

## Week3 friday

**Theorem:** For an alphabet  $\Sigma$ , For each language  $L$  over  $\Sigma$ ,

$$\begin{array}{c} L \text{ is recognized by some DFA} \\ \text{iff} \\ L \text{ is recognized by some NFA} \\ \text{iff} \\ L \text{ is described by some regular expression} \end{array}$$

If (any, hence all) these conditions apply,  $L$  is called **regular**.

**Prove or Disprove:** There is some alphabet  $\Sigma$  for which there is some language recognized by an NFA but not by any DFA.

**Prove or Disprove:** There is some alphabet  $\Sigma$  for which there is some finite language not described by any regular expression over  $\Sigma$ .

**Prove or Disprove:** If a language is recognized by an NFA then the complement of this language is not recognized by any DFA.

| Set  | Cardinality |
|--|-------------|
| $\{0, 1\}$   |             |
| $\{0, 1\}^*$                                       |             |
| $\mathcal{P}(\{0, 1\})$                            |             |
| The set of all languages over $\{0, 1\}$           |             |
| The set of all regular expressions over $\{0, 1\}$ |             |
| The set of all regular languages over $\{0, 1\}$   |             |

**Pumping Lemma** (Sipser Theorem 1.70): If  $A$  is a regular language, then there is a number  $p$  (a *pumping length*) where, if  $s$  is any string in  $A$  of length at least  $p$ , then  $s$  may be divided into three pieces,  $s = xyz$  such that

- $|y| > 0$
- for each  $i \geq 0$ ,  $xy^iz \in A$
- $|xy| \leq p$ .

**True or False:** A pumping length for  $A = \{0, 1\}^*$  is  $p = 5$ .

**True or False:** A pumping length for  $A = \{1, 01, 001, 0001, 00001\}$  is  $p = 4$ .

**True or False:** A pumping length for  $A = \{0^j1 \mid j \geq 0\}$  is  $p = 3$ .

**True or False:** For any language  $A$ , if  $p$  is a pumping length for  $A$  and  $p' > p$ , then  $p'$  is also a pumping length for  $A$ .