Q1 Prove or disprove the following is not a regular language. If regular, write down its regular expression. Otherwise, prove it.

Q1.1 $\{O^n | n$ is a perfect number$\}$. A perfect number is a number whose square root is an integer.

– **Answer:**

Q1.2 $\{a^n b^n | 1 \leq n \leq 2021\}$

Answer: n is a finite number and a finite automata can be used to model the language, with a total of $2 * n$ states. Therefore, the language is regular and its regular expression is $(ab)1, 2021$, where the the strings accepted are the strings with ab repeated between 1 and 2021 times.

Q1.3 $\{ww | w \exists \Sigma^*, \Sigma = \{0, 1\}\}$

Q1.4 $\{0^m 1^n | m \leq n\}$

Answer: Memory is required to store the current number m so that it can be compared to the number n after accepting m 0s. Therefore, the language is not regular since memory is required.

Q2 Prove the following statements.

Q2.1 $\bar{L}$ is regular if L is regular.

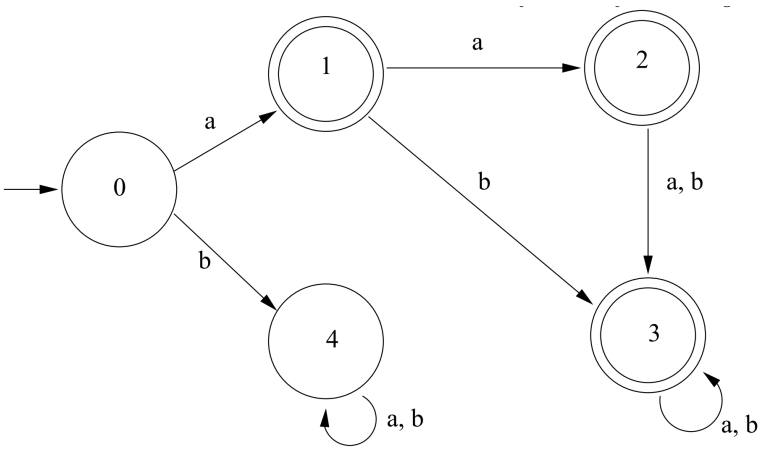
Answer: If we create an finite automata A to represent L, then $\bar{L}$ can be represented by the same automata, with the accepting and non-accepting states flipped B. Any string w is in $L(A)$ if and only if it is not in $L(B)$. Therefore, B can be modeled with finite states and is regular.

Q2.2 $L \cap M$ is regular if L and M are regular.

Answer: If we create a two finite automata that represent both L and M, A and B, we can group the two automata together and evaluate them in parallel. Then we can select accepting states that are pairs of all accepting states of A and B. Finally, we model the union of A and B as a single automaton which can be represented with finite states. Therefore $L \cap M$ is regular.

Q3 Given the DFA:

Q3.1 What states are equivalent to state 3?



Answer: State 2 is equivalent to state 3.

Q3.2 How many states are there in the minimized DFA?

Answer: After combining states 2 and 3, there are 4 states. However, the DFA can be further minimized. The combined state $\{2, 3\}$ is equivalent to state 1. Therefore, the final minimized DFA has 3 states: $[\{0\}, \{1, 2, 3\}, \{4\}]$. The language accepted by the DFA is any string w in the language $L = \{a, b\}$ that starts with a.

Q4 Which of the following conversion has the worst time complexity?

A DFA to NFA

B NFA to DFA

C FA to RE

D RE to FA

Answer: C

Q5 Given the context free grammar: $S \rightarrow aSbS|bSaS|\epsilon$. Is the grammar ambiguous? Yes/No

Answer: Yes.