

# Dãy nhiễu [NOISEQ]

Cho dãy số nguyên  $A = (a_1, a_2, \dots, a_n)$  và số nguyên dương  $R$ , hãy tìm một dãy số nguyên  $B = (b_1, b_2, \dots, b_n)$  thỏa mãn

- $|a_i - b_i| \leq R \forall i$
- Dãy  $B$  có nhiều giá trị phân biệt nhất.

## Dữ liệu

- Dòng 1: hai số nguyên  $n, R$  ( $1 \leq n \leq 10^5; 1 \leq R \leq 10^9$ )
- Dòng 2:  $n$  số nguyên  $a_1, a_2, \dots, a_n$  ( $|a_i| \leq 10^9 \forall i$ )

## Kết quả

- Dòng 1: số nguyên là số giá trị phân biệt trong dãy  $B$  tìm được
- Dòng 2:  $n$  số nguyên  $b_1, b_2, \dots, b_n$

## Ví dụ

NOISEQ.INP	NOISEQ.OUT
5 2	5
1 2 3 4 5	1 2 3 4 5
3 1	3
1 1 1	0 1 2

## Tut

First of all, note that to restore some kind of source array, you need to apply the same noise function to this one. Then we sort the array in ascending order, and when we examine each element, we will try to get the smallest possible from it that we have not yet received it. Let  $x$  be the current element, and  $y$ , the previous value, then three cases are possible:

\begin{itemize}

\item  $x - r > y$ , then assume that the element  $x$  was equal to  $x - r$  in the original array.

\item  $x + r < y$ , then all the numbers from the range  $[x - r; x + r]$  have already been obtained on the previous elements, which means that the  $x$  element does not lead to a unique one.

\item otherwise we can assume that instead of  $x$  we used  $y + 1$  before, and put it in response.

\end{itemize}

Accordingly, the answer  $\sim$  is the sum of the first and third cases, but you could first put the numbers in this way, and then calculate the answer.

To prove the optimality of this approach, consider two numbers  $x_0$  and  $x_1$ , such that  $x_0 < x_1$ . Obviously, the intersection of the ranges  $[x_0 - r; x_0 + r]$  and  $[x_1 - r; x_1 + r]$  is not advantageous to borrow when considering  $x_0$ , as we reduce the number of possible options for  $x_1$ . In the case if we take the minimum number from  $[x_0 - r; x_0 + r]$ , which was not taken before, we will leave the largest possible number of variants for  $x_1$ .

The complexity of the solution  $O(n \log(n) + n)$ .