

Competitive Programming Reference

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Contents

1 Data Structures	3
1.1 Prefix Sums	3
1.1.1 1D	3
1.1.2 2D	3
1.2 Fenwick Trees	3
1.2.1 Point Update Range Query	3
1.2.2 Range Update Point Query	4
1.2.3 Range Update Range Query	4
1.2.4 2D PURQ / RUPQ	4
1.2.5 2D RURQ	5
1.3 Segment Trees	6
1.3.1 Standard	6
1.3.2 Lazy Propagation	6
1.3.3 Lazy Node Creation	7
1.3.4 Maxsum	7
1.3.5 Merge Sort Tree / Order Statistics	8
1.3.6 2D	9
2 Graph Theory	10
2.1 Depth First Search	10
2.2 Breadth First Search	10
2.3 0-1 BFS	10
2.4 Floyd-Warshall	10
2.5 Bellman-Ford	11
2.6 Dijkstra's Algorithm	11
2.7 Shortest Path Faster Algorithm	12
2.8 Prim's Algorithm	12
2.9 Union Find Disjoint Subset	12
2.10 Kruskal's Algorithm	13
2.11 Topological Sort	13
2.12 Floyd's Cycle Finding Algorithm	13
2.13 Maximum Cardinality Bipartite Matching	14
2.13.1 Kun's Algorithm	14
2.13.2 Hopcroft-Karp	15
2.14 Articulation Points and Bridges	16
2.15 Strongly Connected Components	16
2.16 Travelling Salesman Problem	17
2.16.1 Bottom Up	17
2.16.2 Top Down	18
2.17 Trees	18
2.17.1 Pre/Postorder Traversal	18
2.17.2 Subtree to Range	18
2.17.3 Weighted Maximum Independent Set	19
2.17.4 Diameter	19
2.17.5 2^K Decomposition	19

2.17.6	Lowest Common Ancestor	20
2.17.7	Shortest Path	20
2.17.8	Heavy Light Decomposition	20
2.17.9	Centroid Decomposition	21
3	Dynamic Programming	23
3.1	Maxsum	23
3.1.1	1D	23
3.2	Longest Increasing Subsequence	23
3.2.1	N^2 DP	23
3.2.2	Data Structure Speedup	23
3.2.3	$N \log N$ DP	24
3.3	Coin Combinations	24
3.4	Coin Change	24
3.5	Knapsack	24
3.5.1	0-1	24
3.5.2	0-1 No Values	25
3.6	Digit DP	25
3.7	Convex Hull Trick	26
3.8	Li Chao Tree	27
3.9	Divide and Conquer	28
4	Math	29
4.1	Fast Exponentiation	29
4.2	Prime Factorisation	29
4.3	Sieve of Eratosthenes	29
4.4	Greatest Common Divisor	29
4.5	Lowest Common Multiple	30
4.6	Modular Inverse	30
4.7	$\binom{n}{k}$	30
4.8	Fibonacci	30
5	Algorithms	31
5.1	Binary Search	31
5.2	Binary Search using Lifting	31
5.3	Sliding Set	31
5.4	Sliding Window	32
5.5	Set Merging	32
5.6	Discretisation	32
5.7	Meet in the Middle	33
5.8	On the Fly	33
5.9	Square Root Decomposition	33
5.10	Mo's Algorithm	34
6	Miscellaneous	34
6.1	Fast I/O	34
6.2	Superfast I/O	35

1 Data Structures

Data Structure	Precomputation / Update	Query	Memory	Notes
Prefix Sum	$O(N) / X$	$O(1)$	$O(N)$	Associative Functions (+, XOR)
Sparse Table	$O(N \log N) / X$	$O(1)$	$O(N \log N)$	Non-Associative Functions (max, gcd)
Fenwick Tree	$X / O(\log N)$	$O(\log N)$	$O(N)$	Prefix Sum with Updates
Segment Tree	$X / O(\log N)$	$O(\log N)$	$O(4N)$	Allows more Information

Table 1: Quick Summary of Data Structures

1.1 Prefix Sums

1.1.1 1D

$O(N)$ precomputation, $O(1)$ query.

```
//Query - 1-Indexed
int query(int s, int e){
    return ps[e]-ps[s-1];
}

//Precomputation
ps[0] = 0;
for(int i = 1; i <= n; i++) ps[i] = ps[i-1]+a[i];
```

1.1.2 2D

$O(R \cdot C)$ precomputation, $O(1)$ query.

```
//Query - 1-Indexed
int query(int x1, int y1, int x2, int y2){
    return ps[x2][y2]-ps[x1-1][y2]-ps[x2][y1-1]+ps[x1-1][y1-1];
}

//Precomputation
for (int i = 0; i <= r; i++) ps[i][0] = 0;
for (int j = 0; j <= c; j++) ps[0][j] = 0;
for (int i = 1; i <= r; i++) {
    for (int j = 1; j <= c; j++) {
        ps[i][j] = ps[i-1][j]+ps[i][j-1]-ps[i-1][j-1]+a[i][j];
    }
}
```

1.2 Fenwick Trees

1.2.1 Point Update Range Query

$O(\log N)$ update and query.

```
inline int ls(int x){ return (x)&(-x); }

int fw[MAXN]; // 1-Indexed

void pu(int i, int v) {
    for(; i <= n; i += ls(i)) fw[i] += v;
}

int pq(int i) {
    int t = 0;
```

```

        for(; i; i -= ls(i)) t += fw[i];
    return t;
}

int rq(int s, int e) {
    return pq(e) - pq(s - 1);
}

```

1.2.2 Range Update Point Query

$O(\log N)$ update and query.

```

// Requires PURQ Code (PU, PQ)

void ru(int s, int e, int v) {
    pu(s, v);
    pu(e+1, -v);
}

```

1.2.3 Range Update Range Query

$O(\log N)$ update and query.

```

// Requires PURQ Code (PU, PQ)
// Functions need to be modified to take in array parameter
// e.g. int pu(*tree, int i, int v)

void ru(int s, int e, int v) {
    pu(fw1, s, v);
    pu(fw1, e+1, -v);
    pu(fw2, s, -v*(s-1));
    pu(fw2, e+1, v*e);
}

int ps(int i) {
    return pq(fw1, i)*i + pq(fw2, i);
}

int rq(int s, int e) {
    return ps(e) - ps(s - 1);
}

```

1.2.4 2D PURQ / RUPQ

$O(\log N \cdot \log M)$ update and query.

```

inline int ls(int x) { return x & -x; }

int fw[MAXN][MAXN]; // 1-indexed

void pu(int x, int y, int v) {
    for (int i = x; i < MAXN; i += ls(i)) {
        for (int j = y; j < MAXN; j += ls(j)) {
            fw[i][j] += v;
        }
    }
}

```

```

int pq(int x, int y) {
    int ans = 0;
    for (int i = x; i > 0; i -= ls(i)) {
        for (int j = y; j > 0; j -= ls(j)) {
            ans += fw[i][j];
        }
    }
    return ans;
}

// Range Query
int rq(int x1, int y1, int x2, int y2) {
    return pq(x2, y2) - pq(x1 - 1, y2) - pq(x2, y1 - 1) + pq(x1 - 1, y1 - 1);
}

// Range Update, Point Query
void ru(int x1, int y1, int x2, int y2, int v) {
    pu(x1, y1, v);
    pu(x1, y2 + 1, -v);
    pu(x2 + 1, y1, -v);
    pu(x2 + 1, y2 + 1, v);
}

```

1.2.5 2D RURQ

$O(\log N \cdot \log M)$ update and query.

```

// Requires 2D PURQ Code

void ru(int x1, int y1, int x2, int y2, long long v) {
    pu(fw1, x1, y1, v);
    pu(fw1, x1, y2+1, -v);
    pu(fw1, x2+1, y1, -v);
    pu(fw1, x2+1, y2+1, v);

    pu(fw2, x1, y1, v*(x1-1));
    pu(fw2, x1, y2+1, -v*(x1-1));
    pu(fw2, x2+1, y1, -v*x2);
    pu(fw2, x2+1, y2+1, v*x2);

    pu(fw3, x1, y1, v*(y1-1));
    pu(fw3, x1, y2+1, -v*y2);
    pu(fw3, x2+1, y1, -v*(y1-1));
    pu(fw3, x2+1, y2+1, v*y2);

    pu(fw4, x1, y1, v*(x1-1)*(y1-1));
    pu(fw4, x1, y2+1, -v*(x1-1)*y2);
    pu(fw4, x2+1, y1, -v*x2*(y1-1));
    pu(fw4, x2+1, y2+1, v*x2*y2);
}

long long ps(int x, int y) {
    return pq(fw1, x, y)*x*y - pq(fw2, x, y)*y - pq(fw3, x, y)*x + pq(fw4, x, y);
}

long long rq(int x1, int y1, int x2, int y2) {
    return ps(x2, y2) - ps(x1-1, y2) - ps(x2, y1-1) + ps(x1-1, y1-1);
}

```

1.3 Segment Trees

1.3.1 Standard

$O(\log N)$ point update and range query.

```
struct node {
    int s, e, m, v;
    node *l, *r;
    node(int _s, int _e) {
        s = _s; e = _e; m = (s+e)/2; v = 0;
        if (s != e) {
            l = new node(s, m);
            r = new node(m+1, e);
        }
    }
    void pu(int x, int y) {
        if (s == e) { v = y; return; }
        if (x <= m) l->pu(x, y);
        if (x > m) r->pu(x, y);
        v = min(l->v, r->v);
    }
    int rq(int x, int y) {
        if (s == x && e == y) return v;
        if (y <= m) return l->rq(x, y);
        if (x > m) return r->rq(x, y);
        return min(l->rq(x, m), r->rq(m+1, y));
    }
} *root;
root = new node(0, n-1);
```

1.3.2 Lazy Propagation

$O(\log N)$ range update and range query.

```
struct node {
    int s, e, m, v, lazy;
    node *l, *r;
    node(int _s, int _e) {
        s = _s; e = _e; m = (s+e)/2; v = lazy = 0;
        if (s != e) {
            l = new node(s, m);
            r = new node(m+1, e);
        }
    }
    int pu() {
        if (s == e) { v += lazy; lazy = 0; return v; }
        v += lazy;
        l->lazy += lazy; r->lazy += lazy;
        lazy = 0;
        return v;
    }
    void ru(int x, int y, int z) {
        if (s == x && e == y) { lazy += z; return; }
        if (y <= m) l->ru(x, y, z);
        else if (x > m) r->ru(x, y, z);
        else l->ru(x, m, z), r->ru(m+1, y, z);
    }
}
```

```

        v = max(l->pu(), r->pu());
    }

    int rq(int x, int y) {
        pu();
        if (s == x && e == y) return pu();
        if (y <= m) return l->rq(x, y);
        if (x > m) return r->rq(x, y);
        return max(l->rq(x, m), r->rq(m+1, y));
    }
} *root;

root = new node(0, n-1);

```

1.3.3 Lazy Node Creation

```

struct node {
    int s, e, m, v;
    node *l, *r;
    node(int _s, int _e) {
        s = _s; e = _e; m = (s+e)/2; v = 0;
        l = nullptr, r = nullptr;
    }
    void create() {
        if (s != e) {
            l = new node(s, m);
            r = new node(m+1, e);
        }
    }
    void pu(int x, int y) {
        if (s == e) { v = y; return; }
        create();
        if (x <= m) l->pu(x, y);
        if (x > m) r->pu(x, y);
        v = min(l->v, r->v);
    }
    int rq(int x, int y) {
        if (s == x && e == y) return v;
        create();
        if (y <= m) return l->rq(x, y);
        if (x > m) return r->rq(x, y);
        return min(l->rq(x, m), r->rq(m+1, y));
    }
} *root;

root = new node(0, n-1);

```

1.3.4 Maxsum

$O(\log N)$ point update and range query.

```

struct node {
    ll s, e, m, ps, ss, ms, ts;
    node *l, *r;
    node(ll _s, ll _e) {
        s = _s; e = _e; m = (s+e)/2; ps = ss = ms = ts = 0;
        if (s != e) {

```

```

        l = new node(s, m);
        r = new node(m+1, e);
    }
}

void pu(ll x, ll y) {
    if (s == e) { ps = ss = ms = ts = y; return; }
    if (x <= m) l->pu(x, y);
    if (x > m) r->pu(x, y);
    //New Prefix Max -> Left Prefix, Left + Right Prefix
    ps = max(l->ps, l->ts+r->ps);
    //New Suffix Max -> Right Suffix, Left Suffix + Right
    ss = max(r->ss, r->ts+l->ss);
    //Total Sum -> Left + Right
    ts = l->ts+r->ts;
    //Maxsum - Left Suffix + Right Prefix, Left, Right,
    //          Total, Left Maxsum, Right Maxsum
    ms = max({l->ss+r->ps, ps, ss, ts, l->ms, r->ms});
}
ll ans() {
    return ms;
}
} *root;

root = new node(0, n-1);

```

1.3.5 Merge Sort Tree / Order Statistics

Insertion: $O(\log N)$, Building: $O(N \log^2 N)$, Counting: $O(\log^2 N)$, Finding: $O(\log V \cdot \log^2 N)$, Range Max: $O(\log N)$.

```

struct node {
    int s, e, m;
    vector<int> v;
    node *l, *r;
    node(int _s, int _e) {
        s = _s; e = _e; m = (s+e)/2;
        if (s != e) {
            l = new node(s, m);
            r = new node(m+1, e);
        }
    }
    void insert(int x, int y) {
        if (s == e) { v.push_back(y); return; }
        if (x > m) r->insert(x, y);
        if (x <= m) l->insert(x, y);
        v.push_back(y);
    }
    void build(){
        if (s == e) return;
        l->build();
        r->build();
        sort(v.begin(), v.end());
    }
    int countLessEqual(int x, int y, int k) {
        if (x > y) return 0;
        if (s == x && e == y) {
            return upper_bound(v.begin(), v.end(), k)-v.begin();
        }
    }
}

```

```

        if (x > m) return r->countLessEqual(x, y, k);
        if (y <= m) return l->countLessEqual(x, y, k);
        return l->countLessEqual(x, m, k)+r->countLessEqual(m+1, y, k);
    }
    int kthSmallest(int x, int y, int k) {
        int mini = 0, maxi = (1 << 30);
        int ans = mini, gap = maxi;
        while (gap > 0) {
            while (ans + gap <= maxi && countLessEqual(x, y, ans + gap) < k) {
                ans += gap;
            }
            gap >>= 1;
        }
        return ans + 1;
    }
    int rangeMax(int x, int y) {
        if (x > y) return 0;
        if (s == x && e == y) return v.back();
        if (x > m) return r->rangeMax(x, y);
        if (y <= m) return l->rangeMax(x, y);
        return max(l->rangeMax(x, m), r->rangeMax(m+1, y));
    }
}
*root;

root = new node(0, n-1);

```

1.3.6 2D

$O(N \cdot \log N \cdot \log M)$ point update and range query.

```

struct node2D {
    int s, e, m;
    node1D *maxi;
    node2D *l, *r;
    node2D(int a, int b, int c, int d) {
        s = a; e = b; m = (s+e)/2;
        maxi = new node1D(c, d);
        if (s != e) {
            l = new node2D(s, m, c, d);
            r = new node2D(m+1, e, c, d);
        }
    }
    void pu(int a, int b, int v) {
        if (s == e) { maxi->pu(b, v); return; }
        if (a <= m) l->pu(a, b, v);
        else r->pu(a, b, v);
        maxi->pu(b, max(l->maxi->rq(b, b), r->maxi->rq(b, b)));
    }
    int rq(int a, int b, int c, int d) {
        if (s == a && e == b) return maxi->rq(c, d);
        if (b <= m) return l->rq(a, b, c, d);
        if (a > m) return r->rq(a, b, c, d);
        return max(l->rq(a, m, c, d), r->rq(m+1, b, c, d));
    }
}
*root;

root = new node(0, n-1, 0, n-1);

```

2 Graph Theory

2.1 Depth First Search

Runs in $O(V + E)$.

```
// For adjacency lists
void dfs(int x, int p) {
    for (int y : adj[x]) {
        if (y != p) {
            dist[y] = dist[x]+1;
            dfs(y, x);
        }
    }
}
```

2.2 Breadth First Search

Runs in $O(V + E)$.

```
// For adjacency lists
visited[s] = 1;
dist[s] = 0;
q.push(s);
while (!q.empty()) {
    int f = q.front(); q.pop();
    for (int i : adjlist[f]) {
        if (!visited[i]) {
            q.push(i);
            visited[i] = 1;
            dist[i] = dist[f] + 1;
        }
    }
}
```

2.3 0-1 BFS

Runs in $O(V + E)$.

```
// For adjacency lists
deque<int> dq;
dist[s] = 0;
dq.push(s);
while (!dq.empty()) {
    int u = dq.front(); dq.pop();
    for (int e : adjlist[u]) {
        int v = e.first, w = e.second;
        if (dist[v] > dist[u] + w) {
            dist[v] = dist[u] + w;
            if (w == 0) dq.push_front(v);
            else dq.push_back(v);
        }
    }
}
```

2.4 Floyd-Warshall

Runs in $O(N^3)$.

```

// Initialise adjacency matrix
for (int i = 0; i < n; i++) {
    for (int j = 0; j < n; j++) {
        if (i == j) adj[i][j] = 0;
        else adj[i][j] = INF;
    }
}
// Floyd-Warshall
for (int k = 0; k < n; k++) {
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++) {
            adj[i][j] = min(adj[i][j], adj[i][k]+adj[k][j]);
            if (adj[i][i] < 0) negCycle = true;
        }
    }
}

```

2.5 Bellman-Ford

Runs in $O(VE)$.

```

vector<int> dist(n, INF);
dist[s] = 0;
bool negCycle = false;
for (int i = 1; i <= n; i++) {
    bool update = false;
    for (Edge e : edges) {
        if (dist[e.u] < INF && dist[e.v] > dist[e.u] + e.w) {
            dist[e.v] = dist[e.u] + e.w;
            update = true;
        }
    }
    if (!update) break;
    if (update && i == n) negCycle = true;
}

```

2.6 Dijkstra's Algorithm

Runs in $O(E \log V)$.

```

priority_queue<pi, vector<pi>, greater<pi>> pq;
vector<int> dist(n, INF);
dist[s] = 0;
pq.push({0, s});
while (!pq.empty()) {
    pi f = pq.top(); pq.pop();
    int d = f.first, u = f.second;
    if (d != dist[u]) continue;
    for (pi x : adj[u]) {
        int v = x.first, w = x.second;
        if (dist[v] > d + w) {
            dist[v] = d + w;
            pq.push({dist[v], v});
        }
    }
}

```

2.7 Shortest Path Faster Algorithm

Runs in $O(VE)$.

```
vector<int> dist(n, INF);
vector<int> inQueue(n, 0);
queue<int> q;
dist[s] = 0;
q.push(s);
inQueue[s]++;
bool negCycle = false;
while (!q.empty()) {
    int u = q.front(); q.pop();
    inQueue[u]--;
    for (Edge e : adj[u]) {
        if (dist[e.v] > dist[u] + e.w) {
            dist[e.v] = dist[u] + e.w;
            if (inQueue[e.v] == 0) {
                q.push(e.v);
                inQueue[e.v]++;
                if (inQueue[e.v] > n) {
                    negCycle = true;
                    break;
                }
            }
        }
    }
}
if (negCycle) break;
}
```

2.8 Prim's Algorithm

Runs in $O(E \log V)$.

```
priority_queue<pi, vector<pi>, greater<pi>> pq;
vector<int> dist(n, INF);
vector<bool> vis(n, false);
dist[s] = 0;
pq.push({0, s});
while (!pq.empty()) {
    pi f = pq.top(); pq.pop();
    int d = f.first, u = f.second;
    if (vis[u]) continue;
    vis[u] = true;
    for (pi x : adj[u]) {
        int v = x.first, w = x.second;
        if (!vis[v] && dist[v] > w) {
            dist[v] = w;
            pq.push({dist[v], v});
        }
    }
}
```

2.9 Union Find Disjoint Subset

With both path compression and union by rank, runs in $O(\alpha(n))$ (basically constant time).

```
int p[MAXN];
int sz[MAXN];
```

```

int root(int x) {
    if (p[x] == -1) return x;
    return p[x] = root(p[x]);
}

void connect(int x, int y) {
    x = root(x); y = root(y);
    if (x == y) return;
    if (sz[x] < sz[y]) swap(x, y);
    p[y] = x;
    sz[x] += sz[y];
}

fill(p, p+MAXN, -1);
fill(sz, sz+MAXN, 1);

```

2.10 Kruskal's Algorithm

Runs in $O(E \log E)$.

```

sort(edges.begin(), edges.end());
for (Edge e : edges) {
    if (root(e.u) != root(e.v)) {
        connect(e.u, e.v);
        cost += e.w;
    }
}

```

2.11 Topological Sort

Runs in $O(V + E)$.

```

void dfs(int x) {
    if (v[x]) return;
    v[x] = 1;
    for (int y : adj[x]) dfs(y);
    topo.push_back(x);
}
for (int i = 0; i < n; i++) dfs(i);
reverse(topo.begin(), topo.end());

```

2.12 Floyd's Cycle Finding Algorithm

For graphs with outdegree 1, runs in $O(V + E)$.

```

// detect cycle
int slow = s, fast = s;
do {
    slow = nxt[slow];
    fast = nxt[nxt[fast]];
} while (slow != fast);
// find start of cycle
slow = start;
while (slow != fast) {
    slow = nxt[slow];
    fast = nxt[fast];
}

```

```

// collect all nodes in cycle
vector<int> cycle;
int cur = slow;
do {
    cycle.push_back(cur);
    cur = nxt[cur];
} while (cur != slow);

```

2.13 Maximum Cardinality Bipartite Matching

2.13.1 Kun's Algorithm

Runs in $O(V^3)$.

```

int n, mcbms = 0, miss = 0;
vector<int> match;
vector<bool> vis;
vector<vector<int>> adj;
vector<int> left;

// dfs for augmenting path
bool dfs(int u) {
    for (int v : adj[u]) {
        if (!vis[v]) {
            vis[v] = true;
            if (match[v] == -1 || dfs(match[v])) {
                match[v] = u;
                match[u] = v;
                return true;
            }
        }
    }
    return false;
}

void mcbm() {
    match.assign(n, -1);
    mcbms = 0;
    // greedy initial matching
    for (int u : left) {
        for (int v : adj[u]) {
            if (match[v] == -1) {
                match[v] = u;
                match[u] = v;
                mcbms++;
                break;
            }
        }
    }
    // dfs augmenting paths for unmatched
    for (int u : left) {
        if (match[u] == -1) {
            vis.assign(n, false);
            if (dfs(u)) mcbms++;
        }
    }
    miss = n - mcbms;
}

```

2.13.2 Hopcroft-Karp

Runs in $O(E \cdot \sqrt{V})$.

```

int n, m;
vector<vector<int>> adj;
vector<int> pairU, pairV, dist;

bool bfs() {
    queue<int> q;
    for (int u = 0; u < n; u++) {
        if (pairU[u] == -1) {
            dist[u] = 0;
            q.push(u);
        } else {
            dist[u] = INF;
        }
    }
    bool found = false;
    while (!q.empty()) {
        int u = q.front(); q.pop();
        for (int v : adj[u]) {
            if (pairV[v] == -1) {
                found = true;
            } else if (dist[pairV[v]] == INF) {
                dist[pairV[v]] = dist[u] + 1;
                q.push(pairV[v]);
            }
        }
    }
    return found;
}

bool dfs(int u) {
    for (int v : adj[u]) {
        if (pairV[v] == -1 || (dist[pairV[v]] == dist[u] + 1 && dfs(pairV[v]))) {
            pairU[u] = v;
            pairV[v] = u;
            return true;
        }
    }
    dist[u] = INF;
    return false;
}

int hopcroftKarp() {
    pairU.assign(n, -1);
    pairV.assign(m, -1);
    dist.assign(n, 0);
    int mcbm = 0;
    while (bfs()) {
        for (int u = 0; u < n; u++) {
            if (pairU[u] == -1 && dfs(u)) {
                mcbm++;
            }
        }
    }
    return mcbm;
}

```

2.14 Articulation Points and Bridges

Using Tarjan's Algorithm, runs in $O(V + E)$.

```
vector<int> adj[MAXN];
vector<int> dep(MAXN, 0), low(MAXN, 0), par(MAXN, -1);
vector<int> chi(MAXN, 0), atp(MAXN, 0);
vector<bool> vis(MAXN, false);
vector<pi> bridges;

void tarjan(int u, int d) {
    vis[u] = true;
    dep[u] = low[u] = d;
    chi[u] = 0; atp[u] = 1;
    for (int v : adj[u]) {
        if (!vis[v]) {
            par[v] = u;
            chi[u]++;
            tarjan(v, d+1);
            low[u] = min(low[u], low[v]);
            if (low[v] >= dep[u]) atp[u]++;
            if (low[v] > dep[u]) bridges.push_back({u, v});
        } else if (v != par[u]) {
            low[u] = min(low[u], dep[v]);
        }
    }
}

tarjan(0, 0);
// handle root separately since it has no parents
atp[0] = chi[0];
// atp stores number of components separated upon removal
// bridges stores all bridges in the graph
```

2.15 Strongly Connected Components

Using Tarjan's Algorithm, runs in $O(V + E)$.

```
vector<int> adj[MAXN];
vector<int> comps[MAXN];
set<int> adjScc[MAXN];
vector<int> idxs(MAXN, -1), low(MAXN, 0);
vector<bool> onStack(MAXN, false);
stack<int> st;
int dfsidx = 0, sccidx;
int comp[MAXN];

void scc(int u) {
    idxs[u] = low[u] = dfsidx++;
    st.push(u);
    onStack[u] = true;
    for (int v : adj[u]) {
        if (idxs[v] == -1) {
            scc(v);
            low[u] = min(low[u], low[v]);
        } else if (onStack[v]) {
            low[u] = min(low[u], idxs[v]);
        }
    }
}
```

```

    if (low[u] == idxs[u]) {
        // u is a root of an scc
        comps[sccidx].clear();
        int w;
        do {
            w = st.top(); st.pop();
            onStack[w] = false;
            comps[sccidx].push_back(w);
            comp[w] = sccidx;
        } while (w != u);
        sccidx++;
    }
}

for (int i = 1; i <= n; i++) {
    if (idxs[i] == -1) scc(i);
}

for (int u = 1; u <= n; u++) {
    for (int v : adj[u]) {
        if (comp[u] != comp[v]) adjScc[comp[u]].insert(comp[v]);
    }
}

// dfs idxs now range from 0 to dfsidx-1
// scc idxs now range from 0 to sccidx-1

```

2.16 Travelling Salesman Problem

Both of these solutions run in $O(N^2 \cdot 2^N)$ time.

2.16.1 Bottom Up

```

int full = (1 << n) - 1;
for (int mask = 0; mask <= full; mask++) {
    for (int i = 0; i < n; i++) {
        dp[mask][i] = INF;
    }
}
dp[1][0] = 0; // assume source is node 0
for (int mask = 1; mask <= full; mask++) {
    for (int i = 0; i < n; i++) {
        if (!(mask & (1 << i)) || dp[mask][i] >= INF) continue;
        for (int j = 0; j < n; j++) {
            if (mask & (1 << j)) continue;
            if (adj[i][j] >= INF) continue;
            int newMask = mask | (1 << j);
            dp[newMask][j] = min(dp[newMask][j], dp[mask][i] + adj[i][j]);
        }
    }
}
ll ans = INF;
for (int i = 0; i < n; i++) {
    if (adj[i][0] < INF) {
        // close the tour
        ans = min(ans, dp[full][i] + adj[i][0]);
    }
}

```

```
}
```

2.16.2 Top Down

```
ll adj[MAXN][MAXN], dp[1 << MAXN][MAXN];  
  
ll tsp(int mask, int pos) {  
    if (mask == (1 << n) - 1) {  
        // assume node 0 is the start and end  
        return adj[pos][0] < INF ? adj[pos][0] : INF;  
    }  
    if (dp[mask][pos] != -1) return dp[mask][pos];  
    ll ans = INF;  
    for (int i = 0; i < n; i++) {  
        if ((mask & (1 << i)) == 0 && adj[pos][i] < INF) {  
            ans = min(ans, adj[pos][i] + tsp(mask | (1 << i), i));  
        }  
    }  
    return dp[mask][pos] = ans;  
}  
  
memset(dp, -1, sizeof(dp));  
ll res = tsp(1, 0); // initial mask has source visited
```

2.17 Trees

2.17.1 Pre/Postorder Traversal

Runs in $O(V)$.

```
int prec = 0, postc = 0;  
void dfs(int x, int p) {  
    pre[x] = prec++;  
    for(int y : adj[x]) {  
        if (y != p) dfs(y, x);  
    }  
    post[x] = postc++;  
}
```

2.17.2 Subtree to Range

Runs in $O(V)$.

```
int dfs(int x, int p) {  
    pre[x] = c++;  
    rig[pre[x]] = pre[x];  
    for (int y : adj[x]) {  
        if (y != p) {  
            rig[pre[x]] = max(rig[pre[x]], dfs(y, x));  
        }  
    }  
    return rig[pre[x]];  
}  
// Subtree -> pre[x], rig[pre[x]]  
// Node Index -> pre[x]  
// Range of Children -> pre[x]+1, rig[pre[x]]
```

2.17.3 Weighted Maximum Independent Set

Runs in $O(V)$.

```

int dp[MAXN][2];

int mis(int v, bool take, int p) {
    if (dp[v][take] != -1) return dp[v][take];
    int ans = take * c[v];
    for (int u : adj[v]) {
        if (u == p) continue;
        int temp = mis(u, 0, v);
        if (!take) temp = max(temp, mis(u, 1, v));
        ans += temp;
    }
    return dp[v][take] = ans;
}

void ans(int v, bool take, int p) {
    for (int u : adj[v]) {
        if (u == p) continue;
        int temp0 = dp[u][0], temp1 = (take ? -1 : dp[u][1]);
        if (temp0 > temp1) ans(u, 0, v);
        else { a.push_back(u); ans(u, 1, v); }
    }
}

memset(dp, -1, sizeof(dp));
mis(0, 0, -1); // don't take root
mis(0, 1, -1); // take root
if (dp[0][1] > dp[0][0]) { a.push_back(0); ans(0, 1, -1); }
else ans(0, 0, -1);

```

2.17.4 Diameter

Runs in $O(V)$.

```

pi dfs(int x, int p, int d) {
    pi b = {x, d};
    for (pi y : adj[x]) {
        if (y.first != p) {
            pi c = dfs(y.first, x, d + y.second);
            if (c.second > b.second) b = c;
        }
    }
    return b;
}
pi s = dfs(0, -1, 0);
pi e = dfs(s.first, -1, 0);
// e.second gives diameter
// For even diameter, centroid is at e.second / 2
// For odd diameter, centroid is at e.second / 2 and e.second / 2 + 1

```

2.17.5 2^K Decomposition

$O(N \log N)$ precomputation and memory, $O(\log N)$ query.

```

int par(int x, int k) {
    for (int i = MAXLOGN; i >= 0; i--) {
        if (k >= (1 << i)) {

```

```

        if(x == -1) return x;
        x = p[x][i];
        k -= (1 << i);
    }
}
return x;
}

int p[MAXN][MAXLOGN];
memset(p, -1, sizeof(p));
dfs(0); // compute initial parent p[i][0]
for (int k = 1; k <= MAXLOGN; k++) {
    for (int i = 0; i < n; i++) {
        if(p[i][k-1] != -1) p[i][k] = p[p[i][k-1]][k-1];
    }
}

```

2.17.6 Lowest Common Ancestor

Runs in $O(\log N)$.

```

int lca(int x, int y) {
    // make both nodes the same depth
    if (dep[x] < dep[y]) swap(x, y);
    for (int k = MAXLOGN; k >= 0; k--) {
        if (p[x][k] != -1 && dep[p[x][k]] >= dep[y]) x = p[x][k];
    }
    if (x == y) return x;
    // perform binary lifting while parents are different
    for (int k = MAXLOGN; k >= 0; k--) {
        if (p[x][k] != p[y][k]) {
            x = p[x][k];
            y = p[y][k];
        }
    }
    // find the next parent
    return p[x][0];
}

```

2.17.7 Shortest Path

Runs in $O(\log N)$.

```

int distance(int x, int y) {
    return dist[x] + dist[y] - 2 * dist[lca(x, y)];
}

```

2.17.8 Heavy Light Decomposition

$O(N)$ precomputation, $O(\log N)$ query (excluding $O(\log N)$ from data structure).

```

// Requires segment tree, supports path maximum queries

vector<pi> adj[MAXN];
int par[MAXN], dep[MAXN], heavy[MAXN], head[MAXN], pos[MAXN], cpos = 0;
// heavy: heavy child, head: start of chain, pos: position in segment tree

int dfs(int x, int p) {

```

```

    int sz = 1, maxc = 0;
    par[x] = p;
    for (pi y : adj[x]) {
        if (y.s != p) {
            dep[y.s] = dep[x] + 1;
            int csz = dfs(y.s, x);
            sz += csz;
            if (csz > maxc) {
                maxc = csz;
                heavy[x] = y.s;
            }
        }
    }
    return sz;
}

void decomp(int x, int h) {
    head[x] = h; pos[x] = cpos++;
    if (heavy[x] != -1) decomp(heavy[x], h);
    for (pi y : adj[x]) {
        if (y.s == par[x]) continue;
        if (y.s != heavy[x]) decomp(y.s, y.s);
        root->update(pos[y.s], y.f);
    }
}

int query(int a, int b) {
    int res = 0;
    for (; head[a] != head[b]; b = par[head[b]]) {
        // maintain b as deeper
        if (dep[head[a]] > dep[head[b]]) swap(a, b);
        res = max(res, root->query(pos[head[b]], pos[b]));
    }
    // a and b are now on the same chain
    if (dep[a] > dep[b]) swap(a, b);
    res = max(res, root->query(pos[a]+1, pos[b]));
    return res;
}

memset(heavy, -1, sizeof(heavy));
dep[0] = 0;
dfs(0, -1);
decomp(0, 0);

```

2.17.9 Centroid Decomposition

$O(N \log N)$ precomputation, $O(\log N)$ updates and queries.

```

// Example: Xenia and Tree
// Update: Mark a node red
// Query: Find distance to closest red node to query node

int sz[MAXN], par[MAXN], lvl[MAXN];
ll dist[MAXN][MAXLOGN], best[MAXN];

// find subtree sizes
int dfsSize(int x, int p) {
    sz[x] = 1;

```

```

        for (int y : adj[x]) {
            if (lvl[y] != -1 || y == p) continue;
            sz[x] += dfsSize(y, x);
        }
        return sz[x];
    }

    // find centroid of each subtree
    int dfsCentroid(int x, int p, int n) {
        for (int y : adj[x]) {
            if (lvl[y] != -1 || y == p) continue;
            if (sz[y] > n / 2) return dfsCentroid(y, x, n);
        }
        // current node is centroid
        return x;
    }

    void dfsDist(int x, int p, int level, ll d) {
        dist[x][level] = d;
        for (int y : adj[x]) {
            if (lvl[y] != -1 || y == p) continue;
            dfsDist(y, x, level, d + 1);
        }
    }

    void build(int x, int p, int level) {
        // find subtree sizes and centroid
        int size = dfsSize(x, -1);
        int cent = dfsCentroid(x, -1, size);
        if (p == -1) p = cent;
        par[cent] = p; // set parent of centroid to previous centroid
        lvl[cent] = level;
        dfsDist(cent, -1, level, 0);
        // recursively build subtrees
        for (int y : adj[cent]) {
            if (lvl[y] != -1) continue;
            build(y, cent, level + 1);
        }
    }

    void update(int x) {
        int y = x;
        int level = lvl[x];
        while (level != -1) {
            // shortest distance from y to any red node
            // update using new node x and its distance to the centroid y
            best[y] = min(best[y], dist[x][level]);
            y = par[y];
            level--;
        }
    }

    ll query(int x) {
        ll res = INF;
        int y = x;
        int level = lvl[x];
        while (level != -1) {
            // shortest distance to any red node in centroid chain

```

```

        res = min(res, best[y] + dist[x][level]);
        y = par[y];
        level--;
    }
    return res;
}

memset(lvl, -1, sizeof(lvl));
fill(best, best+n, INF);
build(0, -1, 0);
update(0); // adding a node to be considered
query(y); // querying with all considered nodes

```

3 Dynamic Programming

3.1 Maxsum

3.1.1 1D

Kadane's Algorithm. Runs in $O(N)$.

```

int ans = nums[0], cur = nums[0];
for (int i = 1; i < nums.size(); i++) {
    if (cur < 0) cur = 0;
    cur += nums[i];
    ans = max(ans, cur);
}

```

3.2 Longest Increasing Subsequence

3.2.1 N^2 DP

```

int ans = 0, dp[n];
memset(dp, 0, sizeof(dp));
for (int i = 0; i < n; i++) {
    for (int j = 0; j < i; j++) {
        if (a[j] < a[i]) {
            dp[i] = max(dp[i], dp[j]);
        }
    }
    dp[i]++;
    ans = max(ans, dp[i]);
}

```

3.2.2 Data Structure Speedup

```

// Data structure should support point max updates and range max queries
// Discretise values first
for (int i = 0; i < n; i++) {
    t = query(a[i] - 1) + 1;
    update(a[i], t);
    ans = max(ans, t);
}

```

3.2.3 $N \log N$ DP

```
int len = 0, dp[n];
memset(dp, 0, sizeof(dp));
for (int x : a) {
    int pos = lower_bound(dp, dp + len, x) - dp;
    dp[pos] = x;
    if (pos == len) len++;
}
cout << len;
```

3.3 Coin Combinations

Runs in $O(N \cdot V)$.

```
int ways[v+1];
memset(ways, 0, sizeof(ways));
ways[0] = 1;
for (int i = 0; i < n; i++) {
    int c = coins[i];
    for (int sum = c; sum <= v; sum++) {
        ways[sum] = (ways[sum] + ways[sum - c]) % MOD;
    }
}
cout << ways[v];
```

3.4 Coin Change

Runs in $O(N \cdot V)$.

```
const int INF = 1e9;
vector<int> dp(v + 1, INF);
dp[0] = 0;
for (int i = 1; i <= v; i++) {
    for (int j = 0; j < n; j++) {
        if (i >= c[j] && dp[i - c[j]] != INF) {
            dp[i] = min(dp[i], dp[i - c[j]] + 1);
        }
    }
}
cout << dp[v];
```

3.5 Knapsack

3.5.1 0-1

Runs in $O(N \cdot S)$.

```
for (int i = 0; i < n; i++) {
    for (int j = s; j >= w[i]; j--) {
        dp[j] = max(dp[j], dp[j - w[i]] + v[i]);
    }
}
cout << dp[s];
```

3.5.2 0-1 No Values

Runs in $O(N \cdot X/64)$.

```
bitset<MAXX> dp;
dp[0] = 1;
for (int i = 0; i < n; i++) {
    dp |= dp << w[i];
}
cout << dp[x];
```

3.6 Digit DP

Runs in $O(D)$.

```
// Example - Numbers
// Compute the number of palindrome free numbers in a given range

vector<int> num;
ll dp[20][11][11][2][2]; // idx, last1, last2, tight, hasStarted

ll derp(int pos, int last1, int last2, bool tight, bool hasStarted) {
    if(pos == num.size()) return 1; // successfully populated whole number

    if(dp[pos][last1][last2][tight][hasStarted] != -1) {
        // state already visited
        return dp[pos][last1][last2][tight][hasStarted];
    }

    ll res = 0;
    int limit = tight ? num[pos] : 9; // do we need to keep to the range
    for(int d = 0; d <= limit; d++) { // try all next digits
        bool newHasStarted = hasStarted || (d != 0);
        bool newTight = tight && (d == limit);
        // skip palindromes only if the number has started
        if(newHasStarted) {
            if(d == last1) continue; // palindrome length 2
            if(d == last2) continue; // palindrome length 3
        }
        int newLast1 = newHasStarted ? d : 10;
        int newLast2 = hasStarted ? last1 : 10;
        res += derp(pos+1, newLast1, newLast2, newTight, newHasStarted);
    }
    return dp[pos][last1][last2][tight][hasStarted] = res;
}

// convert number to digits
void dcmp(ll x){
    num.clear();
    if(x == 0) num.push_back(0);
    while(x > 0) {
        num.push_back(x % 10);
        x /= 10;
    }
    reverse(num.begin(), num.end());
}

// Total Valid with value <= x
```

```

// Use PIE to get number within [a, b]
ll solve(ll x){
    dcmp(x);
    memset(dp, -1, sizeof(dp));
    return derp(0, 10, 10, true, false);
}

// To compute the kth string satisfying
// Either binary search or build character by character:
int count = 0;
vector<int> ans;
for (int i = 0; i < n; i++) {
    int x = 0;
    for (int j = 1; j < 10; j++) { // adjust to size of alphabet
        if (count + dp(i, j) > k) {
            break;
        }
        x = j;
    }
    count += dp(i, x); // position i is bounded by x
    ans.push_back(x);
}

```

3.7 Convex Hull Trick

Supports insertion and queries in amortised $O(1)$.

```

// Example: Commando
// Partition into contiguous groups with maximal effectiveness
// Group effectiveness is a quadratic function of their sum
// dp(x) = max(dp(i)+f(p(x)-p(i)))
//         = max(dp(i)+a(p(x)-p(i))^2+b(p(x)-p(i))+c)
//         = max(dp(i)+ap(x)^2-2ap(x)p(i)+ap(i)^2+bp(x)-bp(i))+c
//         = max(dp(i)+ap(i)^2-bp(i)-2ap(x)p(i)+ap(x)^2+bp(x)+c
//         = max([dp(i)+ap(i)^2-bp(i)][-2ap(i)][p(x)][+ap(x)^2+bp(x)+c])
//         = max(c(i)+m(i)p(x))+v(x)
// c(i) = dp(i)+ap(i)^2-bp(i)
// m(i) = -2ap(i)
// v(x) = ap(x)^2+bp(x)+c
// Lines have increasing gradients, queries have increasing x

deque<pi> hull;

ll func(pi line, ll x){
    return line.first*x+line.second;
}

ld intersection(ll m1, ll c1, ll m2, ll c2) {
    return (ld) (c2-c1) / (m1-m2);
}

ld intersect(pi x, pi y) {
    return intersection(x.first, x.second, y.first, y.second);
}

// query maximum y at x
ll query(ll x) {
    while (hull.size() > 1) {

```

```

        if (func(hull[0], x) < func(hull[1], x)) {
            hull.pop_front();
        } else break;
    }
    return func(hull[0], x);
}

// insert new line
void insert(ll m, ll c) {
    pi line = pi(m, c);
    while (hull.size() > 1) {
        ll s = hull.size();
        if (intersect(hull[s-1], line) <= intersect(hull[s-2], line)) {
            hull.pop_back();
        } else break;
    }
    hull.push_back(line);
}

insert(0, 0); // dp[0]
for(int i = 1; i <= n; i++){
    dp[i] = query(ps[i])+a*ps[i]*ps[i]+b*ps[i]+c; // max(mx + c) + v
    insert(-2*a*ps[i], dp[i]+a*ps[i]*ps[i]-b*ps[i]); // insert new (m, c)
}

```

3.8 Li Chao Tree

Supports insertion and queries in $O(\log X)$ time where X is the domain.

```

struct Node {
    ll m, c;
    Node *lc, *rc;
    Node(ll _m = 0, ll _c = -INF) {
        m = _m; c = _c;
        lc = nullptr; rc = nullptr;
    }
    ll value(ll x) {
        return m * x + c;
    }
    void insert(ll nm, ll nc, ll l, ll r) {
        ll mid = (l+r)/2;
        bool leftCheck = nm * l + nc > value(l);
        bool midCheck = nm * mid + nc > value(mid);
        if (midCheck) {
            swap(m, nm);
            swap(c, nc);
        }
        if (r-l == 1) return;
        if (leftCheck != midCheck) {
            if (!lc) lc = new Node();
            lc->insert(nm, nc, l, mid);
        } else {
            if (!rc) rc = new Node();
            rc->insert(nm, nc, mid, r);
        }
    }
    ll query(ll x, ll l, ll r) {
        ll res = value(x);

```

```

        if (r-l == 1) return res;
        ll mid = (l+r)/2;
        if (x < mid && lc) return max(res, lc->query(x, l, mid));
        if (x >= mid && rc) return max(res, rc->query(x, mid, r));
        return res;
    }
};

// Queries are on half open [L, R)
// Example: Commando
Node *root = new Node();
root->insert(0, 0, MINX, MAXX);
for (int i = 1; i <= n; i++) {
    // query lct
    ll x = p[i];
    ll best = root->query(x, MINX, MAXX);
    dp[i] = best + a*x*x + b*x + c;
    // update lct
    ll m = -2*a*x;
    ll c = dp[i] + a*x*x - b*x;
    root->insert(m, c, MINX, MAXX);
}

```

3.9 Divide and Conquer

Reduces complexity from $O(N^2 \cdot K)$ to $O(N \log N \cdot K)$.

```

// Example: Guards
// Minimise sum of costs where cost of partition is length * sum
// DP has form dp(i, j) = min dp(i-1, k-1) + C(k, j)
// i is the current layer, k to j is the new group
// Cost function satisfies quadrangle inequality:
// C(a, c) + C(b, d) <= C(a, d) + C(b, c) for a <= b <= c <= d

long long cost(int s, int e) {
    return (ps[e]-ps[s-1])*(e-s+1);
}

void dnc(int s, int e, long long x, int y, int k) {
    if(s > e) return;
    int m = (s+e)/2, best = 0;
    dp[m][k] = INF;
    for (int i = x; (i <= y && i <= m); i++) {
        ll val = dp[i][!k]+cost(i+1, m);
        if (dp[m][k] > val) {
            dp[m][k] = val;
            best = i;
        }
    }
    if (s < m) dnc(s, m-1, x, best, k);
    if (m < e) dnc(m+1, e, best, y, k);
}

// Uses DP on the fly to save space
for (int i = 1; i <= n; i++) dp[i][0] = INF;
for (int i = 1; i <= g; i++) {
    for (int j = 1; j <= n; j++) dp[j][i%2] = INF;
    dnc(0, n, 0, n, i%2);
}

```

}

4 Math

4.1 Fast Exponentiation

Runs in $O(\log b)$.

```
int powmod(int a, int b, int m) {
    int res = 1;
    while (b > 0) {
        if (b & 1) res = (res * a) % m;
        a = (a * a) % m;
        b >>= 1;
    }
    return res % m;
}
```

4.2 Prime Factorisation

Runs in $O(\sqrt{x})$.

```
map<int, int> cnt;
while (x % 2 == 0) {
    cnt[2]++;
    x /= 2;
}
for (int i = 3; i * i <= x; i++) {
    while (x % i == 0) {
        cnt[i]++;
        x /= i;
    }
}
if (x > 1) {
    cnt[x]++;
}
```

4.3 Sieve of Eratosthenes

Runs in $O(n \log \log n)$ with high constant.

```
bitset<MAXN> prime;
prime.set();
prime[0] = prime[1] = 0;
for (int i = 2; i < MAXN; i++) {
    if (prime[i]) {
        for (int j = i*i; j < MAXN; j += i) {
            prime[j] = 0;
        }
    }
}
```

4.4 Greatest Common Divisor

Runs in $O(\log \min(a, b))$.

```

int gcd(int a, int b) {
    if (a > b) swap(a, b);
    while (a != 0) {
        b %= a;
        swap(a, b);
    }
    return b;
}

```

4.5 Lowest Common Multiple

```

int lcm(int a, int b) {
    return a / gcd(a, b) * b;
}

```

4.6 Modular Inverse

For prime modulo.

```

ll modinv(ll a){
    return powmod(a, MOD-2, MOD);
}

```

$$4.7 \quad \binom{n}{k}$$

Precomputation takes $O(MAXN)$ time, queries answered in $O(1)$.

```

ll fac[MAXN+1], modinv[MAXN+1];

ll nck(ll n, ll k) {
    if (n < k) return 0;
    ll res = fac[n];
    res = (res * modinv[k]) % MOD;
    res = (res * modinv[n-k]) % MOD;
    return res;
}

fac[0] = 1;
for(int i = 1; i <= MAXN; i++) {
    fac[i] = fac[i-1] * i % MOD;
}
modinv[MAXN] = powmod(fac[MAXN], MOD-2, MOD);
for(int i = MAXN; i > 0; i--) {
    modinv[i-1] = modinv[i] * i % MOD;
}

```

4.8 Fibonacci

Runs in $O(\log N)$ time.

```

struct Mat {
    ll a, b, c, d; // 2x2 Matrix: [a b; c d]
};

Mat mul(Mat x, Mat y) {

```

```

    return {
        x.a*y.a + x.b*y.c,
        x.a*y.b + x.b*y.d,
        x.c*y.a + x.d*y.c,
        x.c*y.b + x.d*y.d
    };
}

Mat mpow(Mat base, long long exp) {
    Mat res = {1, 0, 0, 1}; // Identity Matrix
    while (exp) {
        if (exp & 1) res = mul(res, base);
        base = mul(base, base);
        exp >>= 1;
    }
    return res;
}

ll fib(long long n) {
    if (n == 0) return 0;
    Mat m = {1, 1, 1, 0}; // Fibonacci Seed Matrix
    return mpow(m, n-1).a;
}

```

5 Algorithms

5.1 Binary Search

Find the cuberoot of n . Runs in $O(\log N)$.

```

long long n; cin >> n;
long long mini = 0, maxi = 1e6, medi;
while (mini < maxi) {
    medi = mini+(maxi-mini)/2;
    if (medi * medi * medi >= n) maxi = medi;
    else mini = medi+1;
}
cout << mini << "\n";

```

5.2 Binary Search using Lifting

Find the cuberoot of n . Runs in $O(\log N)$.

```

long long n; cin >> n;
long long cur = 0, gap = 1e6, next;
while (gap > 0) {
    while (next = cur + gap, next * next * next < n) {
        cur = next;
    }
    gap >>= 1;
}
cout << cur+1 << "\n";

```

5.3 Sliding Set

Speeds up DP from $O(N^2)$ to $O(N \log N)$.

```

// Example - Candymountain
// Jump across with minimax candies
// Populate with initial window
for(int i = 0; i < k; i++) {
    dp[i] = candies[i];
    s.insert(candies[i]);
}
// Sliding Set
for(int i = k; i < n; i++){
    dp[i] = max(candies[i], *s.begin());
    s.erase(s.find(dp[i-k]));
    s.insert(dp[i]);
}

```

5.4 Sliding Window

Runs in $O(N)$.

```

// Example: Count number of subarrays with sum at least k
// All numbers are non-negative
int s = 0, sum = 0, ans = 0;
for (int i = 0; i < n; i++) {
    sum += a[i];
    while (sum >= k) {
        ans += (n-i);
        sum -= a[s];
        s++;
    }
}

```

5.5 Set Merging

Reduces complexity from $O(Q \cdot N \log N)$ to $O(N \log^2 N)$.

```

for (int i = 0; i < q; i++) {
    cin >> a >> b;
    // small to large merging
    if (s[a].size() > s[b].size()) swap(s[a], s[b]);
    for (int x : s[a]) s[b].insert(x);
    s[a].clear();
    cout << s[b].size() << "\n";
}

```

5.6 Discretisation

Runs in $O(N \log N)$.

```

vector<int> a(n);
vector<int> b = a;
sort(b.begin(), b.end());
b.erase(unique(b.begin(), b.end()), b.end());
for (int &i : a) {
    i = lower_bound(b.begin(), b.end(), i) - b.begin() + 1; // 1-indexed
}
// a now holds discretised values

```

5.7 Meet in the Middle

Reduces time complexity from $O(2^N)$ to $O(N \cdot 2^{N/2})$.

```
// Example: Bobek
// Count number of subsets with sum <= tgt
left = n/2; right = n-left;
vector<int> sums;
for (int i = 0; i < (1 << left); i++) {
    int cur = 0;
    for (int j = 0; j < left; j++) {
        if (i & (1 << j)) cur += a[j];
    }
    sums.push_back(cur);
}
sort(sums.begin(), sums.end());
for (int i = 0; i < (1 << right); i++) {
    int cur = 0;
    for (int j = 0; j < right; j++) {
        if (i & (1 << j)) cur += a[left + j];
    }
    ans += upper_bound(sums.begin(), sums.end(), tgt-cur) - sums.begin();
}
```

5.8 On the Fly

Reduces memory usage from $O(N \cdot K)$ to $O(N)$.

```
// Stolen from DNC code
// Use i%2 and !(i%2) for indexing
for (int i = 1; i <= g; i++) {
    for (int j = 1; j <= n; j++) dp[j][i%2] = INF;
    dnc(0, n, 0, n, i%2);
}
```

5.9 Square Root Decomposition

$O(N)$ precomputation, $O(1)$ update, $O(\sqrt{n})$ query.

```
// Point Update, Range Sum Query

vector<ll> a(n);
int blockSize = sqrt(n) + 1;
vector<ll> blocks(blockSize, 0);
// precomputation
for (int i = 0; i < n; i++) {
    blocks[i / blockSize] += a[i];
}
// update
blocks[k/ blockSize] += u - a[k];
a[k] = u;
// query
ll sum = 0;
int s = l / blockSize;
int e = r / blockSize;
if (s == e) {
    for (int i = l; i <= r; i++) sum += a[i];
} else {
    for (int i = l; i < (s + 1) * blockSize; i++) sum += a[i];
```

```

        for (int b = s+1; b < e; b++) sum += blocks[b];
        for (int i = e * blockSize; i <= r; i++) sum += a[i];
    }

```

5.10 Mo's Algorithm

Runs in $O((n + q) \cdot \sqrt{n})$. Does not support online queries or updates.

```

struct Query {
    int l, r, idx;
};

vector<ll> a(n);
int blockSize = sqrt(n) + 1;
ll sum = 0;

bool cmp(const Query &x, const Query &y) {
    int bx = x.l / blockSize;
    int by = y.l / blockSize;
    if (bx != by) return bx < by;
    return (bx & 1) ? (x.r < y.r) : (x.r > y.r);
}

// modify to insert idx
void plus(int idx) {
    sum += a[idx];
}

// modify to remove idx
void minus(int idx) {
    sum -= a[idx];
}

// make sure indexes are 0-indexed
vector<Query> queries(q);
sort(queries.begin(), queries.end(), cmp);
vector<ll> ans(q);
int curL = 0, curR = -1;
for (Query &cq : queries) {
    while (curL > cq.l) plus(--curL);
    while (curR < cq.r) plus(++curR);
    while (curL < cq.l) minus(curL++);
    while (curR > cq.r) minus(curR--);
    ans[cq.idx] = sum;
}

```

6 Miscellaneous

6.1 Fast I/O

Cannot use with `scanf`, `printf`.

```

ios_base::sync_with_stdio(false);
cin.tie(0);

```

6.2 Superfast I/O

Only for non-negative integer input.

```
inline ll ri () {
    ll x = 0;
    char ch = getchar_unlocked();
    while (ch < '0' || ch > '9') ch = getchar_unlocked();
    while (ch >= '0' && ch <= '9') {
        x = (x << 3) + (x << 1) + ch - '0';
        ch = getchar_unlocked();
    }
    return x;
}
```
