Initial Experiments with the Double-Well System

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1 The System

I have written two functions that implement a single double-well system, optionally with added noise and stress, and a system of coupled double-wells that interact according to an adjacency matrix.

1.1 A Single Double-Well System

$$\Delta x = (-(x - r_1)(x - r_2)(x - r_3) + u + \varepsilon)\Delta t \tag{1}$$

$$x_{t+1} = x_t + \Delta x \tag{2}$$

where $r_1 = 1$, $r_2 = 2$, and $r_3 = 5$ are parameters that influence the "shape" the double-well system, $u = 10$ is a constant representing stress being added to the system, and ε is currently implemented as Gaussian noise ($\sim N(0, 10)$).

1.2 Coupled Double-Wells

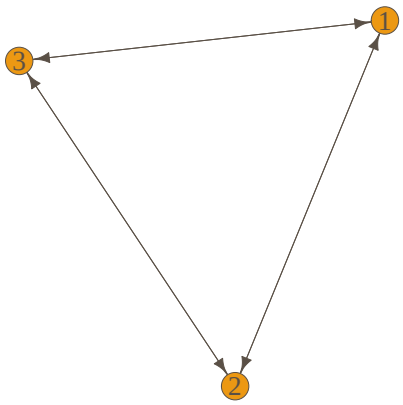
$$\Delta x_i = (-(x_i - r_1)(x_i - r_2)(x_i - r_3) + D\sum_{j=1}^{n} Ax_j + u_i + \varepsilon_i)\Delta t \tag{3}$$

with symbols as before and $D \in (0, 1)$ a coupling strength and A an adjacency matrix.

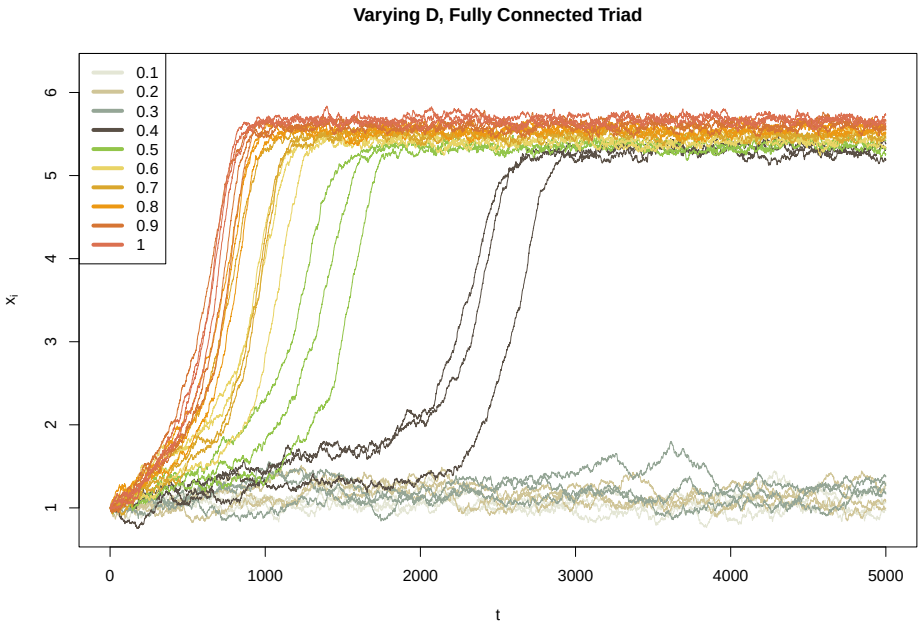
I am focusing on a few particular triads: the complete triad, the "feed-forward" loop or transitive triad, the "feed-back" loop or intransitive triad, and a few variations on the line motif.

2 Initial Results

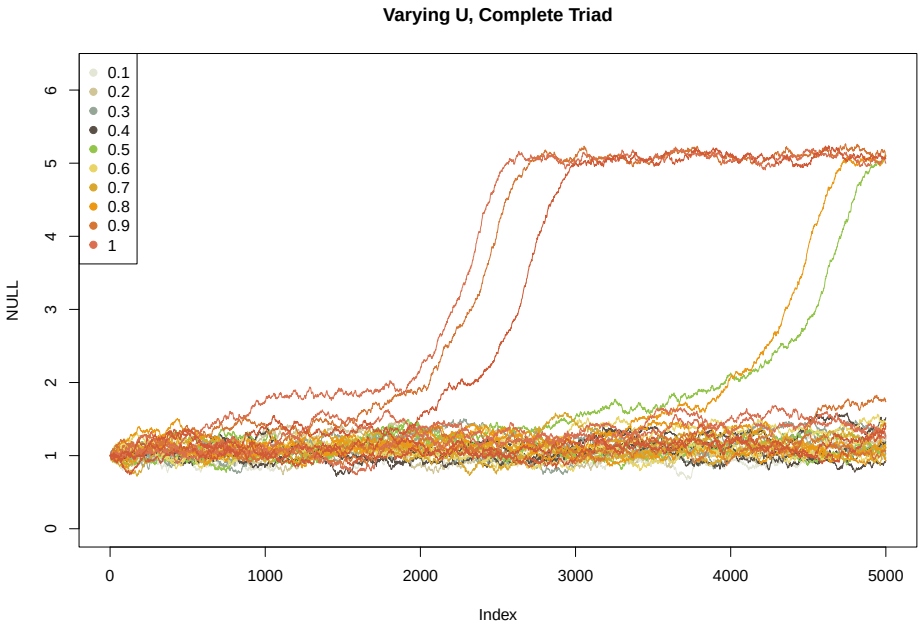
Taking first the complete triad



I vary D, which influences all nodes,



and u. For u, I only add stress u_1 to node x_1.


Varying U, Complete Triad
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
NULL
0
1
2
3
4
5
6
0
1000
2000
3000
4000
5000
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