HW4 : Pushdown Automata and Context-free grammars

CSE105Sp23

Due: May 2nd at 5pm (no penalty late submission until 8am next morning), via Gradescope

In this assignment:

You will practice with the definition of pushdown automata and context-free grammars and reason about regular and context-free languages.

Resources: To review the topics you are working with for this assignment, see the class material from Week 3 through Week 4. We will post frequently asked questions and our answers to them in a pinned Piazza post.

Reading and extra practice problems: Sipser Sections 2.1, 2.2. Chapter 2 exercises 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.9, 2.10, 2.11, 2.12, 2.13, 2.16, 2.17.

Key Concepts: Pushdown automata, stack, context-free grammars, derivations, context-free languages.

For all HW assignments: Weekly homework may be done individually or in groups of up to 3 students. You may switch HW partners for different HW assignments. The lowest HW score will not be included in your overall HW average. Please ensure your name(s) and PID(s) are clearly visible on the first page of your homework submission and then upload the PDF to Gradescope. If working in a group, submit only one submission per group: one partner uploads the submission through their Gradescope account and then adds the other group member(s) to the Gradescope submission by selecting their name(s) in the "Add Group Members" dialog box. You will need to re-add your group member(s) every time you resubmit a new version of your assignment. Each homework question will be graded either for correctness (including clear and precise explanations and justifications of all answers) or fair effort completeness. You may only collaborate on HW with CSE 105 students in your group; if your group has questions about a HW problem, you may ask in drop-in help hours or post a private post (visible only to the Instructors) on Piazza.

All submitted homework for this class must be typed. You can use a word processing editor if you like (Microsoft Word, Open Office, Notepad, Vim, Google Docs, etc.) but you might find it useful to take this opportunity to learn LaTeX. LaTeX is a markup language used widely in computer science and mathematics. The homework assignments are typed using LaTeX and you can use the source files as templates for typesetting your solutions. To generate state diagrams of machines, we recommend using Flap.js or JFLAP. Photographs of clearly hand-drawn diagrams may also be used. We recommend that you submit early drafts to Gradescope so that in case of any technical difficulties, at least some of your work is present. You may update your submission as many times as you'd like up to the deadline.

Integrity reminders

- Problems should be solved together, not divided up between the partners. The homework is designed to give you practice with the main concepts and techniques of the course, while getting to know and learn from your classmates.
- You may not collaborate on homework with anyone other than your group members. You may ask questions about the homework in office hours (of the instructor, TAs, and/or tutors) and on Piazza (as private notes viewable only to the Instructors). You *cannot* use any online resources about the course content other than the class material from this quarter – this is primarily to ensure that we all use consistent notation and definitions (aligned with the textbook) and also to protect the learning experience you will have when the 'aha' moments of solving the problem authentically happen.
- Do not share written solutions or partial solutions for homework with other students in the class who are not in your group. Doing so would dilute their learning experience and detract from their success in the class.

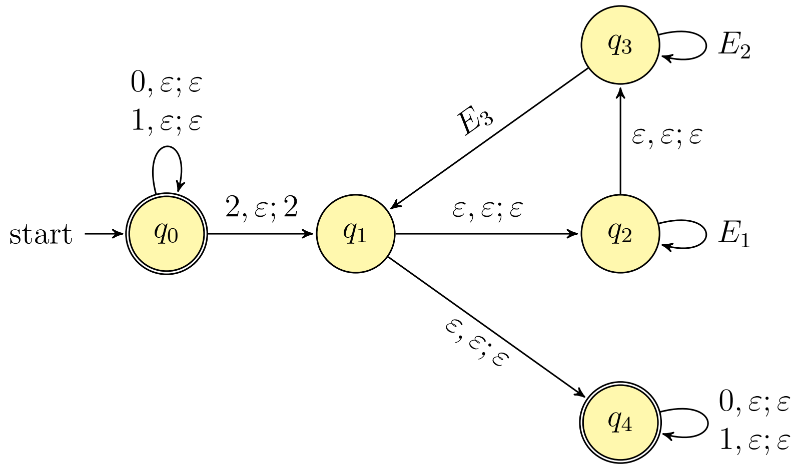
You will submit this assignment via Gradescope (https://www.gradescope.com) in the assignment called "hw4CSE105Sp23".

Requests from your TAs and tutors To help us with grading please

- Start each question on a new page.
- Label the start of each solution with **Answer**.

Assigned questions

1. **A PDA with multiple possibilities** (22 points):
   Consider the PDA with input and stack alphabet $\Gamma = \{0, 1, 2\}$ whose "unfinished" state diagram is given below:



There are three labels (E_1, E_2, and E_3) on the edges that are unspecified. To be precise, each E_i is of the form "$x, y; z$" where $x, y, z \in \Gamma_\varepsilon$ (recall $\Gamma_\varepsilon = \Gamma \cup \{\varepsilon\}$).

(a) (*Graded for correctness*) [1] Prove that (no matter how the labels E_1, E_2, E_3 are specified), the language recognized by this PDA is infinite. A complete solution will include a precise description of an infinite collection of strings each of which is accepted by the PDA, with a precise and clear description of the accepting computation of the PDA on each of these strings.

(b) (*Graded for completeness*) [2] Prove/Disprove: Over all the possible choices for the labels E_1, E_2, E_3, this PDA can only recognize finitely many languages. Justify your solution by referring back to the relevant definitions.

(c) (*Graded for correctness*) Recall that for $L \subseteq \Sigma^*$ with $\Sigma = \{0, 1\}$, we define

$$\begin{aligned}\textsc{Rep}(L) &:= \{w \in \Gamma^* \mid \text{between every pair of successive 2's in } w \text{ is a string in } L\} \\ &= \{w \in \Gamma^* \mid \text{for all } v \in \Sigma^* \text{ if } 2v2 \in \textsc{Substring}(\{w\}), \text{ then } v \in L\}\end{aligned}$$

where for all languages $K \subseteq \Gamma^*$ we let

$$\textsc{Substring}(K) := \{w \in \Gamma^* \mid \text{there exist } a, b \in \Gamma^* \text{ such that } awb \in K\}.$$

Determine how to set the labels E_1, E_2, E_3 so that the language of the PDA is

$$\textsc{Rep}(\{0^n 1^m \mid n \geq 0, m \geq 0\})$$

[1] This means your solution will be evaluated not only on the correctness of your answers, but on your ability to present your ideas clearly and logically. You should explain how you arrived at your conclusions, using mathematically sound reasoning. Whether you use formal proof techniques or write a more informal argument for why something is true, your answers should always be well-supported. Your goal should be to convince the reader that your results and methods are sound.

[2] This means you will get full credit so long as your submission demonstrates honest effort to answer the question. You will not be penalized for incorrect answers. To demonstrate your honest effort in answering the question, we ask that you include your attempt to answer *each* part of the question. If you get stuck with your attempt, you can still demonstrate your effort by explaining where you got stuck and what you did to try to get unstuck.

In addition to specifying each E_i, a complete justification will include a precise description of why this choice of the E_i's means that the PDA recognizes the language indicated.

(d) (*Graded for correctness*) Determine how to set the labels E_1, E_2, E_3 so that the language of the PDA is

$$\text{Rep}(\{0^n 1^n \mid n \geq 0\})$$

In addition to specifying each E_i, a complete justification will include a precise description of why this choice of the E_i's means that the PDA recognizes the language indicated.

2. **Grammar practice** (12 points):
For each of the languages listed below, define a context-free grammar $G = (V, \Sigma, R, S)$ that generates the language. Instead of formally justifying your grammar, illustrate it by giving **two examples** of strings in the language and their derivations using your grammar and **one example** of a string not in the language with an explanation of why it cannot appear on the right side of any derivation in your grammar. Choose your examples so they are different enough to illustrate the role of as many of the variables in your grammar as possible.

   (a) (*Graded for correctness*) $\text{Rep}(\{0^n 1^n \mid n \geq 0\})$

   (b) (*Graded for correctness*) $\{1^n = 1^a + 1^b \in \{1, =, +\}^* \mid a, b, n \geq 1 \text{ such that } a + b = n\}$

3. **Substrings and regularity** (16 points):
For this problem, we fix the alphabet $\Gamma = \{0, 1, 2\}$. Recall the definition of the function Substring from Problem 1.

   (a) (*Graded for correctness*) Prove that $\text{Substring}(\{0^n 1^n \mid n \geq 0\})$ is regular. A complete solution will include a precise definition of a DFA, NFA, or regular expression that recognizes or describes it, along with a brief justification of your construction by explaining the role each state plays in the machine and referring back to relevant definitions.

   (b) (*Graded for correctness*) Prove that $\text{Substring}(\{0^n 1^n 2^n \mid n \geq 0\})$ is not regular.

   (c) (*Graded for completeness*) Is $\text{Substring}(\{0^n 1^n 2^n \mid n \geq 0\})$ context-free? Informally justify your answer, referring to class discussions and/or the textbook.