Solution Sets: Takeaways

Concepts

- An inconsistent system has two or more equations that no solution exists when the augmented matrix is in reduced echelon form.
    - Example of a inconsistent system: $\left[\begin{array}{cc|c} 8 & 4 & 5 \\ 4 & 2 & 5 \end{array}\right]$
- When the determinant is equal to zero, we say the matrix is singular or it contains no inverse.
    - Example of a singular matrix: $\begin{bmatrix} 8 & 4 \\ 4 & 2 \end{bmatrix}$
    - The formula for the determinant of a $2x2$ square matrix is:
    $$\det(A) = ad - bc$$
    - If we substitute in the values, we get a determinant of zero:
    $$\det(A) = ad - bc = 8 \cdot 2 - 4 \cdot 4 = 16 - 16 = 0$$
- A nonhomogenous system is a system where the constants vector ($\vec{b}$) doesn't contain all zeros.
    - Example of a nonhomogenous system: $\left[\begin{array}{cc|c} 8 & 4 & 5 \\ 4 & 2 & 5 \end{array}\right]$
- A homogenous system is a system where the constants vector ($\vec{b}$) is equal to the zero vector.
    - Example of a homogenous system: $\left[\begin{array}{cc|c} 8 & 4 & 0 \\ 4 & 2 & 0 \end{array}\right]$
    - A homogenous system always contains the trivial solution: the zero vector.
- For a nonhomogenous system that contains the same number of rows and columns, there are 3 possible solutions:
    - No solution.
    - A single solution.
    - Infinitely many solutions.
- For rectangular (nonsquare, nonhomogenous) systems, there are two possible solutions:
    - No solution.
    - Infinitely many solutions.
- If $Ax = b$ is a linear system, then every vector $\vec{x}$ which satisfies the system is said to be a solution vector of the linear system. The set of solution vectors of the system is called the solution space of the linear system.
- When the solution is a solution space (and not just a unique set of values), it's common to rewrite it into parametric vector form.
    - Example a vector in parametric vector form: $\vec{x} = x_3 \cdot \begin{bmatrix} 4/3 \\ 0 \\ 1 \end{bmatrix}$

Resources

- [Consistent and Inconsistent equations]
- [Solution Spaces of Homogenous Linear Systems]