Simulating Dynamics on Networks

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sdn is a package for simulating dynamics on networks. The package grew out of a need to do the same tasks with both deterministic and stochastic simulations of the same model, but has expanded as I have come to rely on it more. The current goal is to support numerical investigation of N dynamical equations coupled via a complex network. As such, it supports ordinary and stochastic differential equations, different coupling functions, and phase space investigation.

In this document I intend to discuss a variety of topics related to dynamics on networks and how I address them using this package and the more fundamental packages upon which I most rely: *deSolve* and *igraph*.

Fundamentals

One way to look at dynamics on networks is to imagine that there is some dynamical (i.e., changing in time) process occuring on the nodes of a network, and that each dynamical process influences each other dynamical process in some way through the network edges. We can write this idea generically as

$$\frac{dx_i}{dt} = F(x_i) + G(x_i, x_j), \tag{1}$$

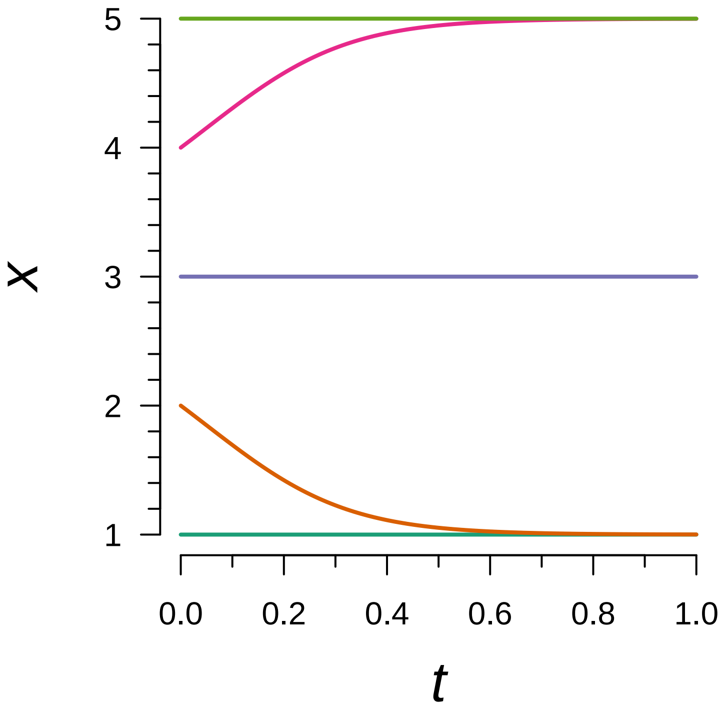
where $F(x_i)$ is the dynamics happening on each node and $G(x_i, x_j)$ is a coupling function. The function $F(x_i)$ defines what the node does in isolation, whereas the function $G(x_i, x_j)$ defines how two nodes influence each other. Thus, the change in state of the ith node is a function of some (often nonlinear) internal process and the state of its network neighbors.

Consider as an example the following dynamical model

$$\frac{dx_i}{dt} = -(x_i - r_1)(x_i - r_2)(x_i - r_3) + D\sum_{j=1}^{N} A_{i,j}x_j. \tag{2}$$

Here, $F(x_i) = -(x_i - r_1)(x_i - r_2)(x_i - r_3)$, where x_i is a numeric state of the ith node and $r_1 < r_2 < r_3$ are parameters which control the position of the equilibria in the absence of coupling. The coupling function $G(x_i, x_j) = D\sum_{j=1}^{N} A_{i,j}x_j$—we'll come back to this below. Before we do, let's get a sense of what's happening with $F(x_i)$.

The node dynamics $F(x_i) = -(x_i - r_1)(x_i - r_2)(x_i - r_3)$ is known as a double-well model. If we plot several possible initial values of x against t we get something like this:


5
4
3
2
1
x
0.0
0.2
0.4
0.6
0.8
1.0
t