Monday - Memorial Day

No class today.

Wednesday

Recall: A is **mapping reducible to** B, written $A \leq_m B$, means there is a computable function $f : \Sigma^* \to \Sigma^*$ such that *for all* strings x in Σ^*,

$$x \in A \qquad \text{if and only if} \qquad f(x) \in B.$$

True or False: $\overline{A_{TM}} \leq_m \overline{HALT_{TM}}$

True or False: $HALT_{TM} \leq_m A_{TM}$.

Theorem (Sipser 5.28): If $A \leq_m B$ and B is recognizable, then A is recognizable.

Proof:

Corollary: If $A \leq_m B$ and A is unrecognizable, then B is unrecognizable.

Strategy:

(i) To prove that a recognizable language R is undecidable, prove that $A_{TM} \leq_m R$.

(ii) To prove that a co-recognizable language U is undecidable, prove that $\overline{A_{TM}} \leq_m U$, i.e. that $A_{TM} \leq_m \overline{U}$.

$$E_{TM} = \{\langle M \rangle \mid M \text{ is a Turing machine and } L(M) = \emptyset\}$$

Example string in E_{TM} is _____ . Example string not in E_{TM} is _____ .

E_{TM} is   decidable / undecidable   and   recognizable / unrecognizable .

$\overline{E_{TM}}$ is   decidable / undecidable   and   recognizable / unrecognizable .

Claim: _____ $\leq_m \overline{E_{TM}}$.

Proof: Need computable function $F : \Sigma^* \to \Sigma^*$ such that $x \in A_{TM}$ iff $F(x) \notin E_{TM}$. Define

   $F = $ " On input x,

   1. Type-check whether $x = \langle M, w \rangle$ for some TM M and string w. If so, move to step 2; if not, output

   2. Construct the following machine M_x':

   3. Output $\langle M_x' \rangle$."

Verifying correctness:

Input string	Output string
$\langle M, w \rangle$ where $w \in L(M)$	
$\langle M, w \rangle$ where $w \notin L(M)$	
x not encoding any pair of TM and string	

Review: Week 9 Wednesday

Please complete the review quiz questions on Gradescope about mapping reductions.

Pre class reading for next time: Introduction to Chapter 7.

Friday

Recall: A is **mapping reducible to** B, written $A \leq_m B$, means there is a computable function $f : \Sigma^* \to \Sigma^*$ such that *for all* strings x in Σ^*,

$$x \in A \qquad \text{if and only if} \qquad f(x) \in B.$$

$$EQ_{TM} = \{\langle M, M' \rangle \mid M \text{ and } M' \text{ are both Turing machines and } L(M) = L(M')\}$$

Example string in EQ_{TM} is _____ . Example string not in EQ_{TM} is _____ .

EQ_{TM} is   decidable / undecidable   and   recognizable / unrecognizable .

$\overline{EQ_{TM}}$ is   decidable / undecidable   and   recognizable / unrecognizable .

To prove, show that _____ $\leq_m EQ_{TM}$ and that _____ $\leq_m \overline{EQ_{TM}}$.

Verifying correctness:

Input string	Output string
$\langle M, w \rangle$ where M halts on w	
$\langle M, w \rangle$ where M loops on w	
x not encoding any pair of TM and string	

In practice, computers (and Turing machines) don't have infinite tape, and we can't afford to wait unboundedly long for an answer. "Decidable" isn't good enough - we want "Efficiently decidable".

For a given algorithm working on a given input, how long do we need to wait for an answer? How does the running time depend on the input in the worst-case? average-case? We expect to have to spend more time on computations with larger inputs.

A language is **recognizable** if _____

A language is **decidable** if _____

A language is **efficiently decidable** if _____

A function is **computable** if _____

A function is **efficiently computable** if _____

Definition (Sipser 7.1): For M a deterministic decider, its **running time** is the function $f : \mathbb{N} \to \mathbb{N}$ given by

$$f(n) = \text{max number of steps } M \text{ takes before halting, over all inputs of length } n$$

Definition (Sipser 7.7): For each function $t(n)$, the **time complexity class** $TIME(t(n))$, is defined by

$$TIME(t(n)) = \{L \mid L \text{ is decidable by a Turing machine with running time in } O(t(n))\}$$

An example of an element of $TIME(1)$ is

An example of an element of $TIME(n)$ is

Note: $TIME(1) \subseteq TIME(n) \subseteq TIME(n^2)$

Definition (Sipser 7.12) : P is the class of languages that are decidable in polynomial time on a deterministic 1-tape Turing machine

$$P = \bigcup_k TIME(n^k)$$

Compare to exponential time: brute-force search.

Theorem (Sipser 7.8): Let $t(n)$ be a function with $t(n) \geq n$. Then every $t(n)$ time deterministic multitape Turing machine has an equivalent $O(t^2(n))$ time deterministic 1-tape Turing machine.

Review: Week 9 Friday

Please complete the review quiz questions on Gradescope about complexity.

Pre class reading for next time: Skim Chapter 7.