

Week3 friday

Theorem: For an alphabet Σ , For each language L over Σ ,

$$\begin{array}{c} L \text{ is recognized by some DFA} \\ \text{iff} \\ L \text{ is recognized by some NFA} \\ \text{iff} \\ L \text{ is described by some regular expression} \end{array}$$

If (any, hence all) these conditions apply, L is called **regular**.

Prove or Disprove: There is some alphabet Σ for which there is some language recognized by an NFA but not by any DFA.

Prove or Disprove: There is some alphabet Σ for which there is some finite language not described by any regular expression over Σ .

Prove or Disprove: If a language is recognized by an NFA then the complement of this language is not recognized by any DFA.

| Set | Cardinality |
|--|-------------|
| $\{0, 1\}$ | |
| $\{0, 1\}^*$ | |
| $\mathcal{P}(\{0, 1\})$ | |
| The set of all languages over $\{0, 1\}$ | |
| The set of all regular expressions over $\{0, 1\}$ | |
| The set of all regular languages over $\{0, 1\}$ | |

Pumping Lemma (Sipser Theorem 1.70): If A is a regular language, then there is a number p (a *pumping length*) where, if s is any string in A of length at least p , then s may be divided into three pieces, $s = xyz$ such that

- $|y| > 0$
- for each $i \geq 0$, $xy^iz \in A$
- $|xy| \leq p$.

True or False: A pumping length for $A = \{0, 1\}^*$ is $p = 5$.

True or False: A pumping length for $A = \{1, 01, 001, 0001, 00001\}$ is $p = 4$.

True or False: A pumping length for $A = \{0^j1 \mid j \geq 0\}$ is $p = 3$.

True or False: For any language A , if p is a pumping length for A and $p' > p$, then p' is also a pumping length for A .