

## Week6 friday

To define a Turing machine, we could give a

- **Formal definition:** the 7-tuple of parameters including set of states, input alphabet, tape alphabet, transition function, start state, accept state, and reject state; or,
- **Implementation-level definition:** English prose that describes the Turing machine head movements relative to contents of tape, and conditions for accepting / rejecting based on those contents.
- **High-level description:** description of algorithm (precise sequence of instructions), without implementation details of machine. As part of this description, can “call” and run another TM as a subroutine.

**Theorem 3.21** A language is Turing-recognizable iff some enumerator enumerates it.

**Proof:**

Assume  $L$  is enumerated by some enumerator,  $E$ , so  $L = L(E)$ . We’ll use  $E$  in a subroutine within a high-level description of a new Turing machine that we will build to recognize  $L$ .

**Goal:** build Turing machine  $M_E$  with  $L(M_E) = L(E)$ .

Define  $M_E$  as follows:  $M_E =$  “On input  $w$ ,

1. Run  $E$ . For each string  $x$  printed by  $E$ .
2. Check if  $x = w$ . If so, accept (and halt); otherwise, continue.”

Assume  $L$  is Turing-recognizable and there is a Turing machine  $M$  with  $L = L(M)$ . We’ll use  $M$  in a subroutine within a high-level description of an enumerator that we will build to enumerate  $L$ .

**Goal:** build enumerator  $E_M$  with  $L(E_M) = L(M)$ .

**Idea:** check each string in turn to see if it is in  $L$ .

*How?* Run computation of  $M$  on each string. *But:* need to be careful about computations that don’t halt.

*Recall* String order for  $\Sigma = \{0, 1\}$ :  $s_1 = \varepsilon$ ,  $s_2 = 0$ ,  $s_3 = 1$ ,  $s_4 = 00$ ,  $s_5 = 01$ ,  $s_6 = 10$ ,  $s_7 = 11$ ,  $s_8 = 000$ , ...

Define  $E_M$  as follows:  $E_M =$  “*ignore any input*. Repeat the following for  $i = 1, 2, 3, \dots$

1. Run the computations of  $M$  on  $s_1, s_2, \dots, s_i$  for (at most)  $i$  steps each
2. For each of these  $i$  computations that accept during the (at most)  $i$  steps, print out the accepted string.”

## Nondeterministic Turing machine

At any point in the computation, the nondeterministic machine may proceed according to several possibilities:  $(Q, \Sigma, \Gamma, \delta, q_0, q_{acc}, q_{rej})$  where

$$\delta : Q \times \Gamma \rightarrow \mathcal{P}(Q \times \Gamma \times \{L, R\})$$

The computation of a nondeterministic Turing machine is a tree with branching when the next step of the computation has multiple possibilities. A nondeterministic Turing machine accepts a string exactly when some branch of the computation tree enters the accept state.

Given a nondeterministic machine, we can use a 3-tape Turing machine to simulate it by doing a breadth-first search of computation tree: one tape is “read-only” input tape, one tape simulates the tape of the nondeterministic computation, and one tape tracks nondeterministic branching. Sipser page 178

Two models of computation are called **equally expressive** when every language recognizable with the first model is recognizable with the second, and vice versa.

**Church-Turing Thesis** (Sipser p. 183): The informal notion of algorithm is formalized completely and correctly by the formal definition of a Turing machine. In other words: all reasonably expressive models of computation are equally expressive with the standard Turing machine.

**Claim:** If two languages (over a fixed alphabet  $\Sigma$ ) are Turing-recognizable, then their union is as well.

**Proof using Turing machines:**

**Proof using nondeterministic Turing machines:**

**Proof using enumerators:**