CS6041 HW2, Noah Gardner, 000843905
Your assignment must be typed in Word or Latex for exact ONE question per page, and turn in PDF files. Non-typed submissions will NOT be graded.

Problem 1. *Design a DFA that accepts the set of decimal strings that are multiples of 5.*

The intution for this DFA is that a decimal string that is a multiple of 5 will end in 0 or 5. Therefore, the DFA only needs the two states q_0 and q_1, where q_0 is the starting and final state of the DFA.

State	$0, 5$	$1, 2, 3, 4, 6, 7, 8, 9$
$* \rightarrow q_0$	q_0	q_1
q_1	q_0	q_1

Figure 1: Transition Table for a DFA that accepts the set of decimal strings that are multiples of 5.

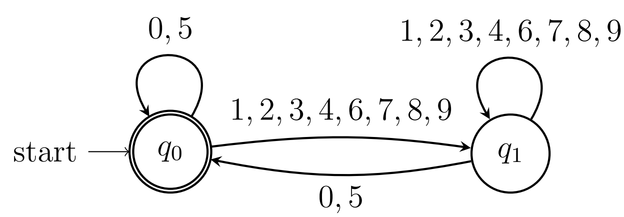

Figure 2: Diagram showing the transition function described in Figure 1.

Let $M = (Q, \sum, \delta, q_0, F)$, where

$$Q = \{q_0,\ q_1\},$$

$$\textstyle\sum = \{0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9\},$$

$$\delta = \{\text{Figure 1}\},$$

$$q_0 = \{q_0\}, \text{ and}$$

$$F = \{q_0\}.$$

Problem 2. *Convert the following NFA to DFA, and describe the language it accepts.*

	0	1
$\to p$	$\{p, q\}$	$\{p\}$
q	$\{r, s\}$	$\{t\}$
r	$\{p, r\}$	$\{t\}$
s	$\emptyset$	$\emptyset$
t	$\emptyset$	$\emptyset$

After subset construction, the states $\{q\}$, $\{r\}$, $\{s\}$, and $\{t\}$ would be unreachable (there is no transition from p). Therefore, the next reachable state would be $\{p, q\}$. The state $\{p, q\}$ can also reach states $*\{p, t\}$ and $*\{p, q, r, s\}$. Thus, the reachable states are:

1. $\{p\}$
2. $\{p, q\}$
3. $*\{p, t\}$
4. $*\{p, q, r, s\}$

	0	1
$\emptyset$	$\emptyset$	$\emptyset$
$\to \{p\}$	$\{p, q\}$	$\{p\}$
$\{p, q\}$	$\{p, q, r, s\}$	$\{p, t\}$
$\{p, t\}$	$\{p, q\}$	$\{p\}$
$\{p, q, r, s\}$	$\{p, q, r, s\}$	$\{p, t\}$

Figure 3: Equivalent DFA transition table for the NFA in the problem description.

This DFA accepts the set of binary strings that either have 00 or 01 adjacent (0 then 0 *or* 0 then 1) in any position.

Problem 3. *Convert the following DFA to a regular expression by finding all $R_{ij}^{(k)}$ in Theorem 3.4. Simplify all $R_{ij}^{(k)}$'s.*

	0	1
$\rightarrow q_1$	q_2	q_1
q_2	q_3	q_1
$*q_3$	q_3	q_2

Find and simplify all $R_{ij}^{(k)}$'s.

R_{ij}^0	Simplified
R_{11}^0	$\epsilon + 1$
R_{12}^0	0
R_{13}^0	$\emptyset$
R_{21}^0	1
R_{22}^0	ϵ
R_{23}^0	0
R_{31}^0	$\emptyset$
R_{32}^0	1
R_{33}^0	$\epsilon + 0$

R_{ij}^1	By direct substituion	Simplified
R_{11}^1	$\epsilon + 1 + (\epsilon + 1)(\epsilon + 1)^*(\epsilon + 1)$	1^*
R_{12}^1	$0 + (\epsilon + 1)(\epsilon + 1)^*(0)$	1^*0
R_{13}^1	$\emptyset + (\epsilon + 1)(\epsilon + 1)^*(\emptyset)$	$\emptyset$
R_{21}^1	$1 + (1)(\epsilon + 1)^*(\epsilon + 1)$	1^*
R_{22}^1	$\epsilon + (1)(\epsilon + 1)^*(0)$	1^*0
R_{23}^1	$0 + (1)(\epsilon + 1)^*(\emptyset)$	0
R_{31}^1	$\emptyset + (\emptyset)(\epsilon + 1)^*(\epsilon + 1)$	$\emptyset$
R_{32}^1	$1 + (\emptyset)(\epsilon + 1)^*(0)$	1
R_{33}^1	$\epsilon + 0 + (\emptyset)(\epsilon + 1)^*(\emptyset)$	$(\epsilon + 0)$

R_{ij}^2	By direct substituion	Simplified
R_{11}^2	$1^* + (1^*0)(1^*0)^*(1^*)$	$1^* + (1^*0)^*1^*$
R_{12}^2	$1^*0 + (1^*0)(1^*0)^*(1^*0)$	$1^*0 + (1^*0)^*1^*0$
R_{13}^2	$\emptyset + (1^*0)(1^*0)^*(0)$	$1^*0(1^*0)^*0$
R_{21}^2	$1^* + (1^*0)(1^*0)^*(1^*)$	$1^* + (1^*0)^*1^*$
R_{22}^2	$1^*0 + (1^*0)(1^*0)^*(1^*0)$	1^*0
R_{23}^2	$0 + (1^*0)(1^*0)^*(0)$	$0 + (1^*0)^*0$
R_{31}^2	$\emptyset + (1)(1^*0)^*(1^*)$	$1(1^*0)^*1^*$
R_{32}^2	$1 + (1)(1^*0)^*(1^*0)$	$1 + 1(1^*0)^*1^*0$
R_{33}^2	$\epsilon + 0 + (1)(1^*0)^*(0)$	$0 + 1(1^*0)^*0$

q_1 is the only starting state and q_3 is the only accepting state. Therefore, the DFA can be described by $(R_{12}^2)(R_{23}^2)$: $(1^*0 + (1^*0)^*1^*0)(0 + (1^*0)^*0)$

Problem 4. *Convert the following DFA to a regular expression, using the state-elimination technique.*

	0	1
$\rightarrow *p$	s	p
q	p	s
r	r	q
s	q	r

Step 1. Simplify loop $p \rightarrow s \rightarrow q \rightarrow p$

$$0(01)^*00 \tag{1}$$

Step 2. Simplify loop $p \rightarrow s \rightarrow r \rightarrow p$

$$01(0)^*10 \tag{2}$$

Step 3. Node p now loops to itself as the only starting and accepting nodes with the original loop ($p \rightarrow p$) or'd with equations 1 and 2.

$$(1 + 0(01)^*00 + 01(0)^*10)^* \tag{3}$$

Problem 5. *Convert the following regular expressions to NFA's with ϵ-transitions.*

- 01^*
- $(0+1)01$
- $00(0+1)^*$

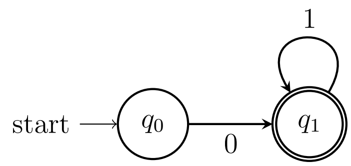

Figure 4: NFA for the regular expression 01^*.

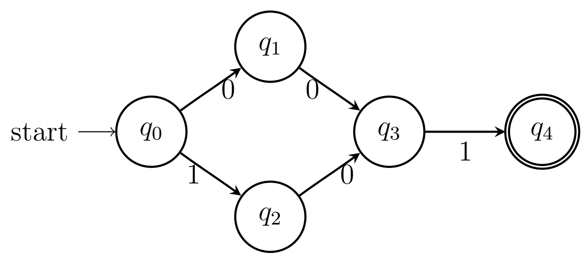

Figure 5: NFA for the regular expression $(0+1)01$.

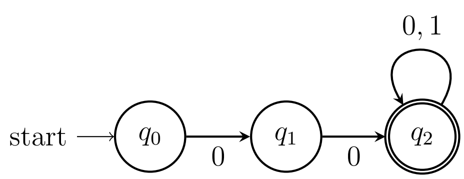

Figure 6: NFA for the regular expression $00(0+1)^*$.