CS6041 HW1, Noah Gardner, 000843905

Your assignment must be typed in Word or Latex for exact ONE question per page, and turn in PDF files. Non-typed submissions will NOT be graded.

Problem 1. *Find the expression that is the contrapositive of* $A \vee \neg B \to C \vee \neg D$.

Apply negation of both sides.

$$\neg(A \vee \neg B) \to \neg(C \vee \neg D) \tag{1}$$

$$\neg A \wedge B \to \neg C \wedge D \tag{2}$$

Reversal of hypothesis and conclusion.

$$\neg C \wedge D \to \neg A \wedge B \tag{3}$$

(3) is the contrapositive of the expression $A \vee \neg B \to C \vee \neg D$.

Problem 2. *Prove the statement* $S(n) = \sum_{i=2}^{n} i = (n+2)(n-1)/2$ *by induction.*

Basis. Show that the statement is true for $n = 2$.

$$S(2) = (2+2)(2-1)/2 = 2 \tag{4}$$

Hypothesis. Basis $n = 2$ is true. Assume the statement is true when n is some value larger than 2, such as $n = k$. That is,

$$S(k) = \sum_{i=2}^{k} i = (k+2)(k-1)/2 \tag{5}$$

Induction. Prove that the statement holds when $n = k+1$, given $k >= 2$.

$$S(k+1) = \sum_{i=2}^{k+1} i = k(k+3)/2 \tag{6}$$

Rule of summation.

$$\sum_{i=2}^{k+1} i = \sum_{i=2}^{k} i + (k+1) \tag{7}$$

Apply hypothesis (5) to (7).

$$\sum_{i=2}^{k} i + (k+1) = (k+2)(k-1)/2 + (k+1) \tag{8}$$

Factor out multiplication.

$$(k+2)(k-1)/2 \to (k^2 + 1k - 2)/2 \tag{9}$$

Apply common denominator of 2.

$$(k+1) \to (2k+2)/2 \tag{10}$$

Add the two components from (9) and (10).

$$(k^2 + 1k - 2)/2 + (2k+2)/2 = (k^2 + 3k)/2 \tag{11}$$

Factor out k.

$$(k^2 + 3k)/2 = k(k+3)/2 \tag{12}$$

(12) is equivalent to (6), so the statement $S(n) = \sum_{i=2}^{n} i = (n+2)(n-1)/2$ is proven by induction.

Problem 3. *Prove the statement $n^n/3^n < n!$ for $n \geq 6$ by induction.*
Basis. Show that the statement is true for $n = 6$.

$$S(6) = 6^6/3^6 < 6! = 2^6 < 720 = 64 < 720 \tag{13}$$

Hypothesis. Basis $n = 6$ is true. Assume the statement is true when n is some value larger than 6, such as $n = k$. That is,

$$S(k) = k^k/3^k < k! \tag{14}$$

If (14) is true, then it also follows that:

$$k^k < 3^k * k! \tag{15}$$

Induction. Prove that the statement holds when $n = k + 1$, given $k \geq 6$.

$$S(k+1) = (k+1)^{(k+1)}/3^{(k+1)} < (k+1)! \tag{16}$$

Multiply both sides by $3^{(k+1)}$, factor out 3.

$$(k+1)^{(k+1)} < 3 * 3^k * (k+1)! \tag{17}$$

Divide both sides by $k + 1$.

$$(k+1)^k < 3 * 3^k * k! \tag{18}$$

Apply hypothesis (15).

$$(k+1)^k < 3 * k^k \tag{19}$$

Raise both sides to power to $1/k$

$$(k+1) < 3k \tag{20}$$

(20) holds true, so the statement $S(n) = n^n/3^n < n!$ for $n \geq 6$ is proven by induction.