2D Heat Equations

IT4110E - Scientific Computing

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Mini Project

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INTRODUCTION

- The heat equation is an important partial differential equation which describes **the distribution of heat** (or variation in temperature) in a given region over time.
- "Heat equations" can be used to quantitatively explain the process of heat conduction.
- They control how the temperature changes over time in a particular system.

INTRODUCTION

Practical application

describe the heat distribution

- Weather Forecasting
- Building Energy Efficiency
- Medical Applications
- Industrial Processes
- Geothermal Energy
- Thermal Management in Electronics

INTRODUCTION

Problem statement

In this report, our group will be presenting the process of heat conduction and simulating the process to show the resulting temperature changes in the object involved.

In the simulation, we make the underlying assumption that the environment we are considering is characterized by ideal conditions, meaning it embodies a set of circumstances that are considered to be perfect or optimal for the purpose of our analysis.

PROBLEM FORMULATION

The heat conduction are analyzed in a solid object by considering a scenario where a metal plate is heated on one side while the other side is at a lower temperature

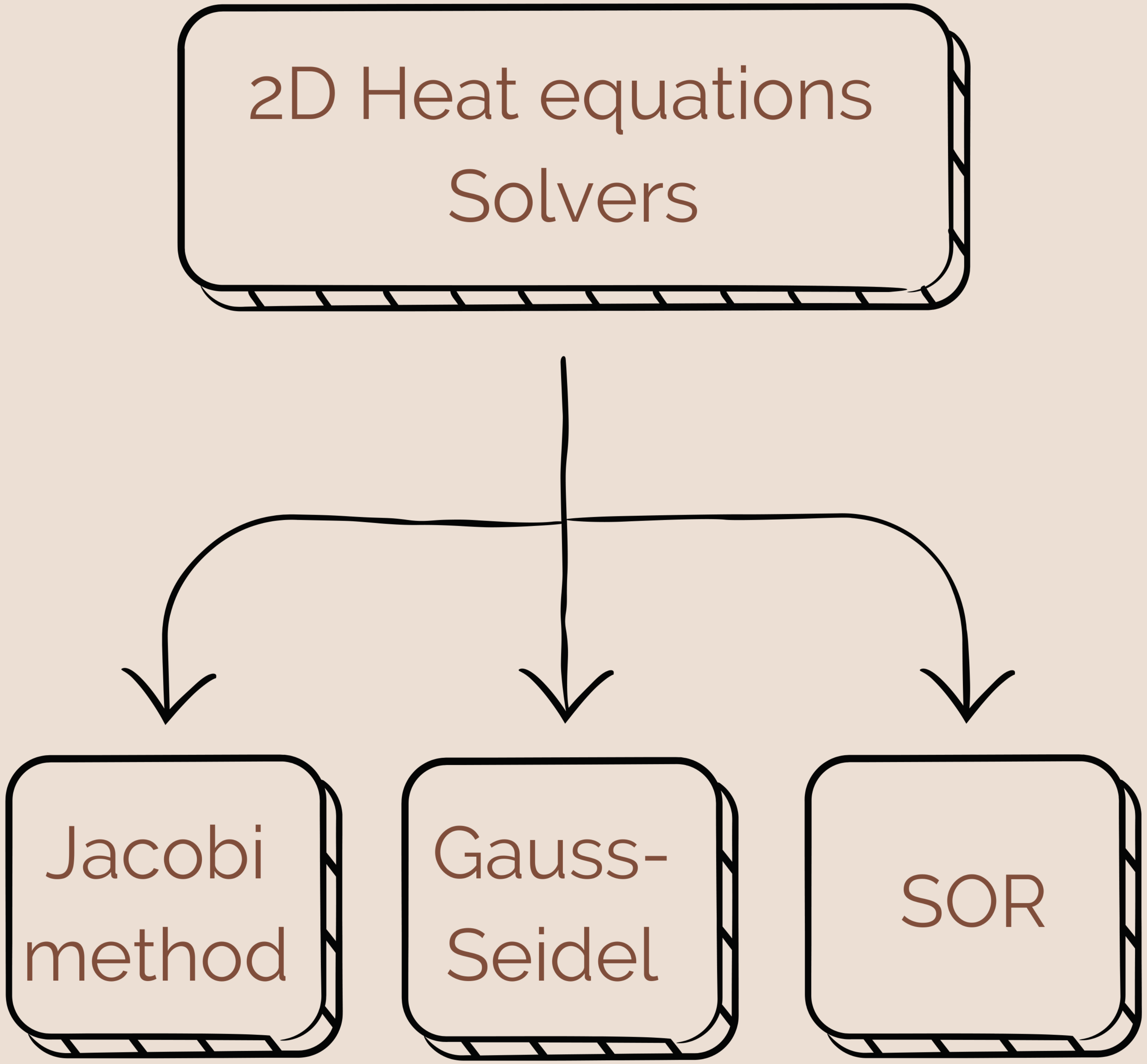
By solving the 2D heat equation below where D is the material-specific quantity, we can determine how temperature distributes across the plate over time.

D: the material-specific quantity

T: the temperature distribution

$$\frac{\partial T}{\partial t} = D\nabla^2 T$$

PROBLEM FORMULATION



PROBLEM FORMULATION

Jacobi method

- For the Jacobi solver, all the values in the right-hand side are taken from the previous iteration.
- The Jacobi iteration are expected to suffer from a relatively large number of iterations needed before convergence..

$$T_{i,j}^{k+1} = \frac{T_{i-1,j}^{k} + T_{i,j-1}^{k} + T_{i+1,j}^{k} + T_{i,j+1}^{k}}{4}$$

PROBLEM FORMULATION

Gauss-Seidel method

The Jacobi iteration method lacks efficiency, which results from the previous iteration always used to update a point, even the new results have been available.

- The Gauss-Seidel iteration method is derived from a simple concept.
- Updating new results as soon as they become available.

$$T_{i,j}^{k+1} = \frac{T_{i-1,j}^{k+1} + T_{i,j-1}^{k+1} + T_{i+1,j}^{k} + T_{i,j+1}^{k}}{4}$$

PROBLEM FORMULATION

Successive over relaxation (SOR)

The final step in the progression from Jacobi and Gauss-Seidel methods involves implementing an additional concept that proves to be highly efficient.

- The SOR solver is a modification of the Gauss-Seidel solver in which the values are updated according to the relaxation parameter.
- ω: parameter which shows how the mixing strength from the previous step affects that of the next step. In our project, we take the parameter **ω = 1.26** to prevent errors.

$$T_{i,j}^{k+1} = (1-\omega)T_{i,j}^{k} + \omega\frac{T_{i-1,j}^{k+1} + T_{i,j-1}^{k+1} + T_{i+1,j}^{k} + T_{i,j+1}^{k}}{4}$$

IMPLEMENTATION DETAILS

Initializing variables and the temperature distribution

Run the solver until the maximum difference falls below the error tolerance

The difference approximation of the Laplacian operator (FD) is computed

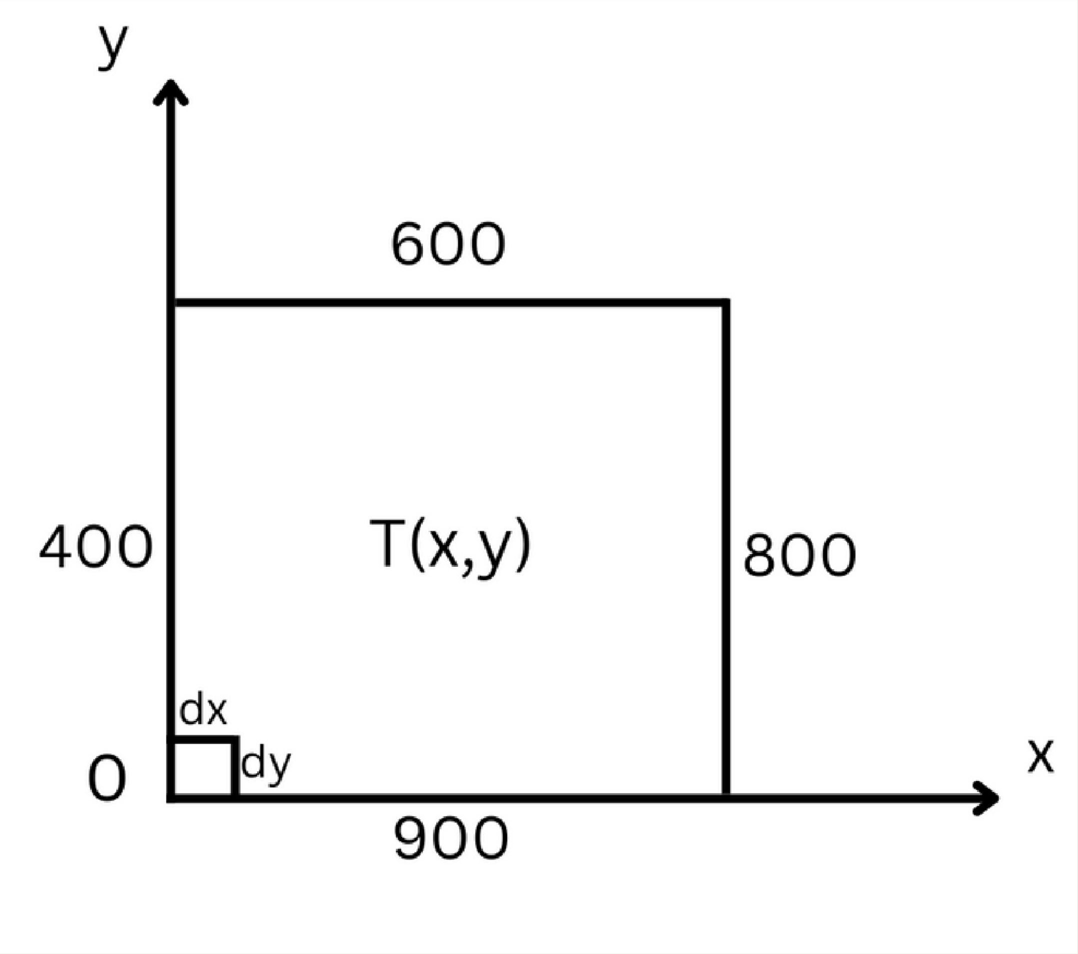
Using Forward Euler method to simulate time evolution of the temperature distribution

> **Results:** The plots are visualized to show the number of iteration times and the value of the center point throughout the process.

RESULTS AND EVALUATION

Experiment

T: temperature
D: the material-specific quantity
m, n: the number of blocks in the x and y directions
TT: the total time of simulation (s)
dt: the time of each step size (s)
dx, dy: the spatial step size (meter)

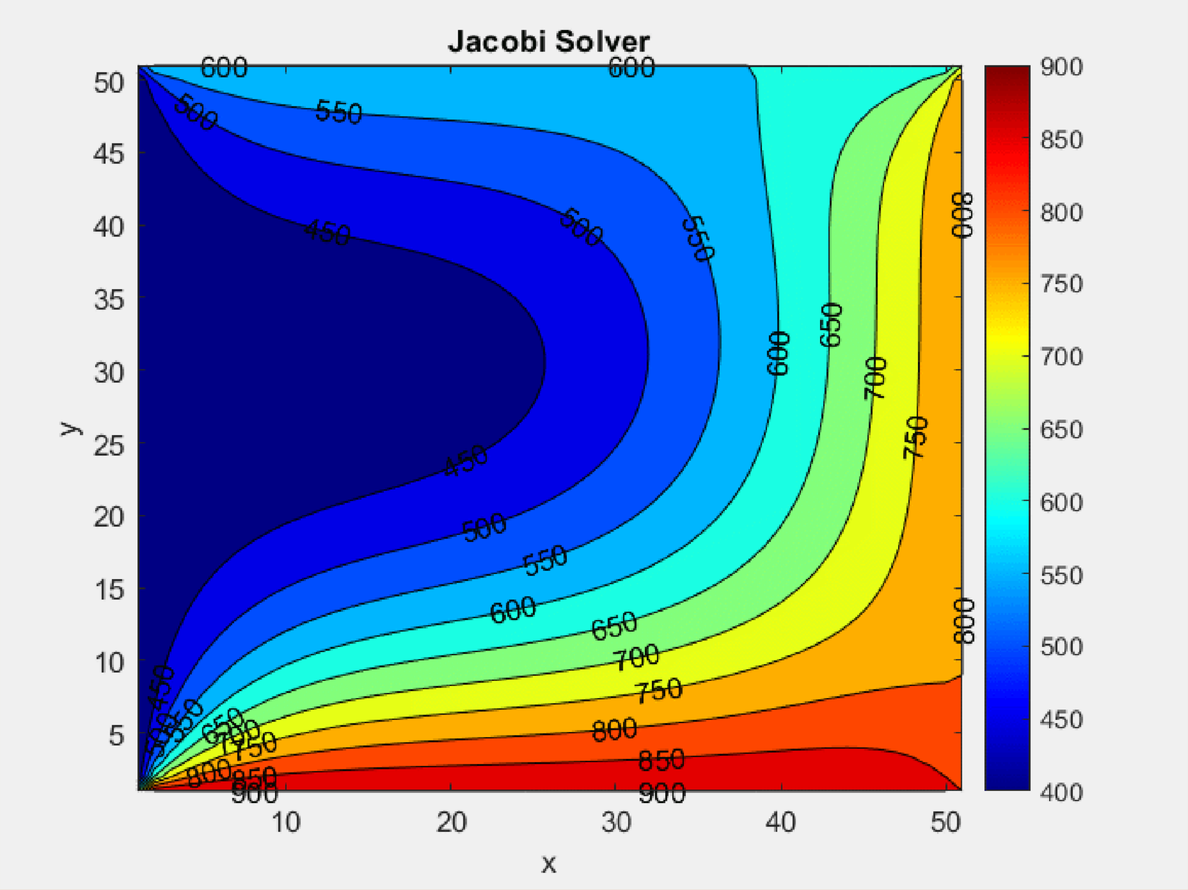


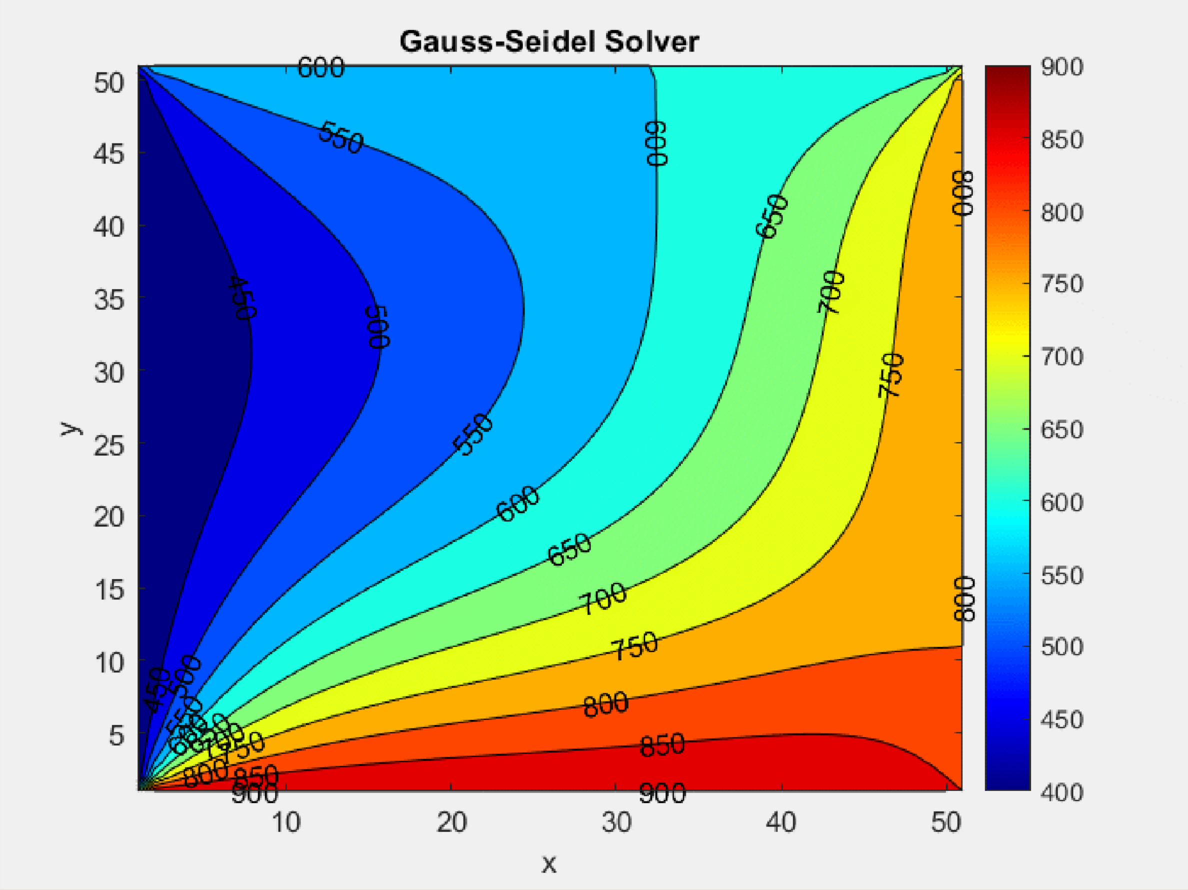
RESULTS AND EVALUATION

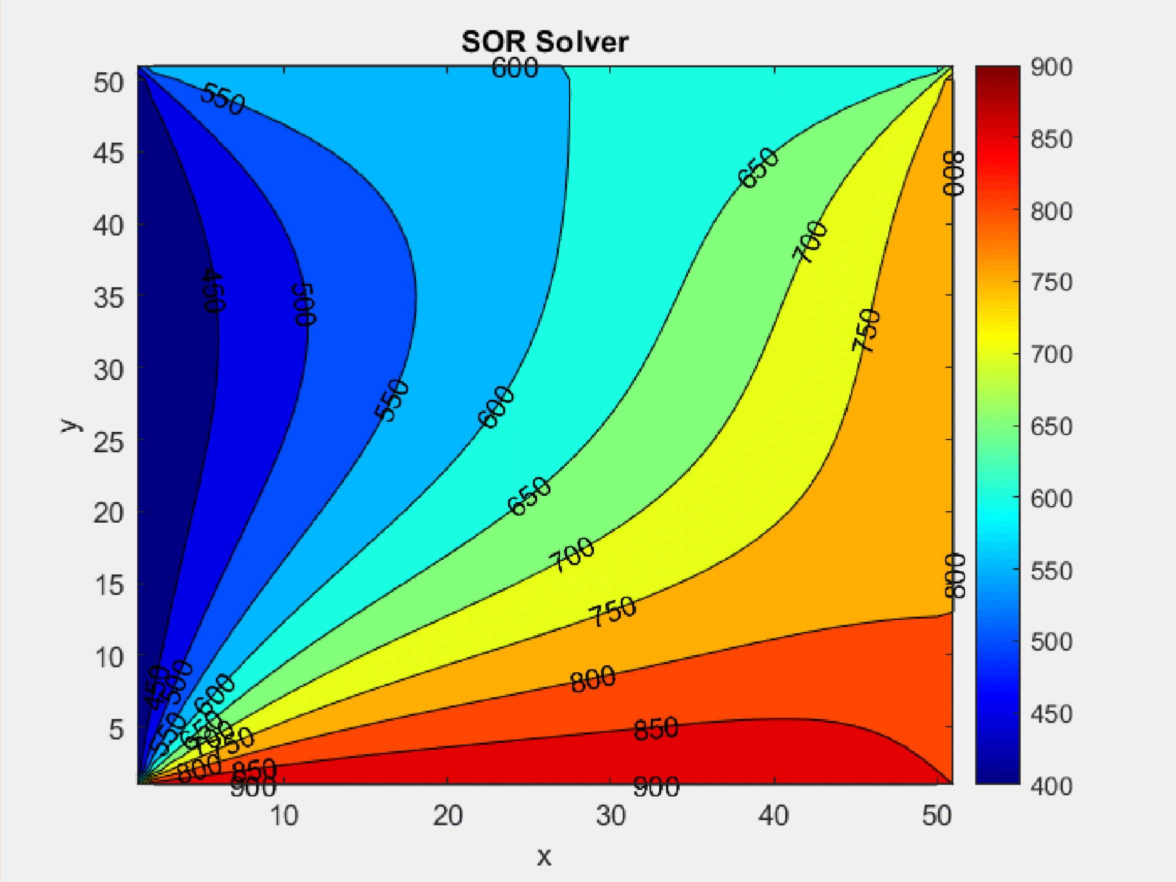
Testcase	Solver	Initial center value	Last center value	Number of iterations
D = 0.1, m = 20, n = 20, TT = 1, dx = 0.1, dt = 0.01	Jacobi solver	635.60	668.03	368
	Gauss-Seidel solver	655.45	673.99	263
	SOR solver	664.11	674.91	209
D = 0.1, m = 50, n = 50, TT = 1, dx = 0.1, dt = 0.02	Jacobi solver	421.45	462.53	789
	Gauss-Seidel solver	548.99	579.43	598
	SOR solver	601.46	626.47	452
D = 0.1, m = 100, n = 100, TT = 1, dx = 0.1, dt = 0.05	Jacobi solver	11.61	675.25	27831
	Gauss-Seidel solver	133.11	150.29	683
	SOR solver	375.45	390.82	790
D = 0.1, m = 100, n = 100, TT = 2, dx = 0.1, dt = 0.05	Jacobi solver	11.61	675.25	72335
	Gauss-Seidel solver	133.11	167.48	703
	SOR solver	375.45	411.21	823
D = 0.1, m = 100, n = 100, TT = 5, dx = 0.1, dt = 0.05	Jacobi solver	11.61	674.75	205848
	Gauss-Seidel solver	133.11	218.01	763
	SOR solver	375.45	467.03	930

Results of all solvers in several testcases

RESULTS AND EVALUATION

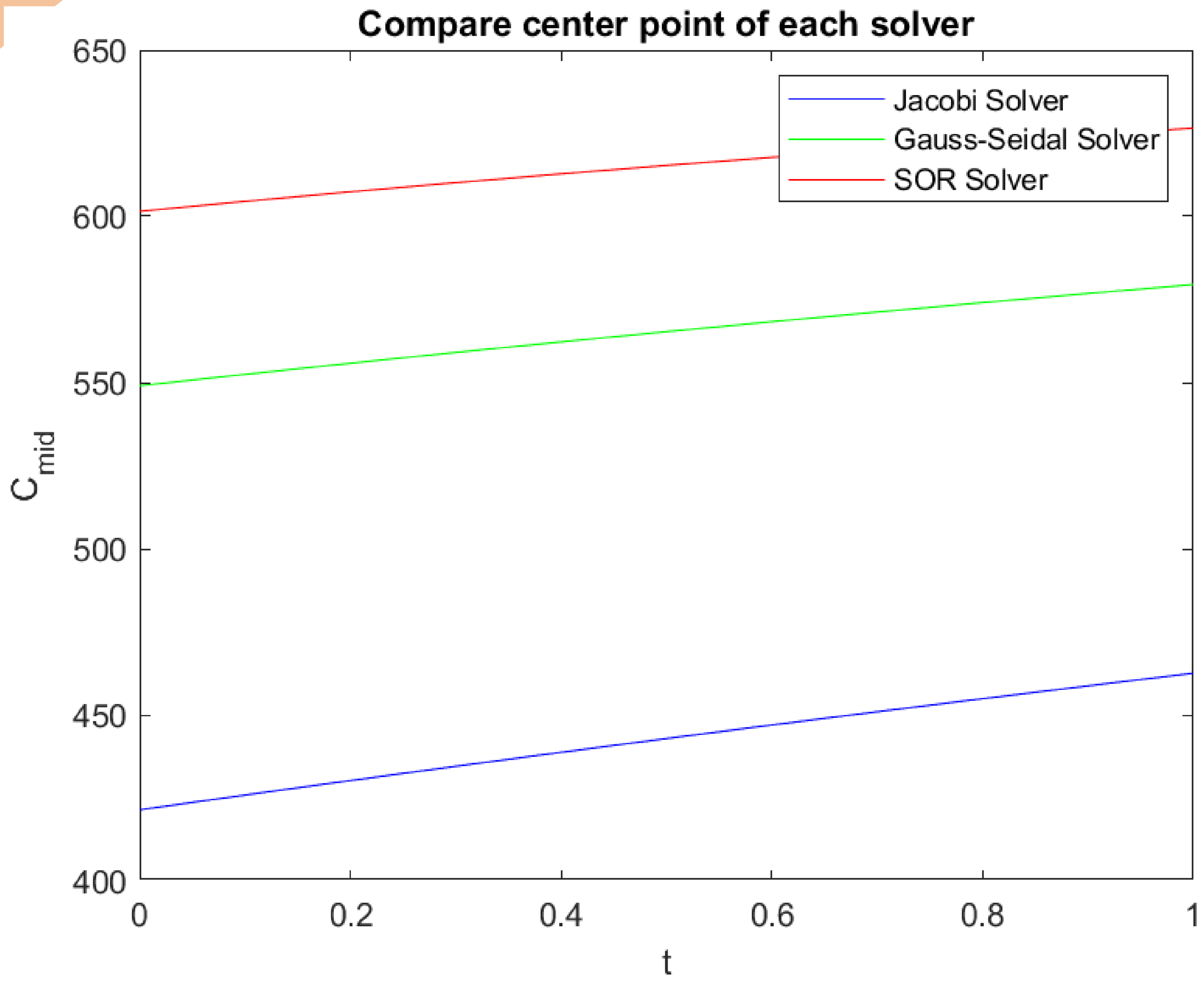






Three different solvers and their results visualization for finding the center point

RESULTS AND EVALUATION



CONCLUSIONS

Our study focused on solving the 2D heat equation using different iterative solvers, in which SOR is the best methods among all.

2D Heat equation have usually been studying as a model of Heat Transfer Analysis, Climate Modeling or Thermal Therapy

Thank you for listening!

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