Competitive Programming Reference

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Last Updated: 10/01/2026

Contents

1 Data Structures

Data Structure	Precomputation	Update	Query	Memory	Notes
Prefix Sum	O(N)	X	O(1)	O(N)	Associative Functions (+, XOR)
Sparse Table	O(N log N)	X	O(1)	O(N log N)	Idempotent Functions (max, gcd)
Fenwick Tree	X	O(log N)	O(log N)	O(N)	Prefix Operations
Segment Tree	X	O(log N)	O(log N)	O(4N)	Very flexible and extendable

1.1 Prefix Sums

1.1.1 1D

$O(N)$ precomputation, $O(1)$ query.

```
//Query - 1-Indexed
int query(int s, int e){
    return ps[e]-ps[s-1];
}

//Precomputation
ps[0] = 0;
for(int i = 1; i <= n; i++) ps[i] = ps[i-1]+a[i];
```

1.1.2 2D

$O(R \cdot C)$ precomputation, $O(1)$ query.

```
//Query - 1-Indexed
int query(int x1, int y1, int x2, int y2){
    return ps[x2][y2]-ps[x1-1][y2]-ps[x2][y1-1]+ps[x1-1][y1-1];
}

//Precomputation
for (int i = 0; i <= r; i++) ps[i][0] = 0;
for (int j = 0; j <= c; j++) ps[0][j] = 0;
for (int i = 1; i <= r; i++) {
    for (int j = 1; j <= c; j++) {
        ps[i][j] = ps[i-1][j]+ps[i][j-1]-ps[i-1][j-1]+a[i][j];
    }
}
```

1.2 Sparse Table

1.2.1 1D

$O(N \log N)$ precomputation, $O(1)$ query.

```
int sp[MAXLOGN][MAXN];

// Query - 0-indexed, inclusive [l, r]
int query(int l, int r){
    r++;
    int p = 31 - __builtin_clz(r - l); // 63 for long long
    return __gcd(sp[p][l], sp[p][r - (1 << p)]);
}

// Precomputation
h = 31 - __builtin_clz(n);
for (int i = 0; i < n; i++) sp[0][i] = a[i];
```

```
for (int i = 1; i <= h; i++) {
    for (int j = 0; j + (1 << i) <= n; j++) {
        sp[i][j] = __gcd(sp[i-1][j], sp[i-1][j + (1 << (i-1))]);
    }
}
```

1.2.2 2D

$O(NM \log N \log M)$ precomputation, $O(1)$ query.

```
int sp[MAXLOGN][MAXLOGM][MAXN][MAXM];

// Query - 0-indexed, inclusive
int query(int x1, int y1, int x2, int y2) {
    x2++; y2++;
    // 63 for long long
    int kx = 31 - __builtin_clz(x2 - x1);
    int ky = 31 - __builtin_clz(y2 - y1);
    int g1 = sp[kx][ky][x1][y1];
    int g2 = sp[kx][ky][x2 - (1 << kx)][y1];
    int g3 = sp[kx][ky][x1][y2 - (1 << ky)];
    int g4 = sp[kx][ky][x2 - (1 << kx)][y2 - (1 << ky)];
    return __gcd(__gcd(g1, g2), __gcd(g3, g4));
}

// Precomputation
// Build single elements
for (int i = 0; i < n; i++) {
    for (int j = 0; j < m; j++) {
        sp[0][0][i][j] = a[i][j];
    }
}
// Build along columns (k2)
for (int k2 = 1; (1 << k2) <= m; k2++) {
    for (int i = 0; i < n; i++) {
        for (int j = 0; j + (1 << k2) <= m; j++) {
            sp[0][k2][i][j] =
                __gcd(sp[0][k2-1][i][j],
                      sp[0][k2-1][i][j + (1 << (k2-1))]);
        }
    }
}
// Build along rows (k1)
for (int k1 = 1; (1 << k1) <= n; k1++) {
    for (int k2 = 0; (1 << k2) <= m; k2++) {
        for (int i = 0; i + (1 << k1) <= n; i++) {
            for (int j = 0; j + (1 << k2) <= m; j++) {
                sp[k1][k2][i][j] =
                    __gcd(sp[k1-1][k2][i][j],
                          sp[k1-1][k2][i + (1 << (k1-1))][j]);
            }
        }
    }
}
```

1.3 Fenwick Trees

1.3.1 Point Update Range Query

$O(\log N)$ update and query.

```
inline int ls(int x){ return (x)&(-x); }

int fw[MAXN]; // 1-Indexed

void pu(int i, int v) {
    for(; i <= n; i += ls(i)) fw[i] += v;
}

int pq(int i) {
    int t = 0;
    for(; i; i -= ls(i)) t += fw[i];
    return t;
}

int rq(int s, int e) {
    return pq(e) - pq(s - 1);
}
```

1.3.2 Range Update Point Query

$O(\log N)$ update and query.

```
// Requires PURQ Code (PU, PQ)

void ru(int s, int e, int v) {
    pu(s, v);
    pu(e+1, -v);
}
```

1.3.3 Range Update Range Query

$O(\log N)$ update and query.

```
// Requires PURQ Code (PU, PQ)
// Functions need to be modified to take in array parameter
// e.g. int pu(*tree, int i, int v)

void ru(int s, int e, int v) {
    pu(fw1, s, v);
    pu(fw1, e+1, -v);
    pu(fw2, s, -v*(s-1));
    pu(fw2, e+1, v*e);
}

int ps(int i) {
    return pq(fw1, i)*i + pq(fw2, i);
}

int rq(int s, int e) {
    return ps(e) - ps(s - 1);
}
```

1.3.4 2D PURQ / RUPQ

$O(\log N \cdot \log M)$ update and query.

```
inline int ls(int x) { return x & -x; }

int fw[MAXN][MAXN]; // 1-indexed

void pu(int x, int y, int v) {
    for (int i = x; i < MAXN; i += ls(i)) {
        for (int j = y; j < MAXN; j += ls(j)) {
            fw[i][j] += v;
        }
    }
}

int pq(int x, int y) {
    int ans = 0;
    for (int i = x; i > 0; i -= ls(i)) {
        for (int j = y; j > 0; j -= ls(j)) {
            ans += fw[i][j];
        }
    }
    return ans;
}

// Range Query
int rq(int x1, int y1, int x2, int y2) {
    return pq(x2, y2) - pq(x1-1, y2) - pq(x2, y1-1) + pq(x1-1, y1-1);
}

// Range Update, Point Query
void ru(int x1, int y1, int x2, int y2, int v) {
    pu(x1, y1, v);
    pu(x1, y2 + 1, -v);
    pu(x2 + 1, y1, -v);
    pu(x2 + 1, y2 + 1, v);
}
```

1.3.5 2D RURQ

$O(\log N \cdot \log M)$ update and query.

```
// Requires 2D PURQ Code

void ru(int x1, int y1, int x2, int y2, long long v) {
    pu(fw1, x1, y1, v);
    pu(fw1, x1, y2+1, -v);
    pu(fw1, x2+1, y1, -v);
    pu(fw1, x2+1, y2+1, v);

    pu(fw2, x1, y1, v*(x1-1));
    pu(fw2, x1, y2+1, -v*(x1-1));
    pu(fw2, x2+1, y1, -v*x2);
    pu(fw2, x2+1, y2+1, v*x2);

    pu(fw3, x1, y1, v*(y1-1));
    pu(fw3, x1, y2+1, -v*y2);
    pu(fw3, x2+1, y1, -v*(y1-1));
```

```
    pu(fw3, x2+1, y2+1, v*y2);

    pu(fw4, x1, y1, v*(x1-1)*(y1-1));
    pu(fw4, x1, y2+1, -v*(x1-1)*y2);
    pu(fw4, x2+1, y1, -v*x2*(y1-1));
    pu(fw4, x2+1, y2+1, v*x2*y2);
}

long long ps(int x,int y) {
    return pq(fw1, x, y)*x*y-pq(fw2, x, y)*y-pq(fw3, x, y)*x+pq(fw4, x, y);
}

long long rq(int x1,int y1,int x2,int y2) {
    return ps(x2, y2)-ps(x1-1, y2)-ps(x2, y1-1)+ps(x1-1, y1-1);
}
```

1.4 Segment Trees

1.4.1 Standard

$O(\log N)$ point update and range query.

```
struct node {
    int s, e, m, v;
    node *l, *r;
    node(int _s, int _e) {
        s = _s; e = _e; m = (s+e)/2; v = 0;
        if (s != e) {
            l = new node(s, m);
            r = new node(m+1, e);
        }
    }
    void pu(int x, int y) {
        if (s == e) { v = y; return; }
        if (x <= m) l->pu(x, y);
        if (x > m) r->pu(x, y);
        v = min(l->v, r->v);
    }
    int rq(int x, int y) {
        if (s == x && e == y) return v;
        if (y <= m) return l->rq(x, y);
        if (x > m) return r->rq(x, y);
        return min(l->rq(x, m), r->rq(m+1, y));
    }
} *root;

root = new node(0, n-1);
```

1.4.2 Lazy Propagation

$O(\log N)$ range update and range query.

```
struct node {
    int s, e, m, v, lazy;
    node *l, *r;
    node(int _s, int _e) {
        s = _s; e = _e; m = (s+e)/2; v = lazy = 0;
        if (s != e) {
```

```
            l = new node(s, m);
            r = new node(m+1, e);
        }
    }

    int pu() {
        if (s == e) { v += lazy; lazy = 0; return v; }
        v += lazy;
        l->lazy += lazy; r->lazy += lazy;
        lazy = 0;
        return v;
    }

    void ru(int x, int y, int z) {
        if (s == x && e == y) { lazy += z; return; }
        if (y <= m) l->ru(x, y, z);
        else if (x > m) r->ru(x, y, z);
        else l->ru(x, m, z), r->ru(m+1, y, z);
        v = max(l->pu(), r->pu());
    }

    int rq(int x, int y) {
        pu();
        if (s == x && e == y) return pu();
        if (y <= m) return l->rq(x, y);
        if (x > m) return r->rq(x, y);
        return max(l->rq(x, m), r->rq(m+1, y));
    }
} *root;

root = new node(0, n-1);
```

1.4.3 Lazy Node Creation

```
struct node {
    int s, e, m, v;
    node *l, *r;
    node(int _s, int _e) {
        s = _s; e = _e; m = (s+e)/2; v = 0;
        l = nullptr, r = nullptr;
    }
    void create() {
        if (s != e) {
            l = new node(s, m);
            r = new node(m+1, e);
        }
    }
    void pu(int x, int y) {
        if (s == e) { v = y; return; }
        create();
        if (x <= m) l->pu(x, y);
        if (x > m) r->pu(x, y);
        v = min(l->v, r->v);
    }
    int rq(int x, int y) {
        if (s == x && e == y) return v;
        create();
```

```
        if (y <= m) return l->rq(x, y);
        if (x > m) return r->rq(x, y);
        return min(l->rq(x, m), r->rq(m+1, y));
    }
} *root;

root = new node(0, n-1);
```

1.4.4 Maxsum

$O(\log N)$ point update and range query.

```
struct node {
    ll s, e, m, ps, ss, ms, ts;
    node *l, *r;
    node(ll _s, ll _e) {
        s = _s; e = _e; m = (s+e)/2; ps = ss = ms = ts = 0;
        if (s != e) {
            l = new node(s, m);
            r = new node(m+1, e);
        }
    }
    void pu(ll x, ll y) {
        if (s == e) { ps = ss = ms = ts = y; return; }
        if (x <= m) l->pu(x, y);
        if (x > m) r->pu(x, y);
        //New Prefix Max -> Left Prefix, Left + Right Prefix
        ps = max(l->ps, l->ts+r->ps);
        //New Suffix Max -> Right Suffix, Left Suffix + Right
        ss = max(r->ss, r->ts+l->ss);
        //Total Sum -> Left + Right
        ts = l->ts+r->ts;
        //Maxsum - Left Suffix + Right Prefix, Left, Right,
        //         Total, Left Maxsum, Right Maxsum
        ms = max({l->ss+r->ps, ps, ss, ts, l->ms, r->ms});
    }
    ll ans() {
        return ms;
    }
} *root;

root = new node(0, n-1);
```

1.4.5 Merge Sort Tree / Order Statistics

Insertion: $O(\log N)$, Building: $O(N \log^2 N)$, Counting: $O(\log^2 N)$, Finding: $O(\log V \cdot \log^2 N)$, Range Max: $O(\log N)$.

```
struct node {
    int s, e, m;
    vector<int> v;
    node *l, *r;
    node(int _s, int _e) {
        s = _s; e = _e; m = (s+e)/2;
        if (s != e) {
            l = new node(s, m);
            r = new node(m+1, e);
        }
```

```
    }
    void insert(int x, int y) {
        if (s == e) { v.push_back(y); return; }
        if (x > m) r->insert(x, y);
        if (x <= m) l->insert(x, y);
        v.push_back(y);
    }
    void build(){
        if (s == e) return;
        l->build();
        r->build();
        sort(v.begin(), v.end());
    }
    int countLessEqual(int x, int y, int k) {
        if (x > y) return 0;
        if (s == x && e == y) {
            return upper_bound(v.begin(), v.end(), k)-v.begin();
        }
        if (x > m) return r->countLessEqual(x, y, k);
        if (y <= m) return l->countLessEqual(x, y, k);
        return l->countLessEqual(x, m, k)+r->countLessEqual(m+1, y, k);
    }
    int kthSmallest(int x, int y, int k) {
        int mini = 0, maxi = (1 << 30);
        int ans = mini, gap = maxi;
        while (gap > 0) {
            while (ans + gap <= maxi && countLessEqual(x, y, ans + gap) < k) {
                ans += gap;
            }
            gap >>= 1;
        }
        return ans + 1;
    }
    int rangeMax(int x, int y) {
        if (x > y) return 0;
        if (s == x && e == y) return v.back();
        if (x > m) return r->rangeMax(x, y);
        if (y <= m) return l->rangeMax(x, y);
        return max(l->rangeMax(x, m), r->rangeMax(m+1, y));
    }
} *root;

root = new node(0, n-1);
```

1.4.6 2D

$O(N \cdot \log N \cdot \log M)$ point update and range query.

```
struct node2D {
    int s, e, m;
    node1D *maxi;
    node2D *l, *r;
    node2D(int a, int b, int c, int d) {
        s = a; e = b; m = (s+e)/2;
        maxi = new node1D(c, d);
        if (s != e) {
            l = new node2D(s, m, c, d);
            r = new node2D(m+1, e, c, d);
```

```
        }
    }
    void pu(int a, int b, int v) {
        if (s == e) { maxi->pu(b, v); return; }
        if (a <= m) l->pu(a, b, v);
        else r->pu(a, b, v);
        maxi->pu(b, max(l->maxi->rq(b, b), r->maxi->rq(b, b)));
    }
    int rq(int a, int b, int c, int d) {
        if (s == a && e == b) return maxi->rq(c, d);
        if (b <= m) return l->rq(a, b, c, d);
        if (a > m) return r->rq(a, b, c, d);
        return max(l->rq(a, m, c, d), r->rq(m+1, b, c, d));
    }
} *root;

root = new node(0, n-1, 0, n-1);
```

2 Graph Theory

Graph Algorithm	Complexity	Notes
DFS	O(V+E)	Flood Fill, Trees
BFS	O(V+E)	Unweighted Shortest Path
0-1 BFS	O(V+E)	Unweighted Shortest Path
Floyd-Warshall	O(N^3)	All Pairs Shortest Path
Bellman-Ford	O(VE)	Negative Single Source Shortest Path
Dijkstra'	O(E log V)	Single Source Shortest Path
SPFA	O(VE)	Negative Single Source Shortest Path
Prim's	O(E log V)	Dijkstra's
UFDS	O($\alpha(n)$)	Connectedness
Kruskal's	O(E log E)	Greedy
Toposort	O(V+E)	Postorder
Floyd's Cycle Finding	O(V+E)	Outdegree 1
Kuhn's	O(V^3)	Bipartite Matching
Hopcroft-Karp	O($E\sqrt{V}$)	Bipartite Matching
Articulation Points	O(V+E)	Find Splitting Nodes
Bridges	O(V+E)	Find Splitting Edges
SCC	O(V+E)	Compress Cycles
TSP	O($N^2 2^N$)	Tour All Nodes
Pre/Postorder	O(V)	Ordering of Nodes
Subtree To Range	O(V)	Ordering Nodes
Weighted MIS	O(V)	Tree DP
Diameter	O(V)	DFS
2^K Decomposition	O($N \log N$) precompute, O($\log N$) query	Find Parent
Lowest Common Ancestor	O($\log N$) query	Find Common Parent
Heavy Light Decomposition	O(N) precompute, O($\log N$) query	Decomposition
Centroid Decomposition	O($N \log N$) precompute, $O(\log N)$ query	Decomposition

2.1 Depth First Search

Runs in $O(V+E)$.

```
// For adjacency lists
void dfs(int x, int p) {
    for (int y : adj[x]) {
```

```
        if (y != p) {
            dist[y] = dist[x]+1;
            dfs(y, x);
        }
    }
}
```

2.2 Breadth First Search

Runs in $O(V + E)$.

```
// For adjacency lists
visited[s] = 1;
dist[s] = 0;
q.push(s);
while (!q.empty()) {
    int f = q.front(); q.pop();
    for (int i : adjlist[f]) {
        if (!visited[i]) {
            q.push(i);
            visited[i] = 1;
            dist[i] = dist[f] + 1;
        }
    }
}
```

2.3 0-1 BFS

Runs in $O(V + E)$.

```
// For adjacency lists
deque<int> dq;
dist[s] = 0;
dq.push(s);
while (!dq.empty()) {
    int u = dq.front(); dq.pop();
    for (int e : adjlist[u]) {
        int v = e.first, w = e.second;
        if (dist[v] > dist[u] + w) {
            dist[v] = dist[u] + w;
            if (w == 0) dq.push_front(v);
            else dq.push_back(v);
        }
    }
}
```

2.4 Floyd-Warshall

Runs in $O(N^3)$.

```
// Initialise adjacency matrix
for (int i = 0; i < n; i++) {
    for (int j = 0; j < n; j++) {
        if (i == j) adj[i][j] = 0;
        else adj[i][j] = INF;
    }
}
// Floyd-Warshall
```

```
for (int k = 0; k < n; k++) {
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++) {
            adj[i][j] = min(adj[i][j], adj[i][k]+adj[k][j]);
            if (adj[i][i] < 0) negCycle = true;
        }
    }
}
```

2.5 Bellman-Ford

Runs in $O(VE)$.

```
vector<int> dist(n, INF);
dist[s] = 0;
bool negCycle = false;
for (int i = 1; i <= n; i++) {
    bool update = false;
    for (Edge e : edges) {
        if (dist[e.u] < INF && dist[e.v] > dist[e.u] + e.w) {
            dist[e.v] = dist[e.u] + e.w;
            update = true;
        }
    }
    if (!update) break;
    if (update && i == n) negCycle = true;
}
```

2.6 Dijkstra's Algorithm

Runs in $O(E \log V)$.

```
priority_queue<pi, vector<pi>, greater<pi>> pq;
vector<int> dist(n, INF);
dist[s] = 0;
pq.push({0, s});
while (!pq.empty()) {
    pi f = pq.top(); pq.pop();
    int d = f.first, u = f.second;
    if (d != dist[u]) continue;
    for (pi x : adj[u]) {
        int v = x.first, w = x.second;
        if (dist[v] > d + w) {
            dist[v] = d + w;
            pq.push({ dist[v], v });
        }
    }
}
```

2.7 Shortest Path Faster Algorithm

Runs in $O(VE)$.

```
vector<int> dist(n, INF);
vector<int> inQueue(n, 0);
queue<int> q;
dist[s] = 0;
q.push(s);
```

```
inQueue[s]++;
bool negCycle = false;
while (!q.empty()) {
    int u = q.front(); q.pop();
    inQueue[u]--;
    for (Edge e : adj[u]) {
        if (dist[e.v] > dist[u] + e.w) {
            dist[e.v] = dist[u] + e.w;
            if (inQueue[e.v] == 0) {
                q.push(e.v);
                inQueue[e.v]++;
                if (inQueue[e.v] > n) {
                    negCycle = true;
                    break;
                }
            }
        }
    }
    if (negCycle) break;
}
```

2.8 Prim's Algorithm

Runs in $O(E \log V)$.

```
priority_queue<pi, vector<pi>, greater<pi>> pq;
vector<int> dist(n, INF);
vector<bool> vis(n, false);
dist[s] = 0;
pq.push({0, s});
while (!pq.empty()) {
    pi f = pq.top(); pq.pop();
    int d = f.first, u = f.second;
    if (vis[u]) continue;
    vis[u] = true;
    for (pi x : adj[u]) {
        int v = x.first, w = x.second;
        if (!vis[v] && dist[v] > w) {
            dist[v] = w;
            pq.push({ dist[v], v });
        }
    }
}
```

2.9 Union Find Disjoint Subset

With both path compression and union by rank, runs in $O(\alpha(n))$ (basically constant time).

```
int p[MAXN];
int sz[MAXN];

int root(int x) {
    if (p[x] == -1) return x;
    return p[x] = root(p[x]);
}

void connect(int x, int y) {
    x = root(x); y = root(y);
```

```
    if (x == y) return;
    if (sz[x] < sz[y]) swap(x, y);
    p[y] = x;
    sz[x] += sz[y];
}

fill(p, p+MAXN, -1);
fill(sz, sz+MAXN, 1);
```

2.10 Kruskal's Algorithm

Runs in $O(E \log E)$.

```
sort(edges.begin(), edges.end());
for (Edge e : edges) {
    if (root(e.u) != root(e.v)) {
        connect(e.u, e.v);
        cost += e.w;
    }
}
```

2.11 Topological Sort

Runs in $O(V + E)$.

```
void dfs(int x) {
    if (v[x]) return;
    v[x] = 1;
    for (int y : adj[x]) dfs(y);
    topo.push_back(x);
}
for (int i = 0; i < n; i++) dfs(i);
reverse(topo.begin(), topo.end());
```

2.12 Floyd's Cycle Finding Algorithm

For graphs with outdegree 1, runs in $O(V + E)$.

```
// detect cycle
int slow = s, fast = s;
do {
    slow = nxt[slow];
    fast = nxt[nxt[fast]];
} while (slow != fast);
// find start of cycle
slow = start;
while (slow != fast) {
    slow = nxt[slow];
    fast = nxt[fast];
}
// collect all nodes in cycle
vector<int> cycle;
int cur = slow;
do {
    cycle.push_back(cur);
    cur = nxt[cur];
} while (cur != slow);
```

2.13 Maximum Cardinality Bipartite Matching

2.13.1 Kuhn's Algorithm

Runs in $O(V^3)$.

```
int n, mcbms = 0, miss = 0;
vector<int> match;
vector<bool> vis;
vector<vector<int>> adj;
vector<int> left;

// dfs for augmenting path
bool dfs(int u) {
    for (int v : adj[u]) {
        if (!vis[v]) {
            vis[v] = true;
            if (match[v] == -1 || dfs(match[v])) {
                match[v] = u;
                match[u] = v;
                return true;
            }
        }
    }
    return false;
}

void mcbm() {
    match.assign(n, -1);
    mcbms = 0;
    // greedy initial matching
    for (int u : left) {
        for (int v : adj[u]) {
            if (match[v] == -1) {
                match[v] = u;
                match[u] = v;
                mcbms++;
                break;
            }
        }
    }
    // dfs augmenting paths for unmatched
    for (int u : left) {
        if (match[u] == -1) {
            vis.assign(n, false);
            if (dfs(u)) mcbms++;
        }
    }
    miss = n - mcbms;
}
```

2.13.2 Hopcroft-Karp

Runs in $O(E \cdot \sqrt{V})$.

```
int n, m;
vector<vector<int>> adj;
vector<int> pairU, pairV, dist;

bool bfs() {
```

```
    queue<int> q;
    for (int u = 0; u < n; u++) {
        if (pairU[u] == -1) {
            dist[u] = 0;
            q.push(u);
        } else {
            dist[u] = INF;
        }
    }
    bool found = false;
    while (!q.empty()) {
        int u = q.front(); q.pop();
        for (int v : adj[u]) {
            if (pairV[v] == -1) {
                found = true;
            } else if (dist[pairV[v]] == INF) {
                dist[pairV[v]] = dist[u] + 1;
                q.push(pairV[v]);
            }
        }
    }
    return found;
}

bool dfs(int u) {
    for (int v : adj[u]) {
        if (pairV[v] == -1 ||
           (dist[pairV[v]] == dist[u] + 1 && dfs(pairV[v]))) {
            pairU[u] = v;
            pairV[v] = u;
            return true;
        }
    }
    dist[u] = INF;
    return false;
}

int hopcroftKarp() {
    pairU.assign(n, -1);
    pairV.assign(m, -1);
    dist.assign(n, 0);
    int mcbm = 0;
    while (bfs()) {
        for (int u = 0; u < n; u++) {
            if (pairU[u] == -1 && dfs(u)) {
                mcbm++;
            }
        }
    }
    return mcbm;
}
```

2.14 Articulation Points and Bridges

Using Tarjan's Algorithm, runs in $O(V + E)$.

```
vector<int> adj[MAXN];
vector<int> dep(MAXN, 0), low(MAXN, 0), par(MAXN, -1);
```

```
vector<int> chi(MAXN, 0), atp(MAXN, 0);
vector<bool> vis(MAXN, false);
vector<pi> bridges;

void tarjan(int u, int d) {
    vis[u] = true;
    dep[u] = low[u] = d;
    chi[u] = 0; atp[u] = 1;
    for (int v : adj[u]) {
        if (!vis[v]) {
            par[v] = u;
            chi[u]++;
            tarjan(v, d+1);
            low[u] = min(low[u], low[v]);
            if (low[v] >= dep[u]) atp[u]++;
            if (low[v] > dep[u]) bridges.push_back({u, v});
        } else if (v != par[u]) {
            low[u] = min(low[u], dep[v]);
        }
    }
}

tarjan(0, 0);
// handle root separately since it has no parents
atp[0] = chi[0];
// atp stores number of components separated upon removal
// bridges stores all bridges in the graph
```

2.15 Strongly Connected Components

Using Tarjan's Algorithm, runs in $O(V + E)$.

```
vector<int> adj[MAXN];
vector<int> comps[MAXN];
set<int> adjscc[MAXN];
vector<int> idxs(MAXN, -1), low(MAXN, 0);
vector<bool> onStack(MAXN, false);
stack<int> st;
int dfsidx = 0, sccidx;
int comp[MAXN];

void scc(int u) {
    idxs[u] = low[u] = dfsidx++;
    st.push(u);
    onStack[u] = true;
    for (int v : adj[u]) {
        if (idxs[v] == -1) {
            scc(v);
            low[u] = min(low[u], low[v]);
        } else if (onStack[v]) {
            low[u] = min(low[u], idxs[v]);
        }
    }
    if (low[u] == idxs[u]) {
        // u is a root of an scc
        comps[sccidx].clear();
        int w;
        do {
```

```
            w = st.top(); st.pop();
            onStack[w] = false;
            comps[sccidx].push_back(w);
            comp[w] = sccidx;
        } while (w != u);
        sccidx++;
    }
}

for (int i = 1; i <= n; i++) {
    if (idxs[i] == -1) scc(i);
}

for (int u = 1; u <= n; u++) {
    for (int v : adj[u]) {
        if (comp[u] != comp[v]) adjscc[comp[u]].insert(comp[v]);
    }
}

// dfs idxs now range from 0 to dfsidx-1
// scc idxs now range from 0 to sccidx-1
```

2.16 Travelling Salesman Problem

Both of these solutions run in $O(N^2 \cdot 2^N)$ time.

2.16.1 Bottom Up

```
int full = (1 << n) - 1;
for (int mask = 0; mask <= full; mask++) {
    for (int i = 0; i < n; i++) {
        dp[mask][i] = INF;
    }
}
dp[1][0] = 0; // assume source is node 0
for (int mask = 1; mask <= full; mask++) {
    for (int i = 0; i < n; i++) {
        if (!(mask & (1 << i)) || dp[mask][i] >= INF) continue;
        for (int j = 0; j < n; j++) {
            if (mask & (1 << j)) continue;
            if (adj[i][j] >= INF) continue;
            int newMask = mask | (1 << j);
            dp[newMask][j] = min(dp[newMask][j], dp[mask][i] + adj[i][j]);
        }
    }
}
ll ans = INF;
for (int i = 0; i < n; i++) {
    if (adj[i][0] < INF) {
        // close the tour
        ans = min(ans, dp[full][i] + adj[i][0]);
    }
}
```

2.16.2 Top Down

```cpp
ll adj[MAXN][MAXN], dp[1 << MAXN][MAXN];

ll tsp(int mask, int pos) {
    if (mask == (1 << n) - 1) {
        // assume node 0 is the start and end
        return adj[pos][0] < INF ? adj[pos][0] : INF;
    }
    if (dp[mask][pos] != -1) return dp[mask][pos];
    ll ans = INF;
    for (int i = 0; i < n; i++) {
        if ((mask & (1 << i)) == 0 && adj[pos][i] < INF) {
            ans = min(ans, adj[pos][i] + tsp(mask | (1 << i), i));
        }
    }
    return dp[mask][pos] = ans;
}

memset(dp, -1, sizeof(dp));
ll res = tsp(1, 0); // initial mask has source visited
```

2.17 Trees

2.17.1 Pre/Postorder Traversal

Runs in $O(V)$.

```cpp
int prec = 0, postc = 0;
void dfs(int x, int p) {
    pre[x] = prec++;
    for(int y : adj[x]) {
        if (y != p) dfs(y, x);
    }
    post[x] = postc++;
}
```

2.17.2 Subtree to Range

Runs in $O(V)$.

```cpp
int dfs(int x, int p) {
    pre[x] = c++;
    rig[pre[x]] = pre[x];
    for (int y : adj[x]) {
        if (y != p) {
            rig[pre[x]] = max(rig[pre[x]], dfs(y, x));
        }
    }
    return rig[pre[x]];
}
// Subtree -> pre[x], rig[pre[x]]
// Node Index -> pre[x]
// Range of Children -> pre[x]+1, rig[pre[x]]
```

2.17.3 Weighted Maximum Independent Set

Runs in $O(V)$.

```
int dp[MAXN][2];

int mis(int v, bool take, int p) {
    if (dp[v][take] != -1) return dp[v][take];
    int ans = take * c[v];
    for (int u : adj[v]) {
        if (u == p) continue;
        int temp = mis(u, 0, v);
        if (!take) temp = max(temp, mis(u, 1, v));
        ans += temp;
    }
    return dp[v][take] = ans;
}
void ans(int v, bool take, int p) {
    for (int u : adj[v]) {
        if (u == p) continue;
        int temp0 = dp[u][0], temp1 = (take ? -1 : dp[u][1]);
        if (temp0 > temp1) ans(u, 0, v);
        else { a.push_back(u); ans(u, 1, v); }
    }
}

memset(dp, -1, sizeof(dp));
mis(0, 0, -1); // don't take root
mis(0, 1, -1); // take root
if(dp[0][1] > dp[0][0]) { a.push_back(0); ans(0, 1, -1); }
else ans(0, 0, -1);
```

2.17.4 Diameter

Runs in $O(V)$.

```
pi dfs(int x, int p, int d) {
    pi b = {x, d};
    for (pi y : adj[x]) {
        if (y.first != p) {
            pi c = dfs(y.first, x, d + y.second);
            if (c.second > b.second) b = c;
        }
    }
    return b;
}
pi s = dfs(0, -1, 0);
pi e = dfs(s.first, -1, 0);
// e.second gives diameter
// For even diameter, centroid is at e.second / 2
// For odd diameter, centroid is at e.second / 2 and e.second / 2 + 1
```

2.17.5 2^K Decomposition

$O(N \log N)$ precomputation and memory, $O(\log N)$ query.

```
int par(int x, int k) {
    for(int i = MAXLOGN; i >= 0; i--) {
        if (k >= (1 << i)) {
            if(x == -1) return x;
            x = p[x][i];
            k -= (1 << i);
```

```
        }
    }
    return x;
}

int p[MAXN][MAXLOGN];
memset(p, -1, sizeof(p));
dfs(0); // compute initial parent p[i][0]
for (int k = 1; k <= MAXLOGN; k++) {
    for (int i = 0; i < n; i++) {
        if(p[i][k-1] != -1) p[i][k] = p[p[i][k-1]][k-1];
    }
}
```

2.17.6 Lowest Common Ancestor

Runs in $O(\log N)$.

```
int lca(int x, int y) {
    // make both nodes the same depth
    if (dep[x] < dep[y]) swap(x, y);
    for (int k = MAXLOGN; k >= 0; k--) {
        if (p[x][k] != -1 && dep[p[x][k]] >= dep[y]) x = p[x][k];
    }
    if (x == y) return x;
    // perform binary lifting while parents are different
    for (int k = MAXLOGN; k >= 0; k--) {
        if (p[x][k] != p[y][k]) {
            x = p[x][k];
            y = p[y][k];
        }
    }
    // find the next parent
    return p[x][0];
}
```

2.17.7 Shortest Path

Runs in $O(\log N)$.

```
int distance(int x, int y) {
    return dist[x] + dist[y] - 2 * dist[lca(x, y)];
}
```

2.17.8 Heavy Light Decomposition

$O(N)$ precomputation, $O(\log N)$ query (excluding $O(\log N)$ from data structure).

```
// Requires segment tree, supports path maximum queries

vector<pi> adj[MAXN];
int par[MAXN], dep[MAXN], heavy[MAXN], head[MAXN], pos[MAXN], cpos = 0;
// heavy: heavy child, head: start of chain, pos: position in segment tree

int dfs(int x, int p) {
    int sz = 1, maxc = 0;
    par[x] = p;
    for (pi y : adj[x]) {
```

```
        if (y.s != p) {
            dep[y.s] = dep[x] + 1;
            int csz = dfs(y.s, x);
            sz += csz;
            if (csz > maxc) {
                maxc = csz;
                heavy[x] = y.s;
            }
        }
    }
    return sz;
}

void decomp(int x, int h) {
    head[x] = h; pos[x] = cpos++;
    if (heavy[x] != -1) decomp(heavy[x], h);
    for (pi y : adj[x]) {
        if (y.s == par[x]) continue;
        if (y.s != heavy[x]) decomp(y.s, y.s);
        root->update(pos[y.s], y.f);
    }
}

int query(int a, int b) {
    int res = 0;
    for (; head[a] != head[b]; b = par[head[b]]) {
        // maintain b as deeper
        if (dep[head[a]] > dep[head[b]]) swap(a, b);
        res = max(res, root->query(pos[head[b]], pos[b]));
    }
    // a and b are now on the same chain
    if (dep[a] > dep[b]) swap(a, b);
    res = max(res, root->query(pos[a]+1, pos[b]));
    return res;
}

memset(heavy, -1, sizeof(heavy));
dep[0] = 0;
dfs(0, -1);
decomp(0, 0);
```

2.17.9 Centroid Decomposition

$O(N \log N)$ precomputation, $O(\log N)$ updates and queries.

```
// Example: Xenia and Tree
// Update: Mark a node red
// Query: Find distance to closest red node to query node

int sz[MAXN], par[MAXN], lvl[MAXN];
ll dist[MAXN][MAXLOGN], best[MAXN];

// find subtree sizes
int dfsSize(int x, int p) {
    sz[x] = 1;
    for (int y : adj[x]) {
        if (lvl[y] != -1 || y == p) continue;
        sz[x] += dfsSize(y, x);
```

```
    }
    return sz[x];
}

// find centroid of each subtree
int dfsCentroid(int x, int p, int n) {
    for (int y : adj[x]) {
        if (lvl[y] != -1 || y == p) continue;
        if (sz[y] > n / 2) return dfsCentroid(y, x, n);
    }
    // current node is centroid
    return x;
}

void dfsDist(int x, int p, int level, ll d) {
    dist[x][level] = d;
    for (int y : adj[x]) {
        if (lvl[y] != -1 || y == p) continue;
        dfsDist(y, x, level, d + 1);
    }
}

void build(int x, int p, int level) {
    // find subtree sizes and centroid
    int size = dfsSize(x, -1);
    int cent = dfsCentroid(x, -1, size);
    if (p == -1) p = cent;
    par[cent] = p; // set parent of centroid to previous centroid
    lvl[cent] = level;
    dfsDist(cent, -1, level, 0);
    // recursively build subtrees
    for (int y : adj[cent]) {
        if (lvl[y] != -1) continue;
        build(y, cent, level + 1);
    }
}

void update(int x) {
    int y = x;
    int level = lvl[x];
    while (level != -1) {
        // shortest distance from y to any red node
        // update using new node x and its distance to the centroid y
        best[y] = min(best[y], dist[x][level]);
        y = par[y];
        level--;
    }
}

ll query(int x) {
    ll res = INF;
    int y = x;
    int level = lvl[x];
    while (level != -1) {
        // shortest distance to any red node in centroid chain
        res = min(res, best[y] + dist[x][level]);
        y = par[y];
        level--;
```

```
    }
    return res;
}

memset(lvl, -1, sizeof(lvl));
fill(best, best+n, INF);
build(0, -1, 0);
update(0); // adding a node to be considered
query(y); // querying with all considered nodes
```

3 Dynamic Programming

DP Algorithm	Complexity	Notes
Maxsum	O(N)	
LIS - Naive	O(N^2)	
LIS - DS / Fast DP	O($N \log N$)	
Coin Combinations	O(NV)	
Coin Change	O(NV)	
Knapsack - 0-1	O(NS)	
Knapsack - 0-1 No Values	O(NX/64)	
Knapsack - 0-K	O($\log K + NS$)	
Digit DP	O(D)	
Convex Hull Trick	O(1) insertion and query	Amortized
Li Chao Tree	O($\log X$) insertion and query	
Divide and Conquer	O($N \log N$)	From O(N^2)
LCS	O(N^2)	
LCS - LIS	O($N \log N$)	

3.1 Maxsum

3.1.1 1D

Kadane's Algorithm. Runs in $O(N)$.

```
int ans = nums[0], cur = nums[0];
for (int i = 1; i < nums.size(); i++) {
    if (cur < 0) cur = 0;
    cur += nums[i];
    ans = max(ans, cur);
}
```

3.2 Longest Increasing Subsequence

3.2.1 N^2 DP

```
int ans = 0, dp[n];
memset(dp, 0, sizeof(dp));
for (int i = 0; i < n; i++) {
    for (int j = 0; j < i; j++) {
        if (a[j] < a[i]) {
            dp[i] = max(dp[i], dp[j]);
        }
    }
    dp[i]++;
    ans = max(ans, dp[i]);
}
```

3.2.2 Data Structure Speedup

```
// Data structure should support point max updates and range max queries
// Discretise values first
for (int i = 0; i < n; i++) {
    t = query(a[i] - 1) + 1;
    update(a[i], t);
    ans = max(ans, t);
}
```

3.2.3 $N \log N$ DP

```
int len = 0, dp[n];
memset(dp, 0, sizeof(dp));
for (int x : a) {
    int pos = lower_bound(dp, dp + len, x) - dp;
    dp[pos] = x;
    if (pos == len) len++;
}
cout << len;
```

3.3 Coin Combinations

Runs in $O(N \cdot V)$.

```
int ways[v+1];
memset(ways, 0, sizeof(ways));
ways[0] = 1;
for (int i = 0; i < n; i++) {
    int c = coins[i];
    for (int sum = c; sum <= v; sum++) {
        ways[sum] = (ways[sum] + ways[sum - c]) % MOD;
    }
}
cout << ways[v];
```

3.4 Coin Change

Runs in $O(N \cdot V)$.

```
const int INF = 1e9;
vector<int> dp(v + 1, INF);
dp[0] = 0;
for (int i = 1; i <= v; i++) {
    for (int j = 0; j < n; j++) {
        if (i >= c[j] && dp[i - c[j]] != INF) {
            dp[i] = min(dp[i], dp[i - c[j]] + 1);
        }
    }
}
cout << dp[v];
```

3.5 Knapsack

3.5.1 0-1

Runs in $O(N \cdot S)$.

```
for (int i = 0; i < n; i++) {
    for (int j = s; j >= w[i]; j--) {
        dp[j] = max(dp[j], dp[j - w[i]] +v[i]);
    }
}
cout << dp[s];
```

3.5.2 0-1 No Values

Runs in $O(N \cdot X/64)$.

```
bitset<MAXX> dp;
dp[0] = 1;
for (int i = 0; i < n; i++) {
    dp |= dp << w[i];
}
cout << dp[x];
```

3.6 Digit DP

Runs in $O(D)$.

```
// Example - Numbers
// Compute the number of palindrome free numbers in a given range

vector<int> num;
ll dp[20][11][11][2][2]; // idx, last1, last2, tight, hasStarted

ll derp(int pos, int last1, int last2, bool tight, bool hasStarted) {
    if(pos == num.size()) return 1; // successfully populated whole number

    if(dp[pos][last1][last2][tight][hasStarted] != -1) {
        // state already visited
        return dp[pos][last1][last2][tight][hasStarted];
    }

    ll res = 0;
    int limit = tight ? num[pos] : 9; // do we need to keep to the range
    for(int d = 0; d <= limit; d++) { // try all next digits
        bool newHasStarted = hasStarted || (d != 0);
        bool newTight = tight && (d == limit);
        // skip palindromes only if the number has started
        if(newHasStarted) {
            if(d == last1) continue; // palindrome length 2
            if(d == last2) continue; // palindrome length 3
        }
        int newLast1 = newHasStarted ? d : 10;
        int newLast2 = hasStarted ? last1 : 10;
        res += derp(pos+1, newLast1, newLast2, newTight, newHasStarted);
    }
    return dp[pos][last1][last2][tight][hasStarted] = res;
}

// convert number to digits
void dcmp(ll x){
    num.clear();
    if(x == 0) num.push_back(0);
```

```
    while(x > 0) {
        num.push_back(x % 10);
        x /= 10;
    }
    reverse(num.begin(), num.end());
}


// Total Valid with value <= x
// Use PIE to get number within [a, b]
ll solve(ll x){
    dcmp(x);
    memset(dp, -1, sizeof(dp));
    return derp(0, 10, 10, true, false);
}

// To compute the kth string satisfying
// Either binary search or build character by character:
int count = 0;
vector<int> ans;
for (int i = 0; i < n; i++) {
    int x = 0;
    for (int j = 1; j < 10; j++) { // adjust to size of alphabet
        if (count + dp(i, j) > k) {
            break;
        }
        x = j;
    }
    count += dp(i, x); // position i is bounded by x
    ans.push_back(x);
}
```

3.7 Convex Hull Trick

Supports insertion and queries in amortised $O(1)$.

```
// Example: Commando
// Partition into contiguous groups with maximal effectiveness
// Group effectiveness is a quadratic function of their sum
// dp(x)  = max(dp(i)+f(p(x)-p(i)))
//        = max(dp(i)+a(p(x)-p(i))^2+b(p(x)-p(i))+c)
//        = max(dp(i)+ap(x)^2-2ap(x)p(i)+ap(i)^2+bp(x)-bp(i))+c
//        = max(dp(i)+ap(i)^2-bp(i)-2ap(x)p(i))+ap(x)^2+bp(x)+c
//        = max([dp(i)+ap(i)^2-bp(i)][-2ap(i)][p(x)])[+ap(x)^2+bp(x)+c]
//        = max(c(i)+m(i)p(x))+v(x)
// c(i) = dp(i)+ap(i)^2-bp(i)
// m(i) = -2ap(i)
// v(x) = ap(x)^2+bp(x)+c
// Lines have increasing gradients, queries have increasing x

deque<pi> hull;

ll func(pi line, ll x){
    return line.first*x+line.second;
}

ld intersection(ll m1, ll c1, ll m2, ll c2) {
    return (ld) (c2-c1) / (m1-m2);
```

```
}

ld intersect(pi x, pi y) {
    return intersection(x.first, x.second, y.first, y.second);
}

// query maximum y at x
ll query(ll x) {
    while (hull.size() > 1) {
        if (func(hull[0], x) < func(hull[1], x)) {
            hull.pop_front();
        } else break;
    }
    return func(hull[0], x);
}

// insert new line
void insert(ll m, ll c) {
    pi line = pi(m, c);
    while (hull.size() > 1) {
        ll s = hull.size();
        if (intersect(hull[s-1], line) <= intersect(hull[s-2], line)) {
            hull.pop_back();
        } else break;
    }
    hull.push_back(line);
}

insert(0, 0); // dp[0]
for(int i = 1; i <= n; i++){
    dp[i] = query(ps[i])+a*ps[i]*ps[i]+b*ps[i]+c; // max(mx + c) + v
    insert(-2*a*ps[i], dp[i]+a*ps[i]*ps[i]-b*ps[i]); // insert new (m, c)
}
```

3.8 Li Chao Tree

Supports insertion and queries in $O(\log X)$ time where X is the domain.

```
struct Node {
    ll m, c;
    Node *lc, *rc;
    Node(ll _m = 0, ll _c = -INF) {
        m = _m; c = _c;
        lc = nullptr; rc = nullptr;
    }
    ll value(ll x) {
        return m * x + c;
    }
    void insert(ll nm, ll nc, ll l, ll r) {
        ll mid = (l+r)/2;
        bool leftCheck = nm * l + nc > value(l);
        bool midCheck = nm * mid + nc > value(mid);
        if (midCheck) {
            swap(m, nm);
            swap(c, nc);
        }
        if (r-l == 1) return;
        if (leftCheck != midCheck) {
```

```
            if (!lc) lc = new Node();
            lc->insert(nm, nc, l, mid);
        } else {
            if (!rc) rc = new Node();
            rc->insert(nm, nc, mid, r);
        }
    }
    ll query(ll x, ll l, ll r) {
        ll res = value(x);
        if (r-l == 1) return res;
        ll mid = (l+r)/2;
        if (x < mid && lc) return max(res, lc->query(x, l, mid));
        if (x >= mid && rc) return max(res, rc->query(x, mid, r));
        return res;
    }
};

// Queries are on half open [L, R)
// Example: Commando
Node *root = new Node();
root->insert(0, 0, MINX, MAXX);
for (int i = 1; i <= n; i++) {
    // query lct
    ll x = p[i];
    ll best = root->query(x, MINX, MAXX);
    dp[i] = best + a*x*x + b*x + c;
    // update lct
    ll m = -2*a*x;
    ll c = dp[i] + a*x*x - b*x;
    root->insert(m, c, MINX, MAXX);
}
```

3.9 Divide and Conquer

Reduces complexity from $O(N^2 \cdot K)$ to $O(N \log N \cdot K)$.

```
// Example: Guards
// Minimise sum of costs where cost of partition is length * sum
// DP has form dp(i, j) = min dp(i-1, k-1) + C(k, j)
// i is the current layer, k to j is the new group
// Cost function satisfies quadrangle inequality:
// C(a, c) + C(b, d) <= C(a, d) + C(b, c) for a <= b <= c <= d

long long cost(int s, int e) {
    return (ps[e]-ps[s-1])*(e-s+1);
}

void dnc(int s, int e, long long x, int y, int k) {
    if(s > e) return;
    int m = (s+e)/2, best = 0;
    dp[m][k] = INF;
    for (int i = x; (i <= y && i <= m); i++) {
        ll val = dp[i][!k]+cost(i+1, m);
        if (dp[m][k] > val) {
            dp[m][k] = val;
            best = i;
        }
    }
```

```
    if (s < m) dnc(s, m-1, x, best, k);
    if (m < e) dnc(m+1, e, best, y, k);
}

// Uses DP on the fly to save space
for (int i = 1; i <= n; i++) dp[i][0] = INF;
for (int i = 1; i <= g; i++) {
    for (int j = 1; j <= n; j++) dp[j][i%2] = INF;
    dnc(0, n, 0, n, i%2);
}
```

4 Math

4.1 Fast Exponentiation

Runs in $O(\log b)$.

```
int powmod(int a, int b, int m) {
    int res = 1;
    while (b > 0) {
        if (b & 1) res = (res * a) % m;
        a = (a * a) % m;
        b >>= 1;
    }
    return res % m;
}
```

4.2 Prime Factorisation

Runs in $O(\sqrt{x})$.

```
map<int, int> cnt;
while (x % 2 == 0) {
    cnt[2]++;
    x /= 2;
}
for (int i = 3; i * i <= x; i++) {
    while (x % i == 0) {
        cnt[i]++;
        x /= i;
    }
}
if (x > 1) {
    cnt[x]++;
}
```

4.3 Sieve of Eratosthenes

Runs in $O(n \log \log n)$ with high constant.

```
bitset<MAXN> prime;
prime.set();
prime[0] = prime[1] = 0;
for (int i = 2; i < MAXN; i++) {
    if (prime[i]) {
        for (int j = i*i; j < MAXN; j += i) {
            prime[j] = 0;
```

```
        }
    }
}
```

4.4 Greatest Common Divisor

Runs in $O(\log \min(a, b))$.

```
int gcd(int a, int b) {
    if (a > b) swap(a, b);
    while (a != 0) {
        b %= a;
        swap(a, b);
    }
    return b;
}
```

4.5 Lowest Common Multiple

```
int lcm(int a, int b) {
    return a / gcd(a, b) * b;
}
```

4.6 Modular Inverse

For prime modulo.

```
ll modinv(ll a){
    return powmod(a, MOD-2, MOD);
}
```

4.7 $\binom{n}{k}$

Precomputation takes $O(MAXN)$ time, queries answered in $O(1)$.

```
ll fac[MAXN+1], modinv[MAXN+1];

ll nck(ll n, ll k) {
    if (n < k) return 0;
    ll res = fac[n];
    res = (res * modinv[k]) % MOD;
    res = (res * modinv[n-k]) % MOD;
    return res;
}

fac[0] = 1;
for(int i = 1; i <= MAXN; i++) {
    fac[i] = fac[i-1] * i % MOD;
}
modinv[MAXN] = powmod(fac[MAXN], MOD-2, MOD);
for(int i = MAXN; i > 0; i--) {
    modinv[i-1] = modinv[i] * i % MOD;
}
```

4.8 Fibonacci

Runs in $O(\log N)$ time.

```
struct Mat {
    ll a, b, c, d; // 2x2 Matrix: [a b; c d]
};

Mat mul(Mat x, Mat y) {
    return {
        x.a*y.a + x.b*y.c,
        x.a*y.b + x.b*y.d,
        x.c*y.a + x.d*y.c,
        x.c*y.b + x.d*y.d
    };
}

Mat mpow(Mat base, long long exp) {
    Mat res = {1, 0, 0, 1}; // Identity Matrix
    while (exp) {
        if (exp & 1) res = mul(res, base);
        base = mul(base, base);
        exp >>= 1;
    }
    return res;
}

ll fib(long long n) {
    if (n == 0) return 0;
    Mat m = {1, 1, 1, 0}; // Fibonacci Seed Matrix
    return mpow(m, n-1).a;
}
```

4.9 Convex Hull

Using Monotone Chain Algorithm, runs in $O(N \log N)$ time.

```
struct pt {
    double x, y;
};

int orientation(pt a, pt b, pt c) {
    double v = a.x*(b.y-c.y)+b.x*(c.y-a.y)+c.x*(a.y-b.y);
    if (v < 0) return -1; // clockwise
    if (v > 0) return +1; // counter-clockwise
    return 0;
}

bool cw(pt a, pt b, pt c, bool includeCollinear) {
    int o = orientation(a, b, c);
    return o < 0 || (includeCollinear && o == 0);
}

bool ccw(pt a, pt b, pt c, bool includeCollinear) {
    int o = orientation(a, b, c);
    return o > 0 || (includeCollinear && o == 0);
}

// Monotone Chain Algorithm
```

```
void convexHull(vector<pt>& a, bool includeCollinear = false) {
    if (a.size() == 1) return;
    sort(a.begin(), a.end(), [](pt a, pt b) {
        return make_pair(a.x, a.y) < make_pair(b.x, b.y);
    });
    pt p1 = a[0], p2 = a.back();
    vector<pt> up, down;
    up.push_back(p1);
    down.push_back(p1);
    for (int i = 1; i < (int)a.size(); i++) {
        if (i == a.size() - 1 || cw(p1, a[i], p2, includeCollinear)) {
            while (up.size() >= 2 &&
                    !cw(up[up.size()-2], up[up.size()-1],
                        a[i], includeCollinear)) {
                up.pop_back();
            }
            up.push_back(a[i]);
        }
        if (i == a.size() - 1 || ccw(p1, a[i], p2, includeCollinear)) {
            while (down.size() >= 2 &&
                    !ccw(down[down.size()-2], down[down.size()-1],
                         a[i], includeCollinear)) {
                down.pop_back();
            }
            down.push_back(a[i]);
        }
    }
    if (includeCollinear && up.size() == a.size()) {
        reverse(a.begin(), a.end());
        return;
    }
    a.clear();
    for (int i = 0; i < up.size(); i++) {
        a.push_back(up[i]);
    }
    for (int i = down.size() - 2; i > 0; i--) {
        a.push_back(down[i]);
    }
}
```

4.10 Furthest Pair of Points

Using Rotating Calipers, runs in $O(H)$ time.

```
struct pt {
    double x, y;
    int id; // original index
};

// Include Convex Hull code here

// Squared Distance
ll dist2(const pt &a, const pt &b) {
    ll dx = (ll)(a.x - b.x);
    ll dy = (ll)(a.y - b.y);
    return dx * dx + dy * dy;
}
```

```
// Rotating Calipers
pair<int, int> farthestPair(const vector<pt> &hull) {
    int n = hull.size();
    if (n == 1) return {hull[0].id, hull[0].id};
    if (n == 2) return {hull[0].id, hull[1].id};

    ll best = 0;
    pair<int, int> ans = {hull[0].id, hull[1].id};

    int j = 1;
    for (int i = 0; i < n; i++) {
        int ni = (i + 1) % n;
        while (true) {
            int nj = (j + 1) % n;
            ll cur = llabs(
                (ll)(hull[ni].x - hull[i].x) * (hull[j].y - hull[i].y)
              - (ll)(hull[ni].y - hull[i].y) * (hull[j].x - hull[i].x)
            );
            ll nxt = llabs(
                (ll)(hull[ni].x - hull[i].x) * (hull[nj].y - hull[i].y)
              - (ll)(hull[ni].y - hull[i].y) * (hull[nj].x - hull[i].x)
            );
            if (nxt > cur) j = nj;
            else break;
        }

        ll d1 = dist2(hull[i], hull[j]);
        if (d1 > best) {
            best = d1;
            ans = {hull[i].id, hull[j].id};
        }

        ll d2 = dist2(hull[ni], hull[j]);
        if (d2 > best) {
            best = d2;
            ans = {hull[ni].id, hull[j].id};
        }
    }
    return ans;
}

convexHull(pts, false);
pair<int, int> ans = farthestPair(pts);
```

4.11 Nearest Pair of Points

Using Shamos-Hoey, runs in $O(N \log N)$ time.

```
struct pt {
    long long x, y;
    int id;
};

struct cmp_x {
    bool operator()(const pt &a, const pt &b) const {
        if (a.x != b.x) return a.x < b.x;
        return a.y < b.y;
    }
```

```cpp
};

struct cmp_y {
    bool operator()(const pt &a, const pt &b) const {
        return a.y < b.y;
    }
};

vector<pt> a, t;
long long mindist;
pair<int, int> best_pair;

// Square Distance
inline long long dist2(const pt &a, const pt &b) {
    long long dx = a.x - b.x;
    long long dy = a.y - b.y;
    return dx * dx + dy * dy;
}

// Compare and update
inline void update(const pt &a, const pt &b) {
    long long d = dist2(a, b);
    if (d < mindist) {
        mindist = d;
        best_pair = {a.id, b.id};
    }
}

void rec(int l, int r) {
    /// Handle small case naively
    if (r - l <= 3) {
        for (int i = l; i < r; i++)
            for (int j = i + 1; j < r; j++)
                update(a[i], a[j]);
        sort(a.begin() + l, a.begin() + r, cmp_y());
        return;
    }

    int m = (l + r) >> 1;
    long long midx = a[m].x;
    rec(l, m);
    rec(m, r);

    merge(a.begin() + l, a.begin() + m,
          a.begin() + m, a.begin() + r,
          t.begin(), cmp_y());
    copy(t.begin(), t.begin() + (r - l), a.begin() + l);

    int tsz = 0;
    for (int i = l; i < r; i++) {
        long long dx = a[i].x - midx;
        if (dx * dx < mindist) {
            for (int j = tsz - 1; j >= 0; j--) {
                long long dy = a[i].y - t[j].y;
                if (dy * dy >= mindist) break;
                update(a[i], t[j]);
            }
            t[tsz++] = a[i];
```

```
        }
    }
}

// Remember to check for duplicates
sort(a.begin(), a.end(), cmp_x());
mindist = LLONG_MAX;
best_pair = {0, 1};
rec(0, N);
```

5 Algorithms

5.1 Binary Search

Find the cuberoot of n. Runs in $O(\log N)$.

```
long long n; cin >> n;
long long mini = 0, maxi = 1e6, medi;
while (mini < maxi) {
    medi = mini+(maxi-mini)/2;
    if (medi * medi * medi >= n) maxi = medi;
    else mini = medi+1;
}
cout << mini << "\n";
```

5.2 Binary Search using Lifting

Find the cuberoot of n. Runs in $O(\log N)$.

```
long long n; cin >> n;
long long cur = 0, gap = 1e6, next;
while (gap > 0) {
    while (next = cur + gap, next * next * next < n) {
        cur = next;
    }
    gap >>= 1;
}
cout << cur+1 << "\n";
```

5.3 Sliding Set

Speeds up DP from $O(N^2)$ to $O(N \log N)$.

```
// Example - Candymountain
// Jump across with minimax candies
// Populate with initial window
for(int i = 0; i < k; i++) {
    dp[i] = candies[i];
    s.insert(candies[i]);
}
// Sliding Set
for(int i = k; i < n; i++){
    dp[i] = max(candies[i], *s.begin());
    s.erase(s.find(dp[i-k]));
    s.insert(dp[i]);
}
```

5.4 Sliding Window

Runs in $O(N)$.

```
// Example: Count number of subarrays with sum at least k
// All numbers are non-negative
int s = 0, sum = 0, ans = 0;
for (int i = 0; i < n; i++) {
    sum += a[i];
    while (sum >= k) {
        ans += (n-i);
        sum -= a[s];
        s++;
    }
}
```

5.5 Set Merging

Reduces complexity from $O(Q \cdot N \log N)$ to $O(N \log^2 N)$.

```
for (int i = 0; i < q; i++) {
    cin >> a >> b;
    // small to large merging
    if (s[a].size() > s[b].size()) swap(s[a], s[b]);
    for (int x : s[a]) s[b].insert(x);
    s[a].clear();
    cout << s[b].size() << "\n";
}
```

5.6 Discretisation

Runs in $O(N \log N)$.

```
vector<int> a(n);
vector<int> b = a;
sort(b.begin(), b.end());
b.erase(unique(b.begin(), b.end()), b.end());
for (int &i : a) {
    i = lower_bound(b.begin(), b.end(), i) - b.begin() + 1; // 1-indexed
}
// a now holds discretised values
```

5.7 Meet in the Middle

Reduces time complexity from $O(2^N)$ to $O(N \cdot 2^{N/2})$.

```
// Example: Bobek
// Count number of subsets with sum <= tgt
left = n/2; right = n-left;
vector<int> sums;
for (int i = 0; i < (1 << left); i++) {
    int cur = 0;
    for (int j = 0; j < left; j++) {
        if (i & (1 << j)) cur += a[j];
    }
    sums.push_back(cur);
}
sort(sums.begin(), sums.end());
for (int i = 0; i < (1 << right); i++) {
```

```
    int cur = 0;
    for (int j = 0; j < right; j++) {
        if (i & (1 << j)) cur += a[left + j];
    }
    ans += upper_bound(sums.begin(), sums.end(), tgt-cur) - sums.begin();
}
```

5.8 On the Fly

Reduces memory usage from $O(N \cdot K)$ to $O(N)$.

```
// Stolen from DNC code
// Use i%2 and !(i%2) for indexing
for (int i = 1; i <= g; i++) {
    for (int j = 1; j <= n; j++) dp[j][i%2] = INF;
    dnc(0, n, 0, n, i%2);
}
```

5.9 Square Root Decomposition

$O(N)$ precomputation, $O(1)$ update, $O(\sqrt{n})$ query.

```
// Point Update, Range Sum Query

vector<ll> a(n);
int blockSize = sqrt(n) + 1;
vector<ll> blocks(blockSize, 0);
// precomputation
for (int i = 0; i < n; i++) {
    blocks[i / blockSize] += a[i];
}
// update
blocks[k/ blockSize] += u - a[k];
a[k] = u;
// query
ll sum = 0;
int s = l / blockSize;
int e = r / blockSize;
if (s == e) {
    for (int i = l; i <= r; i++) sum += a[i];
} else {
    for (int i = l; i < (s + 1) * blockSize; i++) sum += a[i];
    for (int b = s+1; b < e; b++) sum += blocks[b];
    for (int i = e * blockSize; i <= r; i++) sum += a[i];
}
```

5.10 Mo's Algorithm

Runs in $O((N+Q) \cdot \sqrt{N})$. Does not support online queries or updates.

```
struct Query {
    int l, r, idx;
};

vector<ll> a(n);
int blockSize = sqrt(n) + 1;
ll sum = 0;
```

```
bool cmp(const Query &x, const Query &y) {
    int bx = x.l / blockSize;
    int by = y.l / blockSize;
    if (bx != by) return bx < by;
    return (bx & 1) ? (x.r < y.r) : (x.r > y.r);
}

// modify to insert idx
void plus(int idx) {
    sum += a[idx];
}

// modify to remove idx
void minus(int idx) {
    sum -= a[idx];
}

// make sure indexes are 0-indexed
vector<Query> queries(q);
sort(queries.begin(), queries.end(), cmp);
vector<ll> ans(q);
int curL = 0, curR = -1;
for (Query &cq : queries) {
    while (curL > cq.l) plus(--curL);
    while (curR < cq.r) plus(++curR);
    while (curL < cq.l) minus(curL++);
    while (curR > cq.r) minus(curR--);
    ans[cq.idx] = sum;
}
```

5.11 Strings

5.12 Manacher's Algorithm

Runs in $O(N)$.

```
// for odd length palindromes, returns radius of palindrome at each index
vector<int> manacher(const string &s) {
    int n = s.size();
    vector<int> p(n);
    int l = 0, r = -1;
    for (int i = 0; i < n; i++) {
        int k = (i > r) ? 1 : min(p[l + r - i], r - i + 1);
        while (0 <= i - k && i + k < n && s[i - k] == s[i + k]) {
            k++;
        }
        p[i] = k;
        if (i + k - 1 > r) {
            l = i - k + 1;
            r = i + k - 1;
        }
    }
    return p;
}
// surround with #s so we only have to run odd case
string s, t;
for (char c : s) {
    t.push_back('#');
```

```
    t.push_back(c);
}
t.push_back('#');
vector<int> p = manacher(t);
// handle # removal accordingly
```

5.13 Knuth-Morris-Pratt

Runs in $O(N)$.

```
// Returns prefix function
// p[i] is longest prefix of s[0..i] which is also a suffix of it

vector<int> kmp(string s) {
    int n = s.size();
    vector<int> p(n);
    for (int i = 1; i < n; i++) {
        int j = p[i - 1];
        while (j > 0 && s[i] != s[j]) {
            j = p[j - 1];
        }
        if (s[i] == s[j]) j++;
        p[i] = j;
    }
    return p;
}

// Example: Count how many times a pattern occurs in a text
string concat = pattern + '#' + text;
vector<int> p = kmp(concat);
int ans = 0;
for (int i = 0; i < pattern.size() + 1; i < p.size(); i++) {
    if (p[i] == pattern.size()) ans++;
}
```

5.14 Z-Function

Runs in $O(N)$.

```
// Z-Function: Longest strings that is a prefix of S and S starting from i
vector<int> zfunction(string s) {
    int n = s.size();
    vector<int> z(n);
    int l = 0, r = 0;
    for (int i = 1; i < n; i++) {
        if (i < r) {
            z[i] = min(r - i, z[i - 1]);
        }
        while (i + z[i] < n && s[z[i]] == s[i + z[i]]) {
            z[i]++;
        }
        if (i + z[i] > r) {
            l = i;
            r = i + z[i];
        }
    }
    z[0] = n;
    return z;
```

```
}
```

5.15 Suffix Array

Runs in $O(N \log N)$.

```
// Get starting indexes of all suffixes of a given string after sorting

// Sort cyclic shifts of s
vector<int> sort_cyclic_shifts(const string &s) {
    int n = s.size();
    const int alphabet = 256;
    vector<int> p(n), c(n), cnt(max(alphabet, n), 0);

    // k = 0, sort single characters
    for (int i = 0; i < n; i++) cnt[s[i]]++;
    for (int i = 1; i < alphabet; i++) cnt[i] += cnt[i-1];
    for (int i = 0; i < n; i++) p[--cnt[s[i]]] = i;
    c[p[0]] = 0;
    int classes = 1;
    for (int i = 1; i < n; i++) {
        if (s[p[i]] != s[p[i-1]]) classes++;
        c[p[i]] = classes - 1;
    }

    // strings of length (1 << k)
    vector<int> pn(n), cn(n);
    for (int h = 0; (1 << h) < n; h++) {
        for (int i = 0; i < n; i++) {
            pn[i] = p[i] - (1 << h);
            if (pn[i] < 0) pn[i] += n;
        }
        fill(cnt.begin(), cnt.begin() + classes, 0);
        for (int i = 0; i < n; i++) cnt[c[pn[i]]]++;
        for (int i = 1; i < classes; i++) cnt[i] += cnt[i-1];
        for (int i = n-1; i >= 0; i--) p[--cnt[c[pn[i]]]] = pn[i];
        cn[p[0]] = 0;
        classes = 1;
        for (int i = 1; i < n; i++) {
            pair<int,int> cur = {c[p[i]], c[(p[i] + (1 << h)) % n]};
            pair<int,int> prev = {c[p[i-1]], c[(p[i-1] + (1 << h)) % n]};
            if (cur != prev) classes++;
            cn[p[i]] = classes - 1;
        }
        c.swap(cn);
    }
    return p;
}

vector<int> build_suffix_array(string s) {
    s += "$"; // terminal character
    vector<int> sa = sort_cyclic_shifts(s);
    sa.erase(sa.begin()); // remove index of terminal character
    return sa;
}
```

6 Miscellaneous

6.1 Maximum Time complexity v/s N

Complexity	Maximum in 1s
O(1)	Infinite
O(log N)	2^{10^6}
O(sqrt N)	10^{14}
O(N)	10^7
O(N log N)	10^6
O(N sqrt N)	10^5
O(N^2)	10^4
O(N^3)	500
O(N^4)	100
O(2^N)	22
O(N x 2^N)	20
O(N!)	12
O(N x N!)	11

6.2 Fast I/O

Cannot use with `scanf`, `printf`.

```
ios_base::sync_with_stdio(false);
cin.tie(0);
```

6.3 Superfast I/O

Only for non-negative integer input.

```
inline ll ri () {
    ll x = 0;
    char ch = getchar_unlocked();
    while (ch < '0' || ch > '9') ch = getchar_unlocked();
    while (ch >= '0' && ch <= '9') {
        x = (x << 3) + (x << 1) + ch - '0';
        ch = getchar_unlocked();
    }
    return x;
}
```

6.4 Header

Macros, functions, and variables.

```
#pragma GCC optimize("O3")
#pragma GCC optimize("unroll-loops")

#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>

using namespace __gnu_pbds;
using namespace std;

#define F first
#define S second
```

```
#define pf push_front
#define pb push_back
#define pof pop_front
#define pob pop_back
#define ins insert
#define lb lower_bound
#define ub upper_bound
#define sz(a) ((int)(a).size())
#define all(a) begin(a), end(a)
#define FOR(i, a, b) for(int i = a; i <= b; i++)
#define ROF(i, a, b) for(int i = a; i >= b; i--)
#define FOR(i, a) FOR(i, 0, a)
#define ROF(i, a) ROF(i, a, 0)
#define ITER(i, a) for(auto i : a)
#define FAST ios_base::sync_with_stdio(false); cin.tie(0); cout.tie(0);
#define MOD 1000000007
#define MOD2 998244353
#define INF INT_MAX/2
#define EPS 1e-9

typedef long long ll;
typedef pair<ll, ll> pi;
typedef pair<ll, pi> pii;
typedef tree<pi, null_type, less<pi>,
             rb_tree_tag, tree_order_statistics_node_update> ordered_set;

int ls(int x){ return (x)&(-x); }
mt19937 rng(chrono::steady_clock::now().time_since_epoch().count());
inline ll rngr(ll x, ll y) { return (rng()%(y-x+1))+x; }

inline ll ri() {
    ll x = 0;
    char ch = getchar_unlocked();
    while (ch < '0' || ch > '9') ch = getchar_unlocked();
    while (ch >= '0' && ch <= '9'){
        x = (x << 3) + (x << 1) + ch - '0';
        ch = getchar_unlocked();
    }
    return x;
}

struct custom_hash {
    static uint64_t splitmix64(uint64_t x) {
        // http://xorshift.di.unimi.it/splitmix64.c
        x += 0x9e3779b97f4a7c15;
        x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
        x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
        return x ^ (x >> 31);
    }
    size_t operator()(uint64_t x) const {
        static const uint64_t FIXED_RANDOM =
            chrono::steady_clock::now().time_since_epoch().count();
        return splitmix64(x + FIXED_RANDOM);
    }
    size_t operator()(pair<uint64_t,uint64_t> x) const {
        static const uint64_t FIXED_RANDOM =
            chrono::steady_clock::now().time_since_epoch().count();
        return splitmix64(x.first + FIXED_RANDOM)
```

```
                ^(splitmix64(x.second + FIXED_RANDOM) >> 1);
    }
};

typedef unordered_map<int, int, custom_hash> safe_map;
typedef gp_hash_table<int, int, custom_hash> safe_hash_table;

// 4 Directions
int dx[]={0, 0, -1, 1};
int dy[]={-1, 1, 0, 0};
// 8 Directions
int dx[]={0, 0, -1, 1, -1, 1, -1, 1};
int dy[]={-1, 1, -1, 1, 0, 0, 1, -1};
// Knight Moves
int dx[]={-1, -2, 1, 2, 2, 1, -2, 1};
int dy[]={-2, -1, -2, -1, 1, 2, 1, 2};
```

6.5 Compile Commands

```
g++ -c "%f" -std=c++20 // Editor Compile
g++ -o "%e" "%f" -std=c++20 // Editor Build
g++ "file.cpp" -o "file" -std=c++20 // Simple for command line
```

6.6 Pruning

Kill execution after a specific time duration.

```
#define LIMIT 2.9
auto start = chrono::high_resolution_clock::now();
auto end = chrono::high_resolution_clock::now();
auto elapse = chrono::duration<double>(end-start);
if(elapse.count() > LIMIT) break;
```

6.7 STL Data Structures and Functions

```
//File I/O
freopen("test.in", "r", stdin);
freopen("test.out", "w", stdout);

// Variable / Array Functions
min(a, b);
max(a, b);
__gcd(a, b);
swap(a, b);

lower_bound(a, a+n, b); // Value >= b
upper_bound(a, a+n, b); // Value > b

fiil(a, a+n, 0);
memset(a, 0, sizeof(a));
copy(a, a+n, b);

sort(a, a+n);
stable_sort(a, a+n); // If a = b, indexes will remain same
reverse(a, a+n);
random_shuffle(a, a+n, rng);
```

```cpp
max_element(a, a+n);

// Math Functions (Low Accuracy)
pow(1, 2);
sqrt(1);
cbrt(1);
floor(1);
ceil(1);
abs(1);
log(1);
log10(1);

// Limits
INT_MAX
LLONG_MAX
LDBL_MAX

// Others
to_string(0);
stoll("0");

vector<int> v;
v.push_back(0);
v.front();
v.back();
v.pop_back();

queue<int> q;
q.push(0);
q.front();
q.pop();

priority_queue<int> pq;
pq.push(0)
pq.top();
pq.pop();

deque<int> dq;
dq.push_front(0);
dq.front();
dq.push_back(1);
dq.back();
dq.pop_front();
dq.pop_back();

set<int> s;
unordered_set<int> us;
s.insert(0);
s.find(0); // Pointer to 0's position
s.erase(0);

multiset<int> ms;
ms.insert(0);
ms.insert(0);
ms.count(0);
ms.find(0); // Pointer to a 0's position
ms.erase(ms.find(0)); // Erase a 0
ms.erase(0); // Erase all 0s
```

```cpp
map<int, int> m;
unordered_map<int, int> um;
m[0] = 1;
m[1] = 2;

stack<int> st;
st.push(0);
st.top(); //0
st.pop();

// GNU PBDS
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>

typedef tree<int, null_type, less<int>,
             rb_tree_tag, tree_order_statistics_node_update> ordered_set;

typedef tree<pi, null_type, less<pi>, //Use a pair to simulate multisets
             rb_tree_tag, tree_order_statistics_node_update> ordered_set;

ordered_set os;
os.insert(1);
os.insert(2);
os.insert(4);
os.insert(8);
os.insert(16);
//Find by order - Kth largest (starting from 0)
os.find_by_order(1); // 2
os.find_by_order(2); // 4
os.find_by_order(4); // 16
//Order of key - How many elements are < K
os.order_of_key(-5);  // 0
os.order_of_key(1);   // 0
os.order_of_key(3);   // 2
os.order_of_key(4);   // 2
os.order_of_key(400); // 5
```