Week4 monday

Recap so far: In DFA, the only memory available is in the states. Automata can only "remember" finitely far in the past and finitely much information, because they can have only finitely many states. If a computation path of a DFA visits the same state more than once, the machine can't tell the difference between the first time and future times it visits this state. Thus, if a DFA accepts one long string, then it must accept (infinitely) many similar strings.

Definition A positive integer p is a **pumping length** of a language L over Σ means that, for each string $s \in \Sigma^*$, if $|s| \ge p$ and $s \in L$, then there are strings x, y, z such that

$$s = xyz$$

and

$$|y| > 0$$
, for each $i \ge 0$, $xy^i z \in L$, and $|xy| \le p$.

Negation: A positive integer p is **not a pumping length** of a language L over Σ iff

$$\exists s \ (\ |s| \ge p \land s \in L \land \forall x \forall y \forall z \ (\ (s = xyz \land |y| > 0 \land |xy| \le p \) \rightarrow \exists i (i \ge 0 \land xy^iz \notin L)) \)$$

Informally:

Restating **Pumping Lemma**: If L is a regular language, then it has a pumping length.

Contrapositive: If L has no pumping length, then it is nonregular.

The Pumping Lemma cannot be used to prove that a language is regular.

The Pumping Lemma can be used to prove that a language is not regular.

Extra practice: Exercise 1.49 in the book.

Proof strategy: To prove that a language L is **not** regular,

- Consider an arbitrary positive integer p
- Prove that p is not a pumping length for L
- Conclude that L does not have any pumping length, and therefore it is not regular.

Example: $\Sigma = \{0, 1\}, L = \{0^n 1^n \mid n \ge 0\}.$

Fix p an arbitrary positive integer. List strings that are in L and have length greater than or equal to p:

 ${\rm Pick}\ s =$

Suppose s = xyz with $|xy| \le p$ and |y| > 0.

Then when i =

 $, xy^{i}z =$

Example: $\Sigma = \{0, 1\}, L = \{ww^{\mathcal{R}} \mid w \in \{0, 1\}^*\}.$

Fix p an arbitrary positive integer. List strings that are in L and have length greater than or equal to p:

Pick s =

Suppose s = xyz with $|xy| \le p$ and |y| > 0.

Then when i =

 $, xy^iz =$

Example: $\Sigma = \{0, 1\}, L = \{0^j 1^k \mid j \ge k \ge 0\}.$

Fix p an arbitrary positive integer. List strings that are in L and have length greater than or equal to p:

Pick s =

Suppose s = xyz with $|xy| \le p$ and |y| > 0.

Then when i =

 $, xy^{i}z =$

Example: $\Sigma = \{0, 1\}, L = \{0^n 1^m 0^n \mid m, n \ge 0\}.$

Fix p an arbitrary positive integer. List strings that are in L and have length greater than or equal to p:

Pick s =

Suppose s = xyz with $|xy| \le p$ and |y| > 0.

Then when i =

 $, xy^iz =$

Extra practice:

Language	$s \in L$	$s \notin L$	Is the language regular or nonregular?
$\{a^nb^n\mid 0\leq n\leq 5\}$			
$\{b^na^n\mid n\geq 2\}$			
$\{a^mb^n\mid 0\leq m\leq n\}$			
$\{a^mb^n\mid m\geq n+3, n\geq 0\}$			
$\{b^ma^n\mid m\geq 1, n\geq 3\}$			
$\{w \in \{a, b\}^* \mid w = w^{\mathcal{R}}\}$			
$\{ww^{\mathcal{R}} \mid w \in \{a, b\}^*\}$			

Week3 friday

Theorem: For an alphabet Σ , For each language L over Σ ,

L is recognized by some DFA iff L is recognized by some NFA iff L is described by some regular expression

If (any, hence all) these conditions apply, L is called **regular**.

Prove or Disprove: There is some alphabet Σ for which there is some language recognized by an NFA but not by any DFA.

Prove or **Disprove**: There is some alphabet Σ for which there is some finite language not described by any regular expression over Σ .

Prove or Disprove: If a language is recognized by an NFA then the complement of this language is not recognized by any DFA.

Set	Cardinality
$\{0,1\}$	
$\{0,1\}^*$	
$\mathcal{P}(\{0,1\})$	
The set of all languages over $\{0,1\}$	
The set of all regular expressions over $\{0,1\}$	
The set of all regular languages over $\{0,1\}$	

Pumping Lemma (Sipser Theorem 1.70): If A is a regular language, then there is a number p (a pumping length) where, if s is any string in A of length at least p, then s may be divided into three pieces, s = xyz such that

- |y| > 0
- for each $i \ge 0$, $xy^iz \in A$
- $|xy| \leq p$.

True or False: A pumping length for $A = \{0, 1\}^*$ is p = 5.

True or False: A pumping length for $A = \{1, 01, 001, 0001, 00001\}$ is p = 4.

True or False: A pumping length for $A = \{0^j 1 \mid j \ge 0\}$ is p = 3.

True or False: For any language A, if p is a pumping length for A and p' > p, then p' is also a pumping length for A.