Week1 monday

We will use vocabulary that should be familiar from your discrete math and introduction to proofs classes. Some of the notation conventions may be a bit different: we will use the notation from this class' textbook[1].

Write out in words the meaning of the symbols below:

$$\{a, b, c\}$$

$$|\{a, b, a\}| = 2$$

$$|aba| = 3$$

$$(a, 3, 2, b, b)$$

Term	Typical symbol	Meaning
Alphabet	Σ, Γ	A non-empty finite set
Symbol over Σ	σ, b, x	An element of the alphabet Σ
String over Σ	u, v, w	A finite list of symbols from Σ
The set of all strings over Σ	Σ^*	The collection of all possible strings formed from symbols from Σ
(Some) language over Σ	L	(Some) set of strings over Σ
Empty string	ε	The string of length 0
Empty set	$\emptyset$	The empty language
Natural numbers	$\mathcal{N}$	The set of positive integers
Finite set		The empty set or a set whose distinct elements can be counted by a natural number
Infinite set		A set that is not finite.
Pages 3, 4, 13, 14		

[1]Page references are to the 3rd edition (International) of Siper's Introduction to the Theory of Computation, available at the campus bookstore for under $20. Copies of the book are also available for those who can't access the book to borrow from the course instructor, while supplies last (minnes@eng.ucsd.edu)

Term	Notation	Meaning
Reverse of a string w	$w^{\mathcal{R}}$	write w in the opposite order, if $w = w_1 \cdots w_n$ then $w^{\mathcal{R}} = w_n \cdots w_1$. Note: $\varepsilon^{\mathcal{R}} = \varepsilon$
Concatenating strings x and y	xy	take $x = x_1 \cdots x_m$, $y = y_1 \cdots y_n$ and form $xy = x_1 \cdots x_m y_1 \cdots y_n$
String z is a substring of string w		there are strings u, v such that $w = uzv$
String x is a prefix of string y		there is a string z such that $y = xz$
String x is a proper prefix of string y		x is a prefix of y and $x \neq y$
Shortlex order, also known as string order over alphabet Σ		Order strings over Σ first by length and then according to the dictionary order, assuming symbols in Σ have an ordering.

Pages 13, 14

Circle the correct choice:

A **string** over an alphabet Σ is <u>an element of Σ^* OR a subset of Σ^*</u>.

A **language** over an alphabet Σ is <u>an element of Σ^* OR a subset of Σ^*</u>.

Extra examples for practice:

With $\Sigma_1 = \{0, 1\}$ and $\Sigma_2 = \{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z\}$ and $\Gamma = \{0, 1, x, y, z\}$

An example of a string of length 3 over Σ_1 is _______

An example of a string of length 1 over Σ_2 is _____

The number of distinct strings of length 2 over Γ is ______

An example of a language over Σ_1 of size 1 is _______

An example of an infinite language over Σ_1 is ______

An example of a finite language over Γ is _________

True or **False**: $\varepsilon \in \Sigma_1$

True or **False**: ε is a string over Σ_1

True or **False**: ε is a language over Σ_1

True or **False**: ε is a prefix of some string over Σ_1

True or **False**: There is a string over Σ_1 that is a proper prefix of ε

The first five strings over Σ_1 in string order, using the ordering $0 < 1$:

The first five strings over Σ_2 in string order, using the usual alphabetical ordering for single letters:

Week1 wednesday

Our motivation in studying sets of strings is that they encode problems.

We need to describe the collection of all strings that match the pattern or property of a problem.

Let's start by thinking about how we can describe a language (a set of strings from a given alphabet).

Definition 1.52: A **regular expression** over alphabet Σ is a syntactic expression that can describe a language over Σ. The collection of all regular expressions is defined recursively:

Basis steps of recursive definition

a is a regular expression, for $a \in \Sigma$
ε is a regular expression
$\emptyset$ is a regular expression

Recursive steps of recursive definition

$(R_1 \cup R_2)$ is a regular expression when R_1, R_2 are regular expressions
$(R_1 \circ R_2)$ is a regular expression when R_1, R_2 are regular expressions
(R_1^*) is a regular expression when R_1 is a regular expression

The *semantics* (or meaning) of the syntactic regular expression is the **language described by the regular expression**. The function that assigns a language to a regular expression over Σ is defined recursively, using familiar set operations:

Basis steps of recursive definition

The language described by a, for $a \in \Sigma$, is $\{a\}$ and we write $L(a) = \{a\}$
The language described by ε is $\{\varepsilon\}$ and we write $L(\varepsilon) = \{\varepsilon\}$
The language described by $\emptyset$ is $\{\}$ and we write $L(\emptyset) = \emptyset$.

Recursive steps of recursive definition

When R_1, R_2 are regular expressions, the language described by the regular expression $(R_1 \cup R_2)$ is the union of the languages described by R_1 and R_2, and we write

$$L(\ (R_1 \cup R_2)\ ) = L(R_1) \cup L(R_2) = \{w \mid w \in L(R_1) \vee w \in L(R_2)\}$$

When R_1, R_2 are regular expressions, the language described by the regular expression $(R_1 \circ R_2)$ is the concatenation of the languages described by R_1 and R_2, and we write

$$L(\ (R_1 \circ R_2)\ ) = L(R_1) \circ L(R_2) = \{uv \mid u \in L(R_1) \wedge v \in L(R_2)\}$$

When R_1 is a regular expression, the language described by the regular expression (R_1^*) is the **Kleene star** of the language described by R_1 and we write

$$L(\ (R_1^*)\ ) = (\ L(R_1)\ )^* = \{w_1 \cdots w_k \mid k \geq 0 \text{ and each } w_i \in L(R_1)\}$$

For the following examples assume the alphabet is $\Sigma_1 = \{0, 1\}$:

The language described by the regular expression 0 is $L(0) = \{0\}$

The language described by the regular expression 1 is $L(1) = \{1\}$

The language described by the regular expression ε is $L(\varepsilon) = \{\varepsilon\}$

The language described by the regular expression $\emptyset$ is $L(\emptyset) = \emptyset$

The language described by the regular expression $((0 \cup 1) \cup 1)$ is $L(\ ((0 \cup 1) \cup 1)\ ) =$

The language described by the regular expression 1^+ is $L(\ (1)^+\ ) =$

The language described by the regular expression $\Sigma_1^* 1$ is $L(\ \Sigma_1^* 1\ ) =$

The language described by the regular expression $(\Sigma_1\Sigma_1\Sigma_1\Sigma_1\Sigma_1)^*$ is $L(\ (\Sigma_1\Sigma_1\Sigma_1\Sigma_1\Sigma_1)^*\ ) =$

A regular expression that describes the language $\{00, 01, 10, 11\}$ is

A regular expression that describes the language $\{0^n 1 \mid n \text{ is even}\}$ is

Shorthand and conventions

Assuming Σ is the alphabet, we use the following conventions	
Σ	regular expression describing language consisting of all strings of length 1 over Σ
$*$ then $\circ$ then $\cup$	precedence order, unless parentheses are used to change it
$R_1 R_2$	shorthand for $R_1 \circ R_2$ (concatenation symbol is implicit)
R^+	shorthand for $R^* \circ R$
R^k	shorthand for R concatenated with itself k times, where k is a natural number
Pages 63 - 65	

Caution: many programming languages that support regular expressions build in functionality that is more powerful than the "pure" definition of regular expressions given here.

Regular expressions are everywhere (once you start looking for them).

Software tools and languages often have built-in support for regular expressions to describe **patterns** that we want to match (e.g. Excel/ Sheets, grep, Perl, python, Java, Ruby).

Under the hood, the first phase of **compilers** is to transform the strings we write in code to tokens (keywords, operators, identifiers, literals). Compilers use regular expressions to describe the sets of strings that can be used for each token type.

Next time: we'll start to see how to build machines that decide whether strings match the pattern described by a regular expression.

Extra examples for practice:

Which regular expression(s) below describe a language that includes the string a as an element?

a^*b^*

$a(ba)^*b$

$a^* \cup b^*$

$(aaa)^*$

$(\varepsilon \cup a)b$

Week1 friday

Review: Determine whether each statement below about regular expressions over the alphabet $\{a, b, c\}$ is true or false:

True or False: $a \in L(\ (a \cup b)\ \cup c)$

True or False: $ab \in L(\ (a \cup b)^*\ )$

True or False: $ba \in L(\ a^* b^*\ )$

True or False: $\varepsilon \in L(a \cup b \cup c)$

True or False: $\varepsilon \in L(\ (a \cup b)^*\ )$

True or False: $\varepsilon \in L(\ a^* b^*\ )$

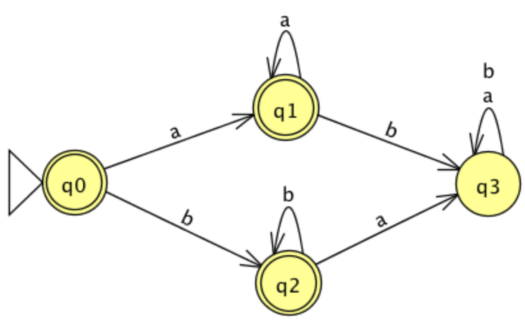
From the pre-class reading, pages 34-36: A deterministic finite automaton (DFA) is specified by $M = (Q, \Sigma, \delta, q_0, F)$. This 5-tuple is called the **formal definition** of the DFA. The DFA can also be represented by its state diagram: with nodes for the state, labelled edges specifying the transition function, and decorations on nodes denoting the start and accept states.

> Finite set of states Q can be labelled by any collection of distinct names. Often we use default state labels $q0, q1, \ldots$
>
> The alphabet Σ determines the possible inputs to the automaton. Each input to the automaton is a string over Σ, and the automaton "processes" the input one symbol (or character) at a time.
>
> The transition function δ gives the next state of the DFA based on the current state of the machine and on the next input symbol.
>
> The start state q_0 is an element of Q. Each computation of the machine starts at the start state.
>
> The accept (final) states F form a subset of the states of the DFA, $F \subseteq Q$. These states are used to flag if the machine accepts or rejects an input string.
>
> The computation of a machine on an input string is a sequence of states in the machine, starting with the start state, determined by transitions of the machine as it reads successive input symbols.
>
> The DFA M accepts the given input string exactly when the computation of M on the input string ends in an accept state. M rejects the given input string exactly when the computation of M on the input string ends in a nonaccept state, that is, a state that is not in F.

The language of M, $L(M)$, is defined as the set of all strings that are each accepted by the machine M. Each string that is rejected by M is not in $L(M)$. The language of M is also called the language recognized by M.

What is **finite** about all deterministic finite automata? (Select all that apply)

- ☐ The size of the machine (number of states, number of arrows)
- ☐ The number of strings that are accepted by the machine
- ☐ The length of each computation of the machine

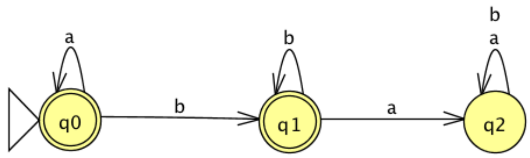


The formal definition of this DFA is

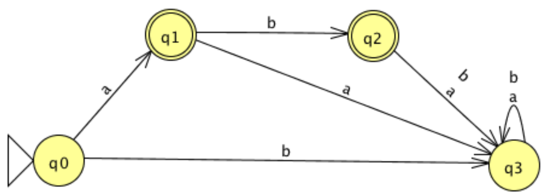
Classify each string $a, aa, ab, ba, bb, \varepsilon$ as accepted by the DFA or rejected by the DFA.

Why are these the only two options?

The language recognized by this DFA is



The language recognized by this DFA is



The language recognized by this DFA is