# Ngoc Ha

# ST 557 - HW 5

## Problem 1 ¶

In [1]:
```
track <- read.csv('TrackData.csv')
head(track)
```

| Country | Abbrev | X100m.s | X200m.s | X400m.s | X800m.m | X1500m.m | X5000m.m | X10000m.m | M |
|---|---|---|---|---|---|---|---|---|---|
| Argentina | ARG | 10.39 | 20.81 | 46.84 | 1.81 | 3.70 | 14.04 | 29.36 | |
| Australia | AUL | 10.31 | 20.06 | 44.84 | 1.74 | 3.57 | 13.28 | 27.66 | |
| Austria | AUS | 10.44 | 20.81 | 46.82 | 1.79 | 3.60 | 13.26 | 27.72 | |
| Belgium | BEL | 10.34 | 20.68 | 45.04 | 1.73 | 3.60 | 13.22 | 27.45 | |
| Bermuda | BER | 10.28 | 20.58 | 45.91 | 1.80 | 3.75 | 14.68 | 30.55 | |
| Brazil | BRA | 10.22 | 20.43 | 45.21 | 1.73 | 3.66 | 13.62 | 28.62 | |

### (1a)

```r
In [2]: distances <- track[,3:10]
S <- cov(distances)
R <- cor(distances)
S
R
```

| | X100m.s | X200m.s | X400m.s | X800m.m | X1500m.m | X5000m.m | X10000m. |
|---|---|---|---|---|---|---|---|
| **X100m.s** | 0.12350249 | 0.20902182 | 0.43069956 | 0.016920438 | 0.03836684 | 0.17441020 | 0.40184 |
| **X200m.s** | 0.20902182 | 0.41557024 | 0.79905603 | 0.033115455 | 0.07788771 | 0.35913859 | 0.81171 |
| **X400m.s** | 0.43069956 | 0.79905603 | 2.12290020 | 0.080743131 | 0.18974209 | 0.90887976 | 2.07341 |
| **X800m.m** | 0.01692044 | 0.03311545 | 0.08074313 | 0.004055758 | 0.00911532 | 0.04406209 | 0.10004 |
| **X1500m.m** | 0.03836684 | 0.07788771 | 0.18974209 | 0.009115320 | 0.02430774 | 0.11592929 | 0.26343 |
| **X5000m.m** | 0.17441020 | 0.35913859 | 0.90887976 | 0.044062088 | 0.11592929 | 0.64185811 | 1.41154 |
| **X10000m.m** | 0.40184545 | 0.81171145 | 2.07341549 | 0.100049327 | 0.26343721 | 1.41154798 | 3.26789 |
| **Marathon.m** | 1.68601222 | 3.54620963 | 9.47785704 | 0.473903333 | 1.24516296 | 6.89104852 | 15.73218 |

| | X100m.s | X200m.s | X400m.s | X800m.m | X1500m.m | X5000m.m | X10000m.m | Ma |
|---|---|---|---|---|---|---|---|---|
| **X100m.s** | 1.0000000 | 0.9226384 | 0.8411468 | 0.7560278 | 0.7002382 | 0.6194618 | 0.6325389 | 0 |
| **X200m.s** | 0.9226384 | 1.0000000 | 0.8507270 | 0.8066265 | 0.7749513 | 0.6953770 | 0.6965391 | 0 |
| **X400m.s** | 0.8411468 | 0.8507270 | 1.0000000 | 0.8701714 | 0.8352694 | 0.7786139 | 0.7872045 | 0 |
| **X800m.m** | 0.7560278 | 0.8066265 | 0.8701714 | 1.0000000 | 0.9180442 | 0.8635939 | 0.8690489 | 0 |
| **X1500m.m** | 0.7002382 | 0.7749513 | 0.8352694 | 0.9180442 | 1.0000000 | 0.9281140 | 0.9346970 | 0 |
| **X5000m.m** | 0.6194618 | 0.6953770 | 0.7786139 | 0.8635939 | 0.9281140 | 1.0000000 | 0.9746354 | 0 |
| **X10000m.m** | 0.6325389 | 0.6965391 | 0.7872045 | 0.8690489 | 0.9346970 | 0.9746354 | 1.0000000 | 0 |
| **Marathon.m** | 0.5199490 | 0.5961837 | 0.7049905 | 0.8064764 | 0.8655492 | 0.9321884 | 0.9431763 | 1 |

$R$ is more appropriate to use for PCA, as all the variables are on the same scale.

## (1b) Eigendecomposition of S

In [3]:
```r
eiDecS <- eigen(S)
eiDecS
```

```
eigen() decomposition
$values
[1] 8.991362e+01 1.412626e+00 2.598442e-01 1.094203e-01 2.730060e-02
[6] 1.273280e-02 2.243554e-03 4.455645e-04

$vectors
             [,1]        [,2]         [,3]         [,4]         [,5]
[1,] -0.019865407 -0.21068958 -0.029041979 -0.358784470  0.190181784
[2,] -0.041554499 -0.35892579 -0.018390126 -0.833534544 -0.048582165
[3,] -0.110631838 -0.82786251 -0.377669011  0.396041212 -0.012020033
[4,] -0.005487699 -0.02317490  0.005341591 -0.009568087 -0.011107487
[5,] -0.014386822 -0.04465255  0.050004337 -0.015981502 -0.043222520
[6,] -0.079308444 -0.12996134  0.336448522  0.018873808 -0.909186992
[7,] -0.181098994 -0.29885393  0.848722695  0.134662690  0.364239482
[8,] -0.972787446  0.18080736 -0.141872114 -0.028425488  0.006575083
             [,6]         [,7]          [,8]
[1,]  0.886865894 -0.052444908 -0.0139585779
[2,] -0.409969944  0.062270182 -0.0037828046
[3,] -0.047663812  0.020389912 -0.0094695712
[4,] -0.007204523 -0.261227847  0.9648302746
[5,] -0.067333230 -0.959092660 -0.2622644611
[6,]  0.184076191  0.052548542 -0.0001130819
[7,] -0.068113893  0.045771467  0.0045055042
[8,]  0.003532208 -0.001055127 -0.0008700758
```

## (1c) Eigendecomposition of R

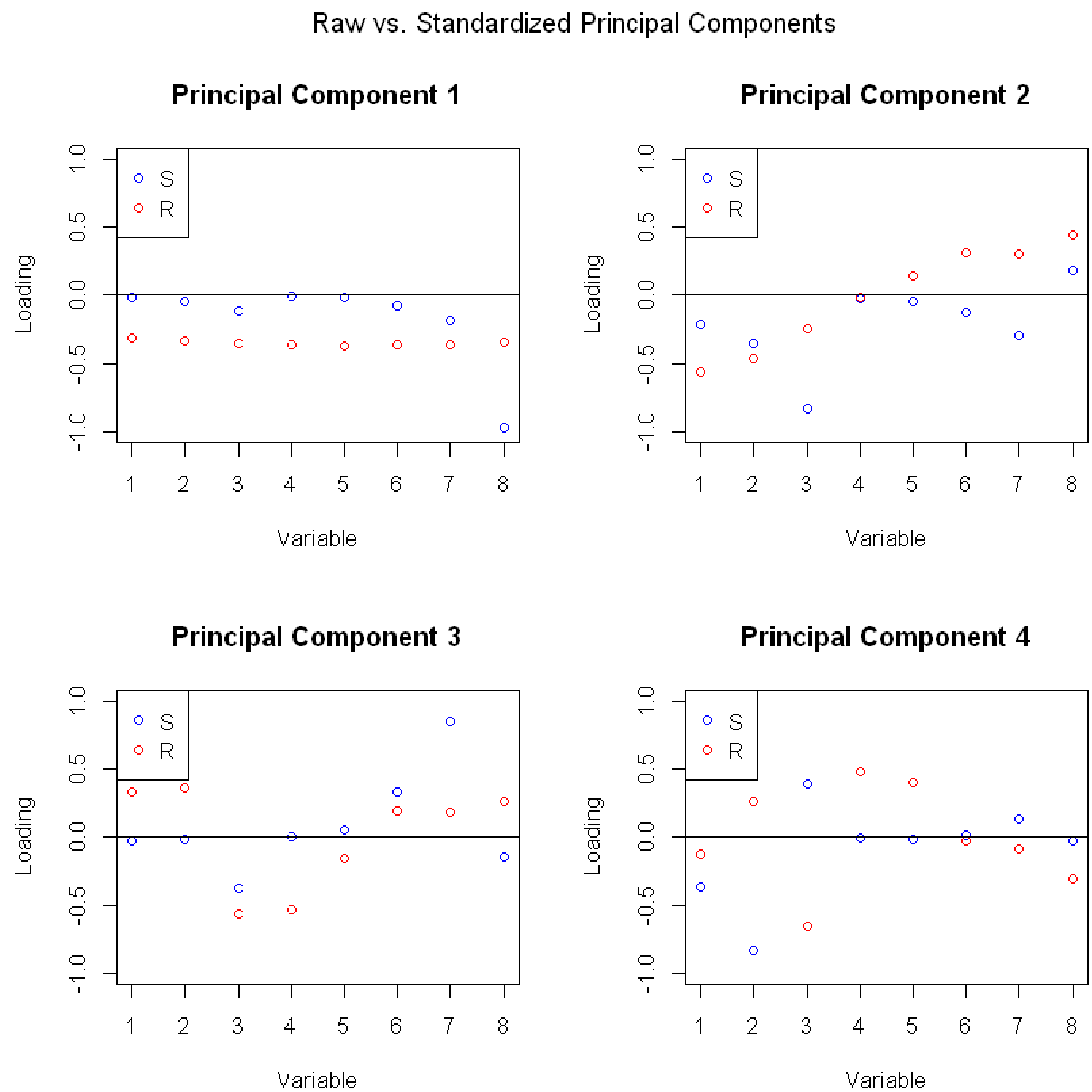
In [4]:
```r
eiDecR <- eigen(R)
eiDecR
```

```
eigen() decomposition
$values
[1] 6.62214613 0.87761829 0.15932114 0.12404939 0.07988027 0.06796515 0.04641
953
[8] 0.02260010

$vectors
           [,1]        [,2]       [,3]        [,4]       [,5]       [,6]
[1,] -0.3175565 -0.56687750  0.3322620 -0.12762827  0.2625555 -0.5937042
[2,] -0.3369792 -0.46162589  0.3606567  0.25911576 -0.1539571  0.6561367
[3,] -0.3556454 -0.24827331 -0.5604674 -0.65234077 -0.2183229  0.1566252
[4,] -0.3686841 -0.01242993 -0.5324823  0.47999895  0.5400528 -0.0146918
[5,] -0.3728099  0.13979665 -0.1534427  0.40451039 -0.4877151 -0.1578430
[6,] -0.3643741  0.31203045  0.1897643 -0.02958755 -0.2539792 -0.1412987
[7,] -0.3667726  0.30685985  0.1817517 -0.08006862 -0.1331764 -0.2190168
[8,] -0.3419261  0.43896267  0.2632087 -0.29951213  0.4979283  0.3152849
             [,7]          [,8]
[1,]  0.136241260 -0.1055416752
[2,] -0.112639528  0.0960543222
[3,] -0.002853707  0.0001272032
[4,] -0.238016094  0.0381651151
[5,]  0.610011482 -0.1392909844
[6,] -0.591298850 -0.5466969221
[7,] -0.176871021  0.7967952190
[8,]  0.398822209 -0.1581638575
```

### (1d)

```
In [5]: par(mfrow=c(2,2), oma=c(0,0,2,0))
for(i in 1:4){
    plot(1:8, eiDecS$vec[,i], xlab="Variable", ylab="Loading", main=paste("Principal Component ", i, sep=""), ylim=c(-1, 1), col = 'blue')
    points(1:8, eiDecR$vec[,i], xlab="Variable", ylab="Loading", main=paste("Principal Component ", i, sep=""), ylim=c(-1, 1), col='red')
    legend("topleft", legend=c("S", "R"), col=c("blue", "red"), pch=21)
        abline(h=0)
}
mtext("Raw vs. Standardized Principal Components", outer=T)
```




## (1e)

For $S$: marathon times (variable 8) dominate the covariances among running times, which makes sense a marathon is much longer than other formats.

### (1f)

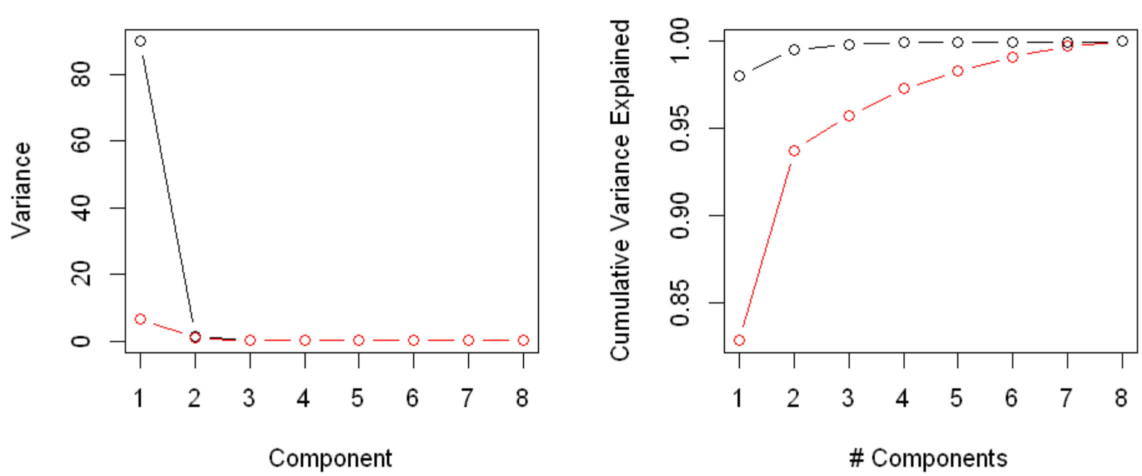
For $R$: the standardized loadings are roughly equal. This principal component shows how fast or slow a country is in general.

### (1g)

The second principal components of $R$ seems to show the contrast between fast marathoners and fast sprinters.

### (1h)

In [6]:
```
options(repr.plot.width=8, repr.plot.height=4)
par(mfrow=c(1,2), oma=c(0,0,0,0))
plot(1:8, eiDecS$val, type="b", xlab="Component", ylab="Variance")
lines(1:8, eiDecR$val, type="b", col='red')
plot(1:8, cumsum(eiDecR$val)/sum(eiDecR$val), type="b", col='red', xlab="# Com
ponents", ylab="Cumulative Variance Explained")
lines(1:8, cumsum(eiDecS$val)/sum(eiDecS$val), type="b")
```




### (1i)

I'd want to keep the first 3 principal components of R, as they explain 95% of the variance, and the variances seem to taper off starting from the 4th principal component.

# Problem 2

In [7]:
```
nyse <- read.csv('NYSEData.csv')
head(nyse)
```

| JPMorgan | Citibank | WellsFargo | RoyalDutchShell | ExxonMobil |
|---|---|---|---|---|
| 0.0130338 | -0.0078431 | -0.0031889 | -0.0447693 | 0.0052151 |
| 0.0084862 | 0.0166886 | -0.0062100 | 0.0119560 | 0.0134890 |
| -0.0179153 | -0.0086393 | 0.0100360 | 0.0000000 | -0.0061428 |
| 0.0215589 | -0.0034858 | 0.0174353 | -0.0285917 | -0.0069534 |
| 0.0108225 | 0.0037167 | -0.0101345 | 0.0291900 | 0.0409751 |
| 0.0101713 | -0.0121978 | -0.0083768 | 0.0137083 | 0.0029895 |

## (2a)

In [10]:
```
S <- cov(nyse)
eiDecS <- eigen(sampCov)
eiDecS
```

```
eigen() decomposition
$values
[1] 0.0013676780 0.0007011596 0.0002538024 0.0001426026 0.0001188868

$vectors
          [,1]       [,2]        [,3]       [,4]        [,5]
[1,] 0.2228228  0.6252260  0.32611218 -0.6627590  0.11765952
[2,] 0.3072900  0.5703900 -0.24959014  0.4140935 -0.58860803
[3,] 0.1548103  0.3445049 -0.03763929  0.4970499  0.78030428
[4,] 0.6389680 -0.2479475 -0.64249741 -0.3088689  0.14845546
[5,] 0.6509044 -0.3218478  0.64586064  0.2163758 -0.09371777
```

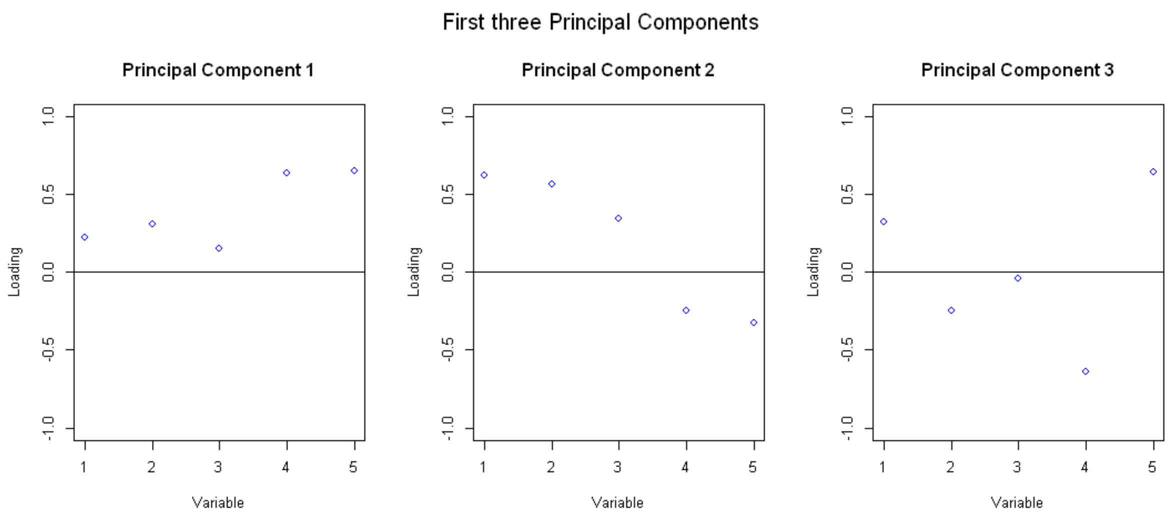
## (2b)

In [11]:
```
propVar <- sum(eiDecS$values[1:3])/sum(eiDecS$values)
cat(propVar, "of total variance is explained by the first 3 principal componen
ts")
```

```
0.8988095 of total variance is explained by the first 3 principal components
```

## (2c)

```
In [14]: options(repr.plot.width=9, repr.plot.height=4)
par(mfrow=c(1,3), oma=c(0,0,2,0))
for(i in 1:3){
    plot(1:5, eiDecS$vec[,i], xlab="Variable", ylab="Loading", main=paste("Pri
ncipal Component ", i, sep=""), ylim=c(-1, 1), col = 'blue')
        abline(h=0)
}
mtext("First three Principal Components", outer=T)
```




First PC demonstrates the performance of the market in general. Second PC shows the contrast between stock performance of Finance sector and Energy sector. Third PC focuses on stocks with higher market capitalization (JP Morgan, Shell, Exxon).

# Problem 4

```
In [1]: corr <- as.matrix(read.csv('PhysioData.csv'))
head(corr)
```

| | weight | height | physact | ldl | alb | crt | |
|---|---|---|---|---|---|---|---|
| **weight** | 1.000000000 | 0.54775881 | -0.02803745 | 0.003570772 | 0.04673810 | 0.25399164 | -0.14944 |
| **height** | 0.547758814 | 1.00000000 | 0.06493509 | -0.156549811 | 0.08819927 | 0.36485715 | -0.29476 |
| **physact** | -0.028037453 | 0.06493509 | 1.00000000 | -0.031756690 | 0.01476370 | -0.02632625 | -0.00900 |
| **ldl** | 0.003570772 | -0.15654981 | -0.03175669 | 1.000000000 | 0.12453340 | -0.13135504 | 0.19678 |
| **alb** | 0.046738095 | 0.08819927 | 0.01476370 | 0.124533399 | 1.00000000 | 0.04428462 | -0.06429 |
| **crt** | 0.253991639 | 0.36485715 | -0.02632625 | -0.131355039 | 0.04428462 | 1.00000000 | -0.15481 |

## (4a)

```
In [2]: eiDecCor <- eigen(corr)
```

**i. m = 2**

```
In [3]: L2 <- cbind(eiDecCor$val[1]*eiDecCor$vec[,1], eiDecCor$val[2]*eiDecCor$vec[,2]
        )
        errorCov2 <- corr - L2%*%t(L2)
        specVars2 <- diag(errorCov2)
        L2_table <- data.frame(L2,specVars2)
        colnames(L2_table) <- c("Factor 1 Loadings","Factor 2 Loadings","Specific Vari
        ances")
        L2_table
```

| | Factor 1 Loadings | Factor 2 Loadings | Specific Variances |
|---|---|---|---|
| **weight** | -1.0452745 | -0.1403353 | -0.11229274 |
| **height** | -1.3346010 | -0.1595277 | -0.80660894 |
| **physact** | -0.1265833 | 0.1814451 | 0.95105432 |
| **ldl** | 0.3651604 | 0.1599542 | 0.84107255 |
| **alb** | -0.2255166 | 0.0941445 | 0.94027907 |
| **crt** | -0.8112691 | -0.5135738 | 0.07808445 |
| **plt** | 0.7143848 | 0.1338270 | 0.47174474 |
| **sbp** | 0.2741563 | -0.7725324 | 0.32803205 |
| **aai** | -0.4325669 | 0.8560635 | 0.08004119 |
| **fev** | -1.1450629 | 0.2331262 | -0.36551677 |
| **dsst** | -0.1974087 | 0.7093896 | 0.45779616 |
| **atrophy** | -0.2404455 | -0.4284885 | 0.75858351 |

**Factor 1**: height, weight and fev contribute the most to factor 1. Interpretation: factor 1 correlates with the size of a person.

**Factor 2** shows the contrast between **systolic blood pressure** vs. **ankle-to-arm sbp ratio** and **coginite performance**.

**ii. m = 3**

In [4]:
```
L3 <- cbind(eiDecCor$val[1]*eiDecCor$vec[,1], eiDecCor$val[2]*eiDecCor$vec[,2]
, eiDecCor$val[3]*eiDecCor$vec[,3])
errorCov3 <- corr - L3%*%t(L3)
specVars3 <- diag(errorCov3)
L3_table <- data.frame(L3,specVars3)
colnames(L3_table) <- c("Factor 1 Loadings","Factor 2 Loadings","Factor 3 Load
ings","Specific Variances")
L3_table
```

| | Factor 1 Loadings | Factor 2 Loadings | Factor 3 Loadings | Specific Variances |
|---|---|---|---|---|
| **weight** | -1.0452745 | -0.1403353 | 0.254226398 | -0.17692380 |
| **height** | -1.3346010 | -0.1595277 | 0.041405944 | -0.80832339 |
| **physact** | -0.1265833 | 0.1814451 | -0.351258636 | 0.82767169 |
| **ldl** | 0.3651604 | 0.1599542 | 0.756781166 | 0.26835482 |
| **alb** | -0.2255166 | 0.0941445 | 0.606733197 | 0.57215390 |
| **crt** | -0.8112691 | -0.5135738 | -0.007366511 | 0.07803019 |
| **plt** | 0.7143848 | 0.1338270 | 0.299009230 | 0.38233822 |
| **sbp** | 0.2741563 | -0.7725324 | 0.023902569 | 0.32746072 |
| **aai** | -0.4325669 | 0.8560635 | -0.156188604 | 0.05564631 |
| **fev** | -1.1450629 | 0.2331262 | 0.027012246 | -0.36624644 |
| **dsst** | -0.1974087 | 0.7093896 | 0.239375966 | 0.40049530 |
| **atrophy** | -0.2404455 | -0.4284885 | 0.326469376 | 0.65200126 |

**Factor 3** correlates with the **cholesterol level** in the subject's blood.

## (4b)

**i. m = 2**

In [5]:
```
res2 <- corr - (L2%*%t(L2)+errorCov2)
res2
```

| | weight | height | physact | ldl | alb | crt |
|---|---|---|---|---|---|---|
| **weight** | 0.000000e+00 | 0.000000e+00 | 1.040834e-17 | -3.903128e-18 | 1.387779e-17 | 0.000000e+00 |
| **height** | 0.000000e+00 | 0.000000e+00 | 0.000000e+00 | 0.000000e+00 | 0.000000e+00 | -5.551115e-17 |
| **physact** | 1.040834e-17 | 0.000000e+00 | 0.000000e+00 | 0.000000e+00 | 0.000000e+00 | 0.000000e+00 |
| **ldl** | -3.903128e-18 | 0.000000e+00 | 0.000000e+00 | 0.000000e+00 | 1.387779e-17 | 0.000000e+00 |
| **alb** | 1.387779e-17 | 0.000000e+00 | 0.000000e+00 | 1.387779e-17 | 0.000000e+00 | 0.000000e+00 |
| **crt** | 0.000000e+00 | -5.551115e-17 | 0.000000e+00 | 0.000000e+00 | 0.000000e+00 | 0.000000e+00 |
| **plt** | 2.775558e-17 | 0.000000e+00 | 3.469447e-18 | 0.000000e+00 | 0.000000e+00 | 0.000000e+00 |
| **sbp** | 5.204170e-18 | 0.000000e+00 | -1.040834e-17 | 0.000000e+00 | 3.469447e-18 | -7.806256e-18 |
| **aai** | 0.000000e+00 | -1.387779e-17 | 0.000000e+00 | 0.000000e+00 | 1.040834e-17 | 0.000000e+00 |
| **fev** | 0.000000e+00 | 0.000000e+00 | 0.000000e+00 | 0.000000e+00 | 0.000000e+00 | 5.551115e-17 |
| **dsst** | 0.000000e+00 | 6.938894e-18 | 6.938894e-18 | 3.361027e-18 | 0.000000e+00 | 0.000000e+00 |
| **atrophy** | 0.000000e+00 | -2.775558e-17 | 0.000000e+00 | 1.387779e-17 | 0.000000e+00 | 0.000000e+00 |

**ii. m = 3**

In [6]:
```
res3 <- corr - (L3%*%t(L3)+errorCov3)
res3
```

| | weight | height | physact | ldl | alb | crt |
|---|---|---|---|---|---|---|
| **weight** | 0.000000e+00 | 0.000000e+00 | -3.469447e-18 | -3.903128e-18 | 1.387779e-17 | 0.000000e+00 |
| **height** | 0.000000e+00 | 0.000000e+00 | 0.000000e+00 | 0.000000e+00 | 0.000000e+00 | -5.551115e-17 |
| **physact** | -3.469447e-18 | 0.000000e+00 | 0.000000e+00 | -6.938894e-18 | 1.040834e-17 | 0.000000e+00 |
| **ldl** | -3.903128e-18 | 0.000000e+00 | -6.938894e-18 | 0.000000e+00 | 0.000000e+00 | -2.775558e-17 |
| **alb** | 1.387779e-17 | 0.000000e+00 | 1.040834e-17 | 0.000000e+00 | 0.000000e+00 | 0.000000e+00 |
| **crt** | 0.000000e+00 | -5.551115e-17 | 0.000000e+00 | -2.775558e-17 | 0.000000e+00 | 0.000000e+00 |
| **plt** | 2.775558e-17 | 0.000000e+00 | -1.040834e-17 | 0.000000e+00 | 0.000000e+00 | 0.000000e+00 |
| **sbp** | 5.204170e-18 | 0.000000e+00 | -1.040834e-17 | 0.000000e+00 | 3.469447e-18 | -7.806256e-18 |
| **aai** | 0.000000e+00 | -1.387779e-17 | 0.000000e+00 | 0.000000e+00 | -3.469447e-18 | 0.000000e+00 |
| **fev** | 0.000000e+00 | 0.000000e+00 | 0.000000e+00 | 0.000000e+00 | 0.000000e+00 | -5.551115e-17 |
| **dsst** | -6.938894e-18 | 6.938894e-18 | 6.938894e-18 | -3.577867e-18 | 1.387779e-17 | 0.000000e+00 |
| **atrophy** | 2.775558e-17 | -2.775558e-17 | 0.000000e+00 | 0.000000e+00 | 6.938894e-18 | 0.000000e+00 |

## (4c)

### i. m = 2

```
In [9]: mlfa2 <- factanal(covmat=corr, factors=2, rotation="none")
mlfa2
```

```
Call:
factanal(factors = 2, covmat = corr, rotation = "none")

Uniquenesses:
 weight  height physact     ldl     alb     crt     plt     sbp     aai      f
ev
  0.675   0.084   0.988   0.974   0.990   0.828   0.903   0.801   0.526   0.5
69
   dsst atrophy
  0.883   0.960

Loadings:
        Factor1 Factor2
weight   0.570
height   0.956
physact
ldl     -0.160
alb
crt      0.385  -0.154
plt     -0.310
sbp             -0.438
aai      0.105   0.681
fev      0.610   0.241
dsst             0.340
atrophy  0.121  -0.160

               Factor1 Factor2
SS loadings      1.930   0.889
Proportion Var   0.161   0.074
Cumulative Var   0.161   0.235

The degrees of freedom for the model is 43 and the fit was 0.1927
```

**Factor 1** correlates with the **size** of the subject. Factor 2 shows a contrast between **systolic blood pressure** and **arm-to-length blood pressure ratio**.

**ii. m = 3**

```
In [10]: mlfa3 <- factanal(covmat=corr, factors=3, rotation="none")
         mlfa3
```

```
Call:
factanal(factors = 3, covmat = corr, rotation = "none")

Uniquenesses:
 weight  height physact     ldl     alb     crt     plt     sbp     aai       f
ev
  0.659   0.097   0.988   0.005   0.969   0.821   0.881   0.800   0.517   0.5
64
   dsst atrophy
  0.884   0.960

Loadings:
        Factor1 Factor2 Factor3
weight   0.584
height   0.935  -0.165
physact
ldl              0.997
alb      0.119   0.124
crt      0.369  -0.135  -0.158
plt     -0.281   0.200
sbp                     -0.437
aai      0.102           0.685
fev      0.613           0.235
dsst                     0.337
atrophy  0.119          -0.160

               Factor1 Factor2 Factor3
SS loadings      1.858   1.106   0.891
Proportion Var   0.155   0.092   0.074
Cumulative Var   0.155   0.247   0.321

The degrees of freedom for the model is 33 and the fit was 0.1187
```

**Factor 1** correlates with the **size** of the subject. Factor 2 correlates with **cholesterol level** in the subject's blood. Factor 3 shows a contrast between **systolic blood pressure** and **arm-to-length blood pressure ratio**.

## (4d)

#### i. m = 2

In [11]:
```
mle2Fit <- mlfa2$load %*% t(mlfa2$load) + diag(mlfa2$uni)
mle2Res <- corr - mle2Fit
mle2Res
```

| | weight | height | physact | ldl | alb | crt | plt |
|---|---|---|---|---|---|---|---|
| **weight** | -2.295501e-06 | 2.316174e-03 | -6.600738e-02 | 9.474229e-02 | -6.364201e-03 | 3.444910e-02 | 2.744664e-02 |
| **height** | 2.316174e-03 | 5.795176e-08 | 4.071739e-03 | -3.665135e-03 | 4.081529e-04 | -8.229091e-03 | 1.002223e-03 |
| **physact** | -6.600738e-02 | 4.071739e-03 | -4.414328e-07 | -2.110953e-02 | 5.157061e-03 | -3.857461e-02 | 1.400327e-02 |
| **ldl** | 9.474229e-02 | -3.665135e-03 | -2.110953e-02 | 1.515341e-07 | 1.394198e-01 | -6.982532e-02 | 1.472004e-01 |
| **alb** | -6.364201e-03 | 4.081529e-04 | 5.157061e-03 | 1.394198e-01 | -3.679141e-07 | 1.442844e-02 | -3.435965e-02 |
| **crt** | 3.444910e-02 | -8.229091e-03 | -3.857461e-02 | -6.982532e-02 | 1.442844e-02 | 2.907133e-06 | -3.956903e-02 |
| **plt** | 2.744664e-02 | 1.002223e-03 | 1.400327e-02 | 1.472004e-01 | -3.435965e-02 | -3.956903e-02 | 2.774056e-06 |
| **sbp** | 5.876081e-02 | -8.040394e-03 | 4.604408e-02 | -4.377930e-02 | 8.276838e-03 | -3.556760e-02 | -7.107719e-03 |
| **aai** | 2.805056e-02 | -4.595808e-03 | 1.118192e-02 | -4.186461e-02 | -5.241403e-03 | 1.837698e-02 | -2.818814e-02 |
| **fev** | -1.168937e-02 | 1.341863e-03 | 4.041874e-02 | 3.369120e-02 | -7.839537e-04 | 2.507144e-02 | 1.605121e-02 |
| **dsst** | 3.688456e-02 | -2.267373e-03 | -6.381315e-02 | 6.538309e-03 | 5.216409e-02 | -1.080193e-01 | 3.696668e-02 |
| **atrophy** | -4.485594e-03 | 7.105641e-05 | -7.647922e-02 | 7.147184e-04 | 4.267490e-02 | 8.309776e-02 | -2.852986e-02 |

### ii. m = 3

In [12]:
```
mle3Fit <- mlfa3$load %*% t(mlfa3$load) + diag(mlfa3$uni)
mle3Res <- corr - mle3Fit
mle3Res
```

| | weight | height | physact | ldl | alb | crt | plt |
|---|---|---|---|---|---|---|---|
| **weight** | 1.769072e-06 | 1.715552e-03 | -6.367751e-02 | 4.930838e-05 | -2.203323e-02 | 3.741820e-02 | 1.466025e-02 |
| **height** | 1.715552e-03 | -1.434591e-07 | 5.151534e-03 | -8.841224e-06 | -7.031566e-04 | -8.562872e-03 | -1.919501e-04 |
| **physact** | -6.367751e-02 | 5.151534e-03 | -9.850525e-07 | 1.740750e-05 | 8.126802e-03 | -3.980536e-02 | 1.700254e-02 |
| **ldl** | 4.930838e-05 | -8.841224e-06 | 1.740750e-05 | -1.488487e-07 | 7.889531e-05 | -3.531039e-05 | 5.173403e-06 |
| **alb** | -2.203323e-02 | -7.031566e-04 | 8.126802e-03 | 7.889531e-05 | 2.104358e-07 | 2.320952e-02 | -5.476550e-02 |
| **crt** | 3.741820e-02 | -8.562872e-03 | -3.980536e-02 | -3.531039e-05 | 2.320952e-02 | 2.870971e-08 | -2.811321e-02 |
| **plt** | 1.466025e-02 | -1.919501e-04 | 1.700254e-02 | 5.173403e-06 | -5.476550e-02 | -2.811321e-02 | 2.484733e-07 |
| **sbp** | 6.251392e-02 | -9.205252e-03 | 4.505925e-02 | -9.551266e-05 | 1.464274e-02 | -3.922948e-02 | -2.902823e-04 |
| **aai** | 3.219461e-02 | -4.912088e-03 | 9.644994e-03 | -4.221580e-06 | 3.195008e-04 | 1.661071e-02 | -2.148498e-02 |
| **fev** | -2.015677e-02 | 1.870694e-03 | 4.145989e-02 | 3.735397e-05 | -7.565595e-03 | 2.440522e-02 | 1.214759e-02 |
| **dsst** | 3.581256e-02 | -3.353576e-03 | -6.361512e-02 | -1.814167e-04 | 5.108954e-02 | -1.079476e-01 | 3.622663e-02 |
| **atrophy** | -5.758711e-03 | 4.482889e-04 | -7.637260e-02 | 4.381138e-05 | 4.217393e-02 | 8.238260e-02 | -2.842938e-02 |

## (4e)

In [17]:
```
cat("Determinant of PCA residual matrix:", det(res3))
cat("\nDeterminant of MLE residual matrix:", det(mle3Res))
```

```
Determinant of PCA residual matrix: 3.02803e-200
Determinant of MLE residual matrix: 3.220028e-56
```

PCA Factor Analysis has smaller residuals => better for this problem.

## (4f)

The factors are similar in both methods (PCA and MLE) for both m = 2 and m = 3.

In [ ]:

```

```

3

SS ≡ sum of squared entries

a)

$$SS[S-(\tilde{L}\tilde{L}^T+\tilde{\Psi})] = SS(S-\tilde{L}\tilde{L}^T) - SS(\mathrm{diag}(S-\tilde{L}\tilde{L}^T)) \leqslant SS(S-\tilde{L}\tilde{L}^T)$$ ✓

b)

$$S-\tilde{L}\tilde{L}^T = \hat{\lambda}_{m+1}\hat{e}_{m+1}\hat{e}^T_{m+1}+\ldots+\hat{\lambda}_p\hat{e}_p\hat{e}_p^T = \hat{P}_{(2)}\hat{\Lambda}_{(2)}\hat{P}_{(2)}^T, \text{ where:}$$

$$\hat{P}_{(2)} = [\hat{e}_{m+1}^T, \ldots \hat{e}_p^T],$$

and $$\hat{\Lambda}_{(2)} = \begin{bmatrix} \hat{\lambda}_{m+1} & & 0 \\ 0 & \ddots & \\ & & \hat{\lambda}_p \end{bmatrix}$$

c)

$$SS(S-\tilde{L}\tilde{L}^T) = tr[(S-\tilde{L}\tilde{L}^T)(S-\tilde{L}\tilde{L}^T)^T]$$

$$= tr[\hat{P}_{(2)}\hat{\Lambda}_{(2)}\hat{\Lambda}_{(2)}\hat{P}_{(2)}^T]$$

$$= tr[\hat{\Lambda}_{(2)}\hat{\Lambda}_{(2)}]$$

$$= \hat{\lambda}_{m+1}^2+\ldots+\hat{\lambda}_p^2$$

$$\rightarrow SS[S-(\tilde{L}\tilde{L}^T+\tilde{\Psi})] \leqslant SS(S-\tilde{L}\tilde{L}^T) = \lambda_{m+1}^2+\ldots+\lambda_p^2$$ ✓