Ngoc Ha

HW 2 - ST 557

Problem 5

$$r_Q = \frac{\sum_{i=1}^{n}(x_{(j)}-\bar{x})(q_{(j)}-\bar{q})}{\sqrt{\sum_{i=1}^{n}(x_{(j)}-\bar{x})^2}\sqrt{\sum_{i=1}^{n}(q_{(j)}-\bar{q})^2}}; \text{ where } q_{(j)} = \phi^{-1}\left(\frac{j-\frac{1}{2}}{n}\right)$$

Prepare functions

In [1]:
```
inv_cdf <- function(i,n){
    return(qnorm((i-0.5)/n))
}
```

In [2]:
```
sampDiffVec <- function(sample){
    return(as.matrix(sort(sample)-mean(sample)))
}
```

In [3]:
```
theoDiffVec <- function(n){
    quantiles = rep(0,n)
    for (i in c(1:n)){
        quantiles[i] = inv_cdf(i,n)
    }
    return(as.matrix(quantiles-mean(quantiles)))
}
```

In [4]:
```
r_Q <- function(sampDiffVec, theoDiffVec){
    return((t(sampDiffVec)%*%theoDiffVec)/(norm(sampDiffVec, type='2')*norm(th
eoDiffVec, type='2')))
}
```

(5a)

In [5]:
```
rQVec_a <- rep(0,10000)
for (i in c(1:10000)){
    samp <- runif(10)
    rQVec_a[i] <- r_Q(sampDiffVec(samp),theoDiffVec(length(samp)))
}
```

In [6]:
```
cat("Part a's rejection rate:", length(rQVec_a[rQVec_a<0.9198])/length(rQVec_a
))
```

```
Part a's rejection rate: 0.056
```

(5b)

In [11]:
```
rQVec_b <- rep(0,10000)
for (i in c(1:10000)){
    samp <- rchisq(n=5,df=5)
    rQVec_b[i] <- r_Q(sampDiffVec(samp),theoDiffVec(length(samp)))
}
```

In [12]:
```
cat("Part b's rejection rate:", length(rQVec_b[rQVec_b<0.8788])/length(rQVec_b
))
```

```
Part b's rejection rate: 0.0847
```

(5c)

In [9]:
```
rQVec_c <- rep(0,10000)
for (i in c(1:10000)){
    samp <- rchisq(n=20,df=2)
    rQVec_c[i] <- r_Q(sampDiffVec(samp),theoDiffVec(length(samp)))
}
```

In [10]:
```
cat("Part c's rejection rate:", length(rQVec_c[rQVec_c<0.9508])/length(rQVec_c
))
```

```
Part c's rejection rate: 0.8067
```