Let's get started

We want you to be successful.

We will work together to build an environment in CSE 105 that supports your learning in a way that respects your perspectives, experiences, and identities (including race, ethnicity, heritage, gender, sex, class, sexuality, religion, ability, age, educational background, etc.). Our goal is for you to engage with interesting and challenging concepts and feel comfortable exploring, asking questions, and thriving.

If you or someone you know is suffering from food and/or housing insecurities there are UCSD resources here to help:

Basic Needs Office: https://basicneeds.ucsd.edu/

Triton Food Pantry (in the old Student Center) is free and anonymous, and includes produce:

https://www.facebook.com/tritonfoodpantry/

Mutual Aid UCSD: https://mutualaiducsd.wordpress.com/

Financial aid resources, the possibility of emergency grant funding, and off-campus housing referral resources are available: see your College Dean of Student Affairs.

If you find yourself in an uncomfortable situation, ask for help. We are committed to upholding University policies regarding nondiscrimination, sexual violence and sexual harassment. Here are some campus contacts that could provide this help: Counseling and Psychological Services (CAPS) at 858 534-3755 or http://caps.ucsd.edu; OPHD at 858 534-8298 or ophd@ucsd.edu , http://ophd.ucsd.edu; CARE at Sexual Assault Resource Center at 858 534-5793 or sarc@ucsd.edu , http://care.ucsd.edu.

Please reach out (minnes@ucsd.edu) if you need support with extenuating circumstances affecting CSE 105.

Introductions

Class website: https://canvas.ucsd.edu/courses/51649/

Instructor: Prof. Mia Minnes "Minnes" rhymes with Guinness, minnes@ucsd.edu, http://cseweb.ucsd.edu/ minnes

Our team: One instructor + two TAs and eleven tutors + all of you

Fill in contact info for students around you, if you'd like:

Welcome to CSE 105: Introduction to Theory of Computation in Winter 2024!

CSE 105's Big Questions

- What problems are computers capable of solving?
- What resources are needed to solve a problem?
- Are some problems harder than others?

In this context, a **problem** is defined as: "Making a decision or computing a value based on some input"

Consider the following problems:

- Find a file on your computer
- Determine if your code will compile
- Find a run-time error in your code
- Certify that your system is un-hackable

Which of these is hardest?

In Computer Science, we operationalize "hardest" as "requires most resources", where resources might be memory, time, parallelism, randomness, power, etc.

To be able to compare "hardness" of problems, we use a consistent description of problems

Input: String

Output: Yes/ No, where Yes means that the input string matches the pattern or property described by the problem.

Monday: Terminology and Notation

The CSE 105 vocabulary and notation build on discrete math and introduction to proofs classes. Some of the conventions may be a bit different from what you saw before so we'll draw your attention to them.

For consistency, we will use the notation from this class' textbook[1].

These definitions are on pages 3, 4, 6, 13, 14, 53.

Term	**Typical symbol** or **Notation**	**Meaning**
Alphabet	Σ, Γ	A non-empty finite set
Symbol over Σ	σ, b, x	An element of the alphabet Σ
String over Σ	u, v, w	A finite list of symbols from Σ
(The) empty string	ε	The (only) string of length 0
The set of all strings over Σ	Σ^*	The collection of all possible strings formed from symbols from Σ
(Some) language over Σ	L	(Some) set of strings over Σ
(The) empty language	$\emptyset$	The empty set, i.e. the set that has no strings (and no other elements either)
The power set of a set X	$\mathcal{P}(X)$	The set of all subsets of X
(The set of) natural numbers	$\mathcal{N}$	The set of positive integers
(Some) finite set		The empty set or a set whose distinct elements can be counted by a natural number
(Some) infinite set		A set that is not finite.
Reverse of a string w	$w^{\mathcal{R}}$	write w in the opposite order, if $w = w_1 \cdots w_n$ then $w^{\mathcal{R}} = w_n \cdots w_1$. Note: $\varepsilon^{\mathcal{R}} = \varepsilon$
Concatenating strings x and y	xy	take $x = x_1 \cdots x_m$, $y = y_1 \cdots y_n$ and form $xy = x_1 \cdots x_m y_1 \cdots y_n$
String z is a substring of string w		there are strings u, v such that $w = uzv$
String x is a prefix of string y		there is a string z such that $y = xz$
String x is a proper prefix of string y		x is a prefix of y and $x \neq y$
Shortlex order, also known as string order over alphabet Σ		Order strings over Σ first by length and then according to the dictionary order, assuming symbols in Σ have an ordering

[1]Page references are to the 3rd edition of Sipser's Introduction to the Theory of Computation, available through various sources for approximately \$30. You may be able to opt in to purchase a digital copy through Canvas. Copies of the book are also available for those who can't access the book to borrow from the course instructor, while supplies last (minnes@ucsd.edu)

Write out in words the meaning of the symbols below:

$$\{a, b, c\}$$

$$|\{a, b, a\}| = 2$$

$$|aba| = 3$$

Circle the correct choice:

A **string** over an alphabet Σ is <u> an element of Σ^* OR   a subset of Σ^*</u>.

A **language** over an alphabet Σ is <u> an element of Σ^* OR   a subset of Σ^*</u>.

With $\Sigma_1 = \{0, 1\}$ and $\Sigma_2 = \{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z\}$ and $\Gamma = \{0, 1, x, y, z\}$

True or **False**: $\varepsilon \in \Sigma_1$

True or **False**: ε is a string over Σ_1

True or **False**: ε is a language over Σ_1

True or **False**: ε is a prefix of some string over Σ_1

True or **False**: There is a string over Σ_1 that is a proper prefix of ε

The first five strings over Σ_1 in string order, using the ordering $0 < 1$:

The first five strings over Σ_2 in string order, using the usual alphabetical ordering for single letters:

Week 1 Wednesday

Our motivation in studying sets of strings is that they can be used to encode problems. To calibrate how difficult a problem is to solve, we describe how complicated the set of strings that encodes it is. How do we define sets of strings?

How would you describe the language that has no elements at all?

How would you describe the language that has all strings over $\{0, 1\}$ as its elements?

This definition was in the pre-class reading **Definition 1.52**: A **regular expression** over alphabet Σ is a syntactic expression that can describe a language over Σ. The collection of all regular expressions over Σ is defined recursively:

Basis steps of recursive definition

a is a regular expression, for $a \in \Sigma$
ε is a regular expression
$\emptyset$ is a regular expression

Recursive steps of recursive definition

$(R_1 \cup R_2)$ is a regular expression when R_1, R_2 are regular expressions
$(R_1 \circ R_2)$ is a regular expression when R_1, R_2 are regular expressions
(R_1^*) is a regular expression when R_1 is a regular expression

The *semantics* (or meaning) of the syntactic regular expression is the **language described by the regular expression**. The function that assigns a language to a regular expression over Σ is defined recursively, using familiar set operations:

Basis steps of recursive definition

The language described by a, for $a \in \Sigma$, is $\{a\}$ and we write $L(a) = \{a\}$
The language described by ε is $\{\varepsilon\}$ and we write $L(\varepsilon) = \{\varepsilon\}$
The language described by $\emptyset$ is $\{\}$ and we write $L(\emptyset) = \emptyset$.

Recursive steps of recursive definition

When R_1, R_2 are regular expressions, the language described by the regular expression $(R_1 \cup R_2)$ is the union of the languages described by R_1 and R_2, and we write

$$L(\ (R_1 \cup R_2)\ ) = L(R_1) \cup L(R_2) = \{w \mid w \in L(R_1) \vee w \in L(R_2)\}$$

When R_1, R_2 are regular expressions, the language described by the regular expression $(R_1 \circ R_2)$ is the concatenation of the languages described by R_1 and R_2, and we write

$$L(\ (R_1 \circ R_2)\ ) = L(R_1) \circ L(R_2) = \{uv \mid u \in L(R_1) \wedge v \in L(R_2)\}$$

When R_1 is a regular expression, the language described by the regular expression (R_1^*) is the **Kleene star** of the language described by R_1 and we write

$$L(\ (R_1^*)\ ) = (\ L(R_1)\ )^* = \{w_1 \cdots w_k \mid k \geq 0 \text{ and each } w_i \in L(R_1)\}$$

For the following examples assume the alphabet is $\Sigma_1 = \{0, 1\}$:

The language described by the regular expression 0 is $L(0) = \{0\}$

The language described by the regular expression 1 is $L(1) = \{1\}$

The language described by the regular expression ε is $L(\varepsilon) = \{\varepsilon\}$

The language described by the regular expression $\emptyset$ is $L(\emptyset) = \emptyset$

The language described by the regular expression $(\Sigma_1\Sigma_1\Sigma_1)^*$ is $L(\ (\Sigma_1\Sigma_1\Sigma_1)^*\ ) =$

The language described by the regular expression 1^+ is $L(\ (1^+)\ ) = L(1^* \circ 1) =$

Shorthand and conventions (Sipser pages 63-65)

Assuming Σ is the alphabet, we use the following conventions

Σ	regular expression describing language consisting of all strings of length 1 over Σ
$*$ then $\circ$ then $\cup$	precedence order, unless parentheses are used to change it
R_1R_2	shorthand for $R_1 \circ R_2$ (concatenation symbol is implicit)
R^+	shorthand for $R^* \circ R$
R^k	shorthand for R concatenated with itself k times, where k is a (specific) natural number

Caution: many programming languages that support regular expressions build in functionality that is more powerful than the "pure" definition of regular expressions given here.

Regular expressions are everywhere (once you start looking for them).

Software tools and languages often have built-in support for regular expressions to describe **patterns** that we want to match (e.g. Excel/ Sheets, grep, Perl, python, Java, Ruby).

Under the hood, the first phase of **compilers** is to transform the strings we write in code to tokens (keywords, operators, identifiers, literals). Compilers use regular expressions to describe the sets of strings that can be used for each token type.

Next time: we'll start to see how to build machines that decide whether strings match the pattern described by a regular expression.

Extra examples for practice:

Which regular expression(s) below describe a language that includes the string a as an element?

a^*b^*

$a(ba)^*b$

$a^* \cup b^*$

$(aaa)^*$

$(\varepsilon \cup a)b$

Week 1 Friday

Review: Determine whether each statement below about regular expressions over the alphabet $\{a, b, c\}$ is true or false:

True or False: $ab \in L(\ (a \cup b)^*\ )$

True or False: $ba \in L(\ a^* b^*\ )$

True or False: $\varepsilon \in L(a \cup b \cup c)$

True or False: $\varepsilon \in L(\ (a \cup b)^*\ )$

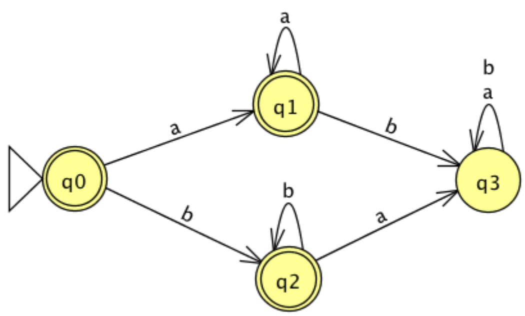
True or False: $\varepsilon \in L(\ aa^* \cup bb^*\ )$

This definition was in the pre-class reading A finite automaton (FA) is specified by $M = (Q, \Sigma, \delta, q_0, F)$. This 5-tuple is called the **formal definition** of the FA. The FA can also be represented by its state diagram: with nodes for the state, labelled edges specifying the transition function, and decorations on nodes denoting the start and accept states.

> Finite set of states Q can be labelled by any collection of distinct names. Often we use default state labels $q0, q1, \ldots$
>
> The alphabet Σ determines the possible inputs to the automaton. Each input to the automaton is a string over Σ, and the automaton "processes" the input one symbol (or character) at a time.
>
> The transition function δ gives the next state of the automaton based on the current state of the machine and on the next input symbol.
>
> The start state q_0 is an element of Q. Each computation of the machine starts at the start state.
>
> The accept (final) states F form a subset of the states of the automaton, $F \subseteq Q$. These states are used to flag if the machine accepts or rejects an input string.
>
> The computation of a machine on an input string is a sequence of states in the machine, starting with the start state, determined by transitions of the machine as it reads successive input symbols.
>
> The finite automaton M accepts the given input string exactly when the computation of M on the input string ends in an accept state. M rejects the given input string exactly when the computation of M on the input string ends in a nonaccept state, that is, a state that is not in F.
>
> The language of M, $L(M)$, is defined as the set of all strings that are each accepted by the machine M. Each string that is rejected by M is not in $L(M)$. The language of M is also called the language recognized by M.

What is **finite** about all finite automata? (Select all that apply)

- ☐ The size of the machine (number of states, number of arrows)
- ☐ The length of each computation of the machine
- ☐ The number of strings that are accepted by the machine

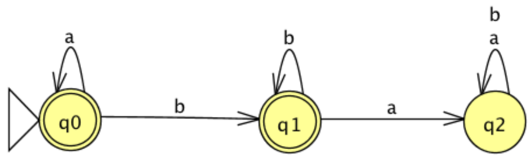


The formal definition of this FA is

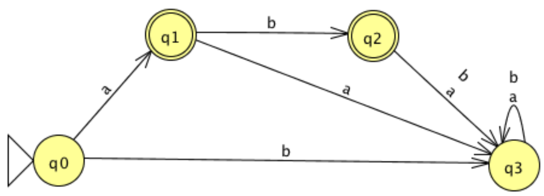
Classify each string $a, aa, ab, ba, bb, \varepsilon$ as accepted by the FA or rejected by the FA.

Why are these the only two options?

The language recognized by this automaton is



The language recognized by this automaton is



The language recognized by this automaton is

Week 1 at a glance

Textbook reading: Chapter 0, Sections 1.3, 1.1

For Monday: Class syllabus https://canvas.ucsd.edu/courses/45073.

For Wednesday: Example 1.51 and Definition 1.52 (definition of regular expressions) on page 64. *Notice*: we are jumping to Section 1.3 and then will come back to Section 1.1 on Friday.

For Friday: Figure 1.4 and Definition 1.5 (definition of finite automata) on pages 34-35. The definition of the union, concatenation, and star operations for languages is given as Definition 1.23 on page 44 and a useful example is Example 1.24.

For Week 2 Wednesday: Pages 41-43 (Figures 1.18, 1.19, 1.20) (examples of automata and languages). *Notice*: Week 2 Monday is a UCSD Holiday in observance of Martin Luther King Jr. day so there is no CSE 105 class.

Textbook references: Within a chapter, each item is numbered consecutively. Figure 1.22 is the twenty-second numbered item in chapter one; it comes right after Example 1.21 and right before Definition 1.23.

Make sure you can:

- Distinguish between alphabet, language, sets, and strings
- Translate a decision problem to a set of strings coding the problem
- Use regular expressions and relate them to languages and automata
  - Write and debug regular expressions using correct syntax
  - Determine if a given string is in the language described by a regular expression
- Use precise notation to formally define the state diagram of finite automata and use clear English to describe computations of finite automata informally.
  - State the formal definition of (deterministic) finite automata
  - Trace the computation of a finite automaton on a given string using its state diagram
  - Translate between a state diagram and a formal definition
  - Determine if a given string is in the language described by a finite automaton

TODO:

#FinAid Assignment on Canvas https://canvas.ucsd.edu/courses/51649/quizzes/158899

Review quizzes based on class material each day.

Homework assignment 1 due next week.