NPRE 449: Homework 7

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1 Question 1

To begin, our differential equations for single phase, 1-D are:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v}{\partial z} = 0 \tag{1a}$$

$$\frac{\partial \rho v}{\partial t} + \frac{\partial}{\partial z}\rho v^2 = -\frac{\partial P}{\partial z} - \tau_F \frac{\xi_w}{A_f} - \rho g \sin(\theta) \tag{1b}$$

$$\frac{\partial \rho h}{\partial t} + \frac{\partial}{\partial z}\rho v h = \frac{q'' \xi_h}{A} + \frac{\partial P}{\partial t} + q''' \tag{1c}$$

Now, we apply our simplifications. Assuming steady state, substituting in $G = \rho v$, the pipe is horizontal (i.e. $\theta = 0$), $h = c_p T$, incompressible, and no heat-generation:

$$\frac{\partial G}{\partial z} = 0 \tag{2a}$$

$$\frac{\partial}{\partial z}\frac{G^2}{\rho} = -\frac{\partial P}{\partial z} - \tau_F \frac{\xi_w}{A_f} \tag{2b}$$

$$c_p \frac{\partial G T}{\partial z} = \frac{q'' \xi_h}{A} \tag{2c}$$

From the mass equation, (2a), we see the mass flux is constant. Further, the definition of τ_F is $\frac{fG^2}{2\rho}$. Thus, the momentum and energy equations simplify further:

$$-\frac{\partial P}{\partial z} = \frac{f G^2 \xi_w}{2 \rho A_f} \tag{3a}$$

$$c_p G \frac{\partial T}{\partial z} = \frac{q'' \xi_h}{A} \tag{3b}$$

Now, solving our energy equation we can find the temperature distribution:

$$\frac{\partial T}{\partial z} = \frac{q'' \xi_h}{c_p G A} \tag{4a}$$

$$\int_0^l \partial T = \int_0^l \frac{q'' \xi_h}{c_p G A} \partial z \tag{4b}$$

$$T(l) - T(0) = \frac{q'' \xi_h}{c_p G A} l \tag{4c}$$

$$\frac{\Delta T c_p G A}{q'' \xi_h} = l \tag{4d}$$

$$\boxed{l = 6.652\ m} \tag{4e}$$

where material properties were evaluated at the average temperature of the fluid, $50/^oC$.

To find the surface temperature at the exit, we utilize newtons law of cooling:

$$q'' = h(T_s(z) - \bar{T}(z)) \tag{5}$$

To find h, we must first determine which correlation to use. Thus we find the Reynolds number, $\frac{GD}{\mu}$, as 598.85 (using material properties at 80 ^oC. Thus the flow is laminar and because it is fully developed in a pipe, we can use $Nu = 4.36$. Thus, using the definition for $Nu = \frac{hD}{k}$, we can solve for h. Using material properties of water at Standard pressure and 80 ^oC, we find h to be 48.468. Thus, plugging in all known values into Eq. 5, and solving for T_s:

$$T_s(l) = \frac{q''}{h} + \bar{T}(l) \tag{6a}$$

$$\boxed{T_s(l) = 121.264} \tag{6b}$$

Finally, fully developed and single phase flow is a poor assumption, as the temperature at the outlet is past the boiling point of water, thus this is actually two phase flow. Further, because it is two-phase, the temperature profile must not be fully developed.

2 Question 2

To begin, our governing equations for the fluid, assuming steady state, are:

$$\frac{\partial G}{\partial z} = 0 \tag{7a}$$

$$G^2 \frac{\partial}{\partial z}\frac{1}{\rho} = -\frac{\partial P}{\partial z} - \tau_F \frac{\xi_w}{A_f} - \rho g \sin(\theta) \tag{7b}$$

$$c_p G \frac{\partial T_f}{\partial z} = \frac{q'' \xi_h}{A} + q''' \tag{7c}$$

Assuming incompressibility, horizontal pipe, and no heat generation in our fluid the momentum and energy equations simplify to:

$$-\frac{\partial P}{\partial z} = \tau_F \frac{\xi_w}{A_f} = \frac{f G^2 \xi_w}{2\rho A_f} \tag{8a}$$

$$c_p G \frac{\partial T_f}{\partial z} = \frac{q'' \xi_h}{A} \tag{8b}$$

Then, through control volume analysis, we can relate q''' in the wall to the heat flux into the fluid:

$$q'' A_s = q''' V_p \tag{9}$$

plugging this back into the energy equation and solving for the temperature distribution:

$$\frac{\partial T_f}{\partial z} = \frac{q''' V_p \xi_h}{c_p G A_f A_s} \tag{10a}$$

$$T_f(z) = \frac{q''' V_p \xi_h}{c_p G A_f A_s} z + T_0 \tag{10b}$$

$$T_f(z) = \frac{q''' \pi h (r_o^2 - r_i^2)(2\pi r_i)}{c_p G (\pi r_i^2)(2\pi r_i h)} z + T_0 \tag{10c}$$

plugging in l for z, we can solve for l:

$$T_f(z) = \frac{q''' (r_o^2 - r_i^2)}{c_p G(\ r_i^2)} z + T_0 \tag{11a}$$

$$l = \frac{\Delta T c_p G (r_i^2)}{q''' (r_o^2 - r_i^2)} \tag{11b}$$

$$\boxed{l = 17.734\ m} \tag{11c}$$

Next to solve for the heat transfer coefficient we simply solve newtons law of cooling for h:

$$q'' = h(T_s - \bar{T}) \tag{12a}$$

$$h = \frac{q''}{T_s - \bar{T}} \tag{12b}$$

$$\boxed{h = 1500} \tag{12c}$$

Next, to solve for the heat transfer coefficient we first solve for the Reynolds: $Re = \frac{GD}{\mu} = 9749.467$. Thus our flow is turbulent, and we will use the Dittus-Boelter correlation to find h:

$$Nu = 0.023 Re^{0.8} Pr^{0.3} \tag{13}$$

Thus our Nusselt number is 55.485, and then solving for h we get:

$$\boxed{h = 2114.197} \tag{14}$$

3 Question 3

First, performing an energy balance, we can determine how much heat was lost by the hot air.

$$\dot{E}_{loss} = \dot{m} c_p \Delta T \tag{15}$$

$$\boxed{\dot{E}_{loss} = 1300.65W} \tag{16}$$

Further, we can relate the heat loss by the air to the heat flux due to convection by:

$$h A_s \Delta T_{air,o} = q'' A_s = \dot{m} c_p \Delta T_{air,i} \tag{17}$$

thus q" is simply:

$$\boxed{q'' = \frac{\dot{m} c_p \Delta T_{air,i}}{A_s} = 552.013} \tag{18}$$

and then, because the ambient air is at o^C, the $\Delta T_{air,o}$ is simply the surface temperature of the duct:

$$\boxed{T_s = \frac{q''}{h} = 92.002} \tag{19}$$

4 Question 4

To begin, the governing equations for this fluid are:

$$\frac{\partial G}{\partial z} = 0 \tag{20a}$$

$$G^2 \frac{\partial}{\partial z}\frac{1}{\rho} = -\frac{\partial P}{\partial z} - \tau_F \frac{\xi_w}{A_f} \tag{20b}$$

$$c_p G \frac{\partial T_f}{\partial z} = \frac{q'' \xi_h}{A} \tag{20c}$$

However, for this problem set-up we only care about the energy equation. Thus, solving this equation:

$$\frac{\partial T_f}{\partial z} = \frac{q'' \xi_h}{c_p A G} \tag{21a}$$

$$\frac{\partial \theta}{\partial z} = \frac{h \xi_h}{c_p A G} \theta(z) \tag{21b}$$

$$\theta(z) = \theta_0 e^{\frac{h\xi_h}{c_p A G} z} \tag{21c}$$

$$T_f(z) = T_s - (T_s - T_{f,in})\, e^{-\frac{h\xi_h}{c_p A G} z} \tag{21d}$$

To find h, we first determine reynolds number. We find Re to be 397.888, and thus it is a laminar pipe. Hence, we use a constant Nusselt number: 3.66. Finally, we find h to simply be: 10.102. Plugging this in, and solving for $T_f(L)$, we find the outlet temperature to be:

$$\boxed{T_f(L) = 24.751^o C} \tag{22}$$

Next, to find the heat transfer rate we utilize $\dot{m} c_p \Delta T$:

$$q = \dot{m} c_p \Delta T \tag{23a}$$

$$\boxed{q = 5062.139\ W} \tag{23b}$$

5 Question 5

To begin, the governing equations for this fluid are:

$$\frac{\partial G}{\partial z} = 0 \tag{24a}$$

$$G^2 \frac{\partial}{\partial z} \frac{1}{\rho} = -\frac{\partial P}{\partial z} - \tau_F \frac{\xi_w}{A_f} \tag{24b}$$

$$c_p G \frac{\partial T_f}{\partial z} = \frac{q'' \xi_h}{A} \tag{24c}$$

Solving the energy equation:

$$\frac{\partial T_f}{\partial z} = \frac{q'' \xi_h}{c_p A G} \tag{25a}$$

$$\frac{\partial \theta}{\partial z} = \frac{h \xi_h}{c_p A G} \theta(z) \tag{25b}$$

$$\theta(z) = \theta_0 e^{\frac{h \xi_h}{c_p A G} z} \tag{25c}$$

$$T_f(z) = T_s - (T_s - T_{f,in})\, e^{-\frac{h \xi_h}{c_p A G} z} \tag{25d}$$

Solving this equation for L, the distance required to heat the pipe to 75 ^oC:

$$T_f(z) = T_s - (T_s - T_{f,in})\, e^{-\frac{h \xi_h}{c_p A G} z} \tag{26a}$$

$$\frac{\theta(z)}{\theta_0} = e^{-\frac{h \xi_h}{c_p A G} L} \tag{26b}$$

$$\ln\left(\frac{\theta(z)}{\theta_0}\right) = \frac{h \xi_h}{c_p A G} L \tag{26c}$$

$$L = \ln\left(\frac{\theta(z)}{\theta_0}\right)\left[\frac{c_p A G}{h \xi_h}\right] \tag{26d}$$

$$h = \frac{Nuk}{D} = \frac{0.023(\frac{GD}{\mu})^{0.8}(\frac{\mu c_p}{k})^{0.4} k}{D} = 6918.149 \tag{26e}$$

$$\boxed{L = 10.563m} \tag{26f}$$

Next, to find the pressure drop, we investigate the momentum equation:

$$-\frac{\partial P}{\partial z} = \tau_f \frac{\xi_w}{A_f} \tag{27a}$$

$$\Delta P = \frac{f G^2 \xi_w l}{2 \rho A_f} \tag{27b}$$

where f is found via investigation of the Reynolds number. Recalling the Reynolds number as

116415.220.

$$f = (0.79 \ln Re - 1.64)^{-2} = \boxed{0.0174} \tag{28}$$

Then:

$$\Delta P = \frac{f G^2 \xi_w L}{2\rho A_f} \tag{29a}$$

$$\Delta P = \frac{f \rho u^2 (\pi D) L}{2(\pi/4D^2)} \tag{29b}$$

$$\Delta P = \frac{2 f \rho u^2 L}{D} \tag{29c}$$

$$Substituting\ in\ \frac{f}{4}\ for\ f:$$

$$\Delta P = \frac{f G^2 L}{2\rho D} \tag{29d}$$

$$\boxed{\Delta P = 5899.008\ kPa} \tag{29e}$$

Then, to solve for the pipe length and pressure drop as a function of $\dot{m}$ and D:

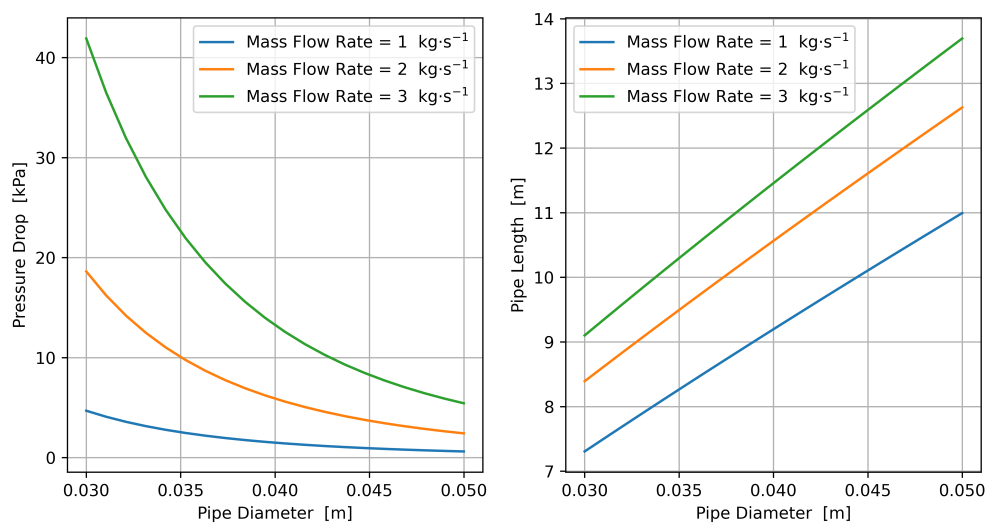

Figure 1: Pressure Drop (left) and Pipe Length (right)

Or in much prettier form, see the appendix.

6 Question 6

To begin, we do a control volume analysis on the fuel rod:

$$\pi D q'' dz = q''' \pi \frac{D^2}{4} dz \tag{30a}$$

$$q''' = \frac{4q'}{\pi D^2} \tag{30b}$$

$$\pi D q'' dz = q' dz \tag{30c}$$

$$\boxed{q'' = \frac{q'}{\xi_h}} \tag{30d}$$

Next, we analyze the mass, momentum, and energy equations:

$$\frac{\partial G}{\partial z} = 0 \tag{31a}$$

$$-\frac{\partial P}{\partial z} = \tau_F \frac{\xi_w}{A_f} + \rho g \tag{31b}$$

$$c_p G \frac{\partial T_f}{\partial z} = \frac{q'' \xi_h}{A} \tag{31c}$$

Solving for q'' for use in the energy equation:

$$q'' = \frac{q'}{\xi_h} \tag{32a}$$

$$q''' = q_0''' \sin \frac{\pi z}{L} \tag{32b}$$

$$q'' = \frac{A_{xs} q_0''' \sin \frac{\pi z}{L}}{\xi_h} \tag{32c}$$

Then subbing in and solving for the temperature distribution:

$$\frac{\partial T_f}{\partial z} = \frac{A_{xs} q_0'''}{c_p G A_f} \sin \frac{\pi z}{L} \tag{33a}$$

$$\int T_f = \int_0^z \frac{A_{xs} q_0'''}{c_p G A_f} \sin \frac{\pi z}{L} \tag{33b}$$

$$\boxed{T_f(z) = \frac{A_{xs} q_0''' L}{c_p G A_f \pi} \left[1 - \cos \frac{\pi z}{L}\right] + T_{f,in}} \tag{33c}$$

Then, finding the maximum surface temperature, we simply differentiate newtons law of cooling:

$$q''(z) = h(T_s(z) - T_f(z)) \tag{34a}$$

$$T_s(z) = \frac{q''}{h} + T_f(z) \tag{34b}$$

$$T_s(z) = \frac{A_{xs} q_0''' \sin\frac{\pi z}{L}}{h\xi_h} + \frac{A_{xs} q_0''' L}{c_p G A_f \pi}\left[1 - \cos\frac{\pi z}{L}\right] + T_{f,in} \tag{34c}$$

$$differentiating\ with\ respect\ to\ z:$$

$$0 = \frac{A_{xs} q_0''' \pi}{h\xi_h L}\cos\frac{\pi z}{L} + \frac{A_{xs} q_0'''}{c_p G A_f}\sin\frac{\pi z}{L} \tag{34d}$$

$$-\frac{\pi c_p G A_f}{h\xi_h L}\cot\frac{\pi z}{L} = 1 \tag{34e}$$

$$\cot\frac{\pi z}{L} = -\frac{h\xi_h L}{\pi c_p G A_f} \tag{34f}$$

$$z = \arctan\left(-\frac{\pi c_p G A_f}{h\xi_h L}\right)\frac{L}{\pi} \tag{34g}$$

7 Appendix

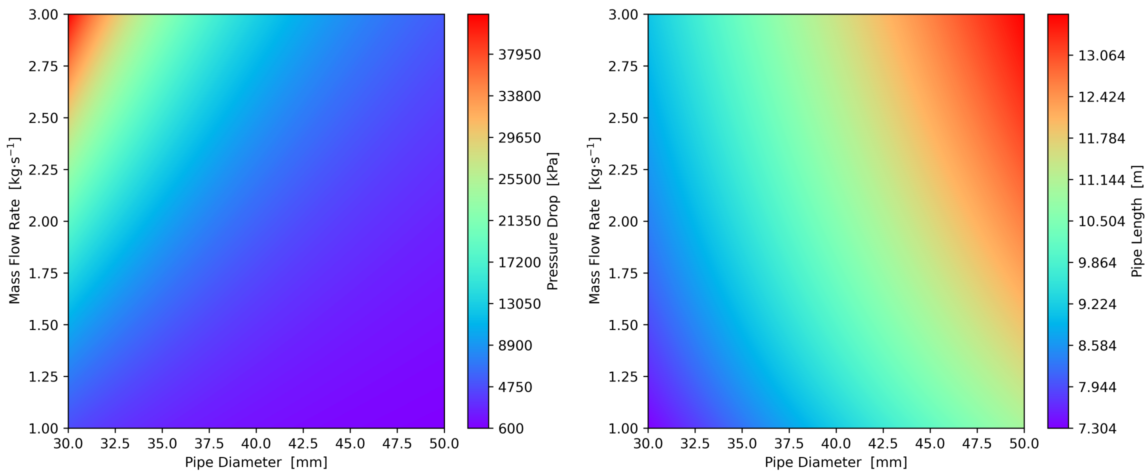

Figure 2: Pressure Drop (left) and Pipe Length (right)