Finding Extreme Points: Takeaways

Concepts

- A derivative is the slope of the tangent line at any point along a curve.
- Let x be a point on the curve and h be the distance between two points, then the mathematical formula for the slope as h approaches zero is given as:
  $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
- Differentiation is the process of finding a function's derivative.
- Finding the derivative of: $f(x) = -(x)^2 + 3x - 1$:
  - $y' = \lim_{h \to 0} \frac{(-(x+h)^2 + 3(x+h) - 1) - (-(x)^2 + 3x - 1)}{h}$
  - $y' = \lim_{h \to 0} \frac{-x^2 - 2xh - h^2 + 3x + 3h - 1 + x^2 - 3x + 1}{h}$
  - $y' = \lim_{h \to 0} \frac{h(-2x - h + 3)}{h}$
  - $y' = \lim_{h \to 0} -2x - h + 3$
  - $y' = -2x + 3$
- Three ways of notating a curve's derivative:
  - $y' = -2x + 3$
  - $f'(x) = -2x + 3$ *Only use if derivative is a function
  - $\frac{d}{dx}[-x^2 + 3x - 1] = -2x + 3$
- A critical point is a point where the slope changes direction from negative slope to positive slope or vice-versa. Critical points represent extreme values, which can be classified as a minimum or extreme value.
- Critical points are found by setting the derivative function to 0 and solving for x
- Critical point classification:
  - When the slope changes direction from positive to negative it can be a maximum value.
  - When the slope changes direction from negative to positive, it can be a minimum value.
  - If the slope doesn't change direction, like at $x = 0$ for $y = x^3$, then it can't be a minimum or maximum value.
- Each maximum or minimum value points are known as local extrema.
- Classifying local extrema:
  - A point is a relative minimum if a critical point is the lowest point in a given interval.
  - A point is a relative maximum if a critical point is the highest point in a given interval.
- Instead of using the definition of the derivative, we can apply derivative rules to easily calculate the derivative functions.
- Derivative rules:
  - Power rule: Let r be some power, then $f'(x) = rx^{r-1}$
    - Example: Let $f(x) = x^2$ In our function, r would be 2. Using the power rule, it's derivative would be $f'(x) = 2x^{2-1}$ or $f'(x) = 2x$
  - Sum rule: $\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)]$
    - Example: $\frac{d}{dx}[-x^3 + x^2] = \frac{d}{dx}[-x^3] + \frac{d}{dx}[x^2] = -3x^2 + 2x$

- Constant factor rule: $\frac{d}{dx}[3x] = 3\frac{d}{dx}x = 3\dot{1} = 3$
- Derivative of x is always 1 and derivative of 1 is always 0.
- Once you found the critical points of a function, you can analyze the direction of the slope around the points using a sign chart to classify the point as a minimum or maximum. We can test points around our points of interest to see if there is a sign change as well as what the change is.

Resources

- Derivative rules
- Sign chart