## Week2 monday

**Review**: Formal definition of DFA:  $M = (Q, \Sigma, \delta, q_0, F)$ 

 $\bullet$  Finite set of states Q

• Start state  $q_0$ 

• Alphabet  $\Sigma$ 

• Accept (final) states F

• Transition function  $\delta$ 

In the state diagram of M, how many outgoing arrows are there from each state?

 $M = (\{q, r, s\}, \{a, b\}, \delta, q, \{s\})$  where  $\delta$  is (rows labelled by states and columns labelled by symbols):

$$\begin{array}{c|cccc}
\delta & a & b \\
\hline
q & r & q \\
r & r & s \\
s & s & s
\end{array}$$

The state diagram for M is

Give two examples of strings that are accepted by M and two examples of strings that are rejected by M:

Add "labels" for states in the state diagram, e.g. "have not seen any of desired pattern yet" or "sink state".





Let n be an arbitrary positive integer. What is a formal definition for a DFA recognizing

 $\{w \mid w \text{ is a string over } \{0,1\} \text{ whose length is not a multiple of } n\}$ ?

## Week2 wednesday

Suppose A is a language over an alphabet  $\Sigma$ . By definition, this means A is a subset of  $\Sigma^*$ . Claim: if there is a DFA M such that L(M) = A then there is another DFA, let's call it M', such that  $L(M') = \overline{A}$ , the complement of A, defined as  $\{w \in \Sigma^* \mid w \notin A\}$ .

Proof idea:

**Proof**:

A useful (optional) bit of terminology: the **iterated transition function** of a DFA  $M = (Q, \Sigma, \delta, q_0, F)$  is defined recursively by

$$\delta^*(\ (q,w)\ ) = \begin{cases} q & \text{if } q \in Q, w = \varepsilon \\ \delta(\ (q,a)\ ) & \text{if } q \in Q, \, w = a \in \Sigma \\ \delta(\ (\delta^*(q,u),a)\ ) & \text{if } q \in Q, \, w = ua \text{ where } u \in \Sigma^* \text{ and } a \in \Sigma \end{cases}$$

Using this terminology, M accepts a string w over  $\Sigma$  if and only if  $\delta^*((q_0, w)) \in F$ .

| Fix $\Sigma = \{a, b\}$ . A state diagram for a DFA that recognizes $\{w \mid w \text{ has } ab \text{ as a substring and is of even length}\}$  |
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| Suppose $A_1, A_2$ are languages over an alphabet $\Sigma$ . Claim: if there is a DFA $M_1$ such that $L(M_1) = A_1$ and DFA $M_2$ such that $L(M_2) = A_2$ , then there is another DFA, let's call it $M$ , such that $L(M) = A_1 \cap A_2$ . |
| Proof idea:  |
| Formal construction:   |
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| <b>Application</b> : When $A_1 = \{w \mid w \text{ has } ab \text{ as a substring}\}$ and $A_2 = \{w \mid w \text{ is of even length}\}.$  |
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| Suppose $A_1, A_2$ are languages over an alphabet $\Sigma$ . Claim: if there is a DFA $M_1$ such that $L(M_1) = A_1$ and DFA $M_2$ such that $L(M_2) = A_2$ , then there is another DFA, let's call it $M$ , such that $L(M) = A_1 \cup A_2$ . Sipser Theorem 1.25, page 45 |
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| Proof idea:   |
| Formal construction:  |
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| <b>Application</b> : A state diagram for a DFA that recognizes $\{w \mid w \text{ has } ab \text{ as a substring or is of even length}\}$ :   |
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## Week2 friday

Nondeterministic finite automaton  $M = (Q, \Sigma, \delta, q_0, F)$ 

Finite set of states Q Can be labelled by any collection of distinct names. Default:  $q0, q1, \ldots$ 

Alphabet  $\Sigma$  Each input to the automaton is a string over  $\Sigma$ .

Arrow labels  $\Sigma_{\varepsilon}$   $\Sigma_{\varepsilon} = \Sigma \cup \{\varepsilon\}.$ 

Arrows in the state diagram are labelled either by symbols from  $\Sigma$  or by  $\varepsilon$ 

Transition function  $\delta$   $\delta: Q \times \Sigma_{\varepsilon} \to \mathcal{P}(Q)$  gives the **set of possible next states** for a transition

from the current state upon reading a symbol or spontaneously moving.

Start state  $q_0$  Element of Q. Each computation of the machine starts at the start state.

Accept (final) states  $F F \subseteq Q$ .

M accepts the input string if and only if there is a computation of M on the input string

that processes the whole string and ends in an accept state.

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The formal definition of the NFA over  $\{0,1\}$  given by this state diagram is:



The language over  $\{0,1\}$  recognized by this NFA is:

Change the transition function to get a different NFA which accepts the empty string.

The state diagram of an NFA over  $\{a,b\}$  is below. The formal definition of this NFA is:



The language recognized by this NFA is:

## Week1 friday

**Review**: Determine whether each statement below about regular expressions over the alphabet  $\{a, b, c\}$  is true or false:

True or False:  $a \in L((a \cup b) \cup c)$ 

True or False:  $ab \in L((a \cup b)^*)$ 

True or False:  $ba \in L(a^*b^*)$ 

True or False:  $\varepsilon \in L(a \cup b \cup c)$ 

True or False:  $\varepsilon \in L(\ (a \cup b)^*\ )$ 

True or False:  $\varepsilon \in L(a^*b^*)$ 

From the pre-class reading, pages 34-36: A deterministic finite automaton (DFA) is specified by  $M = (Q, \Sigma, \delta, q_0, F)$ . This 5-tuple is called the **formal definition** of the DFA. The DFA can also be represented by its state diagram: with nodes for the state, labelled edges specifying the transition function, and decorations on nodes denoting the start and accept states.

Finite set of states Q can be labelled by any collection of distinct names. Often we use default state labels  $q0, q1, \ldots$ 

The alphabet  $\Sigma$  determines the possible inputs to the automaton. Each input to the automaton is a string over  $\Sigma$ , and the automaton "processes" the input one symbol (or character) at a time.

The transition function  $\delta$  gives the next state of the DFA based on the current state of the machine and on the next input symbol.

The start state  $q_0$  is an element of Q. Each computation of the machine starts at the start state.

The accept (final) states F form a subset of the states of the DFA,  $F \subseteq Q$ . These states are used to flag if the machine accepts or rejects an input string.

The computation of a machine on an input string is a sequence of states in the machine, starting with the start state, determined by transitions of the machine as it reads successive input symbols.

The DFA M accepts the given input string exactly when the computation of M on the input string ends in an accept state. M rejects the given input string exactly when the computation of M on the input string ends in a nonaccept state, that is, a state that is not in F.

The language of M, L(M), is defined as the set of all strings that are each accepted by the machine M. Each string that is rejected by M is not in L(M). The language of M is also called the language recognized by M.

What is **finite** about all deterministic finite automata? (Select all that apply)

- ☐ The size of the machine (number of states, number of arrows)
- $\square$  The number of strings that are accepted by the machine
- $\square$  The length of each computation of the machine



The formal definition of this DFA is

Classify each string  $a, aa, ab, ba, bb, \varepsilon$  as accepted by the DFA or rejected by the DFA.

Why are these the only two options?



The language recognized by this DFA is



The language recognized by this DFA is