Monday

Three different CFGs that each generate the language $\{abba\}$

$$(\{S, T, V, W\}, \{a, b\}, \{S \to aT, T \to bV, V \to bW, W \to a\}, S)$$

$$(\{Q\}, \{a, b\}, \{Q \to abba\}, Q)$$

$$(\{X, Y\}, \{a, b\}, \{X \to aYa, Y \to bb\}, X)$$

Design a CFG to generate the language $\{a^n b^n \mid n \geq 0\}$

Sample derivation:

Design a CFG to generate the language $\{a^i b^j \mid j \geq i \geq 0\}$

Sample derivation:

Theorem 2.20: A language is generated by some context-free grammar if and only if it is recognized by some push-down automaton.

Definition: a language is called **context-free** if it is the language generated by a context-free grammar. The class of all context-free language over a given alphabet Σ is called **CFL**.

Consequences:

- Quick proof that every regular language is context free
- To prove closure of the class of context-free languages under a given operation, we can choose either of two modes of proof (via CFGs or PDAs) depending on which is easier
- To fully specify a PDA we could give its 6-tuple formal definition or we could give its input alphabet, stack alphabet, and state diagram. An informal description of a PDA is a step-by-step description of how its computations would process input strings; the reader should be able to reconstruct the state diagram or formal definition precisely from such a descripton. The informal description of a PDA can refer to some common modules or subroutines that are computable by PDAs:
  - PDAs can "test for emptiness of stack" without providing details. *How?* We can always push a special end-of-stack symbol, $, at the start, before processing any input, and then use this symbol as a flag.
  - PDAs can "test for end of input" without providing details. *How?* We can transform a PDA to one where accepting states are only those reachable when there are no more input symbols.

Over $\Sigma = \{a, b\}$, let $L = \{a^n b^m \mid n \neq m\}$. **Goal**: Prove L is context-free.

Suppose L_1 and L_2 are context-free languages over Σ. **Goal**: $L_1 \cup L_2$ is also context-free.

Approach 1: with PDAs

Let $M_1 = (Q_1, \Sigma, \Gamma_1, \delta_1, q_1, F_1)$ and $M_2 = (Q_2, \Sigma, \Gamma_2, \delta_2, q_2, F_2)$ be PDAs with $L(M_1) = L_1$ and $L(M_2) = L_2$.

Define $M =$

Approach 2: with CFGs

Let $G_1 = (V_1, \Sigma, R_1, S_1)$ and $G_2 = (V_2, \Sigma, R_2, S_2)$ be CFGs with $L(G_1) = L_1$ and $L(G_2) = L_2$.

Define $G =$

Suppose L_1 and L_2 are context-free languages over Σ. **Goal**: $L_1 \circ L_2$ is also context-free.

Approach 1: with PDAs

Let $M_1 = (Q_1, \Sigma, \Gamma_1, \delta_1, q_1, F_1)$ and $M_2 = (Q_2, \Sigma, \Gamma_2, \delta_2, q_2, F_2)$ be PDAs with $L(M_1) = L_1$ and $L(M_2) = L_2$.

Define $M =$

Approach 2: with CFGs

Let $G_1 = (V_1, \Sigma, R_1, S_1)$ and $G_2 = (V_2, \Sigma, R_2, S_2)$ be CFGs with $L(G_1) = L_1$ and $L(G_2) = L_2$.

Define $G =$

Summary

Over a fixed alphabet Σ, a language L is **regular**

iff it is described by some regular expression
iff it is recognized by some DFA
iff it is recognized by some NFA

Over a fixed alphabet Σ, a language L is **context-free**

iff it is generated by some CFG
iff it is recognized by some PDA

Fact: Every regular language is a context-free language.

Fact: There are context-free languages that are not nonregular.

Fact: There are countably many regular languages.

Fact: There are countably inifnitely many context-free languages.

Consequence: Most languages are **not** context-free!

Examples of non-context-free languages

$$\{a^n b^n c^n \mid 0 \le n, n \in \mathbb{Z}\}$$
$$\{a^i b^j c^k \mid 0 \le i \le j \le k, i \in \mathbb{Z}, j \in \mathbb{Z}, k \in \mathbb{Z}\}$$
$$\{ww \mid w \in \{0,1\}^*\}$$

(Sipser Ex 2.36, Ex 2.37, 2.38)

There is a Pumping Lemma for CFL that can be used to prove a specific language is non-context-free: If A is a context-free language, there there is a number p where, if s is any string in A of length at least p, then s may be divided into five pieces $s = uvxyz$ where (1) for each $i \ge 0$, $uv^ixy^iz \in A$, (2) $|uv| > 0$, (3) $|vxy| \le p$. *We will not go into the details of the proof or application of Pumping Lemma for CFLs this quarter.*

Review: Week 5 Monday

Recall: Review quizzes based on class material are assigned each day. These quizzes will help you track and confirm your understanding of the concepts and examples we work in class. Quizzes can be submitted on Gradescope as many times (with no penalty) as you like until the quiz deadline: the three quizzes each week are all due on Friday (with no penalty late submission open until Sunday).

Please complete the review quiz questions on Gradescope about context-free grammars

Pre class reading for next time: Figure 3.1 (Pages 165-167)

Wednesday

A set X is said to be **closed** under an operation OP if, for any elements in X, applying OP to them gives an element in X.

True/False	Closure claim
True	The set of integers is closed under multiplication. $\forall x \forall y \, (\, (x \in \mathbb{Z} \wedge y \in \mathbb{Z}) \to xy \in \mathbb{Z} \,)$
True	For each set A, the power set of A is closed under intersection. $\forall A_1 \forall A_2 \, (\, (A_1 \in \mathcal{P}(A) \wedge A_2 \in \mathcal{P}(A) \in \mathbb{Z}) \to A_1 \cap A_2 \in \mathcal{P}(A) \,)$
	The class of regular languages over Σ is closed under complementation.
	The class of regular languages over Σ is closed under union.
	The class of regular languages over Σ is closed under intersection.
	The class of regular languages over Σ is closed under concatenation.
	The class of regular languages over Σ is closed under Kleene star.
	The class of context-free languages over Σ is closed under complementation.
	The class of context-free languages over Σ is closed under union.
	The class of context-free languages over Σ is closed under intersection.
	The class of context-free languages over Σ is closed under concatenation.
	The class of context-free languages over Σ is closed under Kleene star.

Assume $\Sigma = \{0, 1, \#\}$

Σ^*	Regular	/	nonregular and context-free	/	not context-free
$\{0^i \# 1^j \mid i \geq 0, j \geq 0\}$	Regular	/	nonregular and context-free	/	not context-free
$\{0^i 1^j \# 1^j 0^i \mid i \geq 0, j \geq 0\}$	Regular	/	nonregular and context-free	/	not context-free
$\{0^i 1^j \# 0^i 1^j \mid i \geq 0, j \geq 0\}$	Regular	/	nonregular and context-free	/	not context-free

Turing machines: unlimited read + write memory, unlimited time (computation can proceed without "consuming" input and can re-read symbols of input)

- Division betweeen program (CPU, state diagram) and data
- Unbounded memory gives theoretical limit to what modern computation (including PCs, supercomputers, quantum computers) can achieve
- State diagram formulation is simple enough to reason about (and diagonalize against) while expressive enough to capture modern computation

For Turing machine $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$ the **computation** of M on a string w over Σ is:

- Read/write head starts at leftmost position on tape.
- Input string is written on $|w|$-many leftmost cells of tape, rest of the tape cells have the blank symbol. **Tape alphabet** is Γ with $\textvisiblespace \in \Gamma$ and $\Sigma \subseteq \Gamma$. The blank symbol $\textvisiblespace \notin \Sigma$.
- Given current state of machine and current symbol being read at the tape head, the machine transitions to next state, writes a symbol to the current position of the tape head (overwriting existing symbol), and moves the tape head L or R (if possible). Formally, **transition function** is

$$\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$$

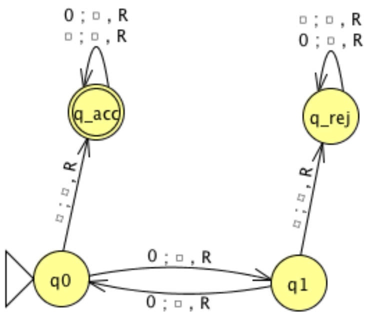
- Computation ends if and when machine enters either the accept or the reject state. This is called **halting**. Note: $q_{accept} \neq q_{reject}$.

The **language recognized by the Turing machine** M, is

$\{w \in \Sigma^* \mid \text{computation of } M \text{ on } w \text{ halts after entering the accept state}\} = \{w \in \Sigma^* \mid w \text{ is accepted by } M\}$

An example Turing machine: $\Sigma =$ $\qquad\qquad\qquad$, $\Gamma =$

$$\delta((q0,0)) =$$



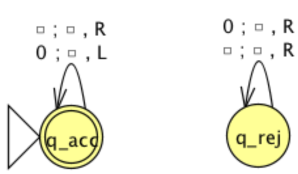
Formal definition:

Sample computation:

$q0 \downarrow$						
0	0	0	␣	␣	␣	␣

The language recognized by this machine is ...

Extra practice:



Formal definition:

Sample computation:

Review: Week 5 Wednesday

Please complete the review quiz questions on Gradescope about context-free languages and Turing machines.

Friday: Midterm exam

The midterm exam policies are below

INSTRUCTIONS — READ THIS NOW

- Write your name, PID, current seat number, exam time, and write out the academic integrity pledge in the indicated space above. Your score **will not be recorded** if any of this identifying information is missing, or if the academic integrity pledge is illegible.
- Write your answers on the designated **answer sheet** in the specified areas, or your work will not be graded. If you need more space, raise your hand.
- We will not be answering questions about the exam during the exam period. If any bugs are found in the exam after the exam period, the affected question(s) will be removed.
- Please show your ID to a proctor when you hand in your exam.
- You may not speak to any other student in the exam room while the exam is in progress (including after you hand in your own exam). You may not share **any information** about the exam with anyone who has not taken it.
- Turn off and put away all cellphones, calculators, and other electronic devices. You may not access any electronic device during the exam period. You may use your one sheet of notes, but no other books, notes, or aids.
- To receive full credit, your answers must be written neatly, legibly, and sufficiently darkly to scan well. Your solution will be evaluated both for correctness and clarity. Read the instructions for each part carefully to determine what is required for full credit.
- This exam is **45 minutes** long. Read all the problems first before you start working on any of them, so you can manage your time wisely.
- This test has 3 problems worth a total of 50 points.