VIII)

Using Eq. 3-70c, estimate the decay heat rate in a 3000 MWth reactor in which 3.2% ^{235}U-enriched UO_2 assemblies are being fed into the core. The burned-up fuel stays in the core for 3 years before being replaced. Consider two cases:

1. The core is replaced in two batches every 18 months.
2. The fuel replacement is so frequent that refueling can be considered a continuous process. (*Note:* The PHWR reactors and some of the water-cooled graphite-moderated reactors in the Soviet Union are effectively continuously refueled.)

Compare the two situations at 1 minute, 1 hour, 1 day, 1 month, and 1 year.

Answers:

	Case 1	Case 2
1 minute:	P = 81.9 MW	P = 81.0 MW
1 hour:	P = 33.2 MW	P = 32.2 MW
1 day:	P = 15.0 MW	P = 14.1 MW
1 month:	P = 4.97 MW	P = 4.26 MW
1 year:	P = 1.28 MW	P = 0.963 MW