

THERMODYNAMICS OF NUCLEAR ENERGY CONVERSION SYSTEMS: NONFLOW AND STEADY FLOW: FIRST AND SECOND LAW APPLICATIONS

I INTRODUCTION

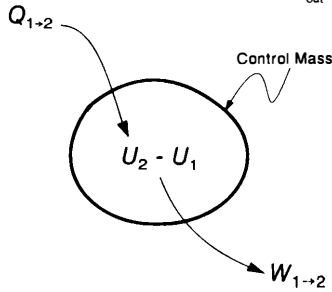
The working forms of the first and second laws for the control mass and control volume approaches are summarized in Table 6-1. There are many applications of these laws to the analysis of nuclear systems. It is of prime importance that the reader not only develop the proficiency to apply these laws to new situations but also recognize which approach (control mass or control volume) is more convenient for the formulation of the solution of specific problems.

The elementary application of these laws avoids the time-dependent prediction of parameters. Usually processes are either modeled as nontransient by specifying the initial and end states or as steady-state processes. This choice is not dictated by inherent limitations in the first and second laws but, rather, by the complexity involved when describing the heat and work rate terms, \dot{Q} and \dot{W} , which appear in these laws. For example, the analytic description of \dot{Q} requires definition of the heat transfer rates which, in many processes, are complex and available in empirical form only.

Nonflow and steady-flow processes are discussed here. Variable-flow processes are discussed in Chapter 7. The examples analyzed in this chapter as nonflow processes are essentially transient processes. However, as for any engineering analysis, it is important to consider the question posed and then to model the process in the simplest form, consistent with the required objective and the information available. Therefore

Table 6-1 Summary of the working forms of the first and second laws of thermodynamics

Parameter	First law	Second law
Control mass	$\dot{U}_{c.m.} = \dot{Q}_{c.m.} - \dot{W}_{c.m.} \quad (\text{Eq. 4-19a})$ <p>For a process involving finite changes between states 1 and 2, Eq. 4-19a becomes</p> $U_2 - U_1 = Q_{1+2} - W_{1+2} \quad (\text{Eq. 6-1})$ <p>if kinetic energy differences are negligible Convention: W_{out} and Q_{in} are positive</p>	$\dot{S}_{c.m.} = \dot{S}_{gen} + \frac{\dot{Q}_{c.m.}}{T_s} \quad (\text{Eq. 4-25b})$ <p>where T_s is the temperature at which heat is supplied</p>



Control volume
(stationary)

$$\begin{aligned} \dot{E}_{c.v.} = \sum_{i=1}^I \dot{m}_i (h_i^o + gz_i) + \dot{Q} + \dot{Q}_{gen} \\ - \dot{W}_{shaft} - \dot{W}_{normal} - \dot{W}_{shear} \end{aligned} \quad (\text{Eq. 4-39})$$

Neglecting shear work and differences in kinetic and potential energy, and treating \dot{Q}_{gen} as part of \dot{U} , Eq. 4-39 becomes

$$\begin{aligned} \dot{U}_{c.v.} = \sum_{i=1}^I \dot{m}_i h_i + \dot{Q} \\ - \dot{W}_{shaft} - \dot{W}_{normal} \end{aligned} \quad (\text{Eq. 6-2})$$

Convention: W_{out} and Q_{in} are positive

$$\dot{S}_{c.v.} = \sum_{i=1}^I \dot{m}_i s_i + \dot{S}_{gen} + \frac{\dot{Q}}{T_s} \quad (\text{Eq. 4-41})$$

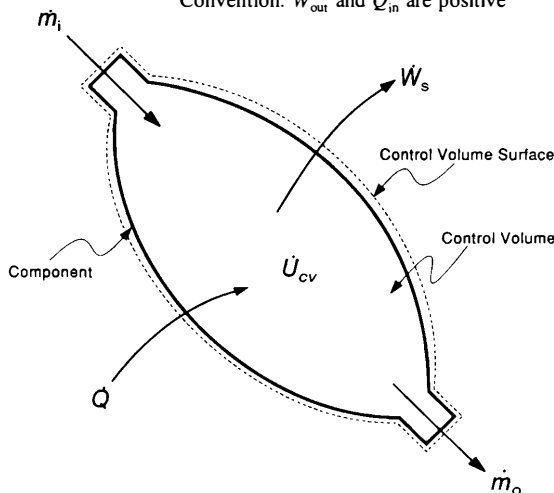


Table 6-2 Examples covered in Chapters 6 and 7

Process	Control mass	Control volume
Nonflow	Ch. 6, Sect. II: expansion work from a fuel-coolant interaction process	—
Steady flow	—	Ch. 6, Sect. IV: reactor system thermodynamic efficiency and irreversibility analysis
Nonsteady flow	Ch. 7, Sect. II: reactor containment pressurization from loss of coolant accident	Ch. 7, Sect. II: reactor containment pressurization from loss of coolant accident
	—	Ch. 7, Sects. III and IV: pressurizer response to load change

when illustrating the control mass and control volume approaches, the examples are modeled in different ways to obviate or include the time-dependent description of the relevant rate processes. In doing so, the differences in the results achieved when complex processes are modeled in these fundamentally different ways are illustrated. Table 6-2 summarizes the approaches and examples covered here and in Chapter 7.

II NONFLOW PROCESS

The essential step in the nonflow approach is to carefully define the control mass and the sign convention for the interactions. A sketch of this control mass, complete with the relevant energy flows is useful. Also desirable is a state diagram using suitable thermodynamic properties of the initial and final equilibrium states of the process the control mass undergoes. Finally, if the control mass undergoes a known reversible process, it is useful to represent the process path on the state diagram.

To determine the energy flows, it is necessary to specify the time base over which the analysis is applicable. Usually a fixed time period is not specified; rather, it is stated or implicitly assumed that the analysis applies over a period sufficient for a desired transition between fixed thermodynamic states for the control mass. In that way we do not explicitly involve the rate processes that would require detailed heat transfer and fluid mechanics information. The price for this simplification is that we do not learn anything of the transient aspects of the process, only the relation between the end states and, if the process path is defined, the work and heat interactions of the control mass.

The first law (Table 6-1) for a control mass undergoing a process involving finite changes between state “1” and state “2” is

$$U_2 - U_1 = Q_{1 \rightarrow 2} - W_{1 \rightarrow 2} \quad (6-1)$$

where positive W = work done by the control mass, and positive Q = heat transferred into the control mass.

As an illustration of application of this law to a control mass, let us examine the thermal interaction between a hot liquid and a more volatile cold liquid. This phe-

nomenon, a molten fuel-coolant interaction, is of interest when evaluating the integrity of the containment under hypothetical accident conditions. It can be postulated to occur in both light-water- and sodium-cooled reactors only in unlikely situations. In both reactors, the hot liquid would consist of the molten fuel, cladding, and structural materials from the partially melted core; and the volatile, cold liquid would be the coolant, water, or sodium. We are interested in evaluating the effect of this interaction on the containment by calculating the expansion work resulting from the mixing of these hot and cold liquids. The relevant properties for these materials are presented in Table 6-3.

To estimate this expansion work, assume that the process occurs in two steps. In the first step, an equilibrium temperature is found for the fuel and coolant under the condition of no expansion by the more volatile coolant or the fuel. This constant-volume condition is reasonable if the thermal equilibration time is small (≤ 1 ms), which, however, is not the case if film boiling occurs when the fuel mixes with the coolant. In the second step, work occurs when the coolant and fuel are assumed to expand reversibly to a prescribed state. This expansion may take place: (1) without heat transfer between the fuel and the coolant; or (2) with heat transfer between the fuel and the coolant so that thermal equilibrium is maintained during the expansion.

With these somewhat artificial prescriptions for the interaction, we have implicitly prescribed time periods for the two steps by defining the intermediate state (i.e., that corresponding to the equilibrium temperature at the initial volume) and the final state. In this manner the truly transient fuel-coolant interaction process has been idealized as a nonflow process. In fact, these time periods are not real because the processes occurring during the two steps are not physically distinct. Furthermore, there are heat transfer and expansion work rates that characterize this process. However, for convenience, our idealization of this process between end states allows us to establish conservative bounds on the expansion work. The analysis of this process that follows generally adopts the approach of Hicks and Menzies [1].

A Fuel-Coolant Thermal Interaction

1 Step I: Coolant and fuel equilibration at constant volume. We may define either one control mass, consisting of the combined mass of the fuel and the coolant, or two control masses, one consisting of the mass of the fuel and the other consisting of the mass of the coolant. We choose the second representation because it is easier to define the T - s diagrams for a one-component (two-phase) fluid (i.e., the coolant or fuel alone) then it is to define the T - s diagram for the two-component coolant-fuel mixture.

The control masses do not provide work to the environment during the constant-volume thermal equilibration process. To analyze the interaction conservatively, we assume that all the heat transferred from the fuel during the process is transferred to the coolant. Finally, because the thermal equilibration process is so fast, the decay heat generation rate is low enough to be neglected during the process. With these assumptions, the control mass and the process representation for the coolant alone may be sketched as in Figure 6-1.

Table 6-3 Properties for fuel-coolant interaction examples

Parameter	Symbol	Units	Sodium	Water	Fuel: UO ₂ or mixed oxide
Mass (typically primary system inventory)	m_c or m_f	kg	3500	4000	40,000
Initial temperature	T_i	°K	600	400	3100
Initial density	ρ_i	kg/m ³	835	945	~8000
Saturation pressure	p_{sat}	MPa	$\ln p_{sat} \text{ (MPa)} = 8.11 - \frac{12,016}{T(K)}$	$\ln p_{sat} \text{ (MPa)} = 10.55 - \frac{4798}{T(K)}$	—
Gas constant	R	J/kg °K	361	462	31
Vapor specific heat ratio	γ	—	1.15	1.3	~1.06 to 1.07
Liquid specific heat at constant volume*	c_{vc} or c_{vf}	J/kg °K	1300	4184	560
Vapor specific heat at constant pressure	c_{pvc} or c_{pvf}	J/kg °K	$\frac{\gamma R}{\gamma - 1} = 2767$	2003	~500
Vapor specific heat at constant volume	c_{vvc} or c_{vvf}	J/kg °K	$\frac{R}{\gamma - 1} = 2410$	1540	~475
Latent heat of vaporization	h_{fg}	J/kg	2.9×10^6	1.9×10^6	$\sim 1.9 \times 10^6$
Coolant critical point properties					
Pressure	p_{crit}	MPa	40.0	22.1	—
Temperature	T_{crit}	°K	2733	647.3	—
Density	ρ_{crit}	kg/m ³	818	317	—
Internal energy	u_{crit}	J/kg	4.29×10^6	2.03×10^6	—

*For nearly incompressible liquids, the constant pressure and constant volume specific heats can be taken as equal.

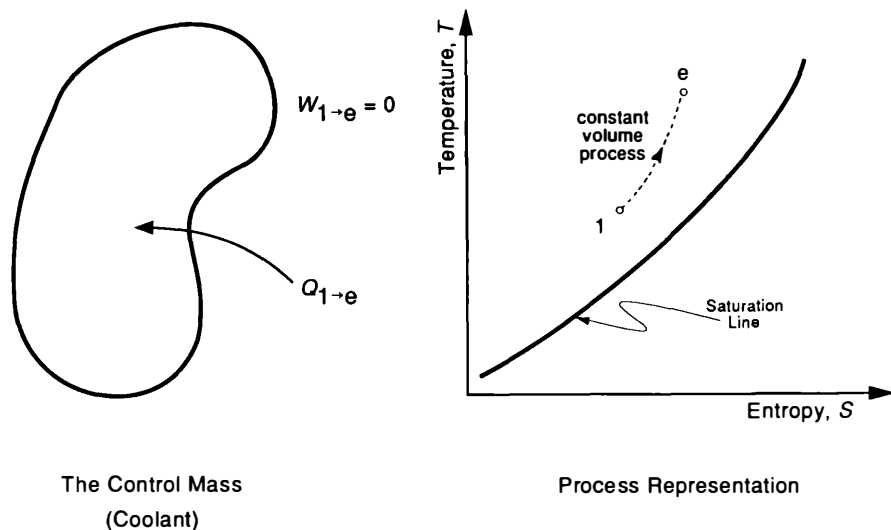


Figure 6-1 Coolant control mass behavior in the constant-volume thermal interaction with fuel: step I.

To evaluate Q , we identify the fuel as another control mass undergoing a constant-volume cooling process to the equilibrium temperature. Analogous sketches for this fuel control mass are shown in Figure 6-2.

Expressing the first law for each control mass, undergoing a change of state from 1 to e, neglecting potential and kinetic energy changes we obtain:

$$\text{Coolant } \Delta E_c = \Delta U_c = Q_{1 \rightarrow e} \quad (6-3a)$$

$$\text{Fuel } \Delta E_f = \Delta U_f = -Q_{1 \rightarrow e} \quad (6-3b)$$

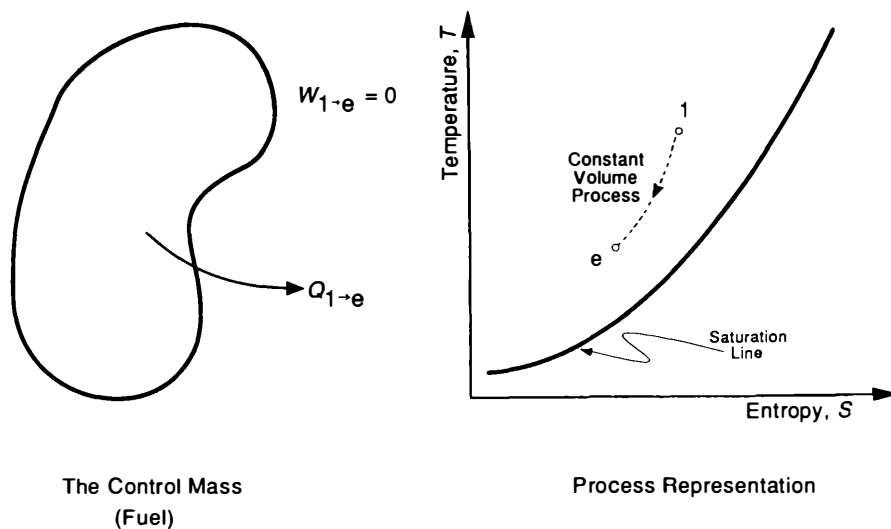


Figure 6-2 Fuel control mass behavior in the constant-volume thermal interaction with coolant: step I.

Using the state equation that expresses internal energy as a function of temperature, we obtain:

$$\Delta U_c = m_c c_{v_c} (T_e - T_{1_c})$$

$$\Delta U_f = m_f c_{v_f} (T_e - T_{1_f})$$

The interaction process of the combined coolant–fuel system is adiabatic.

Hence:

$$m_c c_{v_c} (T_e - T_{1_c}) = m_f c_{v_f} (T_{1_f} - T_e)$$

or:

$$T_e = \frac{\left(\frac{m_f c_{v_f}}{m_c c_{v_c}} \right) T_{1_f} + T_{1_c}}{1 + \frac{m_f c_{v_f}}{m_c c_{v_c}}} \quad (6-4)$$

Because this equilibrium state is to the left of the saturated liquid line on a T – s plot for each component, the static quality of each fluid is by definition either 1.0 or 0 depending on whether T_e is above or below the critical temperature.

Example 6-1 Determination of the equilibrium temperature

PROBLEM Compute the equilibrium temperature achieved by constant volume mixing of (1) sodium and mixed oxide (UO_2 –15 weight percent PuO_2) and (2) water and UO_2 for the parameters of Table 6-3.

SOLUTION For the sodium and mixed oxide Eq. 6-4 yields:

$$T_e = \frac{\left[\frac{40,000 (560)}{3500 (1300)} \right] 3100 + 600}{1 + \frac{40,000 (560)}{3500 (1300)}} = 2678^\circ\text{K}$$

Because the critical temperature of sodium is 2733°K , $(x_{st})_e = 0$. The quality is unsubscripted with regard to coolant versus fuel because the fuel remains a subcooled liquid in the examples of the fuel–coolant interaction in this chapter. For simplicity the subscript st is deleted in the remainder of this section. For the water and UO_2 combination:

$$T_e = \frac{\left[\frac{40,000 (560)}{400 (4184)} \right] 3100 + 400}{1 + \frac{40,000 (560)}{4000 (4184)}} = 1945^\circ\text{K}$$

and $x_e = 1.0$, as T_e is above the critical temperature of water, 647°K .

2 Step II: Coolant and fuel expanded as two independent systems, isentropically and adiabatically. Consider the fuel and the coolant each undergoing an adiabatic, isentropic, and hence reversible expansion. Let us first estimate the work done by the coolant when expanding to 1 atmosphere. The control mass and the process representation for the coolant are shown in Figure 6-3.

Note that when expanding to the final state (state 2) some coolant remains in liquid form. Assume that this liquid occupies negligible volume and is incompressible.

From the first law for our control mass neglecting potential and kinetic energy changes we obtain:

$$W_{e \rightarrow 2} = -\Delta U_{e2} \equiv U_e - U_2 \quad (6-5)$$

In order to obtain an analytic solution for the W_{e2} , the internal energy is next expressed by an approximate equation of state, which allows for expansion into the two-phase region. Thus the change in internal energy is expressed as:

$$\Delta u \equiv c_v \Delta T + \Delta(xu_{fg}) \equiv c_v \Delta T + \Delta[x(h_{fg} - pv_{fg})] \quad (6-6)$$

where $u_{fg} = u_g - u_f$, $h_{fg} = h_g - h_f$, and $v_{fg} = v_g - v_f$; c_v and c_p without further subscript refer to liquid specific heats, whereas vapor specific heats are designated c_{v_v} and c_{p_v} . Considering the change in specific internal energy (Δu) as defined in Eq. 6-6, the work $W_{e \rightarrow 2}$ can be written as:

$$W_{e \rightarrow 2} = -m_c \Delta u = -m_c (u_2 - u_e) \quad (6-7)$$

$$W_{e \rightarrow 2} = m_c [c_v(T_e - T_2) + x_e(h_{fg} - pv_{fg})_e - x_2(h_{fg} - pv_{fg})_2]$$

If we neglect the liquid volume at state 2, this equation can be written as:

$$W_{e \rightarrow 2} = m_c \{c_v(T_e - T_2) + [(xh_{fg})_e - (xh_{fg})_2] - [(xpv_{fg})_e - (xpv_{fg})_2]\} \quad (6-9)$$

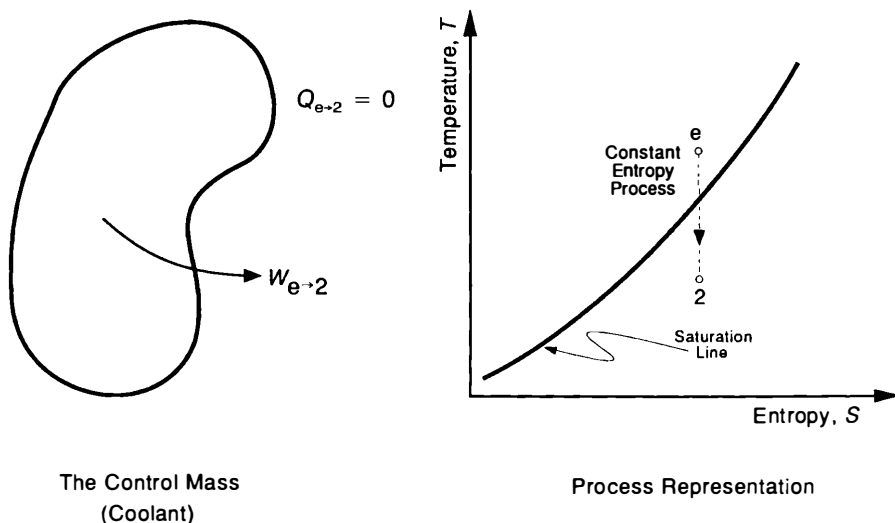


Figure 6-3 Coolant control mass behavior in the adiabatic, isentropic expansion: step II.

To evaluate $W_{e \rightarrow 2}$, we must first determine x_2 , which is done as follows. For a pure substance the following relation exists between thermodynamic properties for two infinitesimally close equilibrium states:

$$Tds = dh - vdp \quad (6-10a)$$

which for an isentropic process can be rewritten as:

$$\left(\frac{dh}{T}\right)_s = v \left(\frac{dp}{T}\right)_s \quad (6-10b)$$

Analogous to the treatment of the internal energy by an approximate equation of state (Eq. 6-7), take h_{fg} and c_p independent of temperature over the range of interest and express the enthalpy change for this isentropic expansion as:

$$dh = c_p dT + h_{fg} dx \quad (6-11)$$

Neglecting the liquid specific volume and applying the perfect gas result:

$$v \equiv v_f + xv_{fg} \approx xv_g \approx x \frac{RT}{p} \quad (6-12)$$

In an isentropic process, the differential of $p(T,s)$ is

$$dp = \left(\frac{\partial p}{\partial T}\right)_s dT$$

If we assume state e is almost at the saturation line, so that the expansion is almost totally under the saturation dome, we can express $\left(\frac{\partial p}{\partial T}\right)_s$ utilizing the Clausius-

Clapeyron relation, which links parameters along the saturation line to the enthalpy and volume of vaporization. Hence:

$$dp = \left(\frac{\partial p}{\partial T}\right)_s dT \approx \left(\frac{\partial p}{\partial T}\right)_{\text{sat}} dT = \frac{h_{fg}}{Tv_{fg}} dT = \frac{ph_{fg}}{RT^2} dT \quad (6-13)$$

where the last step utilizes the assumptions of Eq. 6-12. Substituting Eqs. 6-11, 6-12, and 6-13 into Eq. 6-10 and a rearranging, we obtain:

$$h_{fg} \frac{dx}{T} + c_p \frac{dT}{T} - h_{fg} x \frac{dT}{T^2} = 0$$

which can be written as:

$$d\left(\frac{x}{T}\right) = -\frac{c_p}{h_{fg}} \frac{dT}{T}$$

where h_{fg} and c_p have been taken constant.

Integrating this result between the initial equilibrium temperature of the fuel and coolant and the final coolant temperature, we obtain the expression defining x_2 in terms of the fuel/coolant equilibrium temperature and the final coolant temperature.

$$x_2 = T_2 \left(\frac{x_e}{T_e} + \frac{c_p}{h_{fg}} \ln \frac{T_e}{T_2} \right) \quad (6-14)$$

The work has been calculated for expansion of only the coolant. Next consider the work from the fuel as it reversibly changes volume. The fuel cooldown to a partial pressure which is negligible consistent with a total pressure of one atmosphere would terminate at a temperature different from that achieved in the coolant-only expansion, as Figure 6-4 illustrates. However the work associated with fuel cooldown is small because the fuel remains a subcooled liquid. Hence, it is not evaluated here.

Example 6-2 Determination of final quality and work done by sodium coolant

PROBLEM Evaluate the final quality (x_2) and the work done by the coolant for the sodium case taking the final state (state 2) as the saturation temperature (1154°K) at 1 atmosphere. Use the properties of Table 6-3.

SOLUTION Utilizing Eq. 6-14 and $x_e = 0$ from Example 6-1 yields:

$$x_2 = 1154 \left[0 + \frac{1300}{2.9(10^6)} \ln \frac{2678}{1154} \right] = 0.435$$

Returning to Eq. 6-9, the expansion work done by the coolant is:

$$W = 3500[1300(2678 - 1154) - 0.435(2.9)(10^6) + 0.435(1.013)(10^5)4.11] = 3153 \text{ MJ}$$

where:

$$v_{g_2} \approx \left(\frac{RT}{p} \right)_2 = \frac{361(1154)}{1.013 \times 10^5} = 4.11 \text{ m}^3/\text{kg}$$

and

$$x_e = 0.$$

Examination of the water-fuel case for the properties given in Table 6-3 using Eq. 6-14 shows that the final state is superheated. Because the equilibrium condition has been already shown to be supercritical, use of Eq. 6-7 is inappropriate and Eq. 6-9 for the expansion work must be rederived.

3 Step III: Coolant and fuel expanded as one system in thermal equilibrium, adiabatically and isentropically. Let us now calculate the work considering that the sodium and the fuel remains in thermal equilibrium as the mixture expands adiabatically and isentropically to the final pressure. This process furnishes the upper bound of the expansion work. In this expansion process the coolant-fuel system expands adiabatically and isentropically in thermal equilibrium. Consequently, heat is being removed from the fuel and added to the coolant so that the coolant entropy increases and the fuel entropy decreases an equal amount. Hence the coolant passes into the

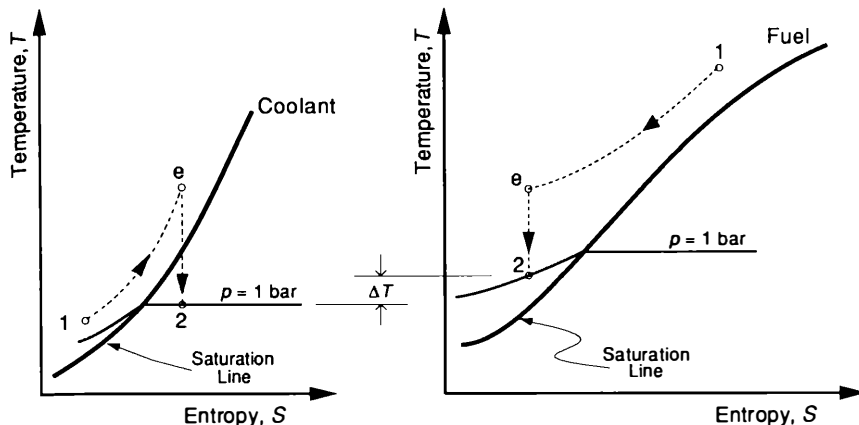


Figure 6-4 Fuel control mass and coolant control mass behavior in the adiabatic isentropic expansion: step II.

two-phase region and may achieve a superheated vapor state. On the other hand, the fuel remains a subcooled liquid throughout the expansion process because the equilibrium temperature is below the fuel boiling point even at 1 atmosphere. The control mass and process representations for the coolant and for the fuel as separate control masses are shown in Figure 6-5, which illustrates the entropy increase of the coolant and the equal entropy decrease of the fuel. For illustration, the final coolant state is shown as superheated vapor. The work (W_{e+2}) is the expansion work performed by the coolant-fuel system. Although the entropy of each component of the mixture changes, the entropy of the mixture is constant, as Figure 6-5 illustrates.

In the analysis we assume that the liquid phase of each component occupies negligible volume and is incompressible. Eq. 6-5 is the relevant first-law formulation but now is applied to the mixture. Considering the mixture specific internal energy (u_m), Eq. 6-5 is written as:

$$W_{e+2} = -m_m \Delta u_m = -(m_c \Delta u_c + m_f \Delta u_f) \quad (6-15)$$

To express the coolant internal energy change (Δu_c), the final state of the coolant (i.e., whether it is a two-phase mixture or a superheated vapor) must be known. Therefore we must evaluate the final coolant quality (x_2) to see if it is less than or greater than unity. (Note that there is no need to subscript the quality as x_{c_2} , as the fuel is always a subcooled liquid.) To do so, we repeat the procedure of Section 2, but now consider the mixture expansion as adiabatic and isentropic.

Expressing Eq. 6-10a for the mixture in this isentropic process:

$$T_m ds_m = dh_m - v_m dp_m = 0 \quad (6-16)$$

From the definition of mixture enthalpy

$$\Delta h_m = \frac{(m_c c_{p_c} + m_f c_{p_f}) \Delta T + m_c \Delta (x h_{fg_c})}{m_c + m_f} \quad (6-17)$$

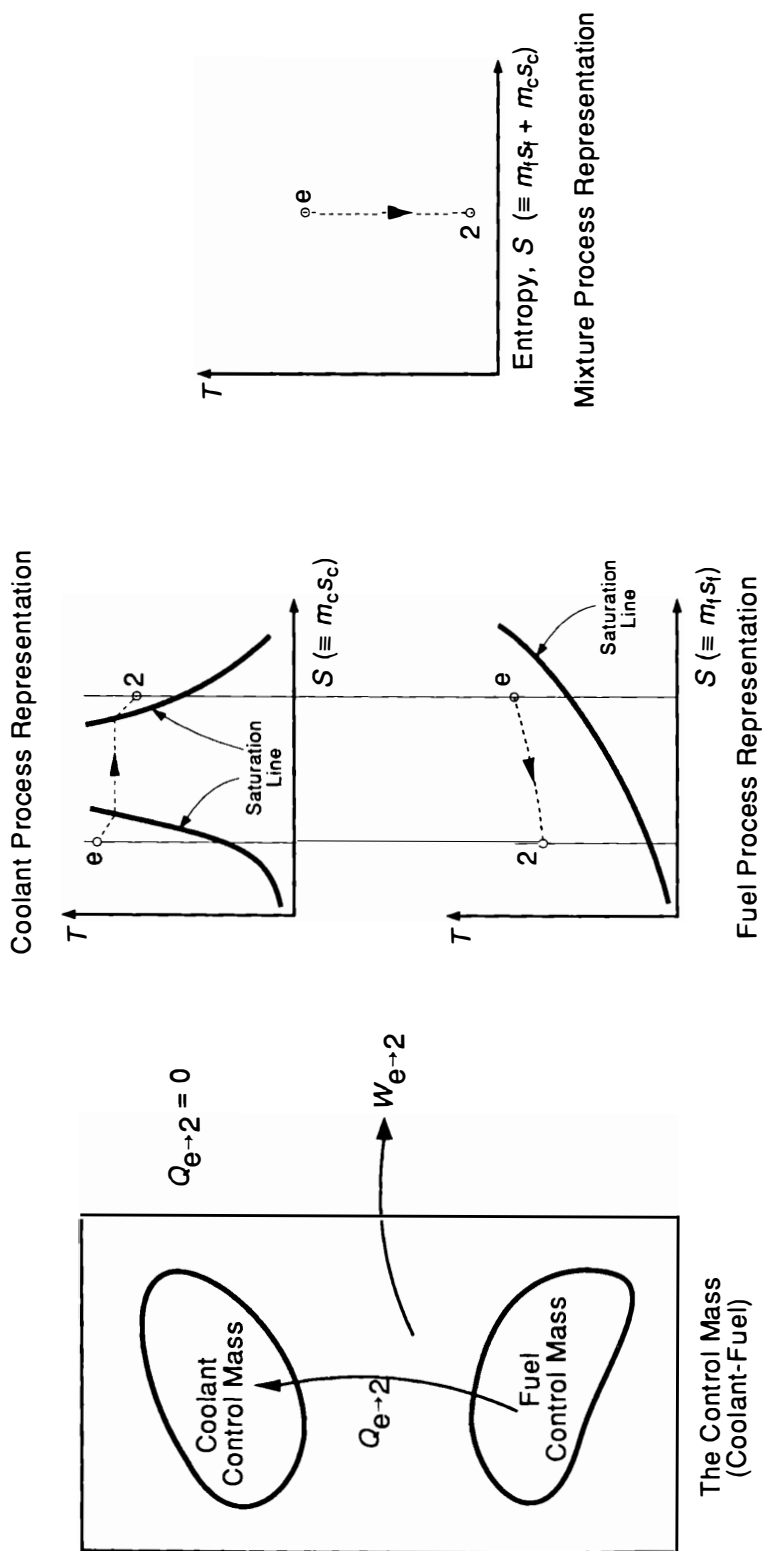


Figure 6-5 Coolant-fuel control mass and coolant-fuel interaction during the adiabatic, isentropic control mass expansion: step II.

Differentiating, assuming h_{fg} constant:

$$dh_m = \frac{(m_c c_{p_c} + m_f c_{p_f})dT + m_c h_{fg} dx}{m_c + m_f} \quad (6-18)$$

Neglecting the fuel volume and the liquid coolant volume:

$$v_m = \frac{x m_c v_{g_c}}{m_c + m_f} = \frac{x R_c T}{p} \left(\frac{m_c}{m_c + m_f} \right) \quad (6-19)$$

Utilizing the Clausius-Clapeyron relation in the same manner as previously, reexpress Eq. 6-13 for the mixture as follows noting that the fuel remains as a subcooled liquid:

$$dp_m = \left(\frac{\partial p}{\partial T} \right)_{\text{sat}} dT = \frac{p h_{fg_c}}{R_c} \frac{dT}{T^2} \quad (6-20)$$

Substituting the above three relations into the state principle (Eq. 6-16) and rearranging, we obtain:

$$(m_c c_{p_c} + m_f c_{p_f}) \frac{dT}{T} + m_c h_{fg_c} d \left(\frac{x}{T} \right) = 0 \quad (6-21)$$

Integrating this result between the initial equilibrium temperature of the fuel and coolant and final mixture temperature, we obtain:

$$x_2 = T_2 \left[\frac{x_e}{T_e} + \left(\frac{m_c c_{p_c} + m_f c_{p_f}}{m_c h_{fg_c}} \right) \ln \frac{T_e}{T_2} \right] \quad (6-22)$$

If evaluation of x_2 indicates that the final state is a superheated vapor, i.e., $x_2 > 1$, Δu_c is written as:

$$\Delta u_c = \Delta u_c (x_e = 0 \text{ to } x = 1) + \Delta u_c (x = 1 \text{ to the actual state 2})$$

which can be written as follows utilizing Eq. 6-7 and the numerical values of x_e and x :

$$\Delta u_c = [c_v(T - T_e) + h_{fg} - p v_{fg}]_c + c_{v_c}(T_2 - T) \quad (6-23)$$

where T = temperature corresponding to x equal to unity, and the subscript on c_v in the second term explicitly indicates that the coolant is in vapor form.

The change in fuel internal energy is

$$\Delta u_f = c_{v_f}(T - T_e) + c_{v_f}(T_2 - T) \quad (6-24)$$

where c_{v_f} is the specific heat at constant volume for liquid fuel.

To evaluate Δu_c and Δu_f , the final temperature (T_2) must be determined in the superheat region. It is done again using the state principle (Eq. 6-16) but now considering only that portion of the expansion process in which the coolant state is a superheated vapor. Therefore Eq. 6-18 is written as:

$$dh_m = \frac{(m_c c_{p_{vc}} + m_f c_{p_f}) dT}{m_c + m_f} \quad (6-25)$$

where again c_{pvc} explicitly indicates that the coolant is in vapor form, and Eq. 6-19 is written for $x = 1$ as:

$$v_m = \frac{R_c T}{p} \left(\frac{m_c}{m_c + m_f} \right) \quad (6-26)$$

Substituting Eqs. 6-25 and 6-26 into Eq. 6-16, we obtain the following relation between mixture temperature and pressure:

$$\left(\frac{m_c c_{p,c} + m_f c_{p,f}}{m_c R_c} \right) \frac{dT}{T} = \frac{dp}{p} \quad (6-27)$$

Integration of Eq. 6-27 to the unknown final state p_2, T_2 yields:

$$T_2 = T \left(\frac{p_2}{p} \right)^{1/n} \quad (6-28)$$

where n = the term in brackets in Eq. 6-27, which can be written in terms of coolant vapor properties as:

$$n \equiv \frac{m_c \frac{\gamma R_c}{\gamma - 1} + m_f c_{p,f}}{m_c R_c} = \frac{m_c c_{p,v} + m_f c_{p,f}}{m_c R_c} \quad (6-29)$$

Although T ($x = 1$) can be determined by expressing Eq. 6-22 for $x = 1$ and this corresponding T , the corresponding p is the coolant partial pressure, not the mixture pressure. However, because the fuel is liquid, its partial pressure is negligible. Therefore we can evaluate T_2 from Eq. 6-28 with negligible error. Finally, returning to evaluation of the expansion work, it can be expressed by applying Eqs. 6-23 and 6-24 to 6-15, yielding:

$$W = W(\text{to } x = 1) + W(x = 1 \text{ to } T_2) = m_c [c_v(T_e - T) - h_{fg} + R_c T]_c + m_f c_{v,f}(T_e - T) + m_c c_{v,c}(T - T_2) + m_f c_{v,f}(T - T_2) \quad (6-30)$$

where the coolant vapor is taken as perfect gas allowing (pv) to be replaced by $R_c T$.

Example 6-3 Determination of final quality, temperature, and expansion work for the sodium-mixed oxide combination

PROBLEM For the sodium-mixed oxide combination, evaluate the final quality and temperature (x_2 and T_2) and the expansion work using the parameters of Table 6-3.

SOLUTION First evaluate x_2 from Eq. 6-22, assuming initially that state 2 is a two-phase state at 1 atmosphere. For this assumption T_2 is the sodium saturation temperature corresponding to 1 atmosphere or 1154°K. The quality (x_2) is obtained from Eq. 6-22 as:

$$x_2 = 1154 \left[0 + \frac{40,000(560) + 3500(1300)}{(3500) 2.9(10^6)} \ln \frac{2678}{1154} \right] = 2.58$$

Because $x_2 > 1$, this final state is superheated, and our original assumption is false. Therefore it becomes necessary to find the state at which $x = 1$. Returning to Eq. 6-22 but defined for the state $x = 1$ and the corresponding temperature T , yielding:

$$1 = T \left[0 + \frac{40,000(560) + 3500(1300)}{(3500)2.9(10^6)} \ln \frac{2678}{T} \right]$$

Solving for T yields 2250°K, which has a corresponding sodium vapor pressure of 158 bars.

T_2 can now be evaluated from Eq. 6-28 utilizing n obtained from Eq. 6-29. These steps yield:

$$T_2 = 2250 \left(\frac{1}{158} \right)^{(1/25.4)} = 1843^\circ\text{K}$$

because:

$$n = \frac{3.5(2767) + 40(560)}{3.5(361)} = 25.4$$

Finally, evaluating the expansion work by Eq. 6-30 yields:

$$\begin{aligned} W &= 3500[1300(2678 - 2250) - 2.9 \times 10^6 + (361)(2250)] \\ &\quad + 40,000(560)(2678 - 2250) + 3500(2410)(2250 - 1843) \\ &\quad + 40,000(560)(2250 - 1843) \\ &= -5360 + 9587 + 3433 + 9117 \\ &= 16,777 \text{ MJ} \end{aligned}$$

Note that the work evaluated by Example 6-3 is greater than that from Example 6-2 because work is dependent on the process path and the initial and end states. Although both processes have the same initial fuel and coolant states and are reversible, the process paths as well as the final states of the fuel and coolant are different. Consequently, there is no reason to expect that expansion work integrals performed to different final states should be the same. For the adiabatic and thermal equilibrium cases, the work is evaluated by Eq. 6-9 and 6-30, respectively. A comparison of the numerical values of the terms of each equation is given in Table 6-4 to illustrate the origin of the differences in net work. In the thermal equilibrium case, the terms involving coolant properties and those involving fuel properties should not be interpreted as the work contributions due to the coolant and the fuel separately.

Table 6-4 Comparison between results of Examples 6-2 and 6-3

Parameter	Fuel and coolant expansion: two independent systems (Eq. 6-9)	Fuel-coolant expansion: one-system (Eq. 6-30)
Terms involving coolant properties	+ 3153 MJ	- 5360 + 3433 = - 1927 MJ
Terms involving fuel properties	0	+ 9587 + 9117 = + 18,704 MJ
Net work	+ 3153 MJ	+ 16,777 MJ

III THERMODYNAMIC ANALYSIS OF NUCLEAR POWER PLANTS

The analysis of nuclear power plants represents a prime application of the methods for thermodynamic analysis of steady-flow processes. The results of such analyses determine the relation between the mixed mean outlet coolant temperatures (or enthalpy for BWR) of the core through the primary and secondary systems to the generation of electricity at the turbine. The variety of possible reactor systems, with their associated coolants, leads to a corresponding multiplicity of primary and secondary system configurations. In addition, because gaseous reactor coolants can be used directly to drive electric turbines, the Brayton cycle can be considered for gas-cooled reactor systems, whereas systems that use steam-driven electric turbines employ a Rankine cycle. The various cycles employed with the principal reactor types have been described in Chapter 1. In this section the cycles used for the various reactor types and the methods of thermodynamic analysis of these cycles are described.

The Rankine and Brayton cycles are constant-pressure heat addition and rejection cycles for steady-flow operation. They differ regarding the phase changes the working fluid undergoes. In the Rankine cycle the working fluid is vaporized and condensed, whereas in the Brayton cycle the working fluid remains a single gaseous phase as it is heated and cooled. In central station nuclear Rankine cycles, water is employed as the working fluid, whereas helium is used in proposed Brayton cycles.

The PWR and BWR employ the water Rankine cycle. Because the PWR limits the reactor coolant to a nominal saturated mixed mean core outlet condition, the vapor that drives the turbine must be generated in a steam generator in a secondary system. A simplified pressurized water reactor two-coolant system is illustrated in Figure 6-6. A steam generator links these primary and secondary systems. Figure 6-7 illustrates the temperature distribution within such a recirculating PWR steam generator. The mixed mean core outlet condition (state 5) is dictated by the allowable core performance, particularly material corrosion limits.

As the temperature is raised, the primary loop pressure must also be raised to maintain state 5 at the nominally saturated condition. Establishment of the optimum

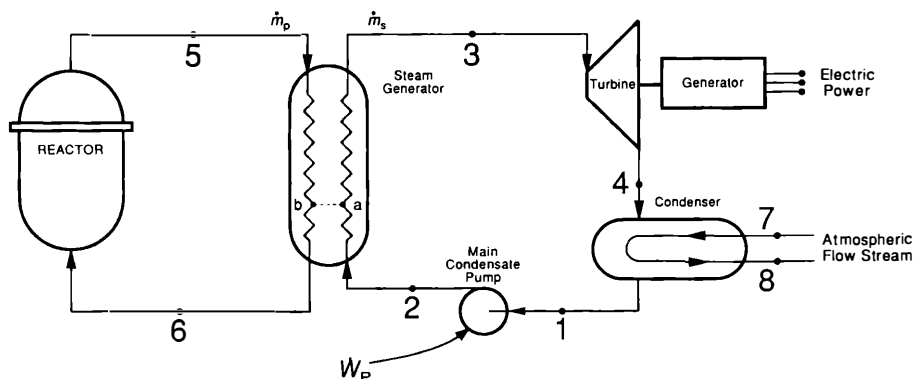


Figure 6-6 Simplified PWR plant.

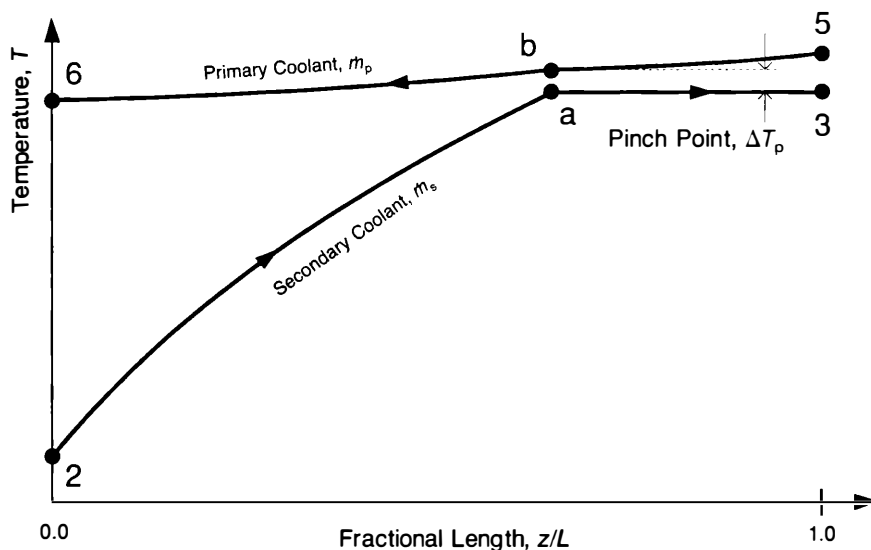


Figure 6-7 Temperature distribution within the steam generator of the simplified PWR plant of Example 6-4.

primary pressure in current PWRs at 2250 psia (15.5 MPa) has involved many considerations, among them the piping and reactor vessel wall dimensions and the pressure dependence of the critical heat flux limit. The secondary side temperature and hence pressure are related to these primary side conditions, as suggested in Figure 6-7. This relation is demonstrated in section IV, below.

The BWR employs a direct Rankine cycle. The reactor is itself the steam generator, so that the mixed mean core outlet is also nominally saturated. The outlet temperature and pressure conditions are thus established by both the allowable core and secondary loop design conditions. As Table 2-1 illustrates, the Rankine working fluid conditions (at turbine entrance) for both PWRs and BWRs are close, i.e., 5.7 MPa and 7.17 MPa, respectively.

The simple Brayton cycle is illustrated in Figure 6-8. There are four components, as in the secondary system of the PWR plant operating under the Rankine cycle. The pictured Brayton cycle compressor and heat exchanger perform functions analogous to those of the Rankine cycle condensate pump and condensor. In practice, the maximum temperature of the Brayton cycle (T_3) is set by turbine blade and gas-cooled reactor core material limits far higher than those for the Rankine cycle, which is set by liquid-cooled reactor core materials limits. The Brayton cycle in its many possible variations is presented in Section VI, below.

It is useful to assess the thermodynamic performance of components and cycles using nondimensional ratios called *efficiencies*. Three definitions for efficiency are considered:

Thermodynamic efficiency (or effectiveness) (ζ)

Isentropic efficiency (η_s)

Thermal efficiency (η_{th})

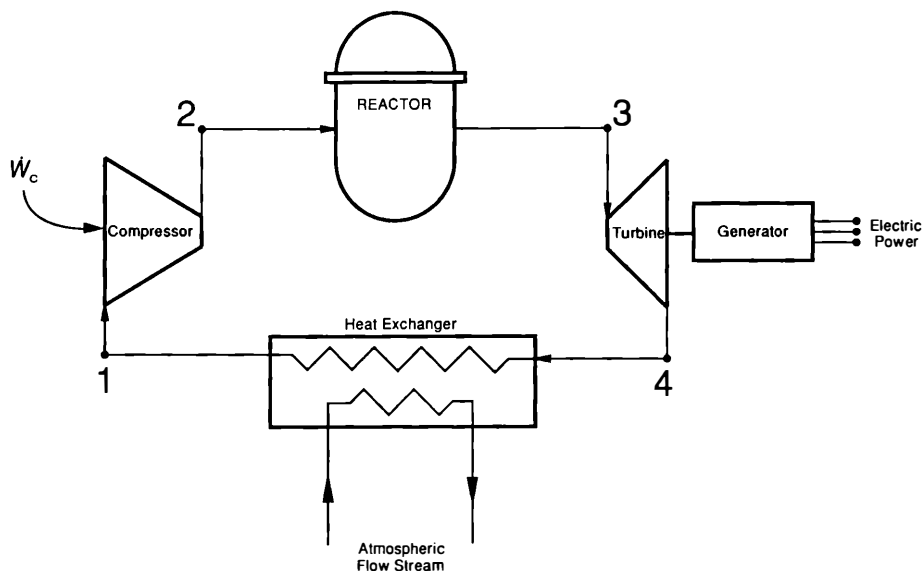


Figure 6-8 Simple Brayton cycle.

The *thermodynamic efficiency* or effectiveness is defined as:

$$\zeta \equiv \frac{\dot{W}_{u, \text{actual}}}{\dot{W}_{u, \text{max}}} \quad (6.31)$$

where $\dot{W}_{u, \text{max}}$ is defined by Eq. 4-46, which for the useful case of a fixed, nondeformable control volume with zero shear work and negligible kinetic and potential energy differences between the inlet and outlet flow streams takes the form:

$$\dot{W}_{u, \text{max}} = - \left[\frac{\partial(U - T_o S)}{\partial t} \right] + \sum_{i=1}^I \dot{m}_i (h - T_o s)_i + \left(1 - \frac{T_o}{T_s} \right) \dot{Q} \quad (6.32)$$

where T_s = temperature at which heat is supplied and $\left(\frac{dQ}{dt} \right)_{\text{gen}}$ is treated as part of $\frac{dU}{dt}$.

The *isentropic efficiency* is defined as:

$$\eta_s = \left(\frac{\dot{W}_{u, \text{actual}}}{\dot{W}_{u, \text{max}}} \right)_{\dot{Q}=0} \quad (6.33)$$

Obviously, for an adiabatic control volume, $\zeta = \eta_s$. The quantity $\dot{W}_{u, \text{max}}|_{\dot{Q}=0}$ is the useful, maximum work associated with a reversible adiabatic (and hence isentropic) process for the control volume. It can be expressed from Eq. 6-32 as:

$$\dot{W}_{u,\max|\dot{Q}=0} = - \left[\frac{\partial U}{\partial t} \right]_{c.v.} + \sum_{i=1}^I \dot{m}_i h_{is} \quad (6-34)$$

which for steady-state conditions becomes:

$$\dot{W}_{u,\max|\dot{Q}=0} = \sum_{i=1}^I \dot{m}_i h_{is} \quad (6-35)$$

The useful, actual work of an adiabatic nondeformable control volume with zero shear and negligible kinetic and potential energy differences between the inlet and outlet flow streams for steady-state conditions from Eq. 4-44 (treating \dot{Q}_{gen} as part of \dot{E}) is:

$$\dot{W}_{u,\text{act}} = \sum_{i=1}^I \dot{m}_i h_i \quad (6-36)$$

and the isentropic efficiency becomes:

$$\eta_s = \frac{\sum_{i=1}^I \dot{m}_i h_i}{\sum_{i=1}^I \dot{m}_i h_{is}} \quad (6-37)$$

Finally, the *thermal efficiency* is defined as:

$$\eta_{\text{th}} = \frac{\dot{W}_{u,\text{act}}}{\dot{Q}_{\text{in}}} \quad (6-38)$$

where \dot{Q}_{in} = rate of heat addition to the control volume. For adiabatic systems the thermal efficiency is not a useful measure of system performance.

These efficiencies are now evaluated for nuclear plants. Typical plants employing the Rankine and Brayton cycles have been presented in Figures 6-6 and 6-8, respectively. Such plants have two interactions with their surroundings: a net work output as electricity and a flow cooling stream that is in mutual equilibrium with the atmosphere. Hence the availability of the inlet and the outlet streams are zero, i.e.,

$$\sum_{i=1}^I \dot{m}_i (h - T_o s)_i = 0$$

Note, however, that the energy and entropy of the stream change as the stream passes through the plant condenser or heat exchanger, but the stream remains at pressure p_o and temperature T_o .

In section IV, each component of typical plants is analyzed using a stationary, nondeformable control volume with zero shear work and negligible kinetic and potential energy differences between the inlet and the outlet flow streams. These results are then utilized for evaluating the entire nuclear plant modeled by one adiabatic control volume, as illustrated in Figure 6-9A. Two other control volume representations of the complete nuclear plant are illustrated in Figure 6-9B,C.

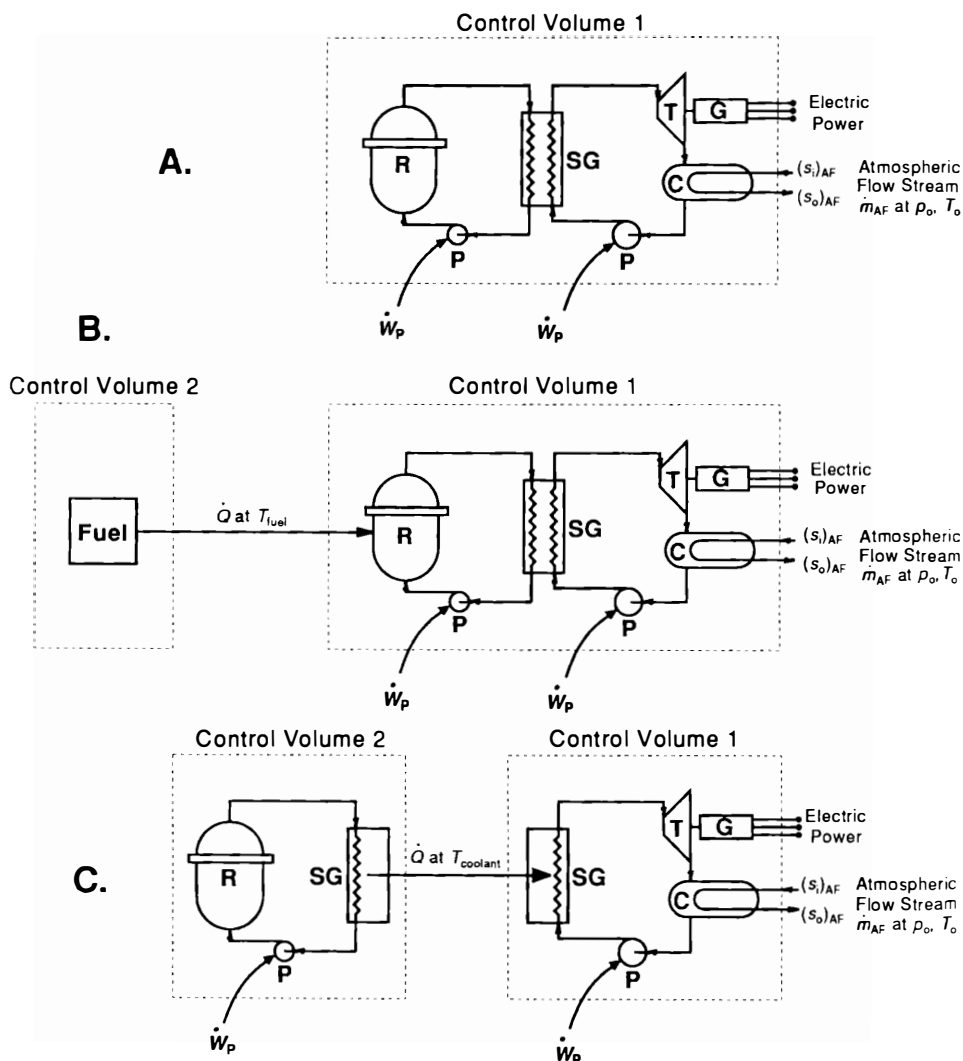


Figure 6-9 Alternative control volume representations of a batch-fueled reactor plant.

Let us now contrast the computed effectiveness and thermal efficiencies for the three nuclear plant representations of Figure 6-9. In section IV, the maximum useful work for control volume 1 of Figure 6-9A is shown equal to the fission rate, which equals the coolant enthalpy rise across the reactor. Hence:

$$\dot{W}_{u,\max} = \dot{m}_p(h_{\text{out}} - h_{\text{in}})_{\text{reactor}}$$

(see Eq. 6-53). The effectiveness of the nuclear plant is then:

$$\zeta = \frac{\dot{W}_{u,\text{actual}}}{\dot{m}_p(h_{\text{out}} - h_{\text{in}})_{\text{reactor}}} \quad (6-39)$$

In Figure 6-9B the same nuclear plant is modeled by two control volumes. Control volume 1 encompasses the reactor plant except for the fuel loading, which is segregated into the second control volume. The energy liberated by fission is transferred from the second control volume to the first control volume as heat (\dot{Q}) at the temperature of the fuel. Because \dot{Q} equals the fission rate:

$$\dot{Q} = \dot{m}_p(h_{\text{out}} - h_{\text{in}})_{\text{reactor}} \quad (6-40)$$

and

$$\eta_{\text{th|c.v.1}} = \frac{\dot{W}_{\text{u,actual}}}{\dot{Q}} = \zeta_{\text{c.v.1}} \quad (6-41)$$

(Fig. 6-9B) (Fig. 6-9A)

Finally, the same nuclear plant is alternatively modeled as shown in Figure 6-9C, with the reactor (including the fuel) and the primary side of the steam generator located within control volume 2. In this case, the same heat transfer rate (\dot{Q}) to control volume 1 occurs across the steam generator but now at the coolant temperature, so that:

$$\eta_{\text{th|c.v.1}} = \eta_{\text{th|c.v.1}} = \zeta_{\text{c.v.1}} \quad (6-42)$$

(Fig. 6-9C) (Fig. 6-9B) (Fig. 6-9A)

Figure 6-9A is the only control volume configuration in which the entire nuclear plant is contained within one control volume. The effectiveness of the portions of the reactor plant contained within the three control volumes designated c.v.1 are not identical because the maximum, useful work associated with each of these control volumes is different. In section IV.B we show how transfer of the fission energy to the reactor coolant may be modeled as a series of three heat interactions that bring the fission energy from T_{fission} to T_{coolant} . Consequently:

$$\dot{W}_{\text{u,max|c.v.1}} > \dot{W}_{\text{u,max|c.v.1}} > \dot{W}_{\text{u,max|c.v.1}} \quad (6-43)$$

(Fig. 6-9A) (Fig. 6-9B) (Fig. 6-9C)

Hence the effectiveness of the portion of the plant within control volume 1 in each of the three configurations are related as:

$$\zeta_{\text{c.v.1}} > \zeta_{\text{c.v.1}} > \zeta_{\text{c.v.1}} \quad (6-44)$$

(Fig. 6-9C) (Fig. 6-9B) (Fig. 6-9A)

In this text, we often refer to the entire nuclear plant, and therefore the representation in Figure 6-9A appears and the term thermodynamic efficiency or effectiveness is used. However, by virtue of the equalities in Eq. 6-42, it is also possible and useful to alternatively refer to the thermal efficiency of the cycle, which is also done. The previous examples illustrate the conditions under which the computation of various efficiencies lead to the same numerical values.

Finally, it is of interest to recall a case in which the effectiveness and the thermal efficiency do not coincide: For the case of reversible cycles operating between a heat source at T_{high} and a reservoir at T_{low} , the thermal efficiency equals the Carnot efficiency. On the other hand, the effectiveness equals unity, as $\dot{W}_{\text{u,max}} = \dot{W}_{\text{u,actual}}$ for these cycles.

IV THERMODYNAMIC ANALYSIS OF A SIMPLIFIED PWR SYSTEM

The steady-state thermodynamic analysis of power plant systems is accomplished by considering in turn the components of these systems and applying to each the control volume form of the first and second laws. Let us examine the simplified PWR two-coolant system shown in Figure 6-6. We wish to develop the procedures for evaluating the thermodynamic states and system flow rates so that the overall plant thermodynamic efficiency and sources of lost work or irreversibilities can be computed from a combined first and second law analysis. At the outset assume for simplicity that pressure changes occur only in the turbine and condensate pumps.

A First Law Analysis of a Simplified PWR System

Consider any one component with multiple inlet and outlet flow streams operating at steady state and surround it with a nondeformable, stationary control volume (Fig. 6-10). Applying the first law (Eq. 6-2) to this control volume we obtain:

$$\sum_{k=1}^I (\dot{m}h)_{in,k} - \sum_{k=1}^I (\dot{m}h)_{out,k} = \dot{W}_{shaft} - \dot{Q} \quad (6-45)$$

where the notation of the summations has been generalized from that of Eq. 4-30b to specifically identify the multiple inlet and outlet flow streams, where for steady state:

$$\dot{m} = \sum_{k=1}^I \dot{m}_{in,k} = \sum_{k=1}^I \dot{m}_{out,k}$$

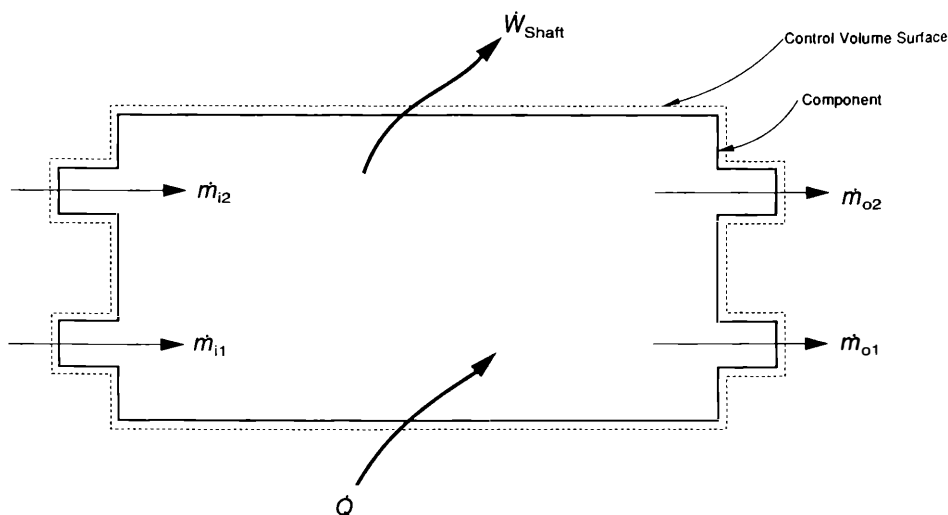


Figure 6-10 Generalized fixed-volume component with surrounding control volume.

Table 6-5 Forms of first law (Eq. 6-2) for PWR plant components

Component	Desired quantity	Assumptions	Resultant equation	Consequence of sign convention on the desired parameter
Turbine	\dot{W}_{shaft}	$\dot{Q}_2 = 0$ $\Delta(\frac{1}{2} v^2) = 0$	$\dot{W}_T^* = [\dot{m}(h_{\text{in}} - h_{\text{out}})]_T$ $= [\dot{m}\eta_T(h_{\text{in}} - h_{\text{out},s})]_T$	\dot{W}_T Positive (work out)
Pump	\dot{W}_{shaft}	$\dot{Q}_2 = 0$ $\Delta(\frac{1}{2} v^2) = 0$	$\dot{W}_P = [\dot{m}(h_{\text{in}} - h_{\text{out}})]_P$ $= \left[\frac{\dot{m}}{\eta_P} (h_{\text{in}} - h_{\text{out},s}) \right]_P$	\dot{W}_P Negative (work in)
Condensor steam generator, any feed water heater	h_o of one stream, e.g., h_{o1}	$\dot{Q} = 0$ $\dot{W}_{\text{shaft}} = 0$ $\Delta(\frac{1}{2} v^2) = 0$	$h_{\text{out},1} = \frac{\sum_{k=1}^I (\dot{m}h)_{\text{in},k} - \sum_{k=2}^I (\dot{m}h)_{\text{out},k}}{\dot{m}_{\text{out},1}}$	$h_{\text{out},1} - h_{\text{in},1}$ positive (stream 1 is heated)

*Because the control volumes are nondeformable, the shaft is dropped on \dot{W} and replaced by a subscript describing the component.

Equation 6-45 can be further specialized for the components of interest. In particular for the components of Figure 6-6, heat addition from an external source can be neglected. For all components except the turbine and the pump, no shaft work exists. Table 6-5 summarizes the results of specializing Eq. 6-45 to the components of Figure 6-6. The adiabatic turbine actual work is related to the ideal work by the component isentropic efficiency defined by Eq. 6-33. From Equation 6-37, which is a specialized form of Eq. 6-33, the turbine efficiency is:

$$\eta_T = \frac{h_{\text{in}} - h_{\text{out}}}{h_{\text{in}} - h_{\text{out},s}} \quad (6-46)$$

where

$$h_{\text{out},s} = f[p(h_{\text{out}}), s_{\text{in}}]. \quad (6-47)$$

Because work is supplied to the pump, the isentropic efficiency of the pump is analogously defined as:

$$\eta_P = \frac{\text{ideal work required}}{\text{actual work required}} = \frac{h_{\text{in}} - h_{\text{out},s}}{h_{\text{in}} - h_{\text{out}}} \quad (6-48)$$

The primary and secondary flow rates are related by application of the first law to the component they both flow through, i.e., the steam generator. Considering only one primary and one secondary flow stream, the first law can be used to express the ratio of these two flow rates. The first law (Eq. 6-45) for the steam generator yields:

$$\frac{\dot{m}_p}{\dot{m}_s} = \frac{(h_{\text{out}} - h_{\text{in}})_s}{(h_{\text{in}} - h_{\text{out}})_p} \quad (6-49)$$

where subscripts p and s = primary and secondary flow streams, respectively.

It is often useful to apply Eq. 6-49 to several portions of the steam generator. In Figure 6-7 temperatures are plotted versus fractional length assuming constant pressure for a recirculating-type PWR steam generator producing saturated steam. The minimum primary to secondary temperature difference occurs at the axial location of the onset of bulk boiling in the secondary side. All temperatures are related to this minimum, or pinch point, temperature difference. The steam generator heat transfer area is inversely related to this minimum temperature difference for a given heat exchange capacity. Also the irreversibility of the steam generator is directly related to this temperature difference. Therefore the specification of the pinch point temperature difference is an important design choice based on tradeoff between cost and irreversibility.

The ratio of flow rates in terms of the pinch point temperature difference (ΔT_p) is, from the first law:

$$\frac{\dot{m}_p}{\dot{m}_s} = \frac{h_3 - h_a}{\bar{c}_p[T_5 - (T_a + \Delta T_p)]} = \frac{h_a - h_2}{\bar{c}_p[(T_a + \Delta T_p) - T_6]} \quad (6-50)$$

where \bar{c}_p = average coolant specific heat over the temperature range of interest.

For the reactor plant, $\dot{W}_{u,\text{actual}}$ is the net rate of work generated, i.e.,

$$\dot{W}_{u,\text{actual}} = \dot{W}_T + \dot{W}_P = [\dot{m}_s(h_{\text{in}} - h_{\text{out}})]_T + [\dot{m}_s(h_{\text{in}} - h_{\text{out}})]_P \quad (6-51)$$

$$= [n_T \dot{m}_s(h_{\text{in}} - h_{\text{out},s})]_T + \left[\frac{\dot{m}_s}{\eta_P} (h_{\text{in}} - h_{\text{out},s}) \right]_P \quad (6-52)$$

For a pump $h_{\text{in}} < h_{\text{out},s}$, whereas for a turbine $h_{\text{in}} > h_{\text{out},s}$. Hence \dot{W}_P is negative, indicating, per our sign convention presented in Tables 6-1 and 6-5, that work is supplied to the pump. Analogously, \dot{W}_T is positive, and the turbine delivers work. The familiar expression of the maximum useful work of the nuclear plant that is derived in Section IV.B is

$$\dot{W}_{u,\text{max}} = \dot{m}_p(h_{\text{out}} - h_{\text{in}})_R = \dot{m}_p(h_5 - h_6) \quad (6-53)$$

for Figure 6-6. It is desirable to reexpress $\dot{W}_{u,\text{max}}$ in terms of secondary side conditions. To do it, apply the first law (Eq. 6-45) to the PWR steam generator of Figure 6-6, yielding:

$$\dot{m}_p(h_{\text{in}} - h_{\text{out}})_{\text{SGp}} = \dot{m}_s(h_{\text{out}} - h_{\text{in}})_{\text{SGs}} \quad (6-54)$$

From Figure 6-6 observe that the primary pump work is neglected so that $(h_{\text{out}})_R$ is identically $(h_{\text{in}})_{\text{SGp}}$ and $(h_{\text{in}})_R$ is identically $(h_{\text{out}})_{\text{SGp}}$. Hence from Eqs. 6-53 and 6-54:

$$\dot{W}_{u,\text{max}} = \dot{m}_p(h_{\text{out}} - h_{\text{in}})_R = [\dot{m}_s(h_{\text{out}} - h_{\text{in}})]_{\text{SGs}} \quad (6-55)$$

or

$$\dot{W}_{u,\text{max}} = \dot{m}_p(h_5 - h_6) = \dot{m}_s(h_3 - h_2) \quad (6-56)$$

for Figure 6-6. Hence utilizing Eqs. 6-31, 6-51 and 6-55, the overall nuclear plant thermodynamic efficiency or effectiveness (ζ) is:

Table 6-6 PWR operating conditions for Example 6-4

State	Temperature °R (°K)	Pressure psia (kPa)	Condition
1	—	1 (6.89)	Saturated liquid
2	—	1124 (7750)	Subcooled liquid
3	—	1124 (7750)	Saturated vapor
4	—	1 (6.89)	Two-phase mixture
5	1078.2 (599)	2250 (15,500)	Subcooled liquid
6	1016.9 (565)	2250 (15,500)	Subcooled liquid
7	—	—	Subcooled liquid
8	—	—	Subcooled liquid
a	—	1124 (7750)	Saturated liquid
b	$T_a + 26$ ($T_a + 14.4$)	2250 (15,500)	Subcooled liquid

$$\zeta = \frac{[\dot{m}_s(h_{in} - h_{out})]_T + [\dot{m}_s(h_{in} - h_{out})]_P}{[\dot{m}_s(h_{out} - h_{in})]_{SGS}} \quad (6-57)$$

For the plant of Figure 6-6, with the entire secondary flow passing through a single turbine, Eq. 6-57 becomes:

$$\zeta = \frac{h_3 - h_4 + h_1 - h_2}{h_3 - h_2} \quad (6-58)$$

Example 6-4 Thermodynamic analysis of a simplified PWR plant

PROBLEM The PWR plant of Figure 6-6 operates under the conditions given in Table 6-6. Assume that the turbine and pump have isentropic efficiencies of 85%.

1. Draw the temperature-entropy (T - s) diagram for this cycle.
2. Compute the ratio of the primary to secondary flow rates.
3. Compute the nuclear plant thermodynamic efficiency.
4. Compute the cycle thermal efficiency.

SOLUTION

1. The T - s diagram is sketched in Figure 6-11. Note that states 4 and 2 reflect the fact that the turbine and the pump are not 100% efficient. Also, states on the primary side of the steam generator are shown at higher temperatures than the corresponding (same x/L position) states on the second side.

The T versus x/L diagram is the same as Figure 6-7.

If a primary system main coolant pump were to be included, the reactor system and its representation on a T - s diagram would be as sketched in Figure 6-12, where the primary system pressure loss is taken as 0.48 MPa (70 psia).

2. From Eq. 6-50:

$$\frac{\dot{m}_p}{\dot{m}_s} = \frac{h_3 - h_a}{\bar{c}_p[T_5 - (T_a + \Delta T_p)]}$$

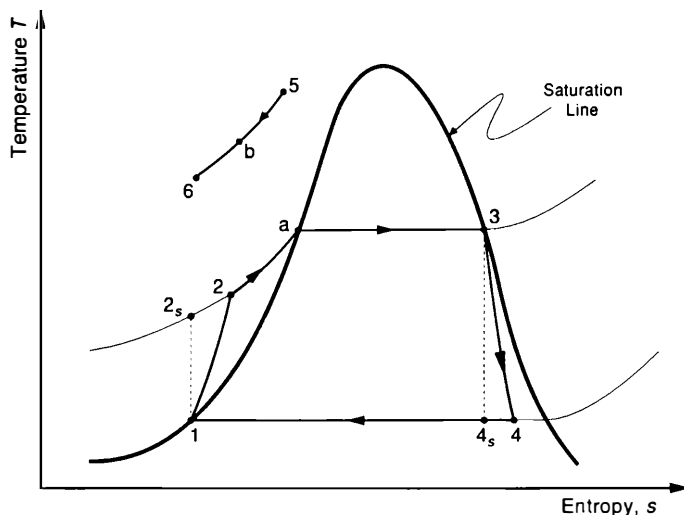


Figure 6-11 T - s diagram for PWR cycle, Example 6-4.

From steam tables:

$$h_3 = h_g(\text{sat. at } 1124 \text{ psia}) = 1187.29 \text{ BTU/lbm } (2.771 \text{ MJ/kg})$$

$$h_a = h_f(\text{sat. at } 1124 \text{ psia}) = 560.86 \text{ BTU/lbm } (1.309 \text{ MJ/kg})$$

$$T_a = \text{sat. liquid at } 1124 \text{ psia} = 1018.8 \text{ R } (566.0^\circ\text{K})$$

Hence:

$$\frac{\dot{m}_p}{\dot{m}_s} = \frac{1187.29 - 560.86}{1.424 [1078.2 - (1018.8 + 26)]} = 13.18$$

in SI:

$$\frac{\dot{m}_p}{\dot{m}_s} = \frac{2.77 \times 10^6 - 1.309 \times 10^6}{5941 [599 - (566 + 14.4)]} = 13.18$$

3. The nuclear plant thermodynamic efficiency (ζ) for the plant within control volume 1 of Figure 6-9A, is given by Eq. 6-39. Draw a control volume around the PWR plant of interest in Figure 6-6. The result is an arrangement identical to that of Figure 6-9A. For this PWR plant, Eq. 6-39 reduces to:

$$\zeta = \frac{h_3 - h_4 + h_1 - h_2}{h_3 - h_2} \quad (6-58)$$

Proceed to obtain the required enthalpies.

From the steam tables:

$$h_1 = h_f(\text{sat. at } 1 \text{ psia}) = 69.74 \text{ BTU/lbm } (0.163 \text{ MJ/kg})$$

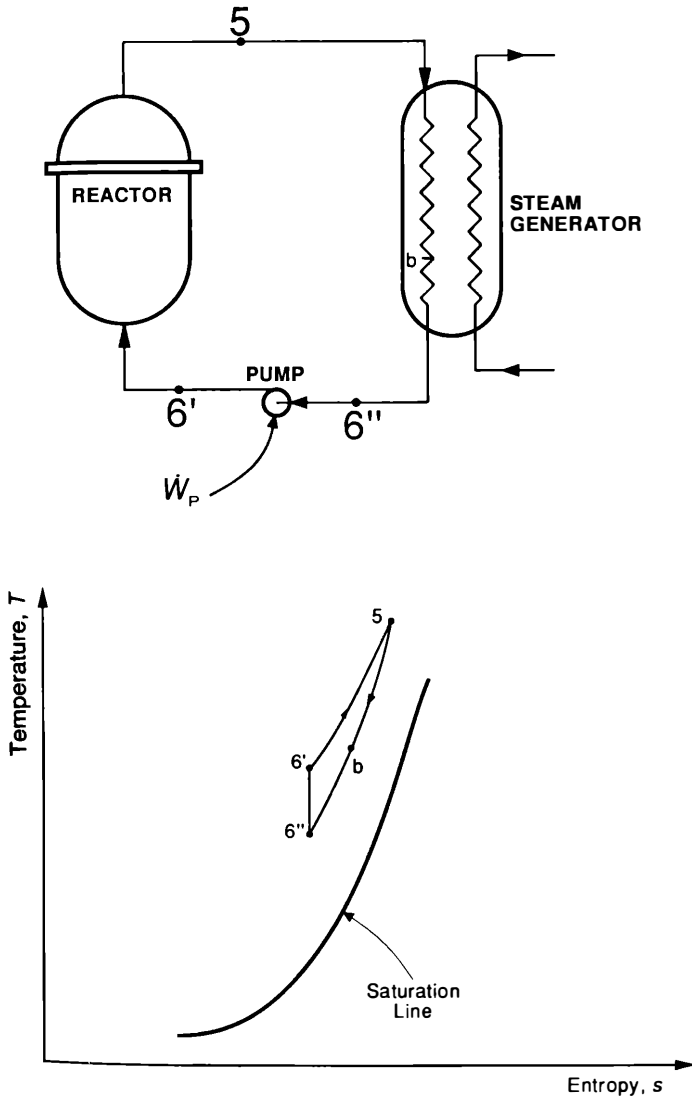


Figure 6-12 Primary system condition of Example 6-4.

From the definition of component isentropic efficiencies:

$$h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_p}$$

$$h_4 = h_3 - \eta_T(h_3 - h_{4s})$$

State 2:

$$s_{2s} = s_1 = 0.13266 \text{ BTU/lbm}^\circ\text{R} \text{ (557 J/kg K)}$$

$$p_{2s} = p_2 = 1124 \text{ psia (7.75MP}_a\text{)}$$

From the subcooled liquid tables by interpolation:

$$\begin{aligned}
 h_{2s} &= 73.09 \text{ BTU/lbm (0.170 MJ/kg)} \\
 h_2 &= 69.74 + \frac{73.09 - 69.74}{0.85} = 73.69 \text{ BTU/lbm} \\
 &= 0.163 + \frac{0.170 - 0.163}{0.85} = 0.171 \text{ MJ/kg}
 \end{aligned}$$

Alternately, considering the liquid specific volume constant and the compression process isothermal, h_2 can be evaluated as:

$$\begin{aligned}
 h_2 &= h_1 + v_1 \frac{(p_2 - p_1)}{\eta_p} \\
 &= 69.74 + \frac{0.016136(1123)}{0.85} \frac{144}{778} = 73.69 \text{ BTU/lbm} \\
 &= 0.163 + \frac{7.75 - 0.0069}{0.85(993)} = 0.171 \text{ MJ/kg}
 \end{aligned}$$

State 4:

$$\begin{aligned}
 s_{4s} &= s_3 = 1.3759 \text{ BTU/lbm (5756 J/kg}^\circ\text{K)} \\
 x_{4s} &= \frac{s_{4s} - s_f}{s_{fg}} = \frac{1.3759 - 0.13266}{1.8453} = 0.674 \\
 h_{4s} &= h_f + x_{4s}h_{fg} = 69.74 + 0.674(1036.0) \\
 &= 768.00 \text{ BTU/lbm (1.79 MJ/kg)} \\
 h_4 &= h_3 - \eta_T(h_3 - h_{4s}) = 1187.29 - 0.85(1187.29 - 768.00) \\
 &= 830.9 \text{ BTU/lbm (1.94 MJ/kg)}
 \end{aligned}$$

Substituting into Eq. 6-58, the thermodynamic efficiency can be evaluated as:

$$\begin{aligned}
 \zeta &= \frac{1187.29 - 830.9 + 69.74 - 73.69}{1187.29 - 73.69} \text{ (English units)} = 0.317 \\
 &= \frac{2.771 - 1.94 + 0.163 - 0.171}{2.771 - 0.171} \text{ (SI units)} = 0.317
 \end{aligned}$$

4. The plant thermal efficiency (η_{th}) is defined by Eq. 6-38 and may be determined for the portion of the plant within control volumes 1 of Figure 6-9B,C. In either case, from Eq. 6-40, \dot{Q} in these figures equals the enthalpy rise of the coolant across the reactor. From Eq. 6-42, the thermal efficiencies of the portions of the plants within control volumes 1 of Figure 6-9B,C are equal to the thermodynamic efficiency of the plant within control volume 1 of Figure 6-9A:

$$\eta_{th|c.v.1} \quad (\text{Fig. 6-9C}) \quad = \quad \eta_{th|c.v.1} \quad (\text{Fig. 6-9B}) \quad = \quad \zeta_{c.v.1} \quad (\text{Fig. 6-9A}) \quad (6-42)$$

The thermal efficiencies of these portions of the plant are commonly referred to as the *cycle thermal efficiency* (because they are numerically equal). This nomenclature

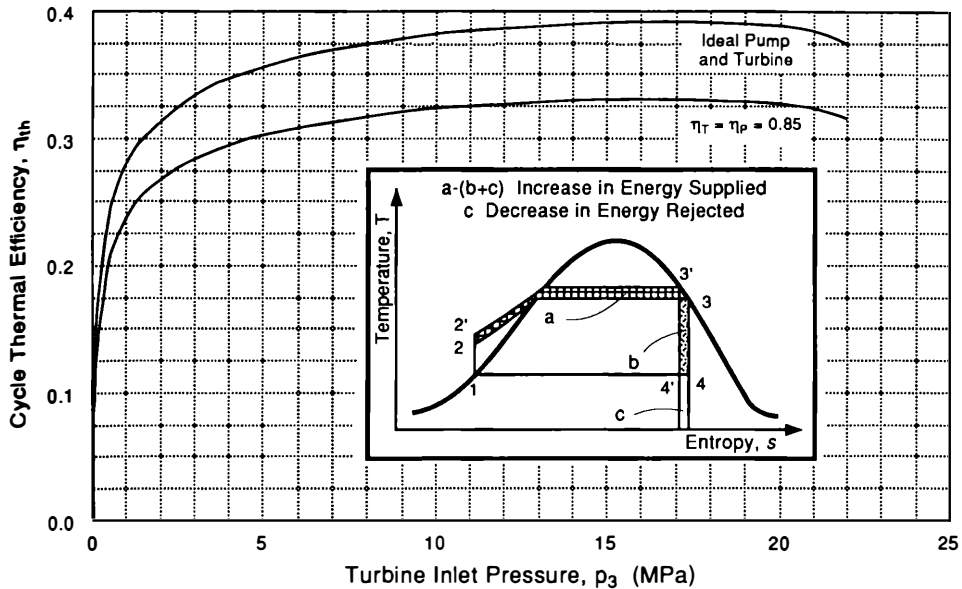


Figure 6-13 Thermal efficiency of Rankine cycle using saturated steam for varying turbine inlet pressure. Turbine inlet: saturated vapor. Exhaust pressure: 7kPa.

is used in the remainder of the chapter for the various reactor plants considered. Because the PWR plant of Figure 6-6 is just a specific case of the general plant illustrated in Figure 6-9, we can directly state that the thermal efficiency of the PWR cycle equals the thermodynamic efficiency of the plant; i.e., $\eta_{th} = \zeta$ for the PWR plant.

Improvements in thermal efficiency for this simple saturated cycle are achievable by (1) increasing the pressure (and temperature) at which energy is supplied to the working fluid in the steam generator (i.e., path 2-3, the steam generator inlet and turbine inlet conditions) and (2) decreasing the pressure (and temperature) at which energy is rejected from the working fluid in the condenser (i.e., path 4-1, the turbine outlet and pump inlet conditions). For the cycle of Figure 6-6, assuming an ideal turbine and condenser, the insets in Figures 6-13 and 6-14 illustrate cycles in which each of these strategies is demonstrated.

The effect on cycle thermal efficiency of increasing the pressure at which energy is supplied as heat and of decreasing the pressure at which energy is rejected as heat is illustrated in Figures 6-13 and 6-14, respectively. The cross-hatched areas in the insets represent changes in energy supplied to and rejected from the working fluid.

B Combined First and Second Law or Availability Analysis of a Simplified PWR System

Consider again the stationary, nondeformable control volume of Figure 6-10 operating at steady state with inlet and outlet flow streams at prescribed states. Neglect shear work and differences between kinetic and potential energies of the inlet and outlet

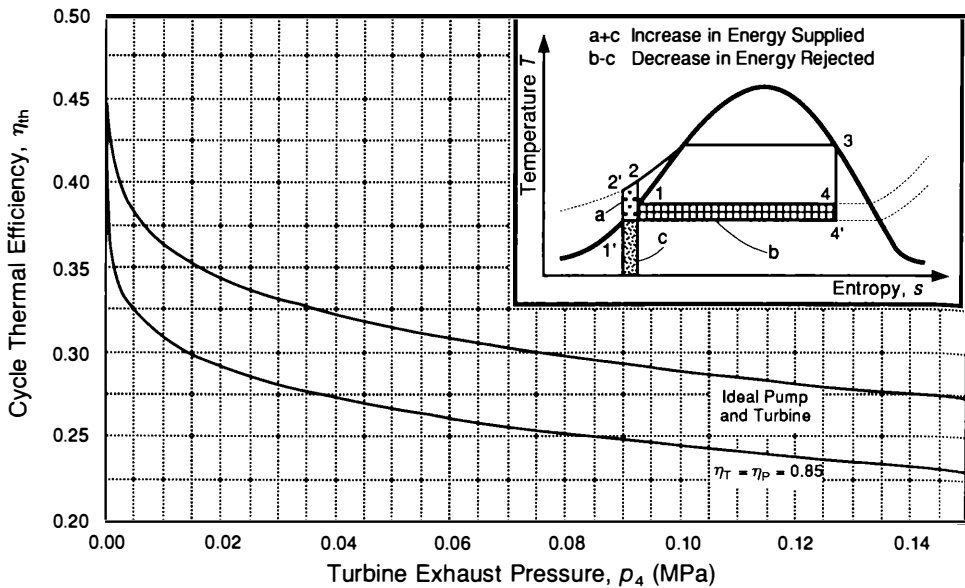


Figure 6-14 Thermal efficiency of Rankine cycle for a saturated turbine inlet state for varying turbine outlet pressure. Turbine inlet: 7.8 MPa saturated vapor.

streams. The maximum useful work obtainable from this control volume, which is in the form of shaft work because the control volume is nondeformable, can be expressed from Eq. 4-46 as follows:

$$\dot{W}_{u,\max} = \sum_{i=1}^I \dot{m}_i (h - T_o s)_i + \left(1 - \frac{T_o}{T_s}\right) \dot{Q} \quad (6-59)$$

where T_s = temperature at which \dot{Q} is supplied and $\left(\frac{dQ}{dt}\right)_{\text{gen}}$ is treated as part of $\frac{dE}{dt}$.

For the same control volume under the same assumptions, the *lost work* or the *irreversibility* is given by Eqs. 4-47 and 4-48 as:

$$\dot{W}_{\text{lost}} \equiv \dot{I} = -T_o \sum_{i=1}^I \dot{m}_i s_i - \frac{T_o}{T_s} \dot{Q} = T_o \dot{S}_{\text{gen}} \quad (6-60)$$

Let us now restrict the above results to adiabatic conditions, i.e., $\dot{Q} = 0$, and examine two cases: a control volume with shaft work and a control volume with no shaft work. Also recall that because the control volume is nondeformable,

$$\left(\frac{dW}{dt}\right)_{\text{normal}} = 0.$$

Case I: $\dot{W}_{\text{shaft}} \neq 0$

From the first law, Eq. 4-39,

$$\dot{W}_{\text{shaft}} = \sum_{i=1}^I \dot{m}_i h_i \quad (6-61a)$$

From Eq. 4-42,

$$\dot{W}_{\text{actual}} = \dot{W}_{\text{shaft}} \quad (6-62a)$$

Now the irreversibility is given by Eq. 6-60 as:

$$\dot{I} = -T_o \sum_{i=1}^I \dot{m}_i s_i = T_o \dot{S}_{\text{gen}} \quad (6-63a)$$

However, the irreversibility is also equal from Eq. 4-47 to:

$$\dot{I} = \dot{W}_{u,\text{max}} - \dot{W}_{\text{actual}} = \sum_{i=1}^I \dot{m}_i (h - T_o s)_i - \dot{W}_{\text{shaft}} \quad (6-63b)$$

where the maximum useful work has been given by Eq. 6-59 with $\dot{Q} = 0$ as:

$$\dot{W}_{u,\text{max}} = \sum_{i=1}^I \dot{m}_i (h - T_o s)_i \quad (6-63c)$$

Case II: $\dot{W}_{\text{shaft}} \equiv 0$

From the first law, Eq. 4-39,

$$\sum_{i=1}^I \dot{m}_i h_i = 0 \quad (6-61b)$$

From Eq. 4-42,

$$\dot{W}_{\text{actual}} = 0 \quad (6-62b)$$

Hence the maximum useful work and the irreversibility, expressed by Eqs. 6-59 and 6-60, respectively, are equal and given by:

$$\dot{W}_{u,\text{max}} = \dot{I} = -T_o \sum_{i=1}^I \dot{m}_i s_i = T_o \dot{S}_{\text{gen}} \quad (6-64)$$

Let us now apply these results to PWR system components.

1 Turbine and pump. The turbine and pump have finite shaft work (case I) and single inlet and exit streams. Hence the irreversibilities and the maximum useful work can be expressed from Eqs. 6-63a and 6-63c, respectively, as:

$$\dot{I} = \dot{m} T_o (s_{\text{out}} - s_{\text{in}}) \quad (6-65)$$

and

$$\dot{W}_{u,\text{max}} = \dot{m} [(h_{\text{in}} - h_{\text{out}}) - T_o (s_{\text{in}} - s_{\text{out}})] \quad (6-66)$$

2 Steam generator and condenser. The steam generator and condenser have zero shaft work (case II) and two inlet and exit streams. Hence the irreversibilities and the maximum useful work can be expressed from Eq. 6-64 as:

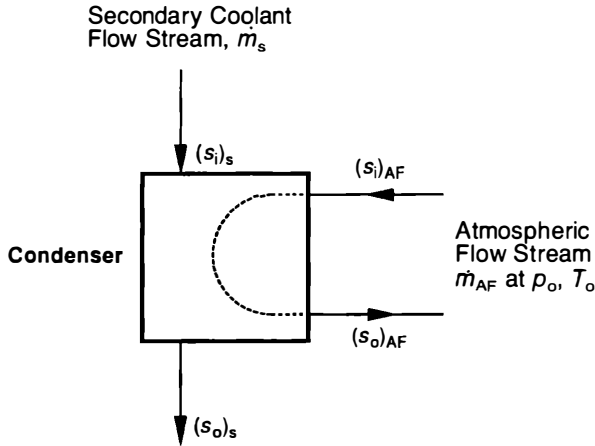


Figure 6-15 Flow streams for a condenser.

$$\dot{i} = \dot{W}_{u,\max} = T_o \left(\sum_{k=1}^2 \dot{m}_{\text{out},k} s_{\text{out},k} - \sum_{k=1}^2 \dot{m}_{\text{in},k} s_{\text{in},k} \right) \quad (6-67)$$

It is useful to expand on this result for the condenser. Figure 6-15 illustrates a condenser cooled by an atmospheric flow stream (\dot{m}_{AF}) that is drawn from and returns to the environment, which acts as a reservoir at p_o , T_o . For the condenser conditions specified in Figure 6-15, the irreversibility of the condenser can be expressed from Eq. 6-67 as:

$$\dot{i} = \dot{m}_{AF} T_o \Delta s_{AF} + \dot{m}_s T_o \Delta s_s \quad (6-68)$$

For this constant pressure atmospheric flow stream interaction, Eq. 6-10a can be written as:

$$T ds = dh \quad (6-10b)$$

which can be expressed for the atmospheric coolant stream as:

$$\dot{m}_{AF} T_o \Delta s_{AF} = \dot{m}_{AF} \Delta h_{AF} \quad (6-69)$$

From an energy balance on the condenser:

$$\dot{m}_{AF} \Delta h_{AF} = - \dot{m}_s \Delta h_s \quad (6-70)$$

Substituting Eq. 6-70 into Eq. 6-69, we obtain the desired result:

$$\dot{m}_{AF} T_o \Delta s_{AF} = - \dot{m}_s \Delta h_s \quad (6-71)$$

The irreversibility can be rewritten using Eq. 6-70 as:

$$\dot{i} = T_o \dot{m}_s \left[- \frac{\Delta h_s}{T_o} + \Delta s_s \right] \quad (6-72a)$$

which, referring to Figure 6-6, becomes:

$$\dot{I} = T_o \dot{m}_s \left[- \left(\frac{h_1 - h_4}{T_o} \right) + s_1 - s_4 \right] \quad (6-72b)$$

The condenser irreversibility also can be obtained directly from an availability balance. The irreversibility of the condenser is simply the difference between the availability of the inlet and exit secondary coolant flow streams because there is no change of availability in the atmospheric flow stream. Note that there is, however, a change of energy and entropy in the atmospheric flow stream. This availability difference is evaluated from the definition of the availability function A given by Eq. 4-27 as:

$$\dot{I} = \dot{m}_{in}(h_{in} - T_o s_{in}) - \dot{m}_{out}(h_{out} - T_o s_{out})$$

For the state of the secondary coolant flow stream, this result is identical to Eq. 6-72b.

3 Reactor irreversibility. To proceed, we first compute the maximum work that could be done by the fissioning fuel. Upon fissioning and with respect to fission products, each fission fragment at steady state may be regarded as having a kinetic energy of about 100 MeV. If it were a perfect gas in a thermodynamic equilibrium state, the fragment would have a temperature T_a such that $\frac{3}{2} kT_a = 100$ MeV. Maximum work is done by the fragment in a reversible process that brings the fragment to a state of mutual equilibrium with the environment at $T_o = 298^\circ\text{K}$ or $kT_o = 0.025$ eV. At this state, the energy of the fragment is $u_o = 0.0375$ eV. Thus the maximum work per unit mass of fission fragments is:

$$W_{u,\max} = h_a - h_o - T_o(s_a - s_o)$$

which can be approximated as:

$$W_{u,\max} \approx u_a - u_o - cT_o \ln \frac{T_a}{T_o} = (u_a - u_o) \left[1 - \frac{kT_o \ln(T_a/T_o)}{k(T_a - T_o)} \right] \quad (6-73)$$

For $T_a \gg T_o$, Eq. 6-73 becomes:

$$W_{u,\max} \approx u_a$$

The maximum work is thus approximately equal to the fission energy (u_a). In other words, all the fission energy may be considered to be available as work.

We compute the rate of lost work in several steps. The maximum useful power available as a result of fission equals the fission power (\dot{u}_a) and is delivered to the fuel rods considered at a constant fuel temperature (T_{fo}). Because this fuel temperature is far below T_a , for the purpose of this computation the fuel temperature and the clad surface temperature are taken to be T_{fo} . The transfer of power from the fission fragments to the fuel rods is highly irreversible, as $T_{fo} \ll T_a$. The maximum power available from the fuel rod at fixed surface temperature T_{fo} is:

$$(\dot{W}_{u,\max})_{\text{fuel}} = (\dot{W}_{u,\max})_{\text{fission}} \left(1 - \frac{T_o}{T_{fo}} \right) \quad (6-74)$$

and therefore the irreversibility is:

$$\dot{I}_{\text{fuel}} = (\dot{W}_{u,\text{max}})_{\text{fission}} - (\dot{W}_{u,\text{max}})_{\text{fuel}} = (\dot{W}_{u,\text{max}})_{\text{fission}} \frac{T_o}{T_{fo}} \quad (6-75)$$

From the fuel rods, all the fission power is transferred to the coolant. The maximum work available from the coolant is:

$$(\dot{W}_{u,\text{max}})_{\text{coolant}} = \dot{m}_p [(h_{\text{out}} - h_{\text{in}})_R - T_o(s_{\text{out}} - s_{\text{in}})_R] \quad (6-76)$$

so here the irreversibility is:

$$\dot{I}_{\text{coolant}} = (\dot{W}_{u,\text{max}})_{\text{fuel}} - (\dot{W}_{u,\text{max}})_{\text{coolant}} = (\dot{W}_{u,\text{max}})_{\text{fission}} \left[\frac{T_o(s_{\text{out}} - s_{\text{in}})_R}{(h_{\text{out}} - h_{\text{in}})_R} - \frac{T_o}{T_{fo}} \right] \quad (6-77)$$

because $(\dot{W}_{u,\text{max}})_{\text{fission}}$ equals the fission power \dot{U}_a , which in turn equals $\dot{m}_p (h_{\text{out}} - h_{\text{in}})_R$, by application of the first law (Eq. 4-39) to the reactor control volume, which is adiabatic, is nondeformable, and has no shaft work—and where kinetic energy, potential energy, and shear work terms are neglected. Hence the irreversibility of the reactor is:

$$\begin{aligned} \dot{I}_R &= \dot{I}_{\text{fission}} + \dot{I}_{\text{fuel}} + \dot{I}_{\text{coolant}} \approx 0 + (\dot{W}_{u,\text{max}})_{\text{fission}} \frac{T_o}{T_{fo}} \\ &\quad + (\dot{W}_{u,\text{max}})_{\text{fission}} \left[\frac{T_o(s_{\text{out}} - s_{\text{in}})_R}{(h_{\text{out}} - h_{\text{in}})_R} - \frac{T_o}{T_{fo}} \right] \end{aligned}$$

Again, because $(\dot{W}_{u,\text{max}})_{\text{fission}}$ equals $\dot{m}_p (h_{\text{out}} - h_{\text{in}})_R$, the above result becomes:

$$\dot{I}_R = \dot{m}_p T_o (s_{\text{out}} - s_{\text{in}})_R \quad (6-78)$$

Eq. 6-78 is a particular form of the general control volume entropy generation equation as expressed by Eq. 6-60. Eq. 6-78 states that for a steady-state, adiabatic reactor control volume the change of entropy of the coolant stream equals the entropy generation within the reactor.

4 Plant irreversibility. Finally, consider the entire nuclear plant within one control volume as illustrated in Figure 6-9A. For this adiabatic, nondeformable, stationary control volume, the maximum useful work, given by Eq. 6-34, is:

$$\dot{W}_{u,\text{max}}^{\text{reactor plant}} = - \left[\frac{\partial(U - T_o S)}{\partial t} \right]_{\text{reactor plant}} + \dot{m}_{\text{AF}} [(h_{\text{in}} - h_{\text{out}}) - T_o(s_{\text{in}} - s_{\text{out}})]_{\text{AF}} \quad (6-79)$$

because $\dot{Q} = 0$ and differences in kinetic and potential energy as well as shear work are neglected. The first term of the right-hand side of Eq. 6-79 involves only the fission process within the reactor, because for all other components the time rate of change of $U - TS$ is zero. Hence:

$$- \left[\frac{\partial(U - T_o S)}{\partial t} \right]_{\text{reactor plant}} = - \left[\frac{\partial(U - T_o S)}{\partial t} \right]_{\text{reactor}} = - \left(\frac{\partial U}{\partial t} \right)_{\text{reactor}} \quad (6-80a)$$

because for the reactor $\dot{I}_R = T_o \dot{S}_{\text{gen}}$ and $\dot{Q} = 0$; hence Eq. 6-79 becomes:

$$\left(\frac{\partial S}{\partial t} \right)_{\text{reactor}} = \dot{m}_p (s_{\text{in}} - s_{\text{out}})_R + \frac{\dot{I}_R}{T_o} = 0$$

utilizing Eq. 6-78 for \dot{I}_R .

Again, the first law (Eq. 4-39) for the reactor control volume, which is adiabatic, is nondeformable, and has no shaft work, becomes (treating \dot{Q}_{gen} as part of \dot{E}):

$$\left(\frac{\partial U}{\partial t} \right)_{\text{reactor}} = \dot{m}_{\text{in}} h_{\text{in}} - \dot{m}_{\text{out}} h_{\text{out}} \quad (6-80b)$$

Hence the change in reactor internal energy can be expressed in terms of steam generator stream enthalpy differences as:

$$- \left(\frac{\partial U}{\partial t} \right)_{\text{reactor}} = \dot{m}_p (h_{\text{in}} - h_{\text{out}})_{\text{SGp}} = \dot{m}_s (h_{\text{out}} - h_{\text{in}})_{\text{SGs}} \quad (6-81)$$

Furthermore, from Eq. 6-69, the second term on the right-hand side of Eq. 6-79 is zero. Hence applying Eq. 6-81 to Eq. 6-79 yields:

$$[\dot{W}_{u,\text{max}}]_{\text{reactor plant}} = \dot{m}_p (h_{\text{in}} - h_{\text{out}})_{\text{SGp}} = \dot{m}_s (h_{\text{out}} - h_{\text{in}})_{\text{SGs}} \quad (6-55)$$

which is the result earlier presented in Eqs. 6-53 through 6-55. The irreversibility from Eq. 4-48 is:

$$\dot{I}_{\text{reactor plant}} = - T_o \sum_{i=1}^I \dot{m}_i s_i = T_o \dot{m}_{\text{AF}} (s_{\text{out}} - s_{\text{in}})_{\text{AF}} \quad (6-82)$$

which by Eqs. 6-69 and 6-70 yields:

$$\dot{I}_{\text{reactor plant}} = - \dot{m}_s \Delta h_s = \dot{m}_s (h_{\text{in}} - h_{\text{out}})_s \quad (6-83)$$

Referring to Figure 6-6, Eq. 6-83 becomes:

$$\dot{I}_{\text{reactor plant}} = \dot{m}_s (h_4 - h_1) \quad (6-84)$$

Example 6-5 Second law thermodynamic analysis of a simplified PWR cycle

PROBLEM Consider the PWR plant of Figure 6-6, which was analyzed in Example 6-4. The plant operating conditions from Example 6-4 are summarized in Table 6-7 together with additional state conditions that can be obtained from the results of Example 6-4. We wish to determine the magnitude of the irreversibility for each

Table 6-7 PWR operating conditions for Example 6-5

Point	Flow rate	Pressure (psia)	Temperature		Enthalpy (BTU/lb)	Entropy (BTU/lb °R)
			°F	°R		
1	1.0	1	101.7	561.3	69.74	0.13266
2	1.0	1124	103.0	562.6	73.69	0.13410
3	1.0	1124	559.1	1018.7	1187.29	1.37590
4	1.0	1	101.7	561.3	830.70	1.48842
5	13.18	2250	618.5	1078.1	640.31	0.83220
6	13.18	2250	557.2	1016.8	555.80	0.75160

*Flow rate relative to 1 lb of steam through the turbine.

component. This analysis is a useful step for assessing the incentive for redesigning components to reduce the irreversibility.

SOLUTION First evaluate the irreversibility of each component. These irreversibilities sum to the irreversibility of the reactor plant, which may be independently determined by considering the entire plant within one control volume. T_o is the reservoir temperature taken at 539.6°R (298.4°K).

For the steam generator, utilizing Eq. 6-67 and dividing by \dot{m}_s :

$$\begin{aligned}
 I_{SG} &= \dot{I}_{SG}/\dot{m}_s = T_o \frac{\dot{m}_p}{\dot{m}_s} (s_6 - s_5) + T_o (s_3 - s_2) \\
 &= 539.6(13.18)(0.7516 - 0.8322) + (539.6)(1.3759 - 0.1341) \quad (6-85) \\
 &= -573.22 + 670.08 \\
 &= 96.85 \text{ BTU/lbm steam (0.226 MJ/kg)}
 \end{aligned}$$

For the turbine, using Eq. 6-65 and dividing by \dot{m}_s :

$$\begin{aligned}
 I_T &= \dot{I}_T/\dot{m}_s = T_o (s_4 - s_3) \\
 &= 539.6 (1.4884 - 1.3759) \\
 &= 60.71 \text{ BTU/lbm steam (0.142 MJ/kg)} \quad (6-86)
 \end{aligned}$$

For the condenser, using Eq. 6-72b and dividing by \dot{m}_s :

$$\begin{aligned}
 I_{CD} &= \frac{\dot{I}_{CD}}{\dot{m}_s} = T_o \left(\frac{h_4 - h_1}{T_o} + s_1 - s_4 \right) \\
 &= 539.6 \left(\frac{830.7 - 69.74}{539.6} + 0.13266 - 1.48842 \right) \quad (6-87) \\
 &= 29.39 \text{ BTU/lbm steam (6.86} \times 10^{-2} \text{ MJ/kg)}
 \end{aligned}$$

For the pump, using Eq. 6-65 and dividing by \dot{m}_s :

$$\begin{aligned}
 I_P &= \frac{\dot{I}_P}{\dot{m}_s} = T_o (s_2 - s_1) \\
 &= 539.6 (0.1341 - 0.13266) \\
 &= 0.78 \text{ BTU/lbm steam (1.81} \times 10^{-2} \text{ MJ/kg)} \quad (6-88)
 \end{aligned}$$

For the reactor, using Eq. 6-78 and dividing by \dot{m}_s :

$$\begin{aligned} I_R &= \frac{\dot{I}_R}{\dot{m}_s} = \frac{\dot{m}_p}{\dot{m}_s} T_o (s_5 - s_6) \\ &= 13.18(539.6) (0.8322 - 0.7516) \\ &= 573.22 \text{ BTU/lbm steam (1.33 MJ/kg)} \end{aligned} \quad (6-89)$$

This reactor irreversibility can be broken into its two nonzero components, \dot{I}_{fuel} and \dot{I}_{coolant} , i.e., irreversibility associated with fission power transfer from the fission products to the fuel rod and from the fuel rod to the coolant. These irreversibility components are expressed using Eqs. 6-75 and 6-77 assuming a fuel surface temperature of 1760°R (977.6°K) as:

$$\begin{aligned} I_{\text{fuel}} &= \frac{\dot{I}_{\text{fuel}}}{\dot{m}_s} = \frac{(\dot{W}_{u,\text{max}})_{\text{fission}}}{\dot{m}_s} \frac{T_o}{T_{fo}} \\ &= \left(\frac{\dot{m}_p}{\dot{m}_s} \right) (h_{\text{out}} - h_{\text{in}})_R \frac{T_o}{T_{fo}} \\ &= (13.18) (640.31 - 555.8) \frac{539.6}{1760} \\ &= 34.14 \text{ BTU/lbm steam (7.97} \times 10^{-2} \text{ MJ/kg)} \\ I_{\text{coolant}} &= \frac{\dot{I}_{\text{coolant}}}{\dot{m}_s} \\ &= \frac{(\dot{W}_{u,\text{max}})_{\text{fission}}}{\dot{m}_s} \left[\left[\frac{T_o(s_{\text{out}} - s_{\text{in}})_R}{(h_o - h_{\text{in}})_R} \right] - \frac{T_o}{T_{fo}} \right] \\ &= \left(\frac{\dot{m}_p}{\dot{m}_s} \right) T_o \left[(s_{\text{out}} - s_{\text{in}})_R - \frac{(h_{\text{out}} - h_{\text{in}})_R}{T_{fo}} \right] \\ &= (13.18) 539.6 \left[0.8322 - 0.7516 - \left(\frac{640.31 - 555.80}{1760} \right) \right] \\ &= 539.08 \text{ BTU/lbm steam (1.25 MJ/kg)} \end{aligned}$$

The sum of the numerical values of the irreversibilities of all components is:

$$I_R = 573.22$$

$$I_{\text{SG}} = 96.85$$

$$I_T = 60.71$$

$$I_{\text{CD}} = 29.39$$

$$I_P = 0.78$$

$$\text{Total} = 760.95 \text{ BTU/lbm steam}$$

which equals (considering round-off error) the irreversibility of the reactor plant evaluated in Eq. 6-84 as:

$$I_{\text{RP}} = \frac{\dot{I}_{\text{RP}}}{\dot{m}_s} = h_4 - h_1 = 830.70 - 69.74 = 760.96 \text{ BTU/lbm steam}$$

The generality of this result may be illustrated by summing the expressions for the irreversibility of each component and showing that the sum is identical to the irreversibility of the reactor plant. Proceeding in this way we obtain:

$$\begin{aligned} \dot{I}_{RP} &= \dot{W}_{u,\max} - \dot{W}_{\text{net}} \\ &= \dot{m}_s [h_3 - h_2 - [h_3 - h_4 - (h_2 - h_1)]] \\ &= \dot{m}_s (h_4 - h_1) \end{aligned} \quad (6-84)$$

$$\dot{I}_R = T_o \dot{m}_p (s_5 - s_6) \quad (6-89)$$

$$\dot{I}_{SG} = T_o \dot{m}_p (s_6 - s_5) + T_o \dot{m}_s (s_3 - s_2) \quad (6-85)$$

$$\dot{I}_T = T_o \dot{m}_s (s_4 - s_3) \quad (6-86)$$

$$\dot{I}_P = T_o \dot{m}_s (s_2 - s_1) \quad (6-88)$$

$$\dot{I}_{CD} = T_o \dot{m}_s (s_1 - s_4) + \dot{m}_s (h_4 - h_1) \quad (6-87)$$

Hence:

$$\dot{I} = \dot{m}_s (h_4 - h_1) = \dot{I}_{RP} \quad (6-84)$$

V MORE COMPLEX RANKINE CYCLES: SUPERHEAT, REHEAT, REGENERATION, AND MOISTURE SEPARATION

Improvements in overall cycle performance can be accomplished by superheat, reheat, and regeneration, as each approach leads to a higher average temperature at which heat is received by the working fluid. Furthermore, to minimize erosion of the turbine blades, the turbine expansion process can be broken into multiple stages with intermediate moisture separation. The superheat and reheat options also allow maintenance of higher exit steam quality, thereby decreasing the liquid fraction in the turbine.

The superheat, reheat, regeneration, and moisture separation options are illustrated in subsequent figures for ideal turbines. In actual expansion processes, the coolant entropy increases, and the exit steam is dryer, although the shaft work generated per unit mass is decreased. Although each process is discussed independently, in practice they are utilized in varying combinations. Superheat is accomplished by heating the working fluid into the superheated vapor region. A limited degree of superheat is being achieved in PWR systems employing once-through versus recirculating steam generators. Figure 6-16 illustrates the heat exchanger and turbine processes in a superheated power cycle.

Reheat is a process whereby the working fluid is returned to a heat exchanger after partial expansion in the turbine. As shown in Figure 6-17, the fluid can be reheated to the maximum temperature and expanded. Actual reheat design conditions may differ with application.

Regeneration is a process by which the colder portion of the working fluid is heated by the hotter portion of the working fluid. Because this heat exchange is internal to the cycle, for single-phase fluids (e.g., Brayton cycle) the average temperature of external heat rejection is reduced and the average temperature of external heat addition

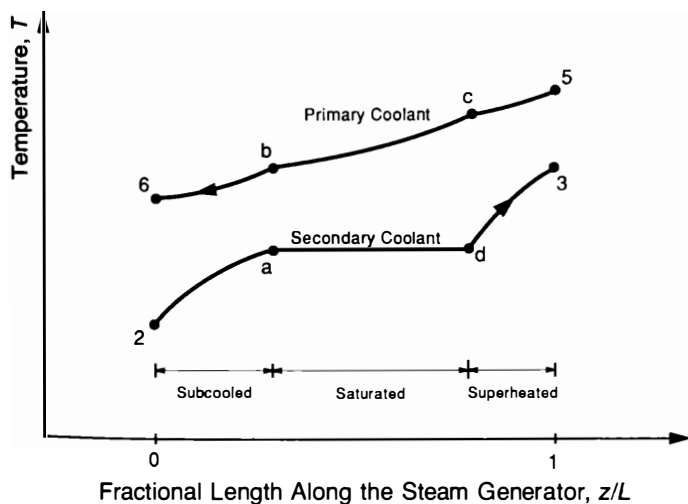
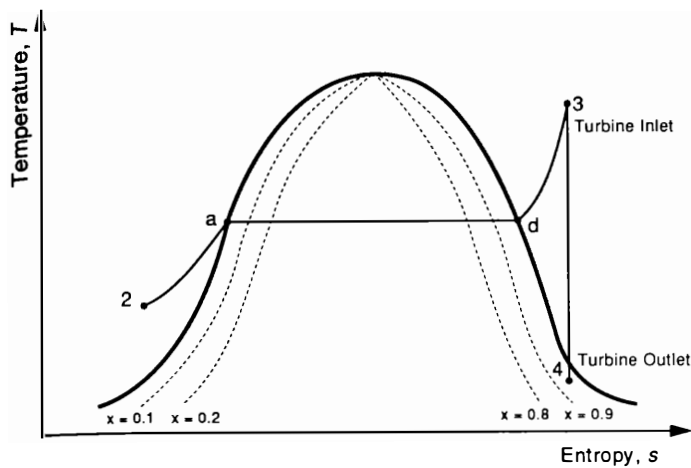


Figure 6-16 Heat exchanger and turbine processes in a superheated power cycle.

is increased. For a two-phase fluid (e.g., Rankine cycle) only the average temperature of external heat addition is changed, i.e., increased. In either case the cycle thermal efficiency increases. For the limiting case of this heat exchange occurring at infinitesimal temperature differences—an ideal regeneration treatment—it can be shown that the cycle thermal efficiency equals the Carnot efficiency of a cycle operating between the same two temperature limits. In practice for a Rankine cycle, this regenerative process is accomplished by extracting steam from various turbine stages and directing it to a series of heaters where the condensate or feedwater is preheated. These feedwater heaters may be open (OFWH), in which the streams are directly

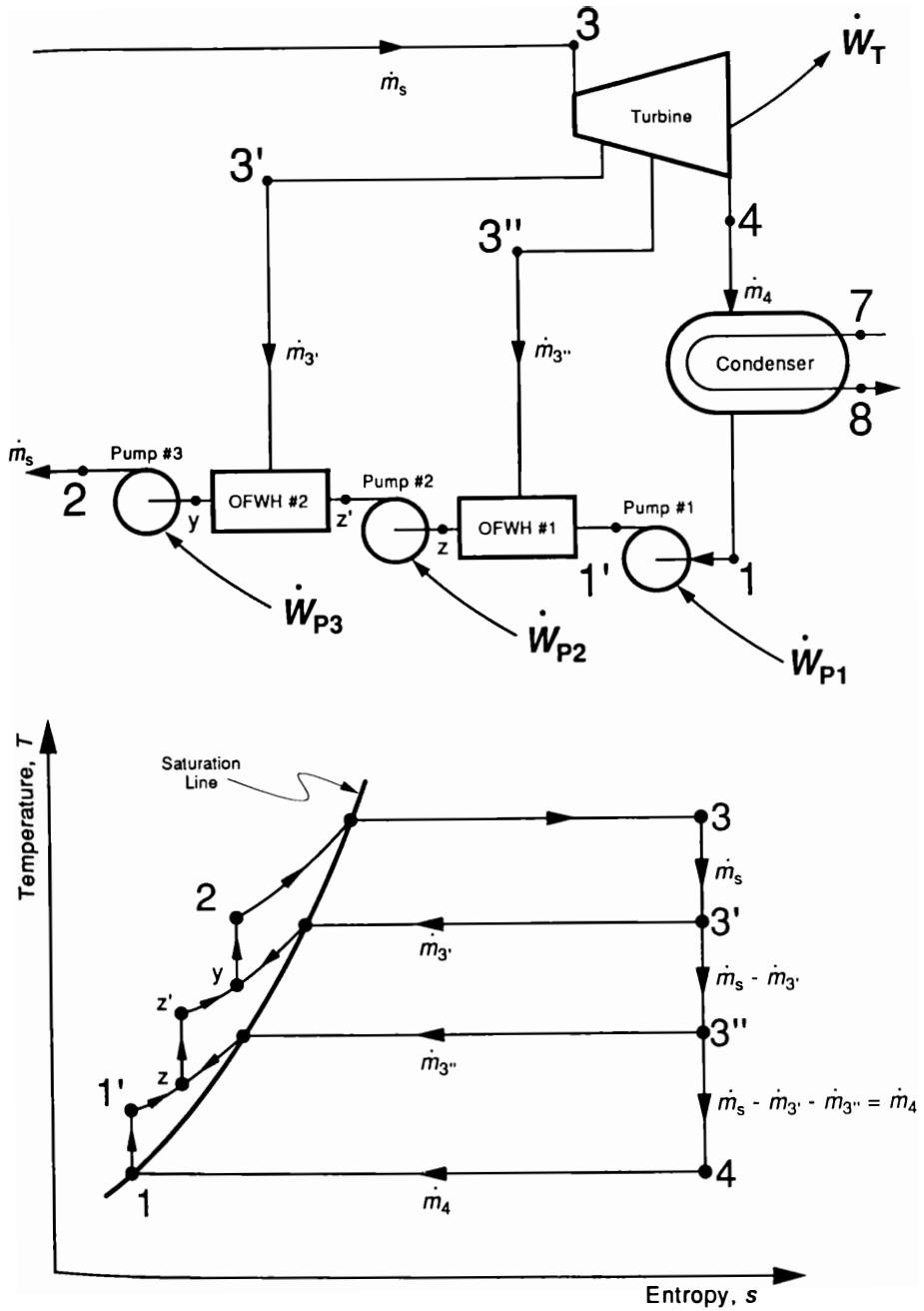


Figure 6-18 Power cycle with open feedwater heaters.

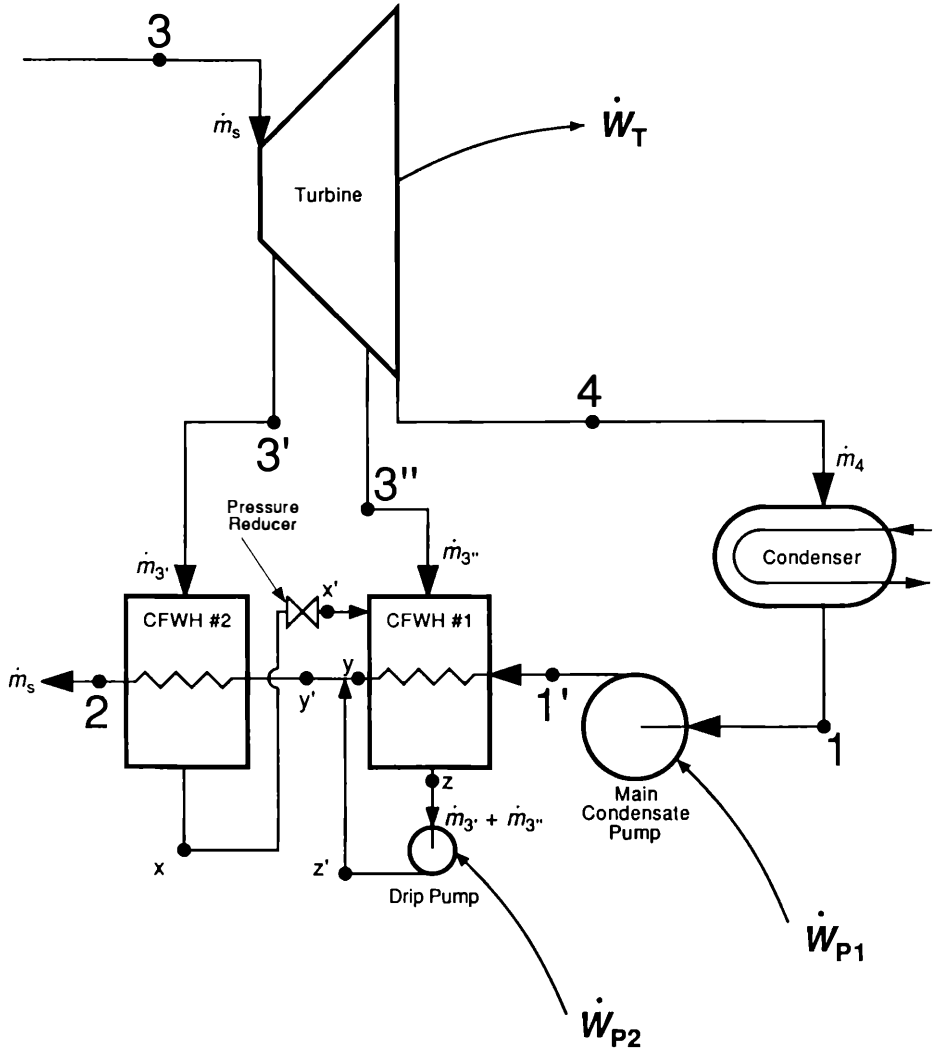


Figure 6-19 Portion of Rankine power cycle with closed feedwater heaters.

For CFWH No. 1:

$$h_y = \frac{\dot{m}_{3'}h_{3''} + \dot{m}_4h_{1'} + \dot{m}_3h_{x'} - (\dot{m}_{3'} + \dot{m}_{3''})h_z}{\dot{m}_4} \quad (6-93)$$

For the mixing tee, at position y' :

$$h_{y'} = \frac{\dot{m}_4h_y + (\dot{m}_{3'} + \dot{m}_{3''})h_{z'}}{\dot{m}_s} \quad (6-94)$$

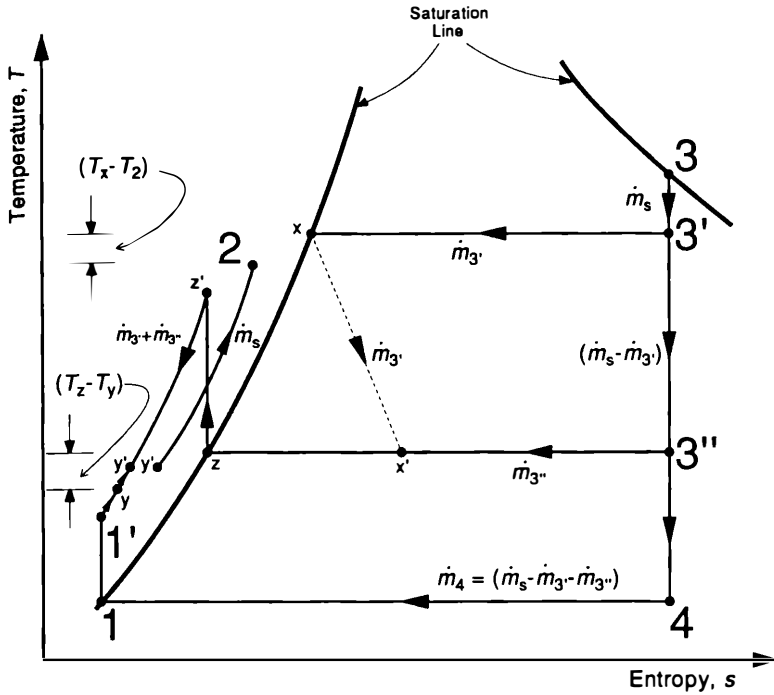


Figure 6-20 Portion of Rankine power cycle with closed feedwater heaters. *Note:* Process y' to 2 offset from constant pressure line $1'y'y'z'2$ for clarity.

For CFWH No. 2:

$$h_2 = \frac{\dot{m}_3(h_{3'} - h_x) + \dot{m}_s h_{y'}}{\dot{m}_s} \quad (6-95)$$

From these feedwater heater illustrations it can be observed that the OFWH approach requires a condensate pump for each heater, whereas the CFWH approach requires only one condensate pump plus a smaller drip pump. However, higher heat transfer rates are achievable with the OFWH. Additionally, the OFWH permits deaeration of the condensate. For most applications closed heaters are favored, but for purposes of feedwater deaeration at least one open heater is provided.

The final process considered is moisture separation. The steam from the high-pressure turbine is passed through a moisture separator, and the separated liquid is diverted to a feedwater heater while the vapor passes to a low-pressure turbine. The portion of the power cycle with a moisture separator and the associated T - s diagram are shown in Figure 6-21. The moisture separator is considered ideal, producing two streams of fluid, a saturated liquid flow rate of magnitude $(1 - x_3)\dot{m}_s$ and enthalpy

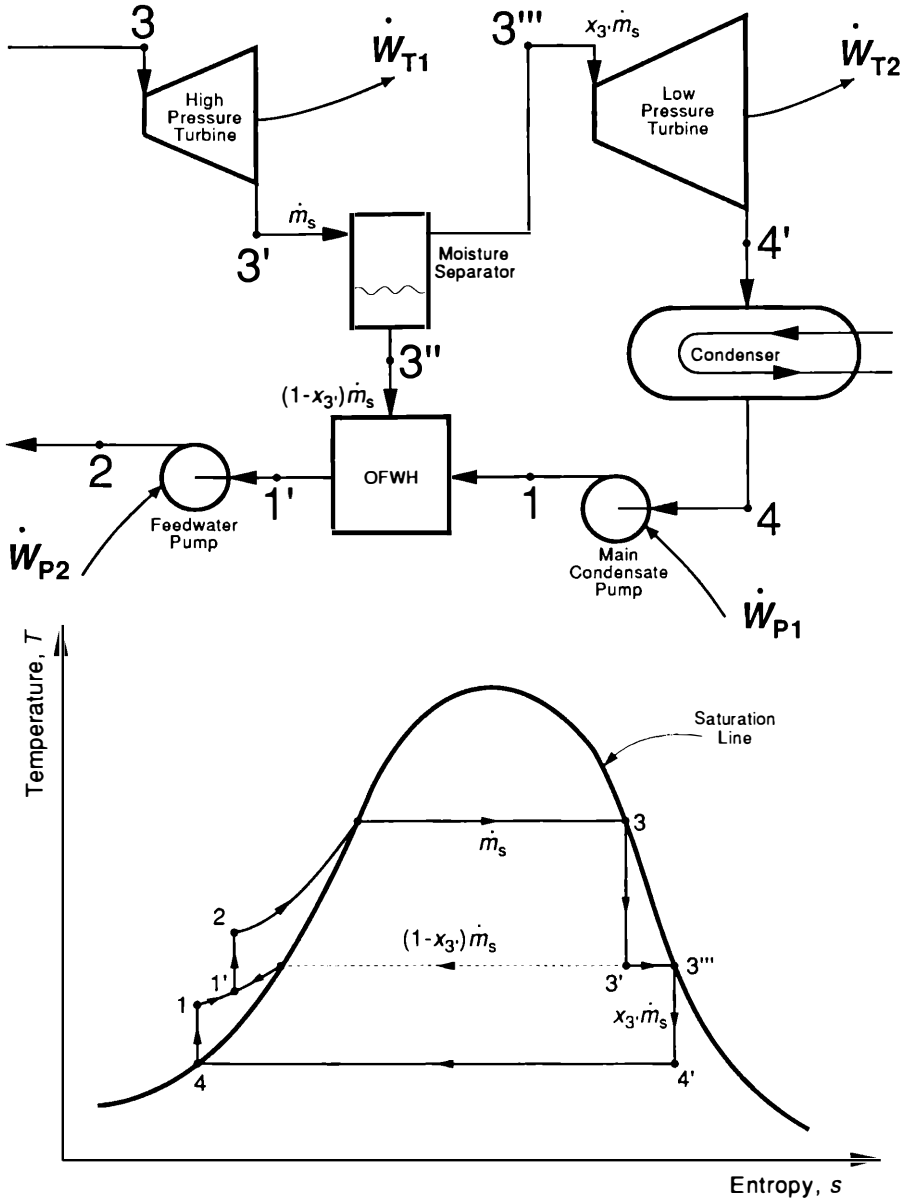


Figure 6-21 Portion of power cycle with ideal moisture separation.

$h_{3''} = h_f$ (at $p_{3'}$), and a saturated vapor flow rate of magnitude $x_3 \dot{m}_s$ and enthalpy $h_{3'''} = h_g$ (at $p_{3'}$). In this case the feedwater heater exit enthalpy is given as

$$h_{1'} = \frac{h_{3''}(1 - x_3) \dot{m}_s + h_{3'''} x_3 \dot{m}_s}{\dot{m}_s} \quad (6-96)$$

Example 6-6 Thermodynamic analysis of a PWR cycle with moisture separation and one stage of feedwater heating

PROBLEM This example demonstrates the advantage in cycle performance gained by adding moisture separation and open feedwater heating to the cycle of Example 6-4. The cycle, as illustrated in Figure 6-22, operates under the conditions given in Table 6-8. All states are identical to those in Example 6-4 except state 2, as states 9 through 13 have been added. The flow rate at state 11, which is diverted to the open feedwater heater, is sufficient to preheat the condensate stream at state 13 to the conditions of state 10. Assume that all turbines and pumps have isentropic efficiencies of 85%.

1. Draw the temperature–entropy (T – s) diagram for this cycle, as well as the temperature–fractional length (T versus z/L) diagram for the steam generator.
2. Compute the ratio of primary to secondary flow rates.
3. Compute the cycle thermal efficiency.

SOLUTION

1. The T – s diagram is sketched in Figure 6-23. The T versus z/L diagram is the same as that in Figure 6-7.

2. $\frac{\dot{m}_p}{\dot{m}_s} = \frac{h_3 - h_a}{\bar{c}_p[T_5 - (T_a + \Delta T_p)]} = 13.18$, as states 3, 5, a, and b were specified the same as in Example 6-4.

$$3. \eta_{th} = \frac{\dot{m}_s(h_3 - h_9) + (1 - f)\dot{m}_g(h_{11} - h_4)}{\dot{m}_s(h_3 - h_2)}, \text{ neglecting pump work.}$$

Table 6-8 PWR operating conditions for Example 6-6

State	Temperature °F (°K)	Pressure psia (MPa)	Condition
1		1 (6.89×10^{-3})	Saturated liquid
2		1124 (7.75)	Subcooled liquid
3		1124 (7.75)	Saturated vapor
4		1 (6.89×10^{-3})	Two-phase mixture
5	618.5 (599)	2250 (15.5)	Subcooled liquid
6	557.2 (565)	2250 (15.5)	Subcooled liquid
7			Subcooled liquid
8			Subcooled liquid
a		1124 (7.75)	Saturated liquid
b	$T_a + 26$ (14.4)	2250 (15.5)	Subcooled liquid
9		50 (0.345)	Two-phase mixture
10		50 (0.345)	Saturated liquid
11		50 (0.345)	Saturated vapor
12		50 (0.345)	Saturated liquid
13		50 (0.345)	Subcooled liquid

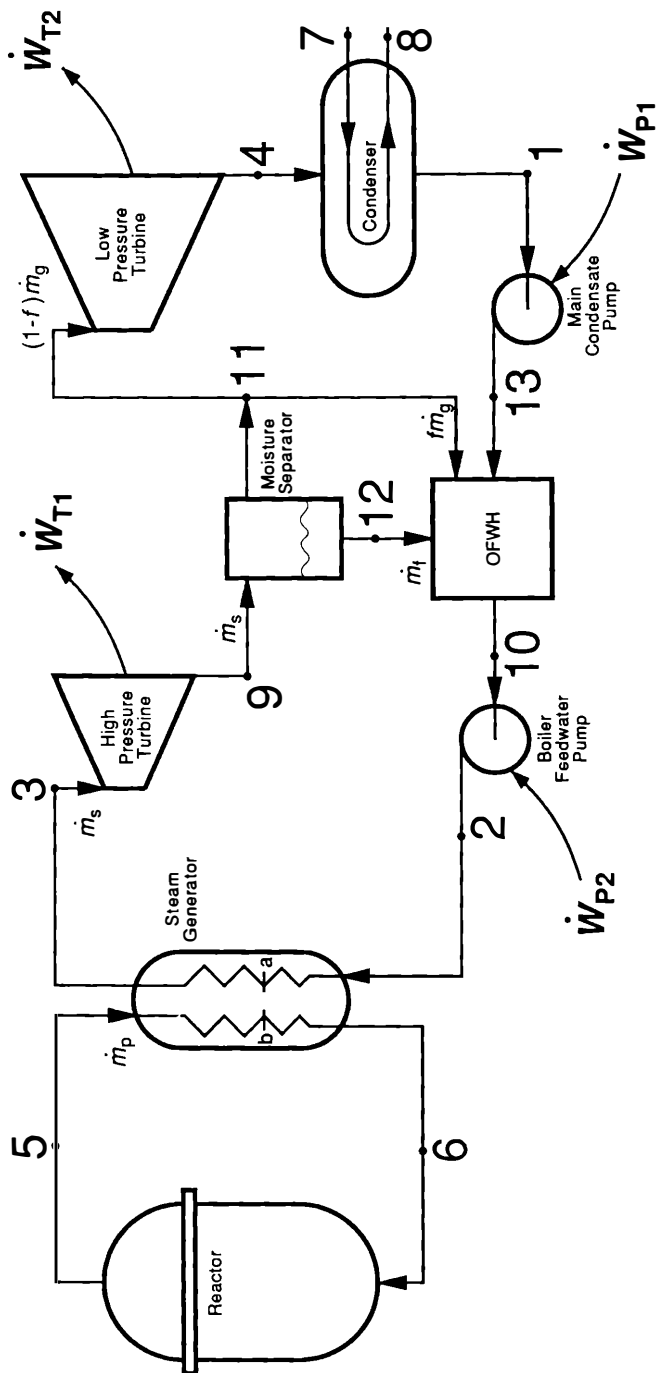


Figure 6-22 PWR cycle analyzed in Example 6-6.

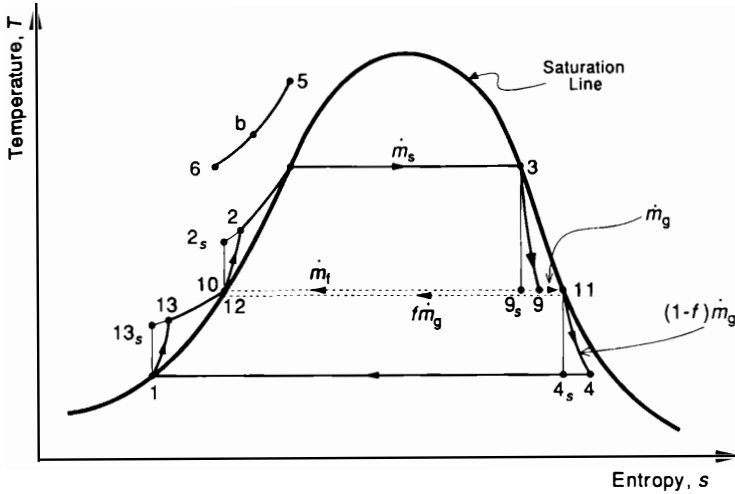


Figure 6-23 PWR cycle with moisture separation and one stage of feedwater heating.

From the problem statement, the following enthalpies can be directly determined:

$$h_3 = 1187.29 \text{ BTU/lbm or } 2.77 \text{ MJ/kg (Example 6-4)}$$

$$h_{11} = 1174.40 \text{ BTU/lbm (2.74 MJ/kg)}$$

$$h_{12} = h_{10} = 250.24 \text{ BTU/lbm (0.584 MJ/kg)}$$

The following parameters must be calculated: h_9 , \dot{m}_g , h_4 , f , and h_2 .

$$h_9 = h_3 - \eta_T(h_3 - h_{9s})$$

where

$$h_{9s} = h_f + x_{9s}h_{fg} = h_f + \left(\frac{s_{9s} - s_f}{s_{fg}} \right) h_{fg}$$

Hence:

$$h_{9s} = 250.24 + \left(\frac{1.3759 - 0.41129}{1.2476} \right) 924.2$$

$$= 964.81 \text{ BTU/lbm (2.25 MJ/kg)}$$

$$h_9 = 1187.29 - 0.85(1187.29 - 964.81)$$

$$= 998.18 \text{ BTU/lbm (2.330 MJ/kg)}$$

$$\dot{m}_g = x_9 \dot{m}_s = \frac{h_9 - h_f}{h_{fg}} \dot{m}_s = \frac{998.18 - 250.24}{924.2} \dot{m}_s = 0.81 \dot{m}_s \text{ (English units)}$$

$$= \frac{2.331 - 0.584}{2.156} \dot{m}_s = 0.81 \dot{m}_s \text{ (SI units)}$$

$$\dot{m}_f = \left(1 - \frac{\dot{m}_g}{\dot{m}_s} \right) \dot{m}_s = 0.19 \dot{m}_s$$

$$\begin{aligned}
 h_4 &= h_{11} - \eta_T(h_{11} - h_{4s}) = 1174.4 - 0.85(1174.4 - 926.61) \\
 &= 963.77 \text{ BTU/lbm (2.25 MJ/kg)}
 \end{aligned}$$

where:

$$\begin{aligned}
 h_{4s} &= h_f + x_{4s}h_{fg} = h_f + \left(\frac{s_{4s} - s_f}{s_{fg}} \right) h_{fg} \\
 &= 69.74 + \left(\frac{1.6589 - 0.13266}{1.8453} \right) 1036.0 \\
 &= 926.61 \text{ BTU/lbm (2.16 MJ/kg)}
 \end{aligned}$$

$$\begin{aligned}
 h_{13} &= h_1 + \frac{v_1(p_{13} - p_1)}{\eta_p} \\
 &= 69.91 \text{ BTU/lbm (0.163 MJ/kg) (same as state 2, Example 6-4)}
 \end{aligned}$$

Applying the first law to the feedwater heater:

$$\begin{aligned}
 \dot{m}_f h_{12} + f \dot{m}_g h_{11} + (1 - f) \dot{m}_g h_{13} &= \dot{m}_s h_{10} \\
 f &= \frac{\dot{m}_s h_{10} - \dot{m}_f h_{12} - \dot{m}_g h_{13}}{\dot{m}_g (h_{11} - h_{13})} = \frac{h_{10} - h_{13}}{h_{11} - h_{13}} = \frac{250.24 - 69.91}{1174.4 - 69.91} = 0.16 \\
 h_2 &= h_{10} + \frac{v_{10}(p_2 - p_{10})}{\eta_p} \\
 &= 250.24 + \frac{0.017269(1124 - 50)}{0.85} \left(\frac{144 \text{ in}^2/\text{ft}^2}{778 \text{ lb}_f\text{-ft/BTU}} \right) \\
 &= 254.28 \text{ BTU/lbm (0.594 MJ/kg)}
 \end{aligned}$$

Hence:

$$\begin{aligned}
 \eta_{th} &= \left[\frac{(1187.29 - 998.18) + (1 - 0.16)0.81(1174.4 - 963.77)}{1187.29 - 254.28} \right] 100 \text{ English units} \\
 &= \left[\frac{2.77 - 2.330 + (1 - 0.16)0.81(2.74 - 2.25)}{2.79 - 0.594} \right] 100 \text{ SI units} \\
 &= 35.6\%
 \end{aligned}$$

VI SIMPLE BRAYTON CYCLE

Reactor systems that employ gas coolants offer the potential for operating as direct Brayton cycles by passing the heated gas directly into a turbine. This Brayton cycle is ideal for single-phase, steady-flow cycles with heat exchange and therefore is the basic cycle for modern gas turbine plants as well as proposed nuclear gas-cooled reactor plants. As with the Rankine cycle, the remaining processes and components are, sequentially, a heat exchanger for rejecting heat, a compressor, and a heat source,

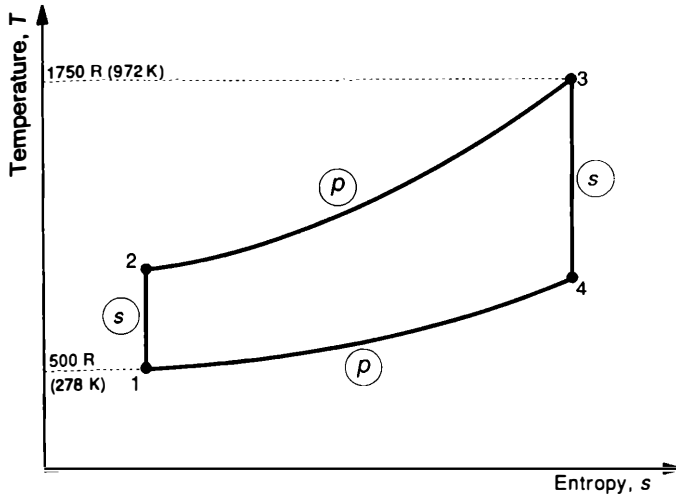


Figure 6-24 Temperature-entropy (T - s) plot of the simple Brayton cycle. (p) Constant pressure process. (s) Constant entropy process.

which in our case is the reactor. The ideal cycle is composed of two reversible constant-pressure heat-exchange processes and two reversible, adiabatic work processes. However, because the working fluid is single phase, no portions of the heat-exchange processes are carried out at constant temperature as in the Rankine cycle. Additionally, because the volumetric flow rate is higher, the compressor work, or “backwork,” is a larger fraction of the turbine work than is the pump work in a Rankine cycle. This large backwork has important ramifications for the Brayton cycle.

The T - s plot of the simple Brayton cycle of Figure 6-8 is shown in Figure 6-24. The pressure or compression ratio of the cycle is defined as:

$$r_p \equiv \frac{P_2}{P_1} = \frac{P_3}{P_4} \quad (6-97)$$

For isentropic processes with a perfect gas, the thermodynamic states are related in the following manner:

$$T v^{\gamma-1} = \text{constant} \quad (6-98)$$

$$T p^{\frac{1-\gamma}{\gamma}} = \text{constant} \quad (6-99)$$

where $\gamma \equiv c_p/c_v$. For a perfect gas, because enthalpy is a function of temperature only and the specific heats are constant:

$$\Delta h = c_p \Delta T \quad (6-100)$$

Applying this result for Δh (Eq. 6-100) and the isentropic relations for a perfect gas (Eqs. 6-98 and 6-99) to the specialized forms of the first law of Table 6-5, the Brayton cycle can be analyzed. The following equations are in terms of the pressure ratio (r_p) and the lowest and highest cycle temperatures (T_1 and T_3) both of which are generally known quantities.

The turbine and compressor works are given as follows. For a perfect gas:

$$\dot{W}_T = \dot{m}c_p(T_3 - T_4) = \dot{m}c_pT_3 \left(1 - \frac{T_4}{T_3}\right) \quad (6-101a)$$

Furthermore, for an isentropic process, application of Eq. 6-99 to Eq. 6-101a yields:

$$\dot{W}_T = \dot{m}c_pT_3 \left[1 - \frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}}}\right] \quad (6-101b)$$

Analogously for the compressor:

$$\dot{W}_{CP} = \dot{m}c_p(T_2 - T_1) = \dot{m}c_pT_1 \left[\frac{T_2}{T_1} - 1\right] = \dot{m}c_pT_1 [(r_p)^{\frac{\gamma-1}{\gamma}} - 1] \quad (6-102)$$

The heat input from the reactor and the heat rejected by the heat exchanger are:

$$\dot{Q}_R = \dot{m}c_p(T_3 - T_2) = \dot{m}c_pT_1 \left[\frac{T_3}{T_1} - (r_p)^{\frac{\gamma-1}{\gamma}}\right] \quad (6-103)$$

$$\dot{Q}_{HX} = \dot{m}c_p(T_4 - T_1) = \dot{m}c_pT_3 \left[\frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}}} - \frac{T_1}{T_3}\right] \quad (6-104)$$

As in Eq. 6-56, the maximum useful work of the reactor plant is equal to the product of the system flow rate and the coolant enthalpy rise across the core. Hence:

$$\dot{W}_{u, \max} \equiv \dot{Q}_R = \dot{m}c_pT_1 \left[\frac{T_3}{T_1} - (r_p)^{\frac{\gamma-1}{\gamma}}\right] \quad (6-105)$$

The Brayton nuclear plant thermodynamic efficiency is then:

$$\begin{aligned} \zeta &= \frac{\dot{W}_T - \dot{W}_{CP}}{\dot{W}_{u, \max}} = \frac{T_3 \left[1 - \frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}}}\right] - T_1 (r_p)^{\frac{\gamma-1}{\gamma}} \left[1 - \frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}}}\right]}{T_1 \left[\frac{T_3}{T_1} - (r_p)^{\frac{\gamma-1}{\gamma}}\right]} \\ &= 1 - \frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}}} \end{aligned} \quad (6-106)$$

It can be further shown that the optimum pressure ratio for maximum net work is:

$$(r_p)_{\text{optimum}} = \left(\frac{T_3}{T_1}\right)^{\frac{\gamma}{2(\gamma-1)}} \quad (6-107)$$

The existence of an optimum compression ratio can also be seen on a T - s diagram by comparing the enclosed areas for cycles operating between fixed temperature limits of T_1 and T_3 .

Example 6-7 First law thermodynamic analysis of a simple Brayton cycle

PROBLEM Compute the cycle efficiency for the simple Brayton cycle of Figures 6-8 and 6-24 for the following conditions:

1. Helium as the working fluid taken as a perfect gas with

$$c_p = 1.25 \text{ BTU/lbm } ^\circ\text{R} \text{ (5230 J/kg } ^\circ\text{K)}$$

$$\gamma = 1.658$$

$$\dot{m} \text{ in lbm/s (English units) or kg/s (SI units)}$$

2. Pressure ratio of 4.0
3. Maximum and minimum temperatures of 1750°R (972°K) and 500°R (278°K), respectively

SOLUTION

$$\begin{aligned} \frac{\dot{W}_T}{\dot{m}} &= c_p T_3 \left[1 - \frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}}} \right] = 1.25 (1750) \left[1 - \frac{1}{(4.0)^{0.397}} \right] \\ &= 925.9 \text{ BTU/lbm (2.150 MJ/kg)} \end{aligned}$$

$$\begin{aligned} \frac{\dot{W}_{CP}}{\dot{m}} &= c_p T_1 [(r_p)^{\frac{\gamma-1}{\gamma}} - 1] = 1.25 (500) [(4.0)^{0.397} - 1] \\ &= 458.67 \text{ BTU/lbm (1.066 MJ/kg)} \end{aligned}$$

$$\begin{aligned} \frac{\dot{W}_{u, \max}}{\dot{m}} &= c_p T_1 \left[\frac{T_3}{T_1} - (r_p)^{\frac{\gamma-1}{\gamma}} \right] = 1.25 (500) \left[\frac{1750}{500} - (4.0)^{0.397} \right] \\ &= 1103.8 \text{ BTU/lbm (2.560 MJ/kg)} \end{aligned}$$

$$\begin{aligned} \zeta &= \frac{(\dot{W}_T - \dot{W}_{CP})/\dot{m}}{\dot{W}_{u, \max}/\dot{m}} = \left(\frac{925.9 - 458.7}{1103.8} \right) 100 \text{ (English units)} \\ &= \left(\frac{2.15 - 1.066}{2.56} \right) 100 \text{ (SI units)} \\ &= 42.3\% \end{aligned}$$

Analogous to the PWR cycle demonstrated in Example 6-5, the Brayton cycle thermal efficiency equals the Brayton nuclear plant thermodynamic efficiency, i.e.,

$$\eta_{th} = \zeta$$

For ideal Brayton cycles, as Eq. 6-106 demonstrates, the thermodynamic efficiency increases with the compression ratio. The thermal efficiency of a Carnot cycle

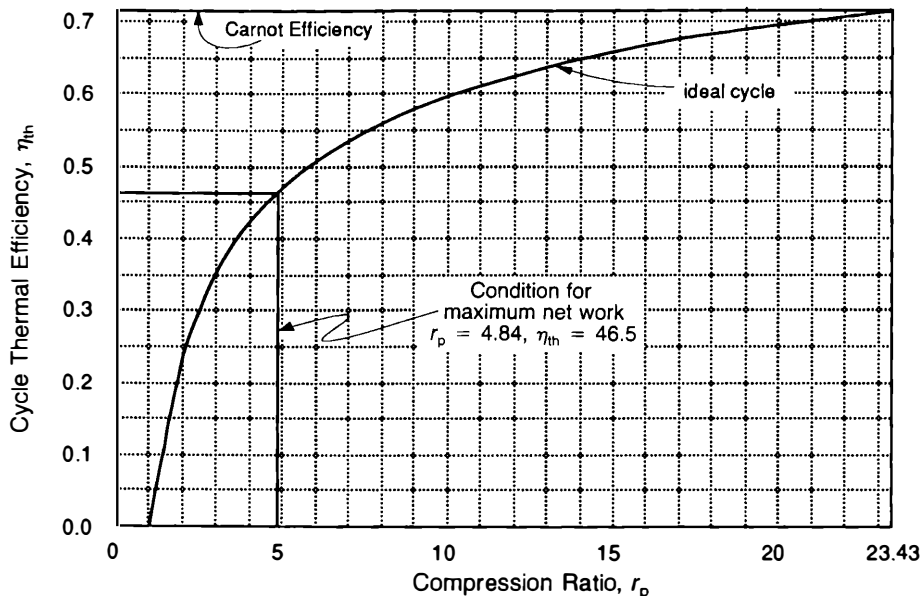


Figure 6-25 Thermal efficiency of an ideal Brayton cycle as a function of the compression ratio. $\gamma = 1.658$.

operating between the maximum and minimum temperatures specified is:

$$\eta_{\text{Carnot}} = \frac{T_{\text{max}} - T_{\text{min}}}{T_{\text{max}}} = \frac{972^{\circ}\text{K} - 278^{\circ}\text{K}}{972^{\circ}\text{K}} = 0.714$$

Figure 6-25 illustrates the dependence of cycle efficiency on the compression ratio for the cycle of Example 6-7. The pressure ratio corresponding to the maximum net work is also identified. This optimum pressure ratio is calculated from Eq. 6-107 for a perfect gas as:

$$(r_p)_{\text{optimum}} = \left(\frac{T_3}{T_1} \right)^{\frac{\gamma}{2(\gamma-1)}} = \left(\frac{972}{278} \right)^{\frac{1.658}{2(0.658)}} = 4.84$$

VII MORE COMPLEX BRAYTON CYCLES

In this section the various realistic considerations are included in the analysis and are illustrated through a number of examples.

Example 6-8 Brayton cycle with real components

PROBLEM Compute the thermal efficiency for the cycle depicted in Figure 6-26 if the isentropic efficiencies of the compressor and the turbine are each 90%. All other conditions of Example 6-7 apply.

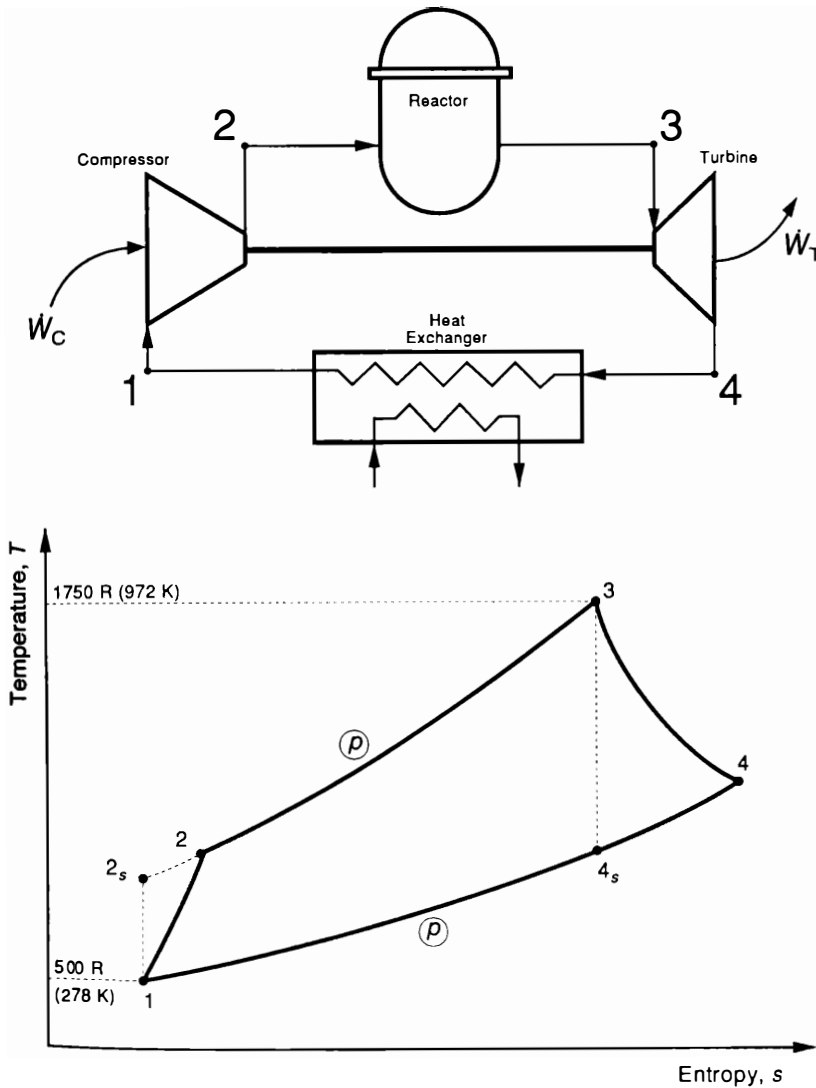


Figure 6-26 Brayton cycle with real components: Example 6-8.

SOLUTION

For \dot{W}_T :

$$\eta_T = \frac{\text{actual work out of turbine}}{\text{ideal turbine work}} = \frac{\dot{W}_T}{\dot{W}_{Ti}} = \frac{\dot{m}c_p(T_3 - T_4)}{\dot{m}c_p(T_3 - T_{4s})}$$

$$\therefore \dot{W}_T = \eta_T \dot{W}_{Ti} = \eta_T \dot{m}c_p(T_3 - T_{4s}) = \eta_T \dot{m}c_p T_3 \left(1 - \frac{T_{4s}}{T_3}\right)$$

$$\begin{aligned}
 &= \eta_T \dot{m} c_p T_3 \left[1 - \frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}}} \right] = \eta_T \dot{m} 925.9 = (0.9)(925.9) \dot{m} \\
 &= 833.3 \dot{m} \text{ BTU/s (1.935 } \dot{m} \text{ MJ/s or MW)}
 \end{aligned}$$

For \dot{W}_{CP} :

$$\begin{aligned}
 \eta_{CP} &= \frac{\text{ideal compressor work}}{\text{actual compressor work}} = \frac{\dot{W}_{CPi}}{\dot{W}_{CP}} = \frac{\dot{m} c_p (T_{2s} - T_1)}{\dot{m} c_p (T_2 - T_1)} \\
 \dot{W}_{CP} &= \frac{\dot{m}}{\eta_{CP}} c_p (T_{2s} - T_1) = \frac{\dot{m}}{\eta_{CP}} c_p T_1 \left(\frac{T_{2s}}{T_1} - 1 \right) = \frac{\dot{m} 458.7}{0.9} \\
 &= 509.7 \dot{m} \text{ BTU/s (1.184 } \dot{m} \text{ MW)} \\
 \dot{W}_{NET} &= \dot{W}_T - \dot{W}_{CP} = \dot{m} (833.3 - 509.7) = \dot{m} 323.6 \text{ BTU/s (} \dot{m} \text{ 0.752 MW)} \\
 \dot{Q}_R &= \dot{m} c_p (T_3 - T_2)
 \end{aligned}$$

To evaluate \dot{Q}_R it is necessary to find T_2 .

In Example 6-7 we found $\dot{W}_{CPi} = \dot{m} c_p (T_{2s} - T_1) = 458.7 \dot{m} \text{ BTU/s} = 1.066 \dot{m} \text{ MW}$, so that from the expressions above for \dot{W}_{CP} and η_{CP} :

$$\begin{aligned}
 T_2 - T_1 &= \frac{\dot{W}_{CP}}{\dot{m} c_p} = \frac{\dot{W}_{CPi}}{\dot{m} c_p \eta_{CP}} = \frac{458.7}{1.25(0.9)} = 407.7^\circ\text{R (226.5}^\circ\text{K)} \\
 T_2 &= 407.7 + T_1 = 407.7 + 500 = 907.7^\circ\text{R (504.3}^\circ\text{K)} \\
 \dot{Q}_R &= (1.25)(T_3 - T_2) \dot{m} = 1.25 (1750 - 907.7) \dot{m} \\
 &= 1052.9 \dot{m} \text{ BTU/s (2.45 } \dot{m} \text{ MW)} \\
 \eta_{th} &= \frac{\dot{W}_{NET}}{\dot{Q}_R} = \left(\frac{323.6}{1052.9} \right) 100 \text{ (English units)} \\
 &= \left(\frac{0.752}{2.45} \right) 100 \text{ (SI units)} \\
 &= 30.7\%
 \end{aligned}$$

Example 6-9 Brayton cycle considering duct pressure losses

PROBLEM Compute the cycle thermal efficiency considering pressure losses in the reactor and heat exchanger processes as well as 90% isentropic turbine and compressor efficiencies. The cycle is illustrated in Figure 6-27. The pressure losses are characterized by the parameter β where:

$$\beta \equiv \left(\frac{p_4 p_2}{p_1 p_3} \right)^{\frac{\gamma-1}{\gamma}} = 1.05$$

All other conditions of Example 6-7 apply.

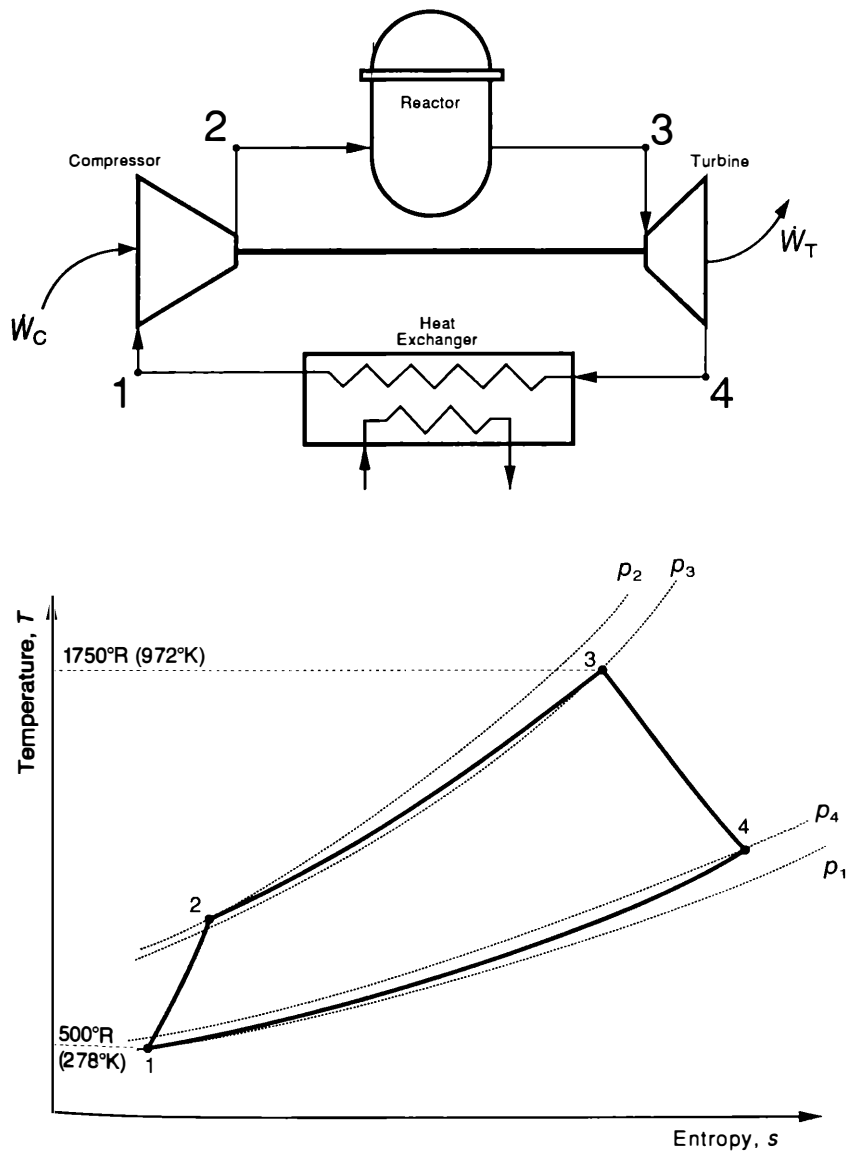


Figure 6-27 Brayton cycle considering duct pressure losses: Example 6-9.

$$\dot{W}_T = \eta_T \dot{m} c_p T_3 \left(1 - \frac{T_{4s}}{T_3} \right) = \eta_T \dot{m} c_p T_3 \left[1 - \frac{1}{\left(\frac{p_3}{p_4} \right)^{\frac{\gamma-1}{\gamma}}} \right]$$

SOLUTION Because β is defined as:

$$\begin{aligned} & \left(\frac{p_4}{p_1} \cdot \frac{p_2}{p_3} \right)^{\frac{\gamma-1}{\gamma}} \\ \therefore \left(\frac{p_4}{p_3} \right)^{\frac{\gamma-1}{\gamma}} &= \frac{\left(\frac{p_4}{p_1} \cdot \frac{p_2}{p_3} \right)^{\frac{\gamma-1}{\gamma}}}{\left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}}} = \frac{\beta}{(r_p)^{\frac{\gamma-1}{\gamma}}} \\ \therefore \dot{W}_T &= \eta_T \dot{m} c_p T_3 \left[1 - \frac{\beta}{(r_p)^{\frac{\gamma-1}{\gamma}}} \right] \\ &= 0.9 \dot{m} (1.25)(1750) \left[1 - \frac{1.05}{(4)^{0.397}} \right] \\ &= 776.5 \dot{m} \text{ BTU/s } (1.803 \dot{m} \text{ MW}) \end{aligned}$$

Again as in Example 6-8:

$$\begin{aligned} \dot{W}_{CP} &= \frac{\dot{m} c_p}{\eta_{CP}} (T_{2s} - T_1) = \dot{m} \frac{c_p T_1}{\eta_{CP}} \left(\frac{T_{2s}}{T_1} - 1 \right) = \frac{\dot{m} c_p T_1}{\eta_{CP}} \left[\left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \\ &= \frac{\dot{m} (1.25)(500)}{0.9} (1.7338 - 1.0) \\ &= 509.7 \dot{m} \text{ BTU/s } (1.184 \dot{m} \text{ MW}) \end{aligned}$$

Now $\dot{Q}_R = \dot{m} c_p (T_3 - T_2)$

We know that

$$\eta_{CP} = \frac{\dot{m} c_p (T_{2i} - T_1)}{\dot{m} c_p (T_2 - T_1)} = \frac{\dot{W}_{CPi}}{\dot{W}_{CP}}$$

where \dot{W}_{CPi} was calculated in Example 6-7.

$$\begin{aligned} \therefore T_2 - T_1 &= \frac{\dot{W}_{CPi}}{c_p \eta_{CP}} = \frac{458.7}{(0.9)(1.25)} = 407.7^\circ\text{R } (226.5^\circ\text{K}) \\ \therefore T_2 &= 500 + 407.7 = 907.7^\circ\text{R } (504.3^\circ\text{K}) \\ \therefore \dot{Q}_R &= \dot{m} c_p (1750 - 907.7) = 1052.9 \dot{m} \text{ BTU/s } (2.45 \dot{m} \text{ MW}) \\ \dot{W}_{NET} &= \dot{W}_T - \dot{W}_{CP} = \dot{m} (776.5 - 509.6) = 266.9 \dot{m} \text{ BTU/s } (0.620 \dot{m} \text{ MW}) \\ \eta_{th} &= \frac{\dot{W}_{NET}}{\dot{Q}_R} = \left(\frac{266.9}{1052.9} \right) 100 \text{ (English units)} \\ &= \left(\frac{0.620}{2.45} \right) 100 \text{ (SI units)} \\ &= 25.3\% \end{aligned}$$

Examples 6-8 and 6-9 demonstrate that consideration of real component efficiencies and pressure losses cause the cycle thermal efficiency to decrease dramatically. Employment of regeneration, if the pressure ratio allows, reverses this trend, as demonstrated next in Example 6-10, which considers both ideal and real turbines and compressors.

Example 6-10 Brayton cycle with regeneration for ideal and then real components

PROBLEM Compute the cycle thermal efficiency first for ideal turbines and compressors but with the addition of a regenerator of effectiveness 0.75. The cycle is illustrated in Figure 6-28. Regenerator effectiveness is defined as the actual preheat temperature change over the maximum possible temperature change, i.e.,

$$\xi = \frac{T_5 - T_2}{T_4 - T_2}$$

All other conditions of Example 6-7 apply.

SOLUTION

$$\begin{aligned}\dot{W}_{\text{Cp}} &= \dot{m}c_p(T_2 - T_1) = \dot{m}c_p T_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \\ &= \dot{m} 458.6 \text{ BTU/sec } (\dot{m} 1.066 \text{ MW})\end{aligned}$$

(as in Example 6-7). Likewise:

$$\begin{aligned}\dot{W}_{\text{T}} &= \dot{m}c_p(T_3 - T_4) = \dot{m}c_p T_3 \left[1 - \frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}}} \right] \\ &= \dot{m} 925.9 \text{ BTU/s } (\dot{m} 2.150 \text{ MW}) \\ \dot{Q}_{\text{R}} &= \dot{m}c_p(T_3 - T_5) \\ \xi_{\text{(effectiveness of regenerator)}} &= \frac{T_5 - T_2}{T_4 - T_2} = 0.75 \\ \therefore T_5 &= (T_4 - T_2)(0.75) + T_2 = 0.75 T_4 + 0.25 T_2\end{aligned}$$

Writing T_4 in terms of T_3 and r_p , T_2 in terms of T_1 and r_p , and noting that processes 1-2 and 3-4 are isentropic, obtain:

$$\begin{aligned}T_5 &= (0.75) \left[\frac{T_3}{(r_p)^{\frac{\gamma-1}{\gamma}}} \right] + 0.25 T_1 (r_p)^{\frac{\gamma-1}{\gamma}} \\ &= (0.75)(0.5767)(1750) + (0.25)(500)(1.7338) \\ &= 973.7^\circ\text{R } (540.9^\circ\text{K})\end{aligned}$$

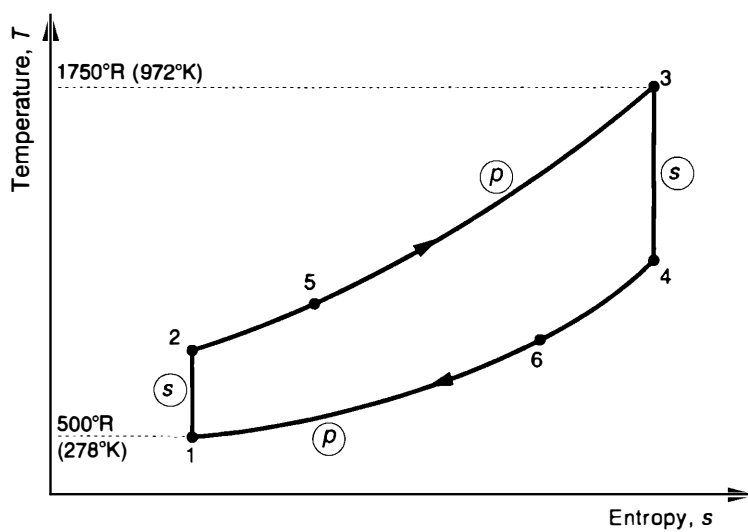
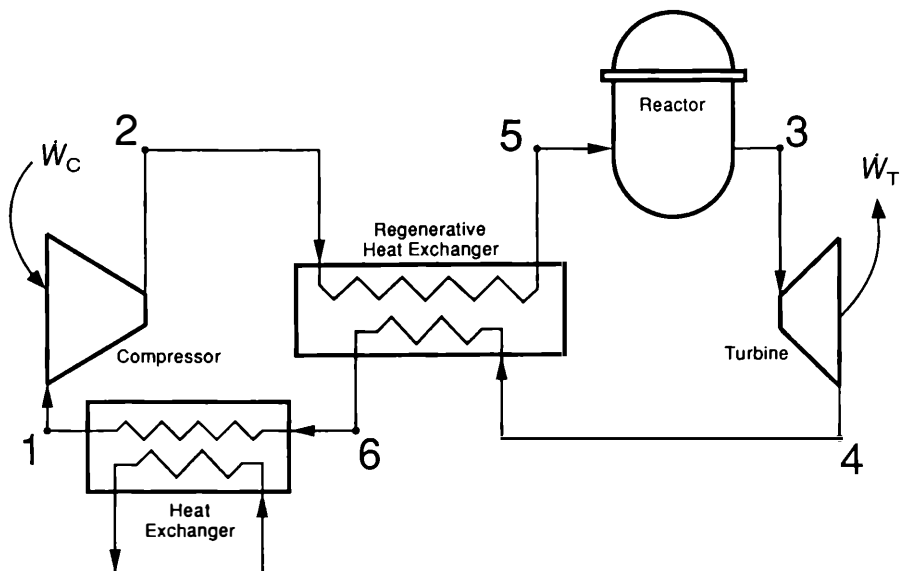


Figure 6-28 Brayton cycle with ideal components and regeneration: Example 6-10.

Hence:

$$\dot{Q}_R = \dot{m}c_p (1750 - 973.7) = 970.4 \dot{m} \text{ BTU/s (2.254 } \dot{m} \text{ MW)}$$

$$\dot{W}_{\text{NET}} = \dot{W}_T - \dot{W}_{\text{CP}} = \dot{m} (925.9 - 458.7) = \dot{m} 467.2 \text{ BTU/s (1.084 } \dot{m} \text{ MW)}$$

$$\begin{aligned}
 \eta_{\text{th}} &= \frac{\dot{W}_{\text{NET}}}{\dot{Q}_{\text{R}}} \\
 &= \left(\frac{467.2}{970.4} \right) 100 \text{ (English units)} \\
 &= \left(\frac{1.084}{2.254} \right) 100 \text{ (SI units)} \\
 &= 48.1\%
 \end{aligned}$$

Repeating these calculations for real compressors and turbines with component isentropic efficiencies of 90%, the thermal efficiency is reduced to 36.4%. Table 6-9 lists the intermediate calculations under Example 6-10B.

It is interesting to compare these results with those of Examples 6-7 and 6-8, which are identical except for regeneration. One sees that the use of regeneration has increased the efficiency for the cases of ideal components (48.1% versus 42.3%) and real components (36.4% versus 30.7%). In each situation regeneration has caused the thermal efficiency of the cycle to increase by decreasing \dot{Q}_{R} (\dot{W}_{NET} being the same), a result that is possible only if T_4 , the turbine exit, is hotter than T_2 , the compressor exit.

This condition is not the case for a sufficiently high pressure ratio (e.g., $p_2 = 8p_1$). In this case (Example 6-11 in Table 6-9), the cycle thermal efficiency is reduced to 38.4%.

A loss of thermal efficiency has resulted, from the introduction of a regenerator compared to the same cycle operated at a lower compression ratio, i.e., Example 6-10A (38.4% versus 48.1%). An additional comparison is between this case and the case of an ideal cycle without regenerator but operated at the same high compression ratio of 8. Again the efficiency of this case is less than this ideal cycle without regeneration (38.4% versus 56.2%—where the value of 56.2% can be obtained from Equation 6-106 or, equivalently, Figure 6-25). Physically both comparisons reflect the fact that for this case (Example 6-11) r_p is so high that T_2 is higher than T_4 ; i.e., the exhaust gases are cooler than those after compression. Heat would thus be transferred to the exhaust gases, thereby requiring \dot{Q}_{R} to increase and η_{th} to decrease. Obviously, for this high compression ratio, inclusion of a regenerator in the cycle is not desirable. Major process variations possible for cycles with high compression ratios are the intermediate extraction and cooling of gases undergoing compression (to reduce compressor work) and the intermediate extraction and heating of gases undergoing expansion (to increase turbine work). These processes are called *intercooling* and *reheat*, respectively, and are analyzed next in Example 6-14. Intermediate cases of intercooling only and reheat only are included in Table 6-9 as Examples 6-12 and 6-13, respectively.

Example 6-14 Brayton cycle with reheat and intercooling

Note: Examples 6-11, 6-12, and 6-13 appear only in Table 6-9.

SOLUTION Calculate the thermal efficiency for the cycle employing both intercooling and reheat as characterized below. The cycle is illustrated in Figure 6-29. All other conditions of Example 6-7 apply.

Table 6-9 Results of Brayton cycle cases of Examples 6-7 through 6-14

Parameter	Ex. 6-7	Ex. 6-8	Ex. 6-9	Ex. 6-10A	Ex. 6-10B	Ex. 6-11	Ex. 6-12	Ex. 6-13	Ex. 6-14
$\beta = \left(\frac{p_2 p_4}{p_3 p_1} \right)^{\frac{\gamma-1}{\gamma}}$	1.0	1.0	1.05	1.0	1.0	1.0	1.0	1.0	1.0
Component isentropic efficiency (η_s)	1.0	0.9	0.9	1.0	0.9	1.0	1.0	1.0	1.0
Regenerator effectiveness (ξ)	—	—	—	0.75	0.75	0.75	—	—	—
Pressure ratio (r_p)	4	4	4	4	4	8	4	4	4
Intercooling	—	—	—	—	—	—	$\frac{p'_1}{p_1} = \frac{1}{2} \frac{p_2}{p_1}$ $T'_1 = T_1$	$\frac{p'_3}{p_3} = \frac{1}{2} \frac{p_2}{p_3}$ $\frac{p'_4}{p_4} = \frac{1}{2} \frac{p_1}{p_4}$ $T'' = T$	$\frac{p'_1}{p_1} = \frac{1}{2} \frac{p_2}{p_1}$ $\frac{p'_3}{p_3} = \frac{1}{2} \frac{p_2}{p_3}$ $\frac{p'_4}{p_4} = \frac{1}{2} \frac{p_1}{p_4}$ $T'' = T$
Reheat	—	—	—	—	—	—	—	$\frac{p'_3}{p_3} = \frac{1}{2} \frac{p_2}{p_3}$ $\frac{p'_4}{p_4} = \frac{1}{2} \frac{p_1}{p_4}$ $T'_3 = T_3$	$\frac{p'_3}{p_3} = \frac{1}{2} \frac{p_2}{p_3}$ $\frac{p'_4}{p_4} = \frac{1}{2} \frac{p_1}{p_4}$ $T'_3 = T_3$
Turbine work (\dot{W}_T/m)									
BTU/lbm	925.9	833.3	776.5	925.9	833.3	1229.4	925.9	1052.5	1052.5
kJ/kg	2.150	1.935	1.803	2.150	1.935	2.855	2.150	2.444	2.444
Compressor work (\dot{W}_c/m)									
BTU/lbm	458.7	509.7	509.7	458.7	509.7	801.9	395.96	458.7	395.96
kJ/kg	1.066	1.184	1.184	1.066	1.184	1.862	0.920	1.066	0.920
Net work (\dot{W}_{NET}/m)									
BTU/lbm	467.2	323.6	266.9	467.2	323.6	427.5	529.9	593.8	656.5
kJ/kg	1.084	0.752	0.620	1.084	0.752	0.993	1.23	1.378	1.524
Heat in (\dot{Q}_H/m)									
BTU/lbm	1103.0	1052.9	1052.9	970.4	888.1	1112.2	1364.5	1630.1	1890.8
kJ/kg	2.560	2.45	2.45	2.254	2.062	2.583	3.169	3.786	4.391
Cycle thermal efficiency (η_{th}) (%)	42.3	30.7	25.3	48.1	36.4	38.4	38.8	36.4	34.7

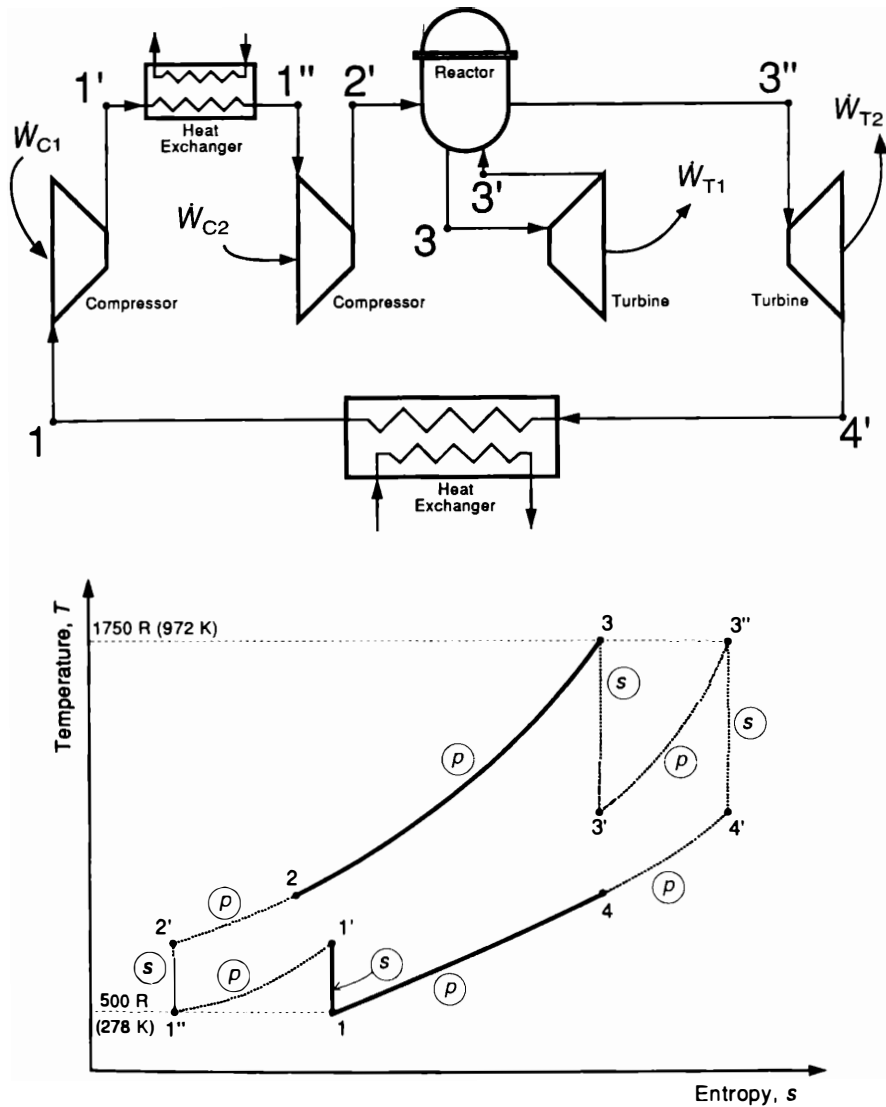


Figure 6-29 Brayton cycle with reheat and intercooling: Example 6-14.

$$\text{Intercooling: } \frac{p_1'}{p_1} = \frac{p_2}{p_1'} = r_p' \quad T_1'' = T_1$$

$$\text{Reheat: } \frac{p_3'}{p_4} = \frac{p_3}{p_3'} = r_p' \quad T_3'' = T_3$$

SOLUTION

$$\dot{W}_{CP} = \dot{m}c_p(T_1' - T_1) + \dot{m}c_p(T_2' - T_1'')$$

$$\begin{aligned}
\dot{W}_{CP} &= \dot{m}c_p T_1 \left(\frac{T'_1}{T_1} - 1 \right) + \dot{m}c_p T''_1 \left(\frac{T'_2}{T''_1} - 1 \right) \\
&= \dot{m}c_p T_1 \left[(r'_p)^{\frac{\gamma-1}{\gamma}} - 1 \right] + \dot{m}c_p T''_1 \left[(r'_p)^{\frac{\gamma-1}{\gamma}} - 1 \right] \\
&= 2\dot{m}c_p T_1 \left[(r'_p)^{\frac{\gamma-1}{\gamma}} - 1 \right] = 2\dot{m}c_p T_1 [(2)^{0.397} - 1] \\
&= 395.96 \dot{m} \text{ BTU/s (0.920 } \dot{m} \text{ MW)} \\
\dot{W}_T &= \dot{m}c_p (T_3 - T'_3) + \dot{m}c_p (T''_3 - T'_4) = \dot{m}c_p T_3 \left(1 - \frac{T'_3}{T_3} \right) + \dot{m}c_p T''_3 \left(1 - \frac{T'_4}{T''_3} \right)
\end{aligned}$$

Again for the isentropic case:

$$\begin{aligned}
\dot{W}_T &= 2\dot{m}c_p T_3 \left[1 - \frac{1}{(r'_p)^{\frac{\gamma-1}{\gamma}}} \right] \\
&= 2\dot{m} (1.25) 1750 \left[1 - \frac{1}{(2)^{0.397}} \right] \\
&= 1052.5 \dot{m} \text{ BTU/s (2.444 } \dot{m} \text{ MW)} \\
\frac{T'_3}{T_3} &= \frac{1}{(r'_p)^{\frac{\gamma-1}{\gamma}}} \\
\therefore T'_3 &= \frac{T_3}{(r'_p)^{\frac{\gamma-1}{\gamma}}} = \frac{1750}{(2)^{0.397}} = \frac{1750}{1.317} \\
&= 1329.0^\circ\text{R (738.3}^\circ\text{K)}
\end{aligned}$$

where $T'_2 = T'_1 (r'_p)^{\frac{\gamma-1}{\gamma}}$ and $T''_1 = T_1 = 500^\circ\text{R (278}^\circ\text{K)}$.

$$\therefore T'_2 = (500^\circ\text{R}) 2^{0.397} = 658.4^\circ\text{R (365.8}^\circ\text{K)}$$

$$\dot{Q}_R = \dot{m}c_p (T_3 - T_2) + \dot{m}c_p (T'_3 - T'_2)$$

$$\dot{Q}_R = \dot{m}c_p [(1750 - 658.4) + (1750 - 1329.0)]$$

$$= 1890.8 \dot{m} \text{ BTU/s (4.391 } \dot{m} \text{ MW)}$$

$$\dot{W}_{NET} = \dot{W}_T - \dot{W}_{CP} = \dot{m} (1052.5 - 395.96) = 656.5 \dot{m} \text{ BTU/s (1.524 MW)}$$

$$\eta_{th} = \frac{\dot{W}_{NET}}{\dot{Q}_R}$$

$$= \left(\frac{656.5}{1890.8} \right) 100 \text{ (English units)}$$

$$= \left(\frac{1.524}{4.391} \right) 100 \text{ (SI units)}$$

$$= 34.7\%$$

Compared with Example 6-7, both the net work and the maximum useful work have increased, but the thermal efficiency has decreased. An increase in the thermal efficiency would result if regeneration were employed. These changes are the combined result of the intercooling and reheating processes. They can best be understood by examining each separately with reference to the results of Examples 6-12 and 6-13, respectively.

In comparison with Example 6-7 (which is the same except for intercooling), we see that intercooling has decreased the compressor work and hence raised the \dot{W}_{NET} . However, the cycle efficiency has decreased to 38.8% from 42.3%, as the cycle with intercooling has a smaller pressure ratio than the basic cycle. However, by lowering inlet temperature to the reactor, i.e., T_2 to T'_2 , a portion of this added energy can be supplied regeneratively, thereby offering the possibility to recoup some of the decrease in efficiency.

As we see in comparison with Example 6-7, reheat also decreases cycle thermal efficiency (36.4% versus 42.3%) because the cycle with reheat has a smaller pressure ratio than the basic cycle itself. However, the turbine exit temperature has been raised from T_4 to T'_4 , offering the possibility of supplying some of the energy addition from states 2 to 3 regeneratively with its associated improvement in cycle thermal efficiency.

REFERENCE

1. Hicks, E. P., and Menzies, D. C. Theoretical studies on the fast reactor maximum accident." In: *Proceedings of the Conference on Safety, Fuels, and Core Design in Large Fast Power Reactors*. ANL-7120, 1965, pp. 654-670.

PROBLEMS

Problem 6-1 Work output of a fuel-water interaction (section II)

Compute the work done by a fuel-water interaction assuming that the 40,000 kg of mixed oxide fuel and 4000 kg of water expand independently and isentropically to 1 atmosphere. Assume that the initial fuel and water conditions are such that equilibrium mixture temperature (T_e) achieved is 1945°K. Other water conditions are as follows: $T_{\text{initial}} = 400^\circ\text{K}$; $\rho_{\text{initial}} = 945 \text{ kg/m}^3$; $c_v = 4184 \text{ J/kgK}$. *Caution:* Eq. 6-9 is inappropriate for these conditions, as the coolant at state e is supercritical.

Answer: $1.67 \times 10^{10} \text{ J}$

Problem 6-2 Evaluation of alternate ideal Rankine cycles (section III)

Three alternative steam cycles illustrated in Figure 6-30 are proposed for a nuclear power station capable of producing either saturated steam or superheated steam at a temperature of 293°C. The condensing steam temperature is 33°C.

1. Assuming ideal machinery, calculate the cycle thermal efficiency and steam rate (kg steam/kWe-hr) for each cycle using the steam tables.
2. Compare the cycle thermal efficiencies calculated with the Carnot efficiency
3. For each cycle, compare the amount of heat added per unit mass of working fluid in the legs $3' \rightarrow 4$ and $4 \rightarrow 1$.
4. Briefly compare advantages and disadvantages of each of these cycles. Which would you use?

Answers: $\eta_{\text{th}} = 38.2\%, 45.9\%, 36.8\%$

Steam rate = 3.60, 5.38, 3.54 kg steam/(kWe-hr)

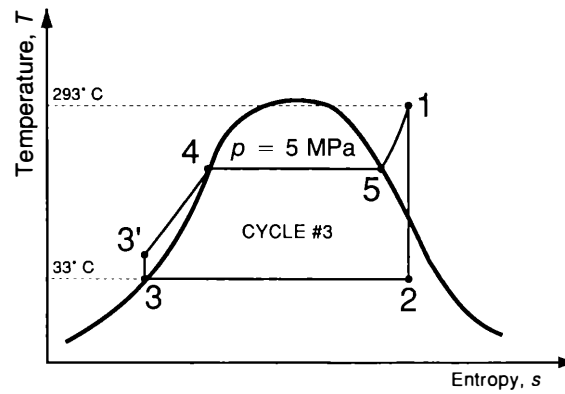
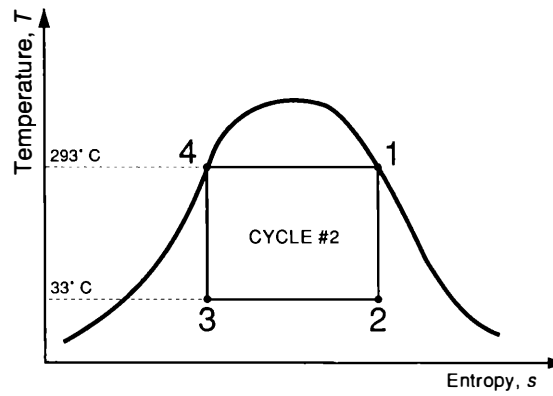
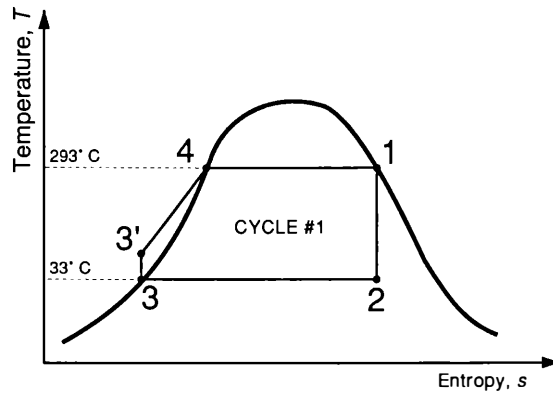


Figure 6-30 Alternate ideal Rankine cycles.

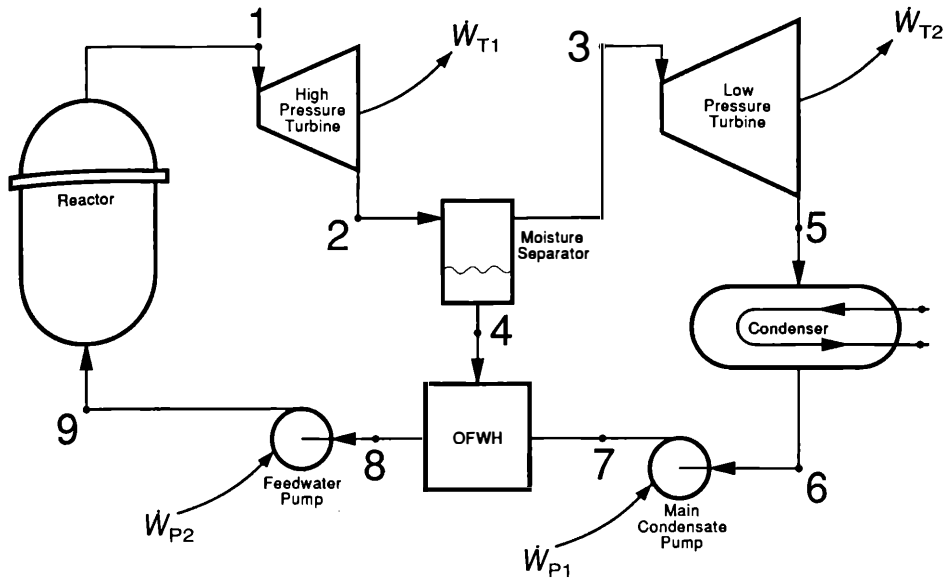


Figure 6-31 BWR plant.

Problem 6-3 Availability analysis of a simplified BWR (section IV)

A BWR system with a one-stage moisture separation is shown in Figure 6-31. The conditions in the table may be assumed.

Points	p(kPa)	Condition
1	6890	Saturated vapor
2	1380	
3	1380	Saturated vapor
4	1380	Saturated liquid
5	6.89	
6	6.89	Saturated liquid
7	1380	
8	1380	
9	6890	

Turbine isentropic efficiency = 90%

Pump isentropic efficiency = 85%

Environmental temperature = 30°C

1. Calculate the cycle thermal efficiency.
2. Recalculate the thermal efficiency of the cycle assuming that the pumps and turbines have isentropic efficiency of 100%.
3. Calculate the lost work due to the irreversibility of each component in the cycle and show numerically that the available work equals the sum of the lost work and the net work.

Answers: $\eta_{th} = 34.2\%$
 $\eta_{th \max} = 37.7\%$
 $\Sigma I = 1.65 \text{ MJ/kg}$
 $\dot{W}_{NET} = 0.86 \text{ MJ/kg}$

Problem 6-4 Analysis of a steam turbine (section IV)

In the test of a steam turbine the following data were observed:

$h_1 = 3000 \text{ kJ/kg}$; $p_1 = 10 \text{ MPa}$; $V_1 = 150 \text{ m/s}$
 $h_2 = 2600 \text{ kJ/kg}$; V_2 is negligible, $p_2 = 0.5 \text{ MPa}$
 $Z_2 = Z_1$ and $W_{1,2} = 384.45 \text{ kJ/kg}$

1. Assume steady flow, and determine the heat transferred to the surroundings per kilogram of a steam.
2. What is the quality of the exit steam?

Answers: $Q = -26.8 \text{ kJ/kg}$
 $x = 92.9\%$

Problem 6-5 Advantages of moisture separation and feedwater heating (section V)

A simplified BWR system with moisture separation and an open feedwater heater is described in Problem 6-3. Compute the improvement in thermal efficiency that results from the inclusion of these two components in the power cycle. Do you think the thermal efficiency improvement from these components is sufficient to justify the capital investment required? Are there other reasons for having moisture separation?

Answers: η_{th} changes from 36.9% to 37.7%

Problem 6-6 Ideal Brayton cycle (section VI)

The Brayton cycle shown in Figure 6-32 operates using CO_2 as a working fluid with compressor and turbine isentropic efficiencies of 1.0. Calculate the thermal efficiency of this cycle when the working fluid is modeled as:

1. A perfect gas of $\gamma = 1.30$.
2. A real fluid (see below for extracted values from Keenan and Kay's gas tables).
3. A real fluid and the compressor and turbine both have isentropic efficiencies of 0.95.

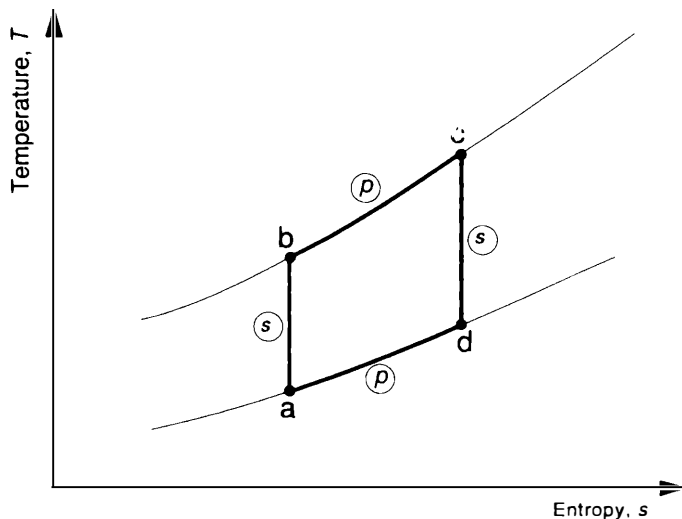


Figure 6-32 Ideal Brayton cycle. State a: $p = 20 \text{ psia}$, $T = 90^\circ\text{F}$. State c: $p = 100 \text{ psia}$, $T = 1000^\circ\text{F}$.

The parameters needed for a real fluid (from Keenan and Kay's gas tables) are shown in the following table.

Parameter	a	b	c	d
T	90		1000	
p	20	100	100	20
p_r	0.16108	0.8054	31.5	6.3
\bar{h}	4146.0	6239.8	14,204	10,077

where T = temperature, °F; p_r = relative pressure*; \bar{h} = enthalpy per mole, BTU/lb-mole.

*The ratio of the pressures p_a and p_b corresponding to the temperatures T_a and T_b , respectively, for an isentropic process is equal to the ratio of the relative pressures p_{ra} and p_{rb} as tabulated for T_a and T_b , respectively. Thus:

$$\left(\frac{p_a}{p_b} \right)_{s = \text{constant}} = \frac{p_{ra}}{p_{rb}}$$

Answers: $\eta_{th} = 31.0\%$, 25.5% , and 21.9% , respectively

Problem 6-7 Complex real Brayton cycle (section VII)

A gas-cooled reactor is designed to heat helium gas to a maximum temperature of 1000°F. The helium flows through a gas turbine, generating power to run the compressors and an electric generator, and then through a regenerative heat exchanger and two stages of compression with precooling to 100°F before entering each compressor. Each compressor and the turbine have an isentropic efficiency of 85%, and the pressure drop factor β is equal to 1.05. Each compression stage has a pressure ratio (r_p) of 1.27. The heat exchanger effectiveness (ξ) is 0.90.

Determine the cycle thermal efficiency. The Brayton cycle system is illustrated in Figure 6-33. The pressure drop factor β is defined as:

$$\beta = \left(\frac{p_4}{p_6} \cdot \frac{p_7}{p_1} \cdot \frac{p_2}{p_3} \right)^{\frac{\gamma-1}{\gamma}} \equiv 1.05$$

Answer: $\eta_{th} = 14.1\%$

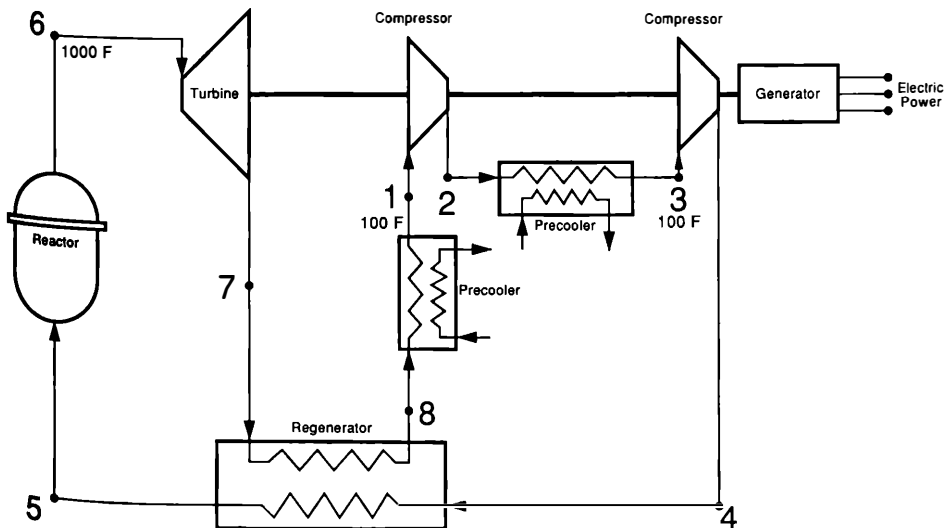


Figure 6-33 Complex Brayton cycle.