

Simulation of Exponential Dist

Ng Ming Chun

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Simulation of Exponential Distribution

In this simulation I am going to investigate the exponential distribution in R and compare it with the Central Limit Theorem.

A distribution of the mean of 40 exponentials with lambda of 0.2 will be used in this simulation.

The 3 things I am going to show: 1. Show the sample mean and compare it to the theoretical mean of the distribution. 2. Show how variable the sample is (via variance) and compare it to the theoretical variance of the distribution. 3. Show that the distribution is approximately normal.

1. Show the sample mean and compare it to the theoretical mean of the distribution.

```
n<-40 # no of exponentials
lambda<-0.2 #rate
nosim<-1000 # no. of simulation

set.seed (123) #set seed to make the simulation reproducible

sim_data_matrix <- matrix(data=rexp(n*nosim, lambda) , nrow=nosim) # this will create nosim simulation
```

The sample mean is

```
simulation_means <- apply(sim_data_matrix, 1, mean) # compute the mean of each simulation
sample_mean <- sum(simulation_means)/nosim # compute the overall sample mean for the nosim simulations
print (sample_mean)
```

```
## [1] 5.011911
```

The theoretical mean of the exponential distribution is

```
theoretical_mean <- 1/lambda
print (theoretical_mean)
```

```
## [1] 5
```

Conclusion: The sample mean is close to the theoretical mean.

2. Show how variable the sample is (via variance) and compare it to the theoretical variance of the distribution.

We use the formula: $\text{variance} = \text{expected value } (x^2) - (\text{expected value } (x))^2$

The sample variance is

```
simulation_means_sq <- sapply(simulation_means, function (x) x^2) # compute the square of each simulate
simulation_means_sq_mean <- sum(simulation_means_sq)/(nosim)
sample_variance <- simulation_means_sq_mean - (sample_mean)^2

print (sample_variance)
```

```
## [1] 0.6082204
```

The sample standard deviation is

```
print (sqrt(sample_variance))
```

```
## [1] 0.7798849
```

The theoretical variance of the exponential distribution is

```
theoretical_variance <- (1/lambda)^2/n

print (theoretical_variance)
```

```
## [1] 0.625
```

The theoretical standard deviation is

```
print (sqrt(theoretical_var))
```

```
## [1] 0.7905694
```

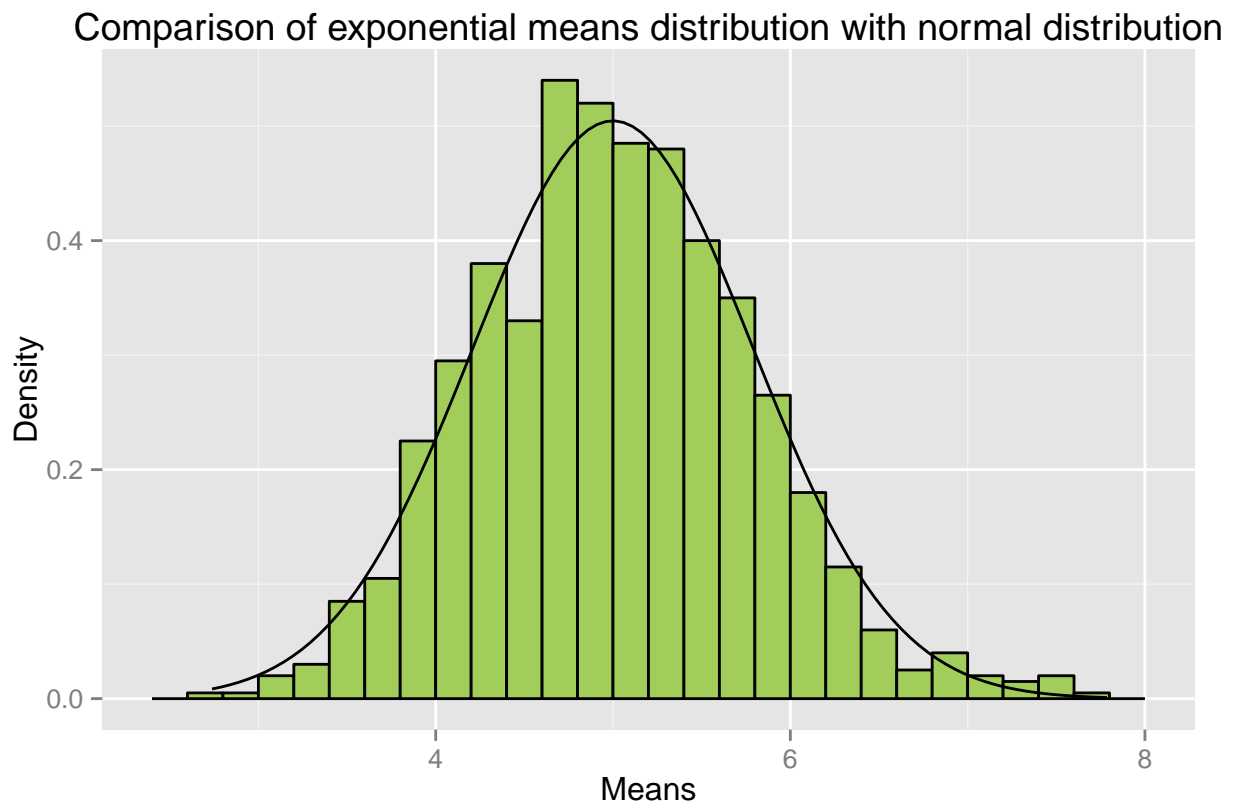
Conclusion: The sample variance is close to the theoretical variance.

3. Show that the distribution is approximately normal.

I plot a histogram of the simulation means and overlay it with a normal distribution.

```
library(ggplot2)

df<-data.frame(x=simulation_means)
ggplot(data = df, aes(x = x)) + geom_histogram(aes(y = ..density..), fill = I("darkolivegreen3"), binwidth = 0.05)
```



Conclusion: The plot shows the sample distribution approximate a normal distribution.