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HW 3 - ST 557

Problem 1

In [14]:
```
scores <- read.csv("TestScores.csv")
head(scores)
```

SocSciHist	Verbal	Science
468	41	26
428	39	26
514	53	21
547	67	33
614	61	27
501	67	29

In [15]:
```
X1 <- scores$SocSciHist
X2 <- scores$Verbal
X3 <- scores$Science
```

(1a) $H_0 : \mu = [500, 50, 30]^T; H_A : \mu \neq [500, 50, 30]^T; \alpha = 0.05$

In [16]:
```
n <- length(scores[,1])
muNull <- c(500, 50, 30)
muVec <- c(mean(X1), mean(X2), mean(X3))
alpha <- 0.05
covar <- var(scores)
covarInv <- solve(covar)
```

In [17]:
```
t2 <- n*t(muVec-muNull)%*%covarInv%*%(muVec-muNull)
t2stat <- (n-3)/((n-1)*3)*t2
Fcrit <- df(0.05, df1 = 3, df2 = n-3)
```

In [18]:
```
cat("Reject Null hypothesis:", t2stat>Fcrit)
```

```
Reject Null hypothesis: TRUE
```

$[500, 50, 30]^T$ lies outside of the 95% Confidence Interval, so there's a reason to believe that the population of students in 2011 is scoring differently from previous years.

(1b) Lengths and directions

In [19]:
```
eiDec <- eigen(covar)
eiDec
```

```
eigen() decomposition
$values
[1] 5878.79165   63.83510   14.59806

$vectors
           [,1]         [,2]         [,3]
[1,] 0.99390539  0.103731534 -0.037307396
[2,] 0.10344339 -0.994589227 -0.009577815
[3,] 0.03809906 -0.005660238  0.999257936
```

In [20]:
```
eiVec1 <- eiDec$vectors[,1]
eiVec2 <- eiDec$vectors[,2]
eiVec3 <- eiDec$vectors[,3]
```

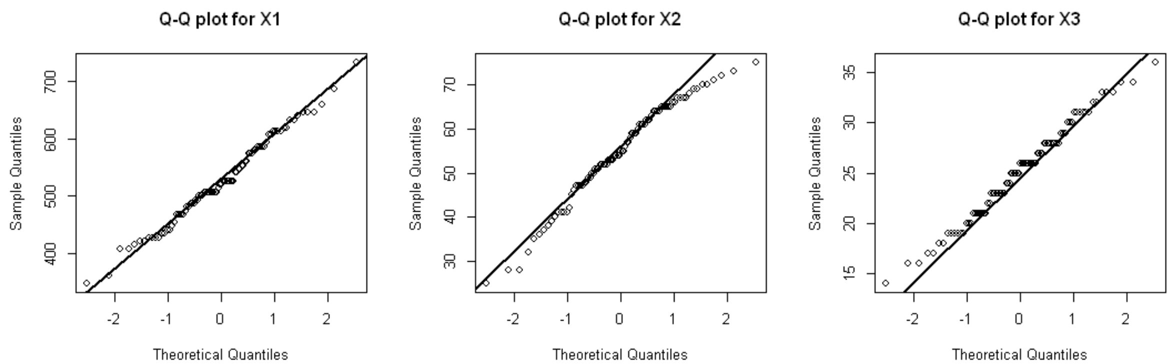
In [21]:
```
scale1 <- sqrt(eiDec$values[1]*3*(n-3)/(n*(n-3))*df(0.05, df1=3, df2=n-3))
scale2 <- sqrt(eiDec$values[2]*3*(n-3)/(n*(n-3))*df(0.05, df1=3, df2=n-3))
scale3 <- sqrt(eiDec$values[3]*3*(n-3)/(n*(n-3))*df(0.05, df1=3, df2=n-3))
```

In [22]:
```
cbind(c(scale1,scale2,scale3),eiDec$vec)
```

9.3660059	0.99390539	0.103731534	-0.037307396
0.9759783	0.10344339	-0.994589227	-0.009577815
0.4667216	0.03809906	-0.005660238	0.999257936

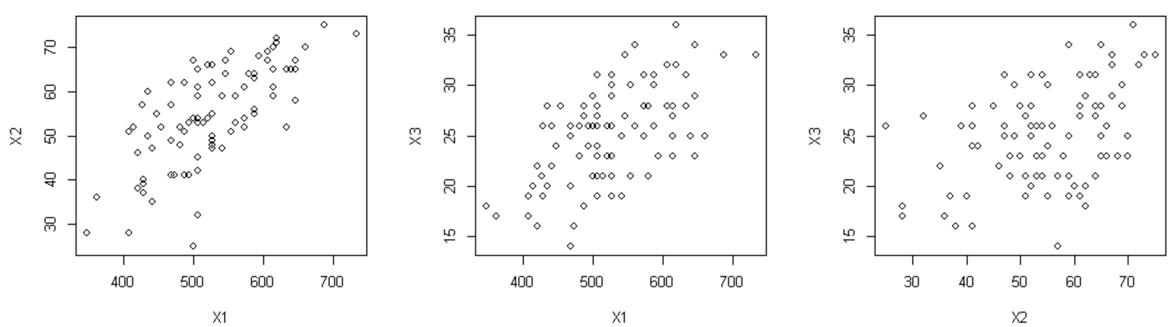
(1c)

In [10]:
```
options(repr.plot.width=9, repr.plot.height=3)
par(mfrow=c(1,3))
qqnorm(X1, main = 'Q-Q plot for X1', ylab = 'Sample Quantiles')
qqline(X1, lwd = 2)
qqnorm(X2, main = 'Q-Q plot for X2', ylab = 'Sample Quantiles')
qqline(X2, lwd = 2)
qqnorm(X3, main = 'Q-Q plot for X3', ylab = 'Sample Quantiles')
qqline(X3, lwd = 2)
```



X1 and X3 appear to be normally distributed. For X2, there's some deviation from normal in the upper tail.

In [11]:
```
options(repr.plot.width=9, repr.plot.height=3)
par(mfrow=c(1,3))
plot(X1,X2)
plot(X1,X3)
plot(X2,X3)
```



Problem 2

In [12]:
```
lumber <- read.csv('LumberData.csv')
head(lumber)
```

Stiffness	Bending
1232	4175
1115	6652
2205	7612
1897	10914
1932	10850
1612	7627

In [13]:
```
X1 <- lumber$Stiffness
X2 <- lumber$Bending
n = 30
p = 2
```

(2a)

In [14]:
```
mu1 <- mean(X1)
mu2 <- mean(X2)
muVec <- c(mu1,mu2)
print(muVec)
```

```
[1] 1860.500 8354.133
```

In [15]:
```
covar <- var(lumber)
eiDec <- eigen(covar)
eiVec1 <- eiDec$vectors[,1]
eiVec2 <- eiDec$vectors[,2]
eiVal1 <- eiDec$values[1]
eiVal2 <- eiDec$values[2]
covar
eiDec
```

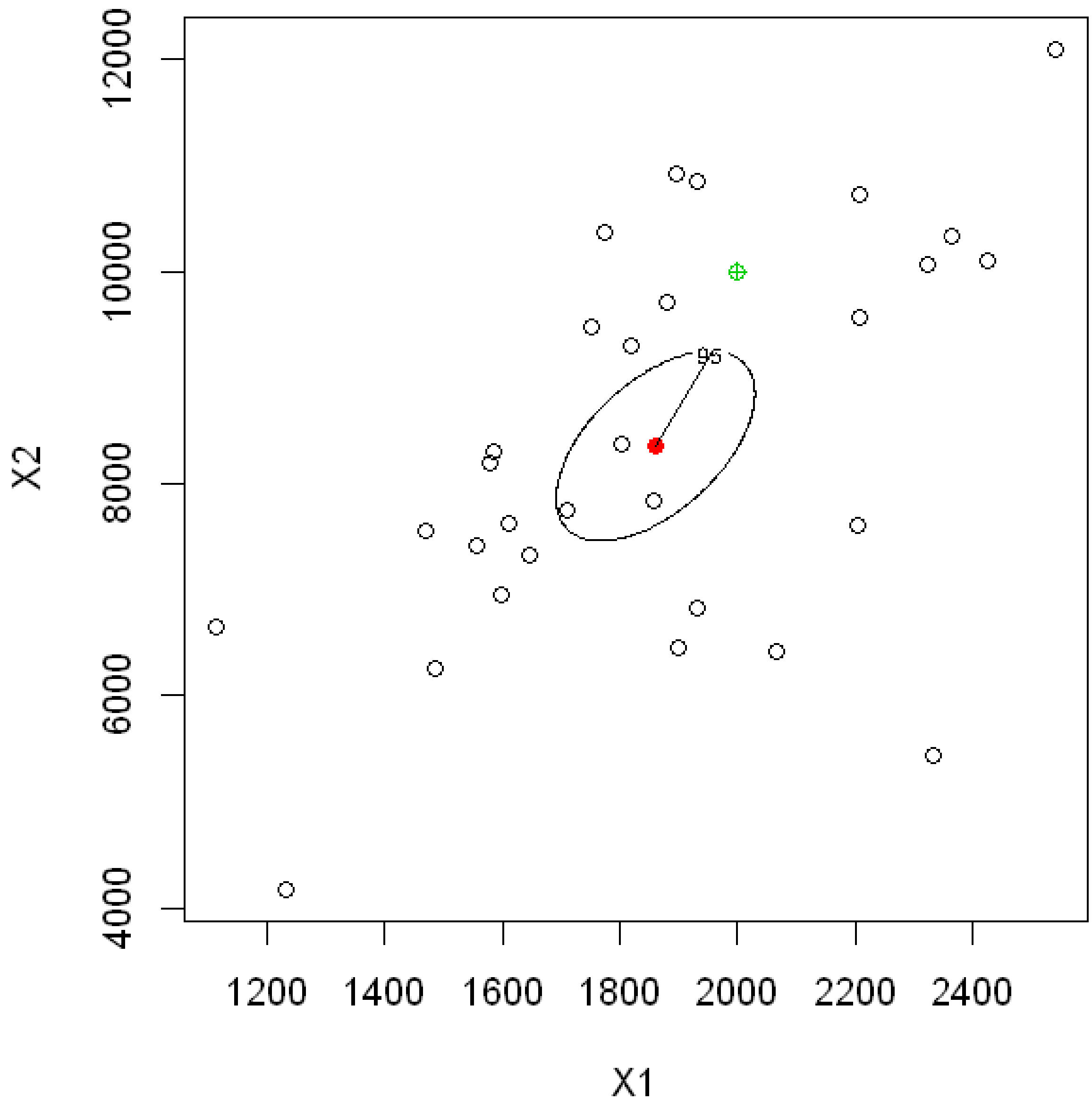
	Stiffness	Bending
Stiffness	124054.7	361620.4
Bending	361620.4	3486333.2

```
eigen() decomposition
$values
[1] 3524786.45   85601.38

$vectors
          [,1]       [,2]
[1,] 0.1057399 -0.9943938
[2,] 0.9943938  0.1057399
```

In [16]:
```
Tstat <- n*t(muVec)%*%(solve(covar))%*%(muVec)
Tstat
```

943.0262

In [28]:
```r
options(repr.plot.width=5, repr.plot.height=5)
muTest.1 <- seq(round(min(X1),-2), round(max(X1),-2), 10)
muTest.2 <- seq(round(min(X2),-2), round(max(X2),-2), 10)
Tstats <- matrix(0, nrow=length(muTest.1), ncol=length(muTest.2))
for(i in 1:length(muTest.1)){
        for(j in 1:length(muTest.2)){
                muTest <- c(muTest.1[i], muTest.2[j])
                Tstats[i,j] <- n*t(muVec - muTest) %*% solve(covar) %*% (muVec
- muTest)
        }
}

par(mar=c(4,4,1,1))

# Plot the data, and superimpose the confidence ellipsoids
# using the contour() function.

plot(lumber, xlab="X1", ylab="X2")
points(mu1, mu2, pch=16, col=2)
points(2000, 10000, pch=10, col=3)
contour(muTest.1, muTest.2, Tstats, levels=(n-1)*p/(n-p)*qf(0.95, p, n-p), dra
wlabels=T, add=T, labels=95)

# Add a line from the sample mean indicating the direction
# and half-length of the major axis of the confidence region.

lines(rbind(muVec, muVec + sqrt(eiVal1*(n-1)*p/(n-p)*qf(0.95, p, n-p)/n)*eiVec
1))
```



(2b) $\mu_0 = [2000, 10000]^T$

μ_0 lies outside of the 95% confidence ellipsoid, so it's not a plausible mean vector for this dataset.

Problem 3

In [34]:
```
bone <- read.csv('BoneMineral.csv')
alpha = 0.05; p = 6; n = dim(bone)[1]
head(bone)
```

dRadius	nRadius	dHumerus	nHumerus	dUlna	nUlna
1.103	1.052	2.139	2.238	0.873	0.872
0.842	0.859	1.873	1.741	0.590	0.744
0.925	0.873	1.887	1.809	0.767	0.713
0.857	0.744	1.739	1.547	0.706	0.674
0.795	0.809	1.734	1.715	0.549	0.654
0.787	0.779	1.509	1.474	0.782	0.571

(3a) 95% Bonferroni intervals

In [36]:
```
# Compute the sample mean vector and sample covariance matrix

sampMean <- apply(bone, 2, mean)
sampVars <- apply(bone, 2, var)
sampCov <- cov(bone)

# Set desired significance level alpha

alpha <- 0.05

# Compute Bonferroni corrected significance level as alpha/p

alphaStar <- alpha/p

# Compute the upper and lower confidence limits for all variables
# simultaneously, using R's vector operations

upperBon <- sampMean + sqrt(sampVars/n)*qt(alphaStar/2, df=n-1, lower.tail=F)
lowerBon <- sampMean - sqrt(sampVars/n)*qt(alphaStar/2, df=n-1, lower.tail=F)
cbind(lowerBon, upperBon)
```

	lowerBon	upperBon
dRadius	0.7782338	0.9093662
nRadius	0.7568766	0.8797634
dHumerus	1.6296774	1.9556826
nHumerus	1.5832656	1.8864144
dUlna	0.6425529	0.7662471
nUlna	0.6346406	0.7530394

(3b) 95% T^2 intervals

In [39]:
```
upperHot <- sampMean + sqrt( p*(n-1)/(n-p)*qf(alpha, p, n-p, lower.tail=F)*sam
pVars/n )
lowerHot <- sampMean - sqrt( p*(n-1)/(n-p)*qf(alpha, p, n-p, lower.tail=F)*sam
pVars/n )
cbind(lowerHot, upperHot)
```

	lowerHot	**upperHot**
dRadius	0.7420179	0.9455821
nRadius	0.7229380	0.9137020
dHumerus	1.5396419	2.0457181
nHumerus	1.4995425	1.9701375
dUlna	0.6083914	0.8004086
nUlna	0.6019414	0.7857386

Hotelling's T^2 simultaneous intervals are wider than Bonferroni's simultaneous intervals.

Problem 4

In [1]:
```
flour <- read.csv('FlourBags.csv')
head(flour)
```

Scale1	**Scale2**	**Scale3**
10.63	10.13	10.23
9.89	9.87	9.73
10.30	10.03	9.67
10.16	10.54	10.76
9.75	9.90	10.10
10.03	10.45	10.17

In [2]:
```
S1 <- flour$Scale1
S2 <- flour$Scale2
S3 <- flour$Scale3
p <- 3; n <- length(flour[,1]); alpha = 0.05
```

(4a) $H_0 : \mu = [10, 10, 10]^T; H_A : \mu \neq [10, 10, 10]^T; \alpha = 0.05$. Hotelling's T^2

In [3]:
```
n <- length(flour[,1])
muNull <- c(10,10,10)
muVec <- c(mean(S1), mean(S2), mean(S3))
alpha <- 0.05
covar <- var(flour)
covarInv <- solve(covar)
```

In [4]:
```
t2 <- n*t(muVec-muNull)%*%covarInv%*%(muVec-muNull)
t2stat <- (n-p)/((n-1)*p)*t2
Fcrit <- df(alpha, df1 = 3, df2 = n-3)
```

In [5]:
```
cat("Reject Null hypothesis:", t2stat>Fcrit)
```

```
Reject Null hypothesis: TRUE
```

(4b) $H_0 : \mu = [10, 10, 10]^T; H_A : \mu \neq [10, 10, 10]^T; \alpha = 0.05$. Bonferroni

In [6]:
```
# Compute the sample mean vector and sample covariance matrix

sampVars <- apply(flour, 2, var)

# Compute Bonferroni corrected significance level as alpha/p

alphaStar <- alpha/p

# Compute the upper and lower confidence limits for all variables
# simultaneously, using R's vector operations

upperBon <- muVec + sqrt(sampVars/n)*qt(alphaStar/2, df=n-1, lower.tail=F)
lowerBon <- muVec - sqrt(sampVars/n)*qt(alphaStar/2, df=n-1, lower.tail=F)
cbind(lowerBon, upperBon)
```

	lowerBon	upperBon
Scale1	9.459005	10.00099
Scale2	9.844532	10.19547
Scale3	9.622842	10.10316

Each measurement lies inside the corresponding 95% confidence interval. Bonferroni's test **fails to reject** the Null.

(4c)

Hotelling's T^2 simultaneous confidence intervals would contain $[10, 10, 10]^T$, because they are more conservative than Bonferroni's simultaneous confidence intervals.

(4d)

In [9]:
```
S_p <- sum(sampVars)/p # pooled variance
df <- p*(n-1)
testStat <- (sum(muVec)/3-10)/sqrt(S_p)
testStat
```

-0.325952581806621

In [11]:
```
tcrit <- dt(1-alpha/2,df=df)
tcrit
```

0.2458615275344

In [12]:
```
cat("Reject Null:", abs(testStat) > tcrit)
```

```
Reject Null: TRUE
```

Problem 5

(5a)

$\sum = \mathbf{I_p} \implies$ elements of $\bar{X}$ are **independent**, because the pairwise covariances are all equal to zero.

(5b)

For each individual test: $P(fails\ to\ reject\ H_0) = 1 - \alpha^*$

$\implies P_{\mu_0}(Reject H_0 : \mu = \mu_0) = 1 - (1 - \alpha^*)^p$

(5c)

$1 - (1 - \alpha^*)^p = \alpha \implies \alpha^* = 1 - \sqrt[p]{1 - \alpha}$

(5d)

In [14]:
```
alpha = 0.05
```

n = 10; p = 4

In [19]:
```
n <- 10; p <- 4
alphaStar <- 1 - (1-alpha)^(1/p)
alphaStarBon <- alpha/p
```

In [20]:
```
cat("Bonferroni longer:", alphaStarBon < alphaStar)
```

```
Bonferroni longer: TRUE
```

n = 10; p = 8

In [21]:
```
n <- 10; p <- 8
alphaStar <- 1 - (1-alpha)^(1/p)
alphaStarBon <- alpha/p
```

In [22]:
```
cat("Bonferroni longer:", alphaStarBon < alphaStar)
```

```
Bonferroni longer: TRUE
```

n = 20; p = 4

In [23]:
```
n <- 20; p <- 4
alphaStar <- 1 - (1-alpha)^(1/p)
alphaStarBon <- alpha/p
```

In [24]:
```
cat("Bonferroni longer:", alphaStarBon < alphaStar)
```

```
Bonferroni longer: TRUE
```

n = 20; p = 8

In [25]:
```
n <- 20; p <- 8
alphaStar <- 1 - (1-alpha)^(1/p)
alphaStarBon <- alpha/p
```

In [26]:
```
cat("Bonferroni longer:", alphaStarBon < alphaStar)
```

```
Bonferroni longer: TRUE
```

In [ ]: