

## Week4 monday

Recap so far: In DFA, the only memory available is in the states. Automata can only “remember” finitely far in the past and finitely much information, because they can have only finitely many states. If a computation path of a DFA visits the same state more than once, the machine can’t tell the difference between the first time and future times it visits this state. Thus, if a DFA accepts one long string, then it must accept (infinitely) many similar strings.

**Definition** A positive integer  $p$  is a **pumping length** of a language  $L$  over  $\Sigma$  means that, for each string  $s \in \Sigma^*$ , if  $|s| \geq p$  and  $s \in L$ , then there are strings  $x, y, z$  such that

$$s = xyz$$

and

$$|y| > 0, \quad \text{for each } i \geq 0, \ xy^i z \in L, \quad \text{and} \quad |xy| \leq p.$$

**Negation:** A positive integer  $p$  is **not a pumping length** of a language  $L$  over  $\Sigma$  iff

$$\exists s ( |s| \geq p \wedge s \in L \wedge \forall x \forall y \forall z ( (s = xyz \wedge |y| > 0 \wedge |xy| \leq p) \rightarrow \exists i (i \geq 0 \wedge xy^i z \notin L) ) )$$

*Informally:*

Restating **Pumping Lemma**: If  $L$  is a regular language, then it has a pumping length.

**Contrapositive:** If  $L$  has no pumping length, then it is nonregular.

The Pumping Lemma *cannot* be used to prove that a language *is* regular.

The Pumping Lemma **can** be used to prove that a language *is not* regular.

*Extra practice:* Exercise 1.49 in the book.

**Proof strategy:** To prove that a language  $L$  is **not** regular,

- Consider an arbitrary positive integer  $p$
- Prove that  $p$  is not a pumping length for  $L$
- Conclude that  $L$  does not have *any* pumping length, and therefore it is not regular.

**Example:**  $\Sigma = \{0, 1\}$ ,  $L = \{0^n 1^n \mid n \geq 0\}$ .

Fix  $p$  an arbitrary positive integer. List strings that are in  $L$  and have length greater than or equal to  $p$ :

Pick  $s =$

Suppose  $s = xyz$  with  $|xy| \leq p$  and  $|y| > 0$ .

Then when  $i =$  ,  $xy^iz =$

**Example:**  $\Sigma = \{0, 1\}$ ,  $L = \{ww^R \mid w \in \{0, 1\}^*\}$ .

Fix  $p$  an arbitrary positive integer. List strings that are in  $L$  and have length greater than or equal to  $p$ :

Pick  $s =$

Suppose  $s = xyz$  with  $|xy| \leq p$  and  $|y| > 0$ .

Then when  $i =$  ,  $xy^iz =$

**Example:**  $\Sigma = \{0, 1\}$ ,  $L = \{0^j1^k \mid j \geq k \geq 0\}$ .

Fix  $p$  an arbitrary positive integer. List strings that are in  $L$  and have length greater than or equal to  $p$ :

Pick  $s =$

Suppose  $s = xyz$  with  $|xy| \leq p$  and  $|y| > 0$ .

Then when  $i =$  ,  $xy^iz =$

**Example:**  $\Sigma = \{0, 1\}$ ,  $L = \{0^n1^m0^n \mid m, n \geq 0\}$ .

Fix  $p$  an arbitrary positive integer. List strings that are in  $L$  and have length greater than or equal to  $p$ :

Pick  $s =$

Suppose  $s = xyz$  with  $|xy| \leq p$  and  $|y| > 0$ .

Then when  $i =$  ,  $xy^iz =$

## Week3 friday

**Theorem:** For an alphabet  $\Sigma$ , For each language  $L$  over  $\Sigma$ ,

$$\begin{array}{c} L \text{ is recognized by some DFA} \\ \text{iff} \\ L \text{ is recognized by some NFA} \\ \text{iff} \\ L \text{ is described by some regular expression} \end{array}$$

If (any, hence all) these conditions apply,  $L$  is called **regular**.

**Prove or Disprove:** There is some alphabet  $\Sigma$  for which there is some language recognized by an NFA but not by any DFA.

**Prove or Disprove:** There is some alphabet  $\Sigma$  for which there is some finite language not described by any regular expression over  $\Sigma$ .

**Prove or Disprove:** If a language is recognized by an NFA then the complement of this language is not recognized by any DFA.

| Set  | Cardinality |
|--|-------------|
| $\{0, 1\}$   |             |
| $\{0, 1\}^*$                                       |             |
| $\mathcal{P}(\{0, 1\})$                            |             |
| The set of all languages over $\{0, 1\}$           |             |
| The set of all regular expressions over $\{0, 1\}$ |             |
| The set of all regular languages over $\{0, 1\}$   |             |

**Pumping Lemma** (Sipser Theorem 1.70): If  $A$  is a regular language, then there is a number  $p$  (a *pumping length*) where, if  $s$  is any string in  $A$  of length at least  $p$ , then  $s$  may be divided into three pieces,  $s = xyz$  such that

- $|y| > 0$
- for each  $i \geq 0$ ,  $xy^iz \in A$
- $|xy| \leq p$ .

**True or False:** A pumping length for  $A = \{0, 1\}^*$  is  $p = 5$ .

**True or False:** A pumping length for  $A = \{1, 01, 001, 0001, 00001\}$  is  $p = 4$ .

**True or False:** A pumping length for  $A = \{0^j1 \mid j \geq 0\}$  is  $p = 3$ .

**True or False:** For any language  $A$ , if  $p$  is a pumping length for  $A$  and  $p' > p$ , then  $p'$  is also a pumping length for  $A$ .