

Competitive Programming Reference

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Contents

1	Data Structures	3
1.1	Prefix Sums	3
1.1.1	1D	3
1.1.2	2D	3
1.2	Fenwick Trees	3
1.2.1	Point Update Range Query	3
1.2.2	Range Update Point Query	4
1.2.3	Range Update Range Query	4
1.2.4	2D PURQ / RUPQ	4
1.2.5	2D RURQ	5
1.3	Segment Trees	6
1.3.1	Standard	6
1.3.2	Lazy Propagation	6
1.3.3	Maxsum	7
1.3.4	Merge Sort Tree / Order Statistics	7
1.3.5	2D	8
2	Graph Theory	9
2.1	Depth First Search	9
2.2	Breadth First Search	9
2.3	0-1 BFS	10
2.4	Floyd-Warshall	10
2.5	Bellman-Ford	10
2.6	Dijkstra's Algorithm	11
2.7	Shortest Path Faster Algorithm	11
2.8	Prim's Algorithm	11
2.9	Union Find Disjoint Subset	12
2.10	Kruskal's Algorithm	12
2.11	Topological Sort	12
2.12	Floyd's Cycle Finding Algorithm	13
2.13	Maximum Cardinality Bipartite Matching	13
2.13.1	Kun's Algorithm	13
2.13.2	Hopcroft-Karp	14
2.14	Trees	15
2.14.1	Pre/Postorder Traversal	15
2.14.2	Subtree to Range	15
2.14.3	Weighted Maximum Independent Set	16
2.14.4	Diameter	16
2.14.5	2^K Decomposition	16
2.14.6	Lowest Common Ancestor	17
2.14.7	Shortest Path	17

3	Dynamic Programming	17
3.1	Maxsum	17
3.1.1	1D	17
3.2	Longest Increasing Subsequence	18
3.2.1	N^2 DP	18
3.2.2	Data Structure Speedup	18
3.2.3	$N \log N$ DP	18
3.3	Coin Combinations	18
3.4	Coin Change	19
3.5	Knapsack	19
3.5.1	0-1	19
3.6	Digit DP	19
4	Math	20
4.1	Fast Exponentiation	20
4.2	Prime Factorisation	21
4.3	Sieve of Eratosthenes	21
4.4	Greatest Common Divisor	21
4.5	Lowest Common Multiple	21
4.6	Modular Inverse	21
4.7	$\binom{n}{k}$	22
4.8	Fibonacci	22
5	Algorithms	23
5.1	Binary Search	23
5.2	Binary Search using Lifting	23
5.3	Sliding Set	23
5.4	Set Merging	23
5.5	Discretisation	24
6	Miscellaneous	24
6.1	Fast I/O	24
6.2	Superfast I/O	24

1 Data Structures

Data Structure	Precomputation / Update	Query	Memory	Notes
Prefix Sum	$O(N) / X$	$O(1)$	$O(N)$	Associative Functions (+, XOR)
Sparse Table	$O(N \log N) / X$	$O(1)$	$O(N \log N)$	Non-Associative Functions (max, gcd)
Fenwick Tree	$X / O(\log N)$	$O(\log N)$	$O(N)$	Prefix Sum with Updates
Segment Tree	$X / O(\log N)$	$O(\log N)$	$O(4N)$	Allows more Information

Table 1: Quick Summary of Data Structures

1.1 Prefix Sums

1.1.1 1D

$O(N)$ precomputation, $O(1)$ query.

```
//Query - 1-Indexed
int query(int s, int e){
    return ps[e]-ps[s-1];
}

//Precomputation
ps[0] = 0;
for(int i = 1; i <= n; i++) ps[i] = ps[i-1]+a[i];
```

1.1.2 2D

$O(R \cdot C)$ precomputation, $O(1)$ query.

```
//Query - 1-Indexed
int query(int x1, int y1, int x2, int y2){
    return ps[x2][y2]-ps[x1-1][y2]-ps[x2][y1-1]+ps[x1-1][y1-1];
}

//Precomputation
for (int i = 0; i <= r; i++) ps[i][0] = 0;
for (int j = 0; j <= c; j++) ps[0][j] = 0;
for (int i = 1; i <= r; i++) {
    for (int j = 1; j <= c; j++) {
        ps[i][j] = ps[i-1][j]+ps[i][j-1]-ps[i-1][j-1]+a[i][j];
    }
}
```

1.2 Fenwick Trees

1.2.1 Point Update Range Query

$O(\log N)$ update and query.

```
inline int ls(int x){ return (x)&(-x); }

int fw[MAXN]; // 1-Indexed

void pu(int i, int v) {
    for(; i <= n; i += ls(i)) fw[i] += v;
}

int pq(int i) {
    int t = 0;
```

```

        for(; i; i -= ls(i)) t += fw[i];
        return t;
    }

    int rq(int s, int e) {
        return pq(e) - pq(s - 1);
    }

```

1.2.2 Range Update Point Query

$O(\log N)$ update and query.

// Requires PURQ Code (PU, PQ)

```

void ru(int s, int e, int v) {
    pu(s, v);
    pu(e+1, -v);
}

```

1.2.3 Range Update Range Query

$O(\log N)$ update and query.

// Requires PURQ Code (PU, PQ)

// Functions need to be modified to take in array parameter

// e.g. int pu(*tree, int i, int v)

```

void ru(int s, int e, int v) {
    pu(fw1, s, v);
    pu(fw1, e+1, -v);
    pu(fw2, s, -v*(s-1));
    pu(fw2, e+1, v*e);
}

int ps(int i) {
    return pq(fw1, i)*i + pq(fw2, i);
}

int rq(int s, int e) {
    return ps(e) - ps(s - 1);
}

```

1.2.4 2D PURQ / RUPQ

$O(\log N \cdot \log M)$ update and query.

inline int ls(int x) { return x & -x; }

int fw[MAXN][MAXN]; // 1-indexed

```

void pu(int x, int y, int v) {
    for (int i = x; i < MAXN; i += ls(i)) {
        for (int j = y; j < MAXN; j += ls(j)) {
            fw[i][j] += v;
        }
    }
}

```

```

int pq(int x, int y) {
    int ans = 0;
    for (int i = x; i > 0; i -= ls(i)) {
        for (int j = y; j > 0; j -= ls(j)) {
            ans += fw[i][j];
        }
    }
    return ans;
}

// Range Query
int rq(int x1, int y1, int x2, int y2) {
    return pq(x2, y2) - pq(x1 - 1, y2) - pq(x2, y1 - 1) + pq(x1 - 1, y1 - 1);
}

// Range Update, Point Query
void ru(int x1, int y1, int x2, int y2, int v) {
    pu(x1, y1, v);
    pu(x1, y2 + 1, -v);
    pu(x2 + 1, y1, -v);
    pu(x2 + 1, y2 + 1, v);
}

```

1.2.5 2D RURQ

$O(\log N \cdot \log M)$ update and query.

// Requires 2D PURQ Code

```

void ru(int x1, int y1, int x2, int y2, long long v) {
    pu(fw1, x1, y1, v);
    pu(fw1, x1, y2+1, -v);
    pu(fw1, x2+1, y1, -v);
    pu(fw1, x2+1, y2+1, v);

    pu(fw2, x1, y1, v*(x1-1));
    pu(fw2, x1, y2+1, -v*(x1-1));
    pu(fw2, x2+1, y1, -v*x2);
    pu(fw2, x2+1, y2+1, v*x2);

    pu(fw3, x1, y1, v*(y1-1));
    pu(fw3, x1, y2+1, -v*y2);
    pu(fw3, x2+1, y1, -v*(y1-1));
    pu(fw3, x2+1, y2+1, v*y2);

    pu(fw4, x1, y1, v*(x1-1)*(y1-1));
    pu(fw4, x1, y2+1, -v*(x1-1)*y2);
    pu(fw4, x2+1, y1, -v*x2*(y1-1));
    pu(fw4, x2+1, y2+1, v*x2*y2);
}

long long ps(int x, int y) {
    return pq(fw1, x, y)*x*y - pq(fw2, x, y)*y - pq(fw3, x, y)*x + pq(fw4, x, y);
}

long long rq(int x1, int y1, int x2, int y2) {
    return ps(x2, y2) - ps(x1-1, y2) - ps(x2, y1-1) + ps(x1-1, y1-1);
}

```

1.3 Segment Trees

1.3.1 Standard

$O(\log N)$ point update and range query.

```
struct node {
    int s, e, m, v;
    node *l, *r;
    node(int _s, int _e) {
        s = _s; e = _e; m = (s+e)/2; v = 0;
        if (s != e) {
            l = new node(s, m);
            r = new node(m+1, e);
        }
    }
    void pu(int x, int y) {
        if (s == e) { v = y; return; }
        if (x <= m) l->pu(x, y);
        if (x > m) r->pu(x, y);
        v = min(l->v, r->v);
    }
    int rq(int x, int y) {
        if (s == x && e == y) return v;
        if (y <= m) return l->rq(x, y);
        if (x > m) return r->rq(x, y);
        return min(l->rq(x, m), r->rq(m+1, y));
    }
}
} *root;

root = new node(0, n-1);
```

1.3.2 Lazy Propagation

$O(\log N)$ range update and range query.

```
struct node {
    int s, e, m, v, lazy;
    node *l, *r;
    node(int _s, int _e) {
        s = _s; e = _e; m = (s+e)/2; v = lazy = 0;
        if (s != e) {
            l = new node(s, m);
            r = new node(m+1, e);
        }
    }
    int pu() {
        if (s == e) { v += lazy; lazy = 0; return v; }
        v += lazy;
        l->lazy += lazy; r->lazy += lazy;
        lazy = 0;
        return v;
    }
    void ru(int x, int y, int z) {
        if (s == x && e == y) { lazy += z; return; }
        if (y <= m) l->ru(x, y, z);
        else if (x > m) r->ru(x, y, z);
        else l->ru(x, m, z), r->ru(m+1, y, z);
    }
}
```

```

        v = max(l->pu(), r->pu());
    }

    int rq(int x, int y) {
        pu();
        if (s == x && e == y) return pu();
        if (y <= m) return l->rq(x, y);
        if (x > m) return r->rq(x, y);
        return max(l->rq(x, m), r->rq(m+1, y));
    }
} *root;

root = new node(0, n-1);

```

1.3.3 Maxsum

$O(\log N)$ point update and range query.

```

struct node {
    ll s, e, m, ps, ss, ms, ts;
    node *l, *r;
    node(ll _s, ll _e) {
        s = _s; e = _e; m = (s+e)/2; ps = ss = ms = ts = 0;
        if (s != e) {
            l = new node(s, m);
            r = new node(m+1, e);
        }
    }
    void pu(ll x, ll y) {
        if (s == e) { ps = ss = ms = ts = y; return; }
        if (x <= m) l->pu(x, y);
        if (x > m) r->pu(x, y);
        //New Prefix Max -> Left Prefix, Left + Right Prefix
        ps = max(l->ps, l->ts+r->ps);
        //New Suffix Max -> Right Suffix, Left Suffix + Right
        ss = max(r->ss, r->ts+l->ss);
        //Total Sum -> Left + Right
        ts = l->ts+r->ts;
        //Maxsum - Left Suffix + Right Prefix, Left, Right,
        //          Total, Left Maxsum, Right Maxsum
        ms = max({l->ss+r->ps, ps, ss, ts, l->ms, r->ms});
    }
    ll ans() {
        return ms;
    }
} *root;

root = new node(0, n-1);

```

1.3.4 Merge Sort Tree / Order Statistics

Insertion: $O(\log N)$, Building: $O(N \log^2 N)$, Counting: $O(\log^2 N)$, Finding: $O(\log V \cdot \log^2 N)$, Range Max: $O(\log N)$.

```

struct node {
    int s, e, m;
    vector<int> v;
    node *l, *r;

```

```

node(int _s, int _e) {
    s = _s; e = _e; m = (s+e)/2;
    if (s != e) {
        l = new node(s, m);
        r = new node(m+1, e);
    }
}

void insert(int x, int y) {
    if (s == e) { v.push_back(y); return; }
    if (x > m) r->insert(x, y);
    if (x <= m) l->insert(x, y);
    v.push_back(y);
}

void build(){
    if (s == e) return;
    l->build();
    r->build();
    sort(v.begin(), v.end());
}

int countLessEqual(int x, int y, int k) {
    if (x > y) return 0;
    if (s == x && e == y) {
        return upper_bound(v.begin(), v.end(), k)-v.begin();
    }
    if (x > m) return r->countLessEqual(x, y, k);
    if (y <= m) return l->countLessEqual(x, y, k);
    return l->countLessEqual(x, m, k)+r->countLessEqual(m+1, y, k);
}

int kthSmallest(int x, int y, int k) {
    int mini = 0, maxi = (1 << 30);
    int ans = mini, gap = maxi;
    while (gap > 0) {
        while (ans + gap <= maxi && countLessEqual(x, y, ans + gap) < k) {
            ans += gap;
        }
        gap >>= 1;
    }
    return ans + 1;
}

int rangeMax(int x, int y) {
    if (x > y) return 0;
    if (s == x && e == y) return v.back();
    if (x > m) return r->rangeMax(x, y);
    if (y <= m) return l->rangeMax(x, y);
    return max(l->rangeMax(x, m), r->rangeMax(m+1, y));
}

} *root;

root = new node(0, n-1);

```

1.3.5 2D

$O(N \cdot \log N \cdot \log M)$ point update and range query.

```

struct node2D {
    int s, e, m;
    node1D *maxi;
    node2D *l, *r;
}

```



```

node2D(int a, int b, int c, int d) {
    s = a; e = b; m = (s+e)/2;
    maxi = new node1D(c, d);
    if (s != e) {
        l = new node2D(s, m, c, d);
        r = new node2D(m+1, e, c, d);
    }
}

void pu(int a, int b, int v) {
    if (s == e) { maxi->pu(b, v); return; }
    if (a <= m) l->pu(a, b, v);
    else r->pu(a, b, v);
    maxi->pu(b, max(l->maxi->rq(b, b), r->maxi->rq(b, b)));
}

int rq(int a, int b, int c, int d) {
    if (s == a && e == b) return maxi->rq(c, d);
    if (b <= m) return l->rq(a, b, c, d);
    if (a > m) return r->rq(a, b, c, d);
    return max(l->rq(a, m, c, d), r->rq(m+1, b, c, d));
}
} *root;

root = new node(0, n-1, 0, n-1);

```

2 Graph Theory

2.1 Depth First Search

Runs in $O(V + E)$.

```

// For adjacency lists
void dfs(int x, int p) {
    for (int y : adj[x]) {
        if (y != p) {
            dist[y] = dist[x]+1;
            dfs(y, x);
        }
    }
}

```

2.2 Breadth First Search

Runs in $O(V + E)$.

```

// For adjacency lists
visited[s] = 1;
dist[s] = 0;
q.push(s);
while (!q.empty()) {
    int f = q.front(); q.pop();
    for (int i : adjlist[f]) {
        if (!visited[i]) {
            q.push(i);
            visited[i] = 1;
            dist[i] = dist[f] + 1;
        }
    }
}
}

```

2.3 0-1 BFS

Runs in $O(V + E)$.

```
// For adjacency lists
deque<int> dq;
dist[s] = 0;
dq.push(s);
while (!dq.empty()) {
    int u = dq.front(); dq.pop();
    for (int e : adjlist[u]) {
        int v = e.first, w = e.second;
        if (dist[v] > dist[u] + w) {
            dist[v] = dist[u] + w;
            if (w == 0) dq.push_front(v);
            else dq.push_back(v);
        }
    }
}
```

2.4 Floyd-Warshall

Runs in $O(N^3)$.

```
// Initialise adjacency matrix
for (int i = 0; i < n; i++) {
    for (int j = 0; j < n; j++) {
        if (i == j) adj[i][j] = 0;
        else adj[i][j] = INF;
    }
}

// Floyd-Warshall
for (int k = 0; k < n; k++) {
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < n; j++) {
            adj[i][j] = min(adj[i][j], adj[i][k] + adj[k][j]);
            if (adj[i][i] < 0) negCycle = true;
        }
    }
}
```

2.5 Bellman-Ford

Runs in $O(VE)$.

```
vector<int> dist(n, INF);
dist[s] = 0;
bool negCycle = false;
for (int i = 1; i <= n; i++) {
    bool update = false;
    for (Edge e : edges) {
        if (dist[e.u] < INF && dist[e.v] > dist[e.u] + e.w) {
            dist[e.v] = dist[e.u] + e.w;
            update = true;
        }
    }
    if (!update) break;
    if (update && i == n) negCycle = true;
}
```

2.6 Dijkstra's Algorithm

Runs in $O(E \log V)$.

```
priority_queue<pi, vector<pi>, greater<pi>> pq;
vector<int> dist(n, INF);
dist[s] = 0;
pq.push({0, s});
while (!pq.empty()) {
    pi f = pq.top(); pq.pop();
    int d = f.first, u = f.second;
    if (d != dist[u]) continue;
    for (pi x : adj[u]) {
        int v = x.first, w = x.second;
        if (dist[v] > d + w) {
            dist[v] = d + w;
            pq.push({ dist[v], v });
        }
    }
}
```

2.7 Shortest Path Faster Algorithm

Runs in $O(VE)$.

```
vector<int> dist(n, INF);
vector<int> inQueue(n, 0);
queue<int> q;
dist[s] = 0;
q.push(s);
inQueue[s]++;
bool negCycle = false;
while (!q.empty()) {
    int u = q.front(); q.pop();
    inQueue[u]--;
    for (Edge e : adj[u]) {
        if (dist[e.v] > dist[u] + e.w) {
            dist[e.v] = dist[u] + e.w;
            if (inQueue[e.v] == 0) {
                q.push(e.v);
                inQueue[e.v]++;
                if (inQueue[e.v] > n) {
                    negCycle = true;
                    break;
                }
            }
        }
    }
}
if (negCycle) break;
}
```

2.8 Prim's Algorithm

Runs in $O(E \log V)$.

```
priority_queue<pi, vector<pi>, greater<pi>> pq;
vector<int> dist(n, INF);
vector<bool> vis(n, false);
dist[s] = 0;
```

```

pq.push({0, s});
while (!pq.empty()) {
    pi f = pq.top(); pq.pop();
    int d = f.first, u = f.second;
    if (vis[u]) continue;
    vis[u] = true;
    for (pi x : adj[u]) {
        int v = x.first, w = x.second;
        if (!vis[v] && dist[v] > w) {
            dist[v] = w;
            pq.push({ dist[v], v });
        }
    }
}

```

2.9 Union Find Disjoint Subset

With both path compression and union by rank, runs in $O(\alpha(n))$ (basically constant time).

```

int p[MAXN];
int sz[MAXN];

int root(int x) {
    if (p[x] == -1) return x;
    return p[x] = root(p[x]);
}

void connect(int x, int y) {
    x = root(x); y = root(y);
    if (x == y) return;
    if (sz[x] < sz[y]) swap(x, y);
    p[y] = x;
    sz[x] += sz[y];
}

fill(p, p+MAXN, -1);
fill(sz, sz+MAXN, 1);

```

2.10 Kruskal's Algorithm

Runs in $O(E \log E)$.

```

sort(edges.begin(), edges.end());
for (Edge e : edges) {
    if (root(e.u) != root(e.v)) {
        connect(e.u, e.v);
        cost += e.w;
    }
}

```

2.11 Topological Sort

Runs in $O(V + E)$.

```

void dfs(int x) {
    if (v[x]) return;
    v[x] = 1;
    for (int y : adj[x]) dfs(y);
}

```

```

        topo.push_back(x);
    }
    for (int i = 0; i < n; i++) dfs(i);
    reverse(topo.begin(), topo.end());

```

2.12 Floyd's Cycle Finding Algorithm

For graphs with outdegree 1, runs in $O(V + E)$.

```

// detect cycle
int slow = s, fast = s;
do {
    slow = nxt[slow];
    fast = nxt[nxt[fast]];
} while (slow != fast);
// find start of cycle
slow = start;
while (slow != fast) {
    slow = nxt[slow];
    fast = nxt[fast];
}
// collect all nodes in cycle
vector<int> cycle;
int cur = slow;
do {
    cycle.push_back(cur);
    cur = nxt[cur];
} while (cur != slow);

```

2.13 Maximum Cardinality Bipartite Matching

2.13.1 Kun's Algorithm

Runs in $O(V^3)$.

```

int n, mcbms = 0, miss = 0;
vector<int> match;
vector<bool> vis;
vector<vector<int>> adj;
vector<int> left;

// dfs for augmenting path
bool dfs(int u) {
    for (int v : adj[u]) {
        if (!vis[v]) {
            vis[v] = true;
            if (match[v] == -1 || dfs(match[v])) {
                match[v] = u;
                match[u] = v;
                return true;
            }
        }
    }
    return false;
}

void mcbm() {
    match.assign(n, -1);

```

```

mcbms = 0;
// greedy initial matching
for (int u : left) {
    for (int v : adj[u]) {
        if (match[v] == -1) {
            match[v] = u;
            match[u] = v;
            mcbms++;
            break;
        }
    }
}
// dfs augmenting paths for unmatched
for (int u : left) {
    if (match[u] == -1) {
        vis.assign(n, false);
        if (dfs(u)) mcbms++;
    }
}
miss = n - mcbms;
}

```

2.13.2 Hopcroft-Karp

Runs in $O(E \cdot \sqrt{V})$.

```

int n, m;
vector<vector<int>> adj;
vector<int> pairU, pairV, dist;

bool bfs() {
    queue<int> q;
    for (int u = 0; u < n; u++) {
        if (pairU[u] == -1) {
            dist[u] = 0;
            q.push(u);
        } else {
            dist[u] = INF;
        }
    }
    bool found = false;
    while (!q.empty()) {
        int u = q.front(); q.pop();
        for (int v : adj[u]) {
            if (pairV[v] == -1) {
                found = true;
            } else if (dist[pairV[v]] == INF) {
                dist[pairV[v]] = dist[u] + 1;
                q.push(pairV[v]);
            }
        }
    }
    return found;
}

bool dfs(int u) {
    for (int v : adj[u]) {
        if (pairV[v] == -1 || (dist[pairV[v]] == dist[u] + 1 && dfs(pairV[v]))) {

```

```

        pairU[u] = v;
        pairV[v] = u;
        return true;
    }
}
dist[u] = INF;
return false;
}

int hopcroftKarp() {
    pairU.assign(n, -1);
    pairV.assign(m, -1);
    dist.assign(n, 0);
    int mcbm = 0;
    while (bfs()) {
        for (int u = 0; u < n; u++) {
            if (pairU[u] == -1 && dfs(u)) {
                mcbm++;
            }
        }
    }
    return mcbm;
}

```

2.14 Trees

2.14.1 Pre/Postorder Traversal

Runs in $O(V)$.

```

int prec = 0, postc = 0;
void dfs(int x, int p) {
    pre[x] = prec++;
    for(int y : adj[x]) {
        if (y != p) dfs(y, x);
    }
    post[x] = postc++;
}

```

2.14.2 Subtree to Range

Runs in $O(V)$.

```

int dfs(int x, int p) {
    pre[x] = c++;
    rig[pre[x]] = pre[x];
    for (int y : adj[x]) {
        if (y != p) {
            rig[pre[x]] = max(rig[pre[x]], dfs(y, x));
        }
    }
    return rig[pre[x]];
}
// Subtree -> pre[x], rig[pre[x]]
// Node Index -> pre[x]
// Range of Children -> pre[x]+1, rig[pre[x]]

```

2.14.3 Weighted Maximum Independent Set

Runs in $O(V)$.

```
int dp[MAXN][2];

int mis(int v, bool take, int p) {
    if (dp[v][take] != -1) return dp[v][take];
    int ans = take * c[v];
    for (int u : adj[v]) {
        if (u == p) continue;
        int temp = mis(u, 0, v);
        if (!take) temp = max(temp, mis(u, 1, v));
        ans += temp;
    }
    return dp[v][take] = ans;
}

void ans(int v, bool take, int p) {
    for (int u : adj[v]) {
        if (u == p) continue;
        int temp0 = dp[u][0], temp1 = (take ? -1 : dp[u][1]);
        if (temp0 > temp1) ans(u, 0, v);
        else { a.push_back(u); ans(u, 1, v); }
    }
}

memset(dp, -1, sizeof(dp));
mis(0, 0, -1); // don't take root
mis(0, 1, -1); // take root
if(dp[0][1] > dp[0][0]) { a.push_back(0); ans(0, 1, -1); }
else ans(0, 0, -1);
```

2.14.4 Diameter

Runs in $O(V)$.

```
pi dfs(int x, int p, int d) {
    pi b = {x, d};
    for (pi y : adj[x]) {
        if (y.first != p) {
            pi c = dfs(y.first, x, d + y.second);
            if (c.second > b.second) b = c;
        }
    }
    return b;
}

pi s = dfs(0, -1, 0);
pi e = dfs(s.first, -1, 0);
// e.second gives diameter
// For even diameter, centroid is at e.second / 2
// For odd diameter, centroid is at e.second / 2 and e.second / 2 + 1
```

2.14.5 2^K Decomposition

$O(N \log N)$ precomputation and memory, $O(\log N)$ query.

```
int par(int x, int k) {
    for(int i = MAXLOGN; i >= 0; i--) {
        if (k >= (1 << i)) {
```



```

        if(x == -1) return x;
        x = p[x][i];
        k -= (1 << i);
    }
}
return x;
}

int p[MAXN][MAXLOGN];
memset(p, -1, sizeof(p));
dfs(0); // compute initial parent p[i][0]
for (int k = 1; k <= MAXLOGN; k++) {
    for (int i = 0; i < n; i++) {
        if(p[i][k-1] != -1) p[i][k] = p[p[i][k-1]][k-1];
    }
}
}

```

2.14.6 Lowest Common Ancestor

Runs in $O(\log N)$.

```

int lca(int x, int y) {
    // make both nodes the same depth
    if (dep[x] < dep[y]) swap(x, y);
    for (int k = MAXLOGN; k >= 0; k--) {
        if (p[x][k] != -1 && dep[p[x][k]] >= dep[y]) x = p[x][k];
    }
    if (x == y) return x;
    // perform binary lifting while parents are different
    for (int k = MAXLOGN; k >= 0; k--) {
        if (p[x][k] != p[y][k]) {
            x = p[x][k];
            y = p[y][k];
        }
    }
    // find the next parent
    return p[x][0];
}

```

2.14.7 Shortest Path

Runs in $O(\log N)$.

```

int distance(int x, int y) {
    return dist[x] + dist[y] - 2 * dist[lca(x, y)];
}

```

3 Dynamic Programming

3.1 Maxsum

3.1.1 1D

Kadane's Algorithm. Runs in $O(N)$.

```

int ans = nums[0], cur = nums[0];
for (int i = 1; i < nums.size(); i++) {
    if (cur < 0) cur = 0;

```

```

    cur += nums[i];
    ans = max(ans, cur);
}

```

3.2 Longest Increasing Subsequence

3.2.1 N^2 DP

```

int ans = 0, dp[n];
memset(dp, 0, sizeof(dp));
for (int i = 0; i < n; i++) {
    for (int j = 0; j < i; j++) {
        if (a[j] < a[i]) {
            dp[i] = max(dp[i], dp[j]);
        }
    }
    dp[i]++;
    ans = max(ans, dp[i]);
}

```

3.2.2 Data Structure Speedup

```

// Data structure should support point max updates and range max queries
// Discretise values first
for (int i = 0; i < n; i++) {
    t = query(a[i] - 1) + 1;
    update(a[i], t);
    ans = max(ans, t);
}

```

3.2.3 $N \log N$ DP

```

int len = 0, dp[n];
memset(dp, 0, sizeof(dp));
for (int x : a) {
    int pos = lower_bound(dp, dp + len, x) - dp;
    dp[pos] = x;
    if (pos == len) len++;
}
cout << len;

```

3.3 Coin Combinations

Runs in $O(N \cdot V)$.

```

int ways[v+1];
memset(ways, 0, sizeof(ways));
ways[0] = 1;
for (int i = 0; i < n; i++) {
    int c = coins[i];
    for (int sum = c; sum <= v; sum++) {
        ways[sum] = (ways[sum] + ways[sum - c]) % MOD;
    }
}
cout << ways[v];

```

3.4 Coin Change

Runs in $O(N \cdot V)$.

```
const int INF = 1e9;
vector<int> dp(v + 1, INF);
dp[0] = 0;
for (int i = 1; i <= v; i++) {
    for (int j = 0; j < n; j++) {
        if (i >= c[j] && dp[i - c[j]] != INF) {
            dp[i] = min(dp[i], dp[i - c[j]] + 1);
        }
    }
}
cout << dp[v];
```

3.5 Knapsack

3.5.1 0-1

Runs in $O(N \cdot S)$.

```
for (int i = 0; i < n; i++) {
    for (int j = s; j >= w[i]; j--) {
        dp[j] = max(dp[j], dp[j - w[i]] + v[i]);
    }
}
cout << dp[s];
```

3.6 Digit DP

Runs in $O(D)$.

```
// Example - Numbers
// Compute the number of palindrome free numbers in a given range

vector<int> num;
ll dp[20][11][11][2][2]; // idx, last1, last2, tight, hasStarted

ll derp(int pos, int last1, int last2, bool tight, bool hasStarted) {
    if(pos == num.size()) return 1; // successfully populated whole number

    if(dp[pos][last1][last2][tight][hasStarted] != -1) {
        // state already visited
        return dp[pos][last1][last2][tight][hasStarted];
    }

    ll res = 0;
    int limit = tight ? num[pos] : 9; // do we need to keep to the range
    for(int d = 0; d <= limit; d++) { // try all next digits
        bool newHasStarted = hasStarted || (d != 0);
        bool newTight = tight && (d == limit);
        // skip palindromes only if the number has started
        if(newHasStarted) {
            if(d == last1) continue; // palindrome length 2
            if(d == last2) continue; // palindrome length 3
        }
        int newLast1 = newHasStarted ? d : 10;
        int newLast2 = hasStarted ? last1 : 10;
```

```

        res += derp(pos+1, newLast1, newLast2, newTight, newHasStarted);
    }
    return dp[pos][last1][last2][tight][hasStarted] = res;
}

// convert number to digits
void dcmp(ll x){
    num.clear();
    if(x == 0) num.push_back(0);
    while(x > 0) {
        num.push_back(x % 10);
        x /= 10;
    }
    reverse(num.begin(), num.end());
}

// Total Valid with value <= x
// Use PIE to get number within [a, b]
ll solve(ll x){
    dcmp(x);
    memset(dp, -1, sizeof(dp));
    return derp(0, 10, 10, true, false);
}

// To compute the kth string satisfying
// Either binary search or build character by character:
int count = 0;
vector<int> ans;
for (int i = 0; i < n; i++) {
    int x = 0;
    for (int j = 1; j < 10; j++) { // adjust to size of alphabet
        if (count + dp(i, j) > k) {
            break;
        }
        x = j;
    }
    count += dp(i, x); // position i is bounded by x
    ans.push_back(x);
}

```

4 Math

4.1 Fast Exponentiation

Runs in $O(\log b)$.

```

int powmod(int a, int b, int m) {
    int res = 1;
    while (b > 0) {
        if (b & 1) res = (res * a) % m;
        a = (a * a) % m;
        b >>= 1;
    }
    return res % m;
}

```

4.2 Prime Factorisation

Runs in $O(\sqrt{x})$.

```
map<int, int> cnt;
while (x % 2 == 0) {
    cnt[2]++;
    x /= 2;
}
for (int i = 3; i * i <= x; i++) {
    while (x % i == 0) {
        cnt[i]++;
        x /= i;
    }
}
if (x > 1) {
    cnt[x]++;
}
```

4.3 Sieve of Eratosthenes

Runs in $O(n \log \log n)$ with high constant.

```
bitset<MAXN> prime;
prime.set();
prime[0] = prime[1] = 0;
for (int i = 2; i < MAXN; i++) {
    if (prime[i]) {
        for (int j = i*i; j < MAXN; j += i) {
            prime[j] = 0;
        }
    }
}
```

4.4 Greatest Common Divisor

Runs in $O(\log \min(a, b))$.

```
int gcd(int a, int b) {
    if (a > b) swap(a, b);
    while (a != 0) {
        b %= a;
        swap(a, b);
    }
    return b;
}
```

4.5 Lowest Common Multiple

```
int lcm(int a, int b) {
    return a / gcd(a, b) * b;
}
```

4.6 Modular Inverse

```

11 modinv(11 a){
    return powmod(a, MOD-2, MOD);
}

```

4.7 $\binom{n}{k}$

Precomputation takes $O(MAXN)$ time, queries answered in $O(1)$.

```

11 fac[MAXN+1], modinv[MAXN+1];

11 nck(11 n, 11 k) {
    if (n < k) return 0;
    11 res = fac[n];
    res = (res * modinv[k]) % MOD;
    res = (res * modinv[n-k]) % MOD;
    return res;
}

fac[0] = 1;
for(int i = 1; i <= MAXN; i++) {
    fac[i] = fac[i-1] * i % MOD;
}
modinv[MAXN] = powmod(fac[MAXN], MOD-2, MOD);
for(int i = MAXN; i > 0; i--) {
    modinv[i-1] = modinv[i] * i % MOD;
}

```

4.8 Fibonacci

Runs in $O(\log N)$ time.

```

struct Mat {
    11 a, b, c, d; // 2x2 Matrix: [a b; c d]
};

Mat mul(Mat x, Mat y) {
    return {
        x.a*y.a + x.b*y.c,
        x.a*y.b + x.b*y.d,
        x.c*y.a + x.d*y.c,
        x.c*y.b + x.d*y.d
    };
}

Mat mpow(Mat base, long long exp) {
    Mat res = {1, 0, 0, 1}; // Identity Matrix
    while (exp) {
        if (exp & 1) res = mul(res, base);
        base = mul(base, base);
        exp >>= 1;
    }
    return res;
}

11 fib(long long n) {
    if (n == 0) return 0;
    Mat m = {1, 1, 1, 0}; // Fibonacci Seed Matrix
}

```

```
        return mpow(m, n-1).a;
    }
```

5 Algorithms

5.1 Binary Search

Find the cuberoot of n . Runs in $O(\log N)$.

```
long long n; cin >> n;
long long mini = 0, maxi = 1e6, medi;
while (mini < maxi) {
    medi = mini+(maxi-mini)/2;
    if (medi * medi * medi >= n) maxi = medi;
    else mini = medi+1;
}
cout << mini << "\n";
```

5.2 Binary Search using Lifting

Find the cuberoot of n . Runs in $O(\log N)$.

```
long long n; cin >> n;
long long cur = 0, gap = 1e6, next;
while (gap > 0) {
    while (next = cur + gap, next * next * next < n) {
        cur = next;
    }
    gap >>= 1;
}
cout << cur+1 << "\n";
```

5.3 Sliding Set

Speeds up DP from $O(N^2)$ to $O(N \log N)$.

```
// Example - Candymountain
// Jump across with minimax candies
// Populate with initial window
for(int i = 0; i < k; i++) {
    dp[i] = candies[i];
    s.insert(candies[i]);
}
// Sliding Set
for(int i = k; i < n; i++){
    dp[i] = max(candies[i], *s.begin());
    s.erase(s.find(dp[i-k]));
    s.insert(dp[i]);
}
```

5.4 Set Merging

Reduces complexity from $O(Q \cdot N \log N)$ to $O(N \log^2 N)$.

```
for (int i = 0; i < q; i++) {
    cin >> a >> b;
    // small to large merging
```

```

    if (s[a].size() > s[b].size()) swap(s[a], s[b]);
    for (int x : s[a]) s[b].insert(x);
    s[a].clear();
    cout << s[b].size() << "\n";
}

```

5.5 Discretisation

Runs in $O(N \log N)$.

```

vector<int> a(n);
vector<int> b = a;
sort(b.begin(), b.end());
b.erase(unique(b.begin(), b.end()), b.end());
for (int &i : a) {
    i = lower_bound(b.begin(), b.end(), i) - b.begin() + 1; // 1-indexed
}
// a now holds discretised values

```

6 Miscellaneous

6.1 Fast I/O

Cannot use with `scanf`, `printf`.

```

ios_base::sync_with_stdio(false);
cin.tie(0);

```

6.2 Superfast I/O

Only for non-negative integer input.

```

inline ll ri () {
    ll x = 0;
    char ch = getchar_unlocked();
    while (ch < '0' || ch > '9') ch = getchar_unlocked();
    while (ch >= '0' && ch <= '9') {
        x = (x << 3) + (x << 1) + ch - '0';
        ch = getchar_unlocked();
    }
    return x;
}

```
