

Week1 monday

We will use vocabulary that should be familiar from your discrete math and introduction to proofs classes. Some of the notation conventions may be a bit different: we will use the notation from this class' textbook¹.

Write out in words the meaning of the symbols below:

$$\{a, b, c\}$$

$$|\{a, b, a\}| = 2$$

$$|aba| = 3$$

$$(a, 3, 2, b, b)$$

¹Page references are to the 3rd edition (International) of Sipser's Introduction to the Theory of Computation, available through various sources for under \$30. You may be able to opt in to purchase a digital copy through Canvas. Copies of the book are also available for those who can't access the book to borrow from the course instructor, while supplies last (minnes@eng.ucsd.edu)

| Term | Typical symbol | Meaning |
|--------------------------------------|------------------|---|
| Alphabet | Σ, Γ | A non-empty finite set |
| Symbol over Σ | σ, b, x | An element of the alphabet Σ |
| String over Σ | u, v, w | A finite list of symbols from Σ |
| The set of all strings over Σ | Σ^* | The collection of all possible strings formed from symbols from Σ |
| (Some) language over Σ | L | (Some) set of strings over Σ |
| Empty string | ε | The string of length 0 |
| Empty set | \emptyset | The empty language |
| The power set of a set X | $\mathcal{P}(X)$ | The set of all subsets of X |
| Natural numbers | \mathcal{N} | The set of positive integers |
| Finite set | | The empty set or a set whose distinct elements can be counted by a natural number |
| Infinite set | | A set that is not finite. |
| <i>Pages 3, 4, 6, 13, 14, 53</i> | | |

| Term | Notation | Meaning |
|---|-------------------|--|
| Reverse of a string w | $w^{\mathcal{R}}$ | write w in the opposite order, if $w = w_1 \cdots w_n$ then $w^{\mathcal{R}} = w_n \cdots w_1$. Note: $\varepsilon^{\mathcal{R}} = \varepsilon$ |
| Concatenating strings x and y | xy | take $x = x_1 \cdots x_m$, $y = y_1 \cdots y_n$ and form $xy = x_1 \cdots x_m y_1 \cdots y_n$ |
| String z is a substring of string w | | there are strings u, v such that $w = uzv$ |
| String x is a prefix of string y | | there is a string z such that $y = xz$ |
| String x is a proper prefix of string y | | x is a prefix of y and $x \neq y$ |
| Shortlex order, also known as string order over alphabet Σ | | Order strings over Σ first by length and then according to the dictionary order, assuming symbols in Σ have an ordering. |

Pages 13, 14

Circle the correct choice:

A **string** over an alphabet Σ is an element of Σ^* OR a subset of Σ^* .

A **language** over an alphabet Σ is an element of Σ^* OR a subset of Σ^* .

Extra examples for practice:

With $\Sigma_1 = \{0, 1\}$ and $\Sigma_2 = \{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z\}$ and $\Gamma = \{0, 1, x, y, z\}$

An example of a string of length 3 over Σ_1 is _____

An example of a string of length 1 over Σ_2 is _____

The number of distinct strings of length 2 over Γ is _____

An example of a language over Σ_1 of size 1 is _____

An example of an infinite language over Σ_1 is _____

An example of a finite language over Γ is _____

True or False: $\varepsilon \in \Sigma_1$

True or False: ε is a string over Σ_1

True or False: ε is a language over Σ_1

True or False: ε is a prefix of some string over Σ_1

True or False: There is a string over Σ_1 that is a proper prefix of ε

The first five strings over Σ_1 in string order, using the ordering $0 < 1$:

The first five strings over Σ_2 in string order, using the usual alphabetical ordering for single letters: