CHAPTER
TWO

THERMAL DESIGN PRINCIPLES

I INTRODUCTION

The general principles of reactor thermal design are introduced in this chapter, with the focus on the parameters, design limits, and figures of merit by which the thermal design process is characterized. We do not attempt to present a procedure for thermal design because nuclear, thermal, and structural aspects are related in a complicated, interactive sequence. Specific design analysis techniques are detailed in Volume II.

II OVERALL PLANT CHARACTERISTICS INFLUENCED BY THERMAL HYDRAULIC CONSIDERATIONS

Thermal hydraulic considerations are important when selecting overall plant characteristics. Primary system temperature and pressure are key characteristics related to both the coolant selection and plant thermal performance. This thermal performance is dictated by the bounds of the maximum allowable primary coolant outlet temperature and the minimum achievable condenser coolant inlet temperature. Because this atmospheric heat sink temperature is relatively fixed, improved thermodynamic performance requires increased reactor coolant outlet temperatures. Figure 2-1 illustrates the relation among reactor plant temperatures for a typical PWR.

Bounds on the achievable primary outlet temperature depend on the coolant type. For liquid metals, in contrast to water, the saturated vapor pressure for a given tem-

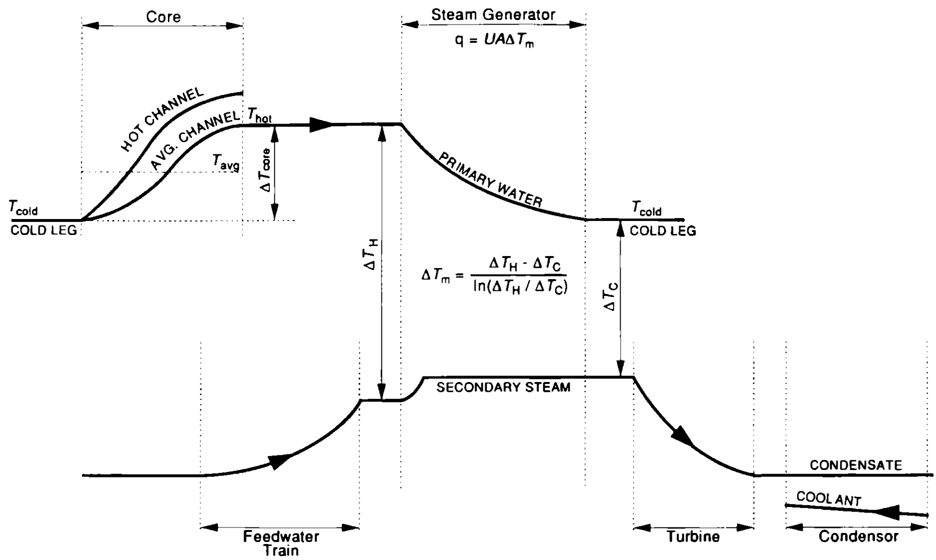


Figure 2-1 Relations among reactor plant temperatures for a typical PWR. (Adapted from Tong [4].)

perature is low, i.e., less than atmospheric pressure at outlet temperatures of interest of 500° to 550°C. Thus the outlet temperature for LMFBRs is not limited by the boiling point of the sodium coolant but, rather, by the creep lifetime characteristics of the stainless steel primary system material. For water-cooled reactors, on the other hand, high primary outlet temperatures require correspondingly high system pressures (7 to 15 MPa), which increases the stored energy in the primary coolant and requires increased structural piping and component wall thicknesses. Single-phase gas coolants offer the potential for high outlet temperatures without such inherently coupled high pressures. For these reactors the system pressure is dictated by the desired core heat transfer capabilities, as gas properties that enter these heat transfer correlations are strongly dependent on pressure. The resulting pressures are moderate, i.e., 4 to 5 MPa, whereas achievable outlet temperatures are high, i.e., 635° to 750°C. The numerical value of the plant thermal efficiency depends on the maximum temperature in the secondary or power-generation system. This temperature is lower than the reactor coolant outlet temperature owing to the temperature difference needed to transfer heat between the primary and secondary systems in the steam generator or intermediate heat exchanger. In the boiling water reactor a direct cycle is employed. The reactor outlet temperature is therefore identical (neglecting losses) to the inlet temperature to the turbine. This outlet temperature is also limited to the saturation condition, as BWRs do not operate under superheat conditions. In a typical BWR, however, the average outlet enthalpy achieved corresponds to an average quality of 15%. The PWR and BWR reactors achieve approximately equal thermal efficiencies, as the turbine steam conditions are comparable even though the primary system pressure and temperature conditions significantly differ (Table 2-1). Note that because of detailed differences in thermodynamic cycles the example PWR plant achieves a slightly higher

Table 2-1 Light-water reactor thermal conditions

Condition	PWR (Sequoyah)	BWR/6
Primary coolant outlet temperature	324°C	288°C
Primary coolant system pressure	15.5 MPa	7.17 MPa
Turbine steam saturation conditions		
Pressure	5.7 MPa	7.17 MPa
Temperature	272.3°C	287.5°C
Plant thermal efficiency	33.5%	32.9%

Source: Table 1-2.

plant thermal efficiency than does the BWR plant even though its steam temperature is lower.

Other plant characteristics are strongly coupled with thermal hydraulic considerations. Some notable examples are as follows.

1. *Primary coolant temperature*
   a. Corrosion behavior, though strongly dependent on water chemistry control, is also temperature-dependent.
   b. The reactor vessel resistance to brittle fracture degrades with accumulated neutron fluence. Vessel behavior under low-temperature, high-pressure transients from operating conditions is carefully evaluated to ensure that the vessel has retained the required material toughness over its lifetime.
2. *Primary system inventory*
   a. The time response during accidents and less severe transients strongly depends on coolant inventories. The reactor vessel inventory above the core is important to behavior in primary system rupture accidents. For a PWR, in particular, the pressurizer and steam generator inventories dictate transient response for a large class of situations.
   b. During steady-state operation the inlet plenum serves as a mixing chamber to homogenize coolant flow into the reactor. The upper plenum serves a similar function in multiple-loop plants with regard to the intermediate heat exchanger/steam generator while at the same time protecting reactor vessel nozzles from thermal shock in transients.
3. *System arrangement*
   a. The arrangement of reactor core and intermediate heat exchanger/steam generator thermal centers is crucial to the plant's capability to remove heat by natural circulation.
   b. Orientation of pump shafts and heat exchanger tubes coupled with support designs and impingement velocities is important relative to prevention of troublesome vibration problems.

The system arrangement issues are sometimes little appreciated if the design of a reactor type has been established for some years. For example, note that the dominant characteristics of the primary coolant system configuration for a typical U.S. designed

PWR (see Fig. 1-7) are cold leg pump placement, vertical orientation of pumps, vertical orientation of the steam generators, and elevation of the steam generator thermal center of gravity above that of the core. The pumps are located in the cold leg to take advantage of the increased subcooling, which increases the margin to onset of pump cavitation. Vertical orientation of the pumps provides for convenient accommodation of loop thermal expansion in a more compact layout and facilitates their serviceability, and the vertical steam generator orientation eases the difficulty in support design as well as eliminates a possible flow stratification problem. The relative vertical arrangements of thermal centers of steam generators above the core is established to provide the capability for natural circulation heat removal in the primary loop. These characteristics lead to creation of a loop seal of varying size in each arrangement between the steam generator exit and the pump inlet. This seal is of significance when considering system behavior under natural circulation, loss of coolant circumstances, or both.

III ENERGY PRODUCTION AND TRANSFER PARAMETERS

Energy production in a nuclear reactor core can be expressed by a variety of terms, reflecting the multidisciplinary nature of the design process. The terms discussed here are the following.

Volumetric energy (or heat) generation rate: $q'''(\vec{r})$
Surface heat flux: $\vec{q}''(S)$
Linear heat-generation rate or power rating: $q'(z)$
Rate of energy generation per pin: $\dot{q}$
Core power: $\dot{Q}$

The two additional parameters below are also commonly used. Because they are figures of merit of core thermal performance, they are discussed later (see section VI) after design limits and design margin considerations are presented.

Core power density: $\dot{Q}/V \equiv Q'''$
Core specific power: $\dot{Q}$/mass of heavy atoms

The relations among these terms must be well understood to ensure communication between the reactor physicist, the thermal designer, and the metallurgist or ceramicist. The reactor physicist deals with fission reaction rates, which lead to the volumetric energy generation rate: $q'''(\vec{r})$. The triple-prime notation represents the fact that it is an energy-generation rate per unit volume of the fuel material. Normally in this text a dot above the symbol is added for a rate, but for simplicity it is deleted here and in other energy quantities with primes. Hence the energy-generation rate is expressed per unit length cubed, as $q'''(\vec{r})$.

The thermal designer must calculate the fuel element surface heat flux, $\vec{q}''(S)$, which is related to $q'''(\vec{r})$ as

$$\iint_S \vec{q}''(S) \cdot \vec{n}\, dS = \iiint_V q'''(\vec{r})dV \tag{2-1}$$

where S is the surface area that bounds the volume (V) within which the heat generation occurs.

Both the thermal and metallurgical designers express some fuel performance characteristics in terms of a linear power rating $q'(z)$ where

$$\int_L q'(z)dz = \iiint_V q'''(\vec{r})dV \tag{2-2}$$

where L is the length of the volume (V) bounded by the surface (S) within which the heat generation occurs.

If the volume V is taken as the entire heat-generating volume of a fuel pin, the quantity ($\dot{q}$), unprimed, is the heat-generation rate in a pin, i.e.,

$$\dot{q} = \iiint_V q'''(\vec{r})dV \tag{2-3}$$

Finally, the core power ($\dot{Q}$) is obtained by summing the heat generated per pin over all the pins in the core (N) assuming that no heat generation occurs in the nonfueled regions.

$$\dot{Q} = \sum_{n=1}^{N} \dot{q}_{\mathrm{n}} \tag{2-4}$$

Actually, as discussed in Chapter 3, approximately 8% of the reactor power is generated directly in the moderator and structural materials.

These general relations take many specific forms, depending on the size of the region over which an average is desired and the specific shape (plate, cylindrical, spherical) of the fuel element. For example, considering a core with N cylindrical fuel pins, each having an active fuel length (L), core average values of the thermal parameters that can be obtained from Eqs. 2-1 to 2-4 are related as

$$\dot{Q} = N \langle \dot{q} \rangle = NL \langle q' \rangle = NL\pi D_{\mathrm{co}} \langle q''_{\mathrm{co}} \rangle = NL\pi R_{\mathrm{fo}}^2 \langle q''' \rangle \tag{2-5}$$

where D_{co} = outside clad diameter; R_{fo} = fuel pellet radius; $\langle\ \rangle$ = a core volume averaged value. A more detailed examination of the above relations and the various factors affecting the heat-generation distribution in the reactor core are discussed in Chapter 3.

IV THERMAL DESIGN LIMITS

The principal design limits for the power reactors discussed in Chapter 1 are detailed here. All reactors except the HTGR employ a metallic clad to hermetically seal the cylindrical fuel. The HTGR uses graphite and silicon carbide barriers around the fuel particles to reduce the rate of diffusion of fission products out of the fuel.

A Fuel Pins with Metallic Cladding

For hermetically sealed fuel pins, thermal design limits are imposed to maintain the integrity of the clad. In theory, these limits should all be expressed in terms of structural design parameters, e.g., strain and fatigue limits for both steady-state and transient operation. However, the complete specification of limits in these terms is presently impractical because of the complex behavior of materials in radiation and thermal environments characteristic of power reactors. For this reason, design limits in power reactors have been imposed directly on certain temperatures and heat fluxes, although the long-term trend should be to transform these limits into more specific structural design terms.

The design limits for reactors that employ cylindrical, metallic clad, oxide fuel pellets are summarized in Table 2-2, which highlights the distinction between conditions that would cause damage (loss of clad integrity) and those that would exceed design limits. Also both PWRs and BWRs have hydrodynamic stability limits. Generally, these limits are not restrictive in the design of these reactors currently, so that such limits are not noted in Table 2-2. The inherent characteristics of light-water reactors (LWRs) limit clad temperatures to a narrow band above the coolant saturation temperature and thus preclude the necessity for a steady-state limit on clad midwall temperature. However, a significant limit on clad average temperature does exist in transient situations, specifically in the loss of coolant accident (LOCA). For this accident a number of design criteria are being imposed, key among which is maintenance of the Zircaloy clad below 2200°F to prevent extensive metal–water reaction from occurring.

Table 2-2 Typical thermal design limits

Characteristic	PWR	BWR	LMFBR
Damage limit	1% Clad strain *or* MDNBR ≤ 1.0	1% Clad strain *or* MCPR ≤ 1.0	0.7% Clad strain
Design limits			
Fuel centerline temperature			
Steady state	—	—	—
Transient	No incipient melt	No incipient melt	No incipient melt
Clad average temperature			
Steady state	—	—	1200–1300°F
Transient	<2200°F (LOCA)	<2200°F (LOCA)	1450°F (788°C) for anticipated transients 1600°F (871°C) for unlikely events
Surface heat flux			
Steady state	—	MCPR ≥ 1.2[a]	—
Transient	MDNBR ≥ 1.3 at 112% power	—	—

LOCA = loss of coolant accident; MDNBR = minimum departure from nucleate boiling ratio; MCPR = minimum critical power ratio.

[a]Corresponding value of minimum critical heat flux ratio is approximately 1.9.

The particular design limit that is governing reactor design varies with the reactor type and the continually evolving state of design methods. For example, for LWRs, fuel centerline temperature is typically maintained well within its design limit due to restrictions imposed by the critical heat flux limit. Furthermore, with the application of improved LOCA analysis methods during the mid-1980s, the LOCA-imposed limit on clad temperature has not been the dominant limit. Finally, for LWRs, the occurrence of excessive mechanical interaction between pellet and clad (pellet–clad interaction) has led to operational restrictions on allowable rates of change of reactor power and the extensive development of fuel and clad materials to alleviate these restrictions on reactor load following ability.

The critical heat flux (CHF) phenomenon results from a relatively sudden reduction of the heat transfer capability of the two-phase coolant. The resulting thermal design limit is expressed in terms of the departure from nucleate boiling condition for PWRs and the critical power condition for BWRs. For fuel rods, where the volumetric energy-generation rate $q'''(r, z, t)$ is the independent parameter, reduction in surface heat transfer capability for nominally fixed bulk coolant temperature (T_b) and heat flux causes the clad temperature to rise, i.e.,

$$T_{co} - T_b = \frac{q''}{h} = \frac{q''' R_{fo}^2}{h D_{co}} \tag{2-6}$$

where h = heat transfer coefficient. Physically, this reduction occurs because of a change in the liquid–vapor flow patterns at the heated surface. At low void fractions typical of PWR operating conditions, the heated surface, which is normally cooled by nucleate boiling, becomes vapor-blanketed, resulting in a clad surface temperature excursion by departure from nucleate boiling (DNB). At high void fractions typical of BWR operating conditions, the heated surface, which is normally cooled by a liquid film, overheats owing to film dryout (DRYOUT). The dryout phenomenon depends significantly on channel thermal hydraulic conditions upstream of the dryout location rather than on the local conditions at the dryout location. Because DNB is a local condition and DRYOUT depends on channel history, the correlations and graphical representations for DNB are in terms of heat flux ratios, whereas for DRYOUT they are in terms of power ratios.

These two mechanisms for the generic critical heat flux phenomenon are shown in Figure 2-2. Correlations have been established for both these conditions in terms of different operating parameters (see Chapter 12). Because these parameters change over the fuel length, different margins exist between the actual operating heat flux and the limiting heat flux for occurrence of DNB or DRYOUT. These differences are illustrated in Figure 2-3 for a typical DNB case. The ratio between the predicted correlation heat flux and the actual operating heat flux is called the departure from nucleate boiling ratio (DNBR). This ratio changes over the fuel length and reaches a minimum value, as shown in Figure 2-3, somewhere downstream of the peak operating heat flux location. An alternative representation in terms of bundle average conditions, which depend on total power input, exists for BWR dryout conditions. This representation is expressed as the critical power ratio (CPR) and is presented in Chapter 12.

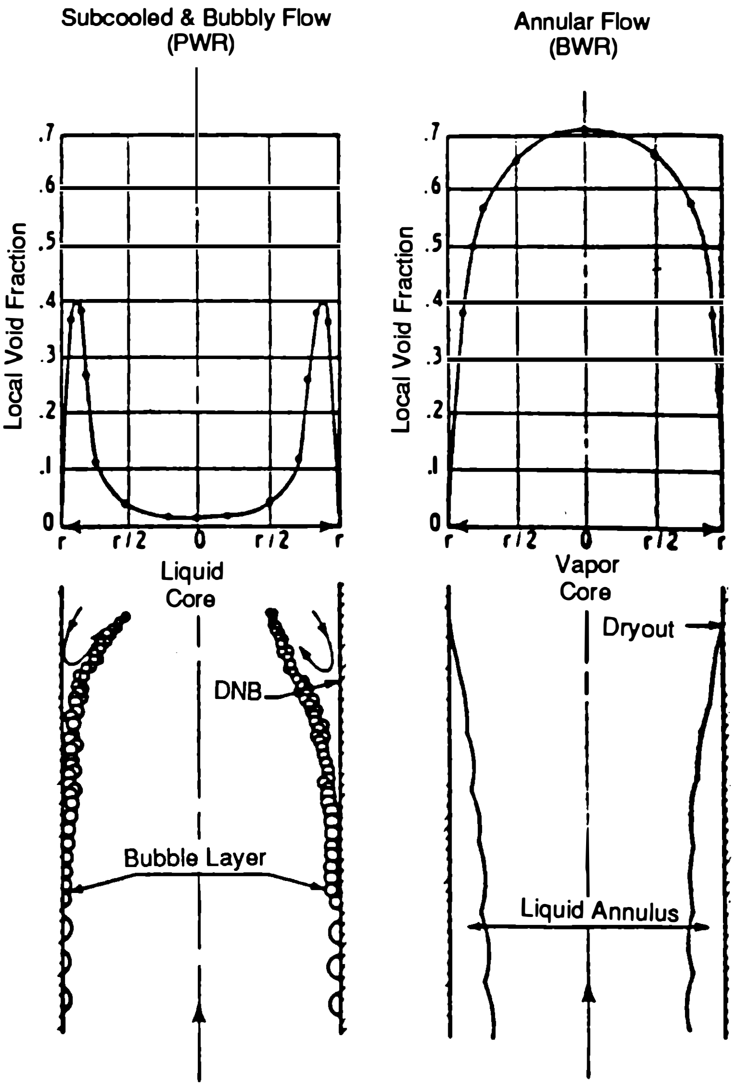


Figure 2-2 Critical heat flux mechanisms for PWR and BWR operating conditions. (From Tong [4].)

Critical condition limits are established for this minimum value of the appropriate ratio, i.e., MDNBR or MCPR (Table 2-2). Furthermore, prior to Brown's Ferry Unit 1, the BWR limit was applied to operational transient conditions, as is the present PWR practice. Subsequently it was applied to the 100% power condition. These limits for a BWR at 100% power and a PWR at 112% power allow for consistent overpower margin, as can be demonstrated for any specific case of prescribed axial heat flux distribution and coolant channel conditions.

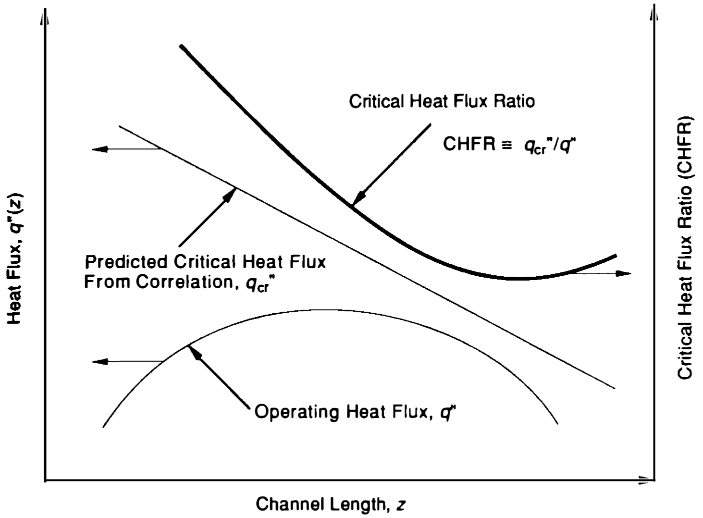
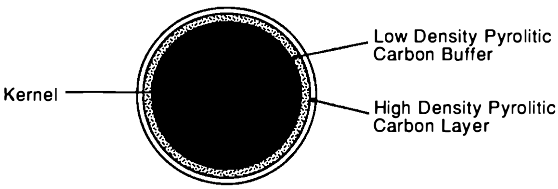


Figure 2-3 Critical heat flux ratio definition.

For the LMFBR, present practice is to require a level of subcooling such that the sodium temperature does not exceed its boiling temperature for transient conditions. Additionally, considerable effort is being applied to ensure that coolant voiding in accident situations can be satisfactorily accommodated. Hence the LMFBR limits are now placed on fuel and clad temperatures. At present, no incipient fuel melting is allowed. However, work is being directed at developing clad strain criteria. These criteria will reflect the fact that the stainless steel clad is operating in the creep regimen and that some degree of center fuel melting can be accommodated without clad failure.

B Graphite-Coated Fuel Pellets

The HTGR fuel is in the form of coated particles deposited in holes symmetrically drilled in graphite matrix blocks to provide passages for helium coolant. Two types of coated particle are used (Fig. 2-4). The BISO type has a fuel kernel surrounded by a low-density pyrolitic carbon buffer region, which is itself surrounded by a high-density, high-strength pyrolitic carbon layer. The TRISO type sandwiches a layer of silicon carbide between the two high-density pyrolitic carbon layers of the BISO type. In both fuel types the inner layer and the fuel kernel are designed to accommodate expansion of the particle and to trap gaseous fission products. The buffer layer acts to attenuate the fission fragment recoils. The laminations described also help to prevent crack propagation. Silicon carbide is used to supply dimensional stability and low diffusion rates. It has a greater thermal expansion rate than the surrounding pyrolitic carbon coating and thus is normally in compression. In both kinds of particles the total coating thickness is about 150μm. The TRISO coating is used for uranium fuel particles that are enriched to 93% ^{235}U. The BISO coating is used for thorium particles, which comprise the fertile material.



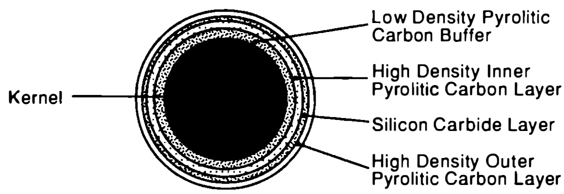


Figure 2-4 High-temperature gas reactor fuel particles. Typical dimensions: kernel diameters, 100 to 300 μm; total coating thickness, 50 to 190 μm.

Because fission gas release occurs in this system by diffusion through the coatings and directly from rupture of the coatings, limits are imposed to control these release rates such that steady-state fission product levels within the primary circuit do not exceed specified levels. These activity levels are established to ensure that radiation doses resulting from accidental release of the primary circuit inventory to the atmosphere are within regulations. These limits are on fuel particle center temperature.

100% power $\simeq$ 1300°C (2372°F)
Peak transient $\simeq$ 1600°C (2822°F) for short term

The full power limit minimizes steady-state diffusion, whereas the transient limit, based on in-pile tests, is imposed to minimize cracking of the protective coatings.

V THERMAL DESIGN MARGIN

A striking characteristic of thermal conditions existing in any core design is the large differences among conditions in different spatial regions of the core. The impact of the variation in thermal conditions for a typical core is shown in Figure 2-5. Starting with a core average condition such as linear power rating ($\langle q' \rangle$, defined by Eq. 2-5), nuclear power peaking factors, overpower factors, and engineering uncertainty factors are sequentially applied, leading to the limiting $\langle q' \rangle$ value. Each condition appearing in Figure 2-5 is clearly defined by a combination of the terms average, peak, nominal,

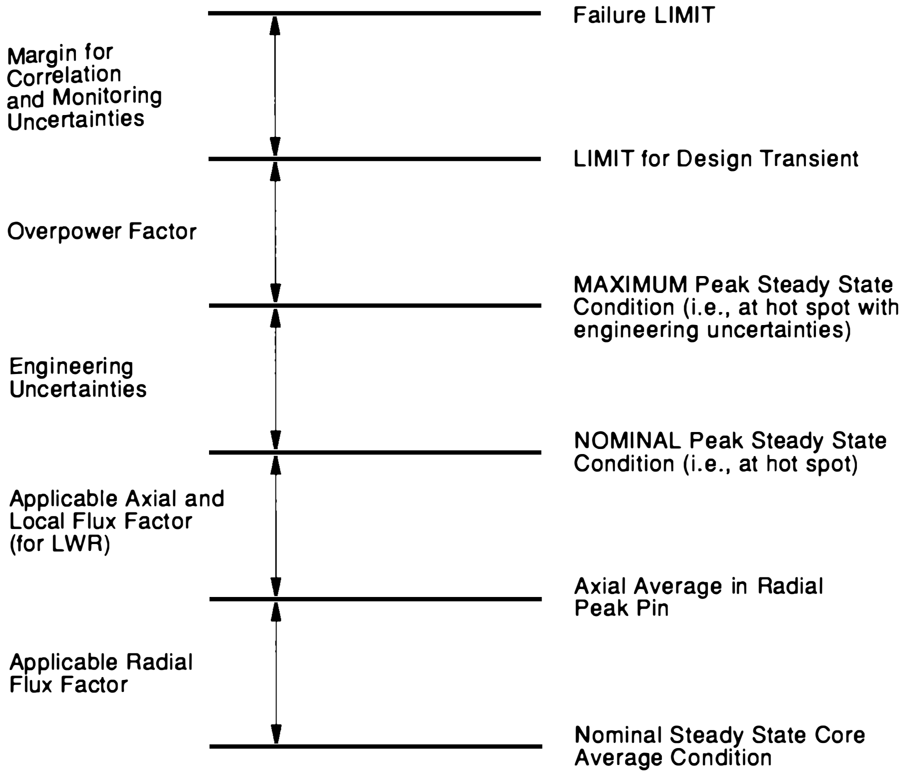


Figure 2-5 Thermal design nomenclature.

and maximum. No consistent set of definitions exists for these terms. However, a consensus of usage, which we adopt, is the following.

Average: Usually a core or pin axially averaged value. The distinction must be made or inferred from the application.

Peak: Sometimes referred to as hot spot; refers exclusively to a physical location at which the extreme value occurs; i.e., peak pin is that pin located where the radial power profile is greatest; peak linear power rating in the peak pin is that axial location in the peak pin of greatest linear power rating.

Nominal: Value of a parameter calculated using variables at their design-specified values.

Maximum: Value of a parameter calculated with allowance provided for deviation of the variables from their design-specified values.

Margin is provided between the transient operating LIMIT and the failure LIMIT (Fig. 2-5). This margin provides for uncertainties in the instrumentation used to monitor the operating condition and uncertainties in the correlations for transient limits.

It is apparent in Figure 2-5 that core power can be enhanced by flattening the shape of the power-generation rate, optimizing the power-to-flow ratio in every radial zone of the core, or both. Power flattening is achieved by a combination of steps involving reflector regions, enrichment zoning, shuffling of fuel assemblies with burnup, and burnable poison and shim control placement. In water reactors, local power peaking effects due to water regions comprise an additional factor that has been addressed by considerable attention to the detailed layout of the fuel/coolant lattice. However, power flattening may not be desirable if neutron leakage from the core is to be minimized.

Optimizing the power-to-flow ratio in the core is important for achieving the desired reactor vessel outlet temperature at maximum net reactor power, i.e., reactor power generation minus pumping power, where pumping power is expressed as:

$$\text{Pumping power} = \text{force through distance per unit time} \tag{2-7}$$

$$\text{Pumping power} = (\Delta p)A_f V \tag{2-8}$$

where Δp = pressure drop through the circuit (F/L^2); A_f = cross-sectional area of the coolant passage (L^2); and V = average coolant velocity (L/T).

Flow control involves establishing coolant flow rates across the core at the levels necessary to achieve equal coolant exit conditions in all assemblies and minimizing the amount of inlet coolant that bypasses the heated core regions. Some bypass flow is required, however, to maintain certain regions (e.g., the inner reactor vessel wall) at design conditions. Variations in energy generation within assemblies with burnup do make it difficult to maintain the power-to-flow ratio near unity. The assembly flow rates are normally adjusted by orificing. Orificing is accomplished by restricting the coolant flow in the assembly, usually at the inlet. Because the orifice devices are not typically designed to be changed during operation, deviations in the power-to-flow ratio from optimum over a full cycle are inevitable. Alternatively, large spatial variations in flow rate can also be accomplished using multiple inlet plena. This design approach, however, is complicated.

VI FIGURES OF MERIT FOR CORE THERMAL PERFORMANCE

The design performance of a power reactor can be characterized by two figures of merit: the power density (Q''') and the specific power. Table 2-3 tabulates the power density for the various power reactor concepts. The specific power can be calculated, as is shown from the other reactor parameters given.

Power density is the measure of the energy generated relative to the core volume. Because the size of the reactor vessel and hence the capital cost are nominally related to the core size, the power density is an indicator of the capital cost of a concept. For propulsion reactors, where weight and hence size are at a premium, power density is a relevant figure of merit.

Table 2-3 Typical core thermal performance characteristics for six reference power reactor types

Characteristic	BWR	PWR(W)	PHWR	HTGR	AGR	LMFBR[a]
Core						
Axis	Vertical	Vertical	Horizontal	Vertical	Vertical	Vertical
No. of assemblies						
Axial	1	1	12	8	8	1
Radial	748	193	380	493	332	364 (C) 233 (BR)
Assembly pitch (mm)	152	215	286	361	460	179
Active fuel height (m)	3.81	3.66	5.94	6.30	8.296	1.0 (C) 1.6 (C + BA)
Equivalent diameter (m)	4.70	3.37	6.29	8.41	9.458	3.66
Total fuel weight (ton)	156 UO_2	101 UO_2	98.4 UO_2	1.72 U 37.5 Th	113.5 UO_2	32 MO_2
Reactor vessel						
Inside dimensions (m)	$6.05D \times 21.6H$	$4.83D \times 13.4H$	$7.6D \times 4L$	$11.3D \times 14.4H$	$20.25D \times 21.87H$	$21D \times 19.5H$
Wall thickness (mm)	152	224	28.6	4.72 m min	5.8 m	25
Material[b]	SS-clad carbon steel	SS-clad carbon steel	Stainless steel	Prestressed concrete	Concrete helical prestressed	Stainless steel
Other features			Pressure tubes	Steel liners	Steel lined	Pool type
Power density core average (kW/L)	54.1	105	12	8.4	2.66	280
Linear heat rate						
Core average (kW/m)	19.0	17.8	25.7	7.87	17.0	29
Core maximum (kW/m)	44.0	42.7	44.1	23.0	29.8	45
Performance						
Equilibrium burnup (MWD/T)	27,500	27,500	7500	95,000	18,000	100,000
Average assembly residence (full-power days)			470	1170	1320	
Refueling						
Sequence	$\frac{1}{4}$ per yr	$\frac{1}{3}$ per yr	Continuous on-line	$\frac{1}{4}$ per yr	Continuous on-line	Variable
Outage time (days)	30	30		14–20		32

Source: Knief [3], except AGR data are from Alderson [1] and Debenham [2].

[a]LMFBR: core (C), radial blanket (BR), axial blanket (BA).

[b]SS = stainless steel.

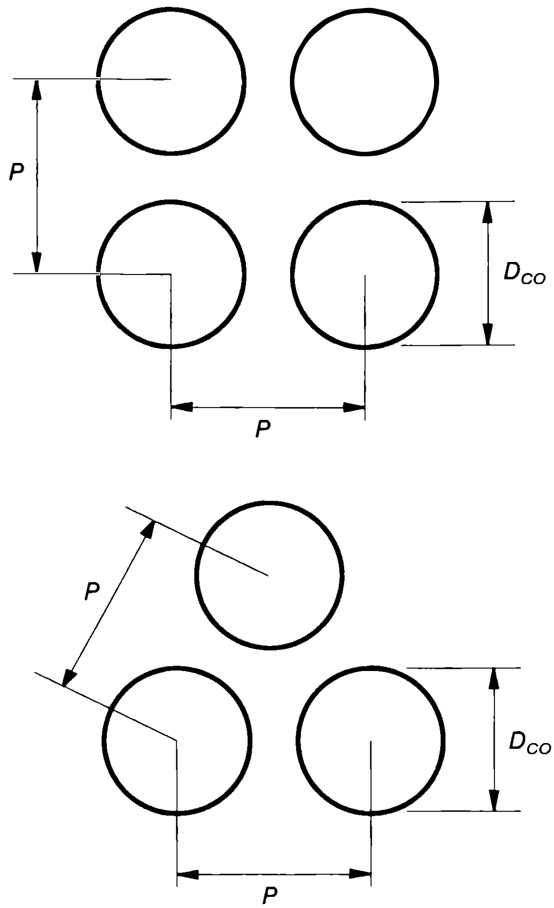


Figure 2-6 Square and triangular rod arrays.

The power density can be varied by changing the fuel pin arrangement in the core. For an infinite square array, shown in Figure 2-6, the power density (Q''') is related to the array pitch (P) as:

$$(Q''')_{\text{square array}} = \frac{4(1/4\ \pi R_{fo}{}^2)q'''\,dz}{P^2\,dz} = \frac{q'}{P^2} \tag{2-9}$$

whereas for an infinite triangular array, the comparable result is:

$$(Q''')_{\text{triangular array}} = \frac{3(1/6\ \pi R_{fo}{}^2)q'''\,dz}{\frac{P}{2}\left(\frac{\sqrt{3}}{2}P\right)dz} = \frac{q'}{\frac{\sqrt{3}}{2}P^2} \tag{2-10}$$

Comparing Eq. 2-9 and 2-10, we observe that the power density of a triangular array is 15.5% greater than that of a square array for a given pitch. For this reason, reactor concepts such as the LMFBR adopt triangular arrays, which are more complicated mechanically than square arrays. For light-water reactors, on the other hand, the simpler square array is more desirable, as the necessary neutron moderation can be provided by the looser-packed square array.

Specific power is the measure of the energy generated per unit mass of fuel material. It is usually expressed as watts per gram of heavy atoms. This parameter has direct implications on the fuel cycle cost and core inventory requirements. For the fuel pellet shown in Figure 2-7, the specific power, (watts per grams of heavy atoms), is:

$$\text{Specific power} = \frac{\dot{Q}}{\text{mass of heavy atoms}} = \frac{q'}{\pi R_{\text{fo}}^2\, \rho_{\text{pellet}} f} = \frac{q'}{\pi (R_{\text{fo}} + \delta_{\text{g}})^2 \rho_{\text{smeared}} f} \tag{2-11}$$

where

$$\rho_{\text{smeared}} = \frac{\pi R_{\text{fo}}^2\, \rho_{\text{pellet}}}{\pi (R_{\text{fo}} + \delta_{\text{g}})^2} \tag{2-12}$$

and

$$f = \text{mass fraction of heavy atoms in the fuel} = \frac{\text{grams of fuel heavy atoms}}{\text{grams of fuel}} \tag{2-13}$$

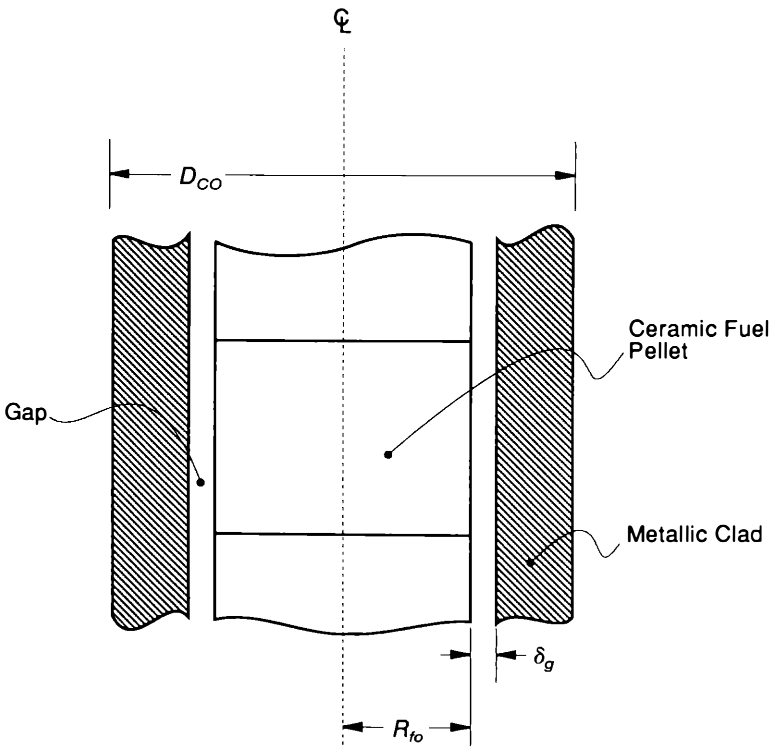


Figure 2-7 Typical power reactor fuel.

This definition of the *mass fraction* (f) is based on the following definition

1. Heavy atoms include all the U, Pu, or Th isotopes and are therefore composed of fissionable atoms (M_{ff}) and nonfissionable atoms (M_{nf}), where M is molecular weight.
2. Fuel is the entire fuel-bearing material, i.e., UO_2 but not the clad.

Thus for oxide fuel:

$$f = \frac{N_{ff}M_{ff} + N_{nf}M_{nf}}{N_{ff}M_{ff} + N_{nf}M_{nf} + N_{O_2}M_{O_2}} \tag{2-14}$$

where N = atomic density.

The enrichment (r) is the mass ratio of fissionable atoms to total heavy atoms, i.e.,

$$r = \frac{N_{ff}M_{ff}}{N_{ff}M_{ff} + N_{nf}M_{nf}} \tag{2-15}$$

and for later convenience:

$$1 - r = \frac{N_{nf}M_{nf}}{N_{ff}M_{ff} + N_{nf}M_{nf}} \tag{2-16}$$

It is useful to express f in terms of molecular weight and the enrichment. It follows from the observation that, for UO_2,

$$N_{O_2} = N_{ff} + N_{nf}$$

and manipulation of Eqs. 2-15 and 2-16 to yield:

$$N_{ff} = \left[r\,\frac{M_{nf}}{M_{ff}}\right]\frac{N_{nf}}{1 - r};\; N_{nf} = \left[(1 - r)\,\frac{M_{ff}}{M_{nf}}\right]\frac{N_{ff}}{r} \tag{2-17a,b}$$

$$N_{ff} + N_{nf} = \left[r\,\frac{M_{nf}}{M_{ff}} + (1 - r)\right]\frac{N_{nf}}{1 - r} \text{ and equally}$$

$$\left[(1 - r)\,\frac{M_{ff}}{M_{nf}} + r\right]\frac{N_{ff}}{r} \tag{2-18a,b}$$

Division of each term of Eq. 2-14 by N_{O_2} and substitution of Eqs. 2-17a,b and 2-18a,b yields the desired result, i.e.,

$$f_{UO_2} = \frac{\dfrac{r}{r + (1 - r)(M_{ff}/M_{nf})}M_{ff} + \dfrac{(1 - r)}{r(M_{nf}/M_{ff}) + (1 - r)}M_{nf}}{\dfrac{r}{r + (1 - r)(M_{ff}/M_{nf})}M_{ff} + \dfrac{(1 - r)}{r(M_{nf}/M_{ff}) + (1 - r)}M_{nf} + M_{O_2}} \tag{2-19}$$

which for the case where $M_{ff} \simeq M_{nf}$ simplifies to:

$$f_{UO_2} = \frac{rM_{ff} + (1 - r)M_{nf}}{rM_{ff} + (1 - r)M_{nf} + M_{O_2}} \tag{2-20}$$

The specific power in Eq. 2-11 has been expressed in terms of both the pellet and the smeared densities. The *smeared density* includes the void that is present as the gap between the fuel pellet and the clad inside diameter. The smeared density is the cold or hot density, depending on whether the gap (δ_g) is taken at the cold or hot condition. Smeared density is an important parameter associated with accommodation of fuel swelling with burnup.

Example 2-1 Power density and specific power for a PWR

PROBLEM Confirm the power density listed in Table 2-3 for the PWR case. Calculate the specific power of the PWR of Tables 2-3 and 1-3.

SOLUTION The power density is given by Eq. 2-9 as:

$$Q'''_{\text{square array}} = \frac{q'}{P^2}$$

From Table 2-3: $\langle q' \rangle_{\text{core average}} = 17.8 \dfrac{\text{kW}}{\text{m}}$

Take $q' = \langle q' \rangle$, which yields a result for Q''' different from that in Table 2-3, as noted below.

From Table 1-3, $P = 12.6$ mm.

$$Q'''_{\text{PWR}} = \frac{17.8}{(12.6 \times 10^{-3})^2} = 0.112 \times 10^6 \frac{\text{kW}}{\text{m}^3} = 112 \frac{\text{kW}}{\text{L}} \text{ or } \frac{\text{MW}}{\text{m}^3}$$

Observe that Table 2-3 lists the average power density as:

$$Q'''_{\text{PWR}} = 105 \frac{\text{kW}}{\text{L}}$$

The difference arises because our calculation is based on the core as an infinite square array, whereas in practice a finite number of pins form each assembly so that edge effects within assemblies must be considered. The specific power is given by Eq. 2-11 as:

$$\frac{\dot{Q}}{\text{Mass heavy atoms}} = \frac{q'}{\pi R_{fo}^2 \, \rho_{\text{pellet}} \, f}$$

Evaluating f using Eq. 2-20 for the enrichment of 2.6% listed in Table 1-3 yields:

$$f = \frac{0.026(235.0439) + 0.974(238.0508)}{0.026(235.0439) + 0.974(238.0508) + 2(15.9944)} = 0.8815$$

For a pellet density of 95% of the UO_2 theoretical density of 10.97 g/cm^3 and a fuel pellet diameter of 8.2 mm, the specific power is:

$$\frac{\dot{Q}}{\text{Mass heavy atoms}} = \frac{1.78 \times 10^4 \ \dfrac{\text{W}}{\text{m}}}{\pi\left(\dfrac{8.2 \times 10^{-3}}{2}\right)^2 \left(\dfrac{0.95\,(10.97)}{10^{-6}}\right) 0.8815\ m^2 \dfrac{\text{g}}{\text{m}^3}}$$

$$= 36.70 \ \frac{\text{W}}{\text{g fuel}}$$

Alternately, Table 2-3 lists the total core loading of fuel material as 101×10^3 kg of UO_2. In this case

$$\frac{\dot{Q}}{\text{Mass heavy atoms}} = \frac{\text{core power}}{\text{fuel loading}} = \frac{\dot{Q}}{fM_{\text{fm}}}$$

If $\dot{Q}$ is evaluated from the PWR dimensions as in Tables 2-3 and 1-3,

$$\dot{Q} = q'LN = .0178 \ \frac{\text{MW}}{\text{m}} [3.66 \text{ m}][193(264)] = 3319 \text{ MWt}$$

Note: A thermal power of 3411 MWt is given in Table 1-2.

Then:

$$\frac{\dot{Q}}{\text{Mass heavy atoms}} = \frac{3319 \text{ MWt}}{0.8815(101 \times 10^3 \text{ kg})} = 37.28 \ \frac{\text{W}}{\text{g fuel}}$$

Unlike the case of power density, specific power is closely estimated, as the fuel mass, not the core volume, is utilized.

REFERENCES

1. Alderson, M. A. H. G. Personal communication, 6 October 1983.
2. Debenham, A. A. Personal communication, 5 August 1988.
3. Knief, R. A. *Nuclear Energy Technology: Theory and Practice of Commercial Nuclear Power.* New York: Hemisphere, 1981, pp. 566–570.
4. Tong, L. S. Heat transfer in water-cooled nuclear reactors. *Nucl. Eng. Design* 6:301–324, 1967.

PROBLEMS

Problem 2-1 Relations among fuel element thermal parameters in various power reactors (section III)

Compute the core average values of the volumetric energy-generation rate in the fuel (q''') and outside surface heat flux (q'') for the reactor types of Table 2-3. Use the core average linear power levels in Table 2-3 and the geometric parameters in Table 1-3.

Answer (for BWR): $q''' = 224 \text{ MW/m}^3$
$q'' = 492.9 \text{ kW/m}^2$

Problem 2-2 Minimum critical heat flux ratio in a PWR for a flow coastdown transient (section IV)

Describe how you would determine the minimum critical heat flux ratio versus time for a flow coastdown transient by drawing the relevant channel operating curves and the CHF limit curves for several time values. Draw your sketches in relative proportion and be sure to state all assumptions.

Problem 2-3 Minimum critical power ratio in a BWR (section IV)

Calculate the minimum critical power ratio for a typical 1000 MWe BWR operating at 100% power using the data in Tables 1-2, 1-3, and 2-3. Assume that:

1. The axial linear power shape can be expressed as

$$q'(z) = q'_{ref}\exp(-\alpha z/L)\sin\frac{\pi z}{L}$$

where $\alpha = 1.96$. Determine q'_{ref} such that $q'_{max} = 44\frac{kW}{m}$

2. The critical bundle power is 9319kW.

Answer: MCPR = 1.54

Problem 2-4 Pumping power for a PWR reactor coolant system (section V)

Calculate the pumping power under steady-state operating conditions for a typical PWR reactor coolant system. Assume the following operating conditions:

Core power = 3817 MWt
ΔT_{core} = 31°C
Reactor coolant system pressure drop = 778 kPa (113 psi)

Answer: Pumping power = 23.3 MW

Problem 2-5 Relations among thermal design conditions in a PWR (section V)

Compute the margin as defined in Figure 2-5 for a typical PWR having a core average linear power rate of 17.8 kW/m. Assume that the failure limit is established by centerline melting of the fuel at 70 kW/m. Use the following multiplication factors:

Radial flux = 1.55
Axial and local flux factor = 1.70
Engineering uncertainty factor = 1.05
Overpower factor = 1.15

Answer: Margin = 1.24