APPLICATION OF INTEGRALS IN CONSUMER'S AND PRODUCER'S SURPLUS

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1 Introduction

While calculating the welfare of the economy in each country, economists have recognized the importance of consumer's and producer's surplus in developing many economic policies, such as the necessities or the calculation of innovation degree. Both concepts can be explained and estimated by the application of integrals. In this paper, we will explain the model of calculating consumer's and producer's surplus using integrals with some real-life applications with real-time data.

2 Application of integrals in consumer and producer surplus

2.1 Some specific things about integral

2.1.1 Definition of antiderivative and integral

In Mathematics, the differential F is the antiderivative function if its derivative is equal to the original function f. Symbolically, we can write : $F' = f$. Along with antiderivative, integration is served as a tool to solve many problems of maths and physics, which include the area of the function under a curve, the length of a function's curve, the volume of a shape that is solid, and other related shapes in formation [Anthony et al., 1996]

Like the derivative, the integral is defined as limits. The basic formula of integrals is calculated by the construction of approximations by finite sums [Anthony et al., 1996]. Moreover, the basic concept of finding the integral is introduced geometrically. That means integrals can be also defined as the area under a function's graph. From the basic formula areas of certain specific plane regions, the theory of integration is developed.

2.1.2 Some basic formula in this project

$$\int kdx = kx + c, \text{ where } k \text{ and } C \text{ is two constants (1)}$$

To calculate the definite integral of a function, we need to find the difference between the value of antiderivative of the two endpoints of the interval [Anthony et al., 1996]:

$$\int_a^b f(x)dx = F(b) - F(a), \text{ where } F(x) \text{ is any anti-derivative of } f(x) \text{ (2)}$$

2.2 Consumer and producer surplus

2.2.1 Terminologies

The supply function or supply curve $S(q)$ is the mathematical model represents the relationship between quantity of an item that producers will supply and the price. **The demand function or demand curve** $D(q)$ is the mathematical model represents the relationship between quantity of an item demanded and the price [Perloff, 2008]. Therefore, when $S(q)$ increases, the more producers are willing to supply and when $D(q)$ decreases, the fewer consumers will buy. **The market equilibrium point** is the intersection point (Q^*, P^*) of the supply and demand curves, where the quantity supplied is equal to the quantity demanded. The numbers Q^* and P^* are represented equilibrium quantity and equilibrium price, respectively.

The Consumer's Surplus (CS) represents the welfare of consumers, which is the extra amount consumers would have been willing to pay but did not. **The Producer's Surplus (PS)** represents the welfare of producers or sellers, which is the extra amount producers received above what they were willing to accept. [Perloff, 2008]

2.2.2 The Mathematical model

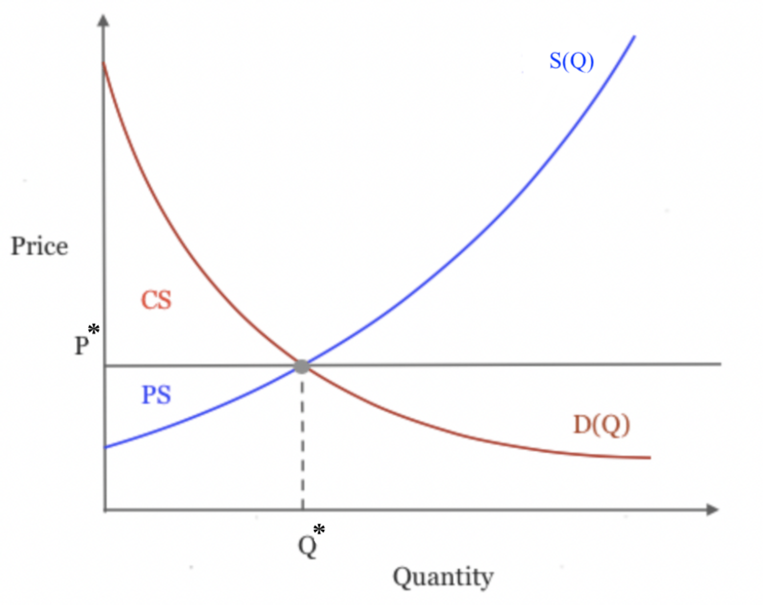


Figure 1: Illustration of consumer's surlus and producer's surplus

Firsly, noting the total the spending amount at equilibrium price: Q^*P^*. Then, we note the total amount paid at maximum price $\int_0^{Q^*} D(q)dx$, and the total amount produced at minimum price $\int_0^{Q^*} S(q)dx$.

Therefore, the Consumer's Surplus and Producer's Surplus are calculated as a definite integral as below by applying (1) and (2):

$$CS = \int_0^{Q^*} (D(Q) - P^*)\, dQ \ = \int_0^{Q^*} D(Q)\, dQ - \int_0^{Q^*} P^*\, dQ = \int_0^{Q^*} D(Q)\, dQ - \int_0^{Q^*} P^*\, dQ = \int_0^{Q^*} D(Q)\, dQ \ - P^*Q^*$$

$$PS = \int_0^{Q^*} (P^* - S(Q))\, dQ \ = \int_0^{Q^*} P^*\, dQ - \int_0^{Q^*} S(Q)\, dQ \ = \int_0^{Q^*} P^*\, dQ - \int_0^{Q^*} S(Q)\, dQ \ = P^*Q^* - \int_0^{Q^*} S(Q)\, dQ$$

Example: Find the equilibrium quantity and price, and the CS and PS given the demand curve $D(q) = -0,8q + 46$ and the supply curve $S(q) = 0.06q^2 + 6$

Calculate the equilibrium quantity and equilibrium amount using: $D(q) = S(q)$

$$-0,8q + 46 = 0.06q^2 + 6$$

$$\Leftrightarrow 0.03q^2 + 0.4q - 20 = 0 \Leftrightarrow (3q + 100)(q - 20) = 0 \Leftrightarrow q = \frac{-100}{3}, q = 20 \Leftrightarrow Q^* = 20$$

$$D(20) = -0.8 * 20 + 46 = 30 \Leftrightarrow P^* = 30$$

Calculate the consumer's and producer's surplus using definite integrals:

$$CS = \int_0^{20} D(Q)\, dQ \ -20 * 30 = 160$$

$$PS = 20 * 30 - \int_0^{20} S(Q)\, dQ = 320$$

3 Real-world application and discussion

3.1 The study of social welfare

Welfare economics studies how the structure of markets determines the overall well-being of society. This idea is closely associated with the utility - value one perceives from a particular good, which can be maximized through the consumption and interactions between buyers and sellers in a competitive market. Consumer's and producer's surplus, as the benefit perceived by the market stakeholders from their expected and actual market price, is an important input for the study.

3.2 Challenges in building hand-on experience

While both producer and consumer surplus are critical inputs to calculations of welfare, producer surplus is particularly more measurable than the other. As manufacturers, service providers, etc. producers are motivated to calculate and communicate their expectations for prices over production quantities through different means, such as algorithms for taxi fares surge, and business quotations can be translated to fairly accurate supply curves.

On the other hand, estimation of consumer surplus from empirical data is extremely challenging. By definition, one needs to integrate the area under the demand curve, in which the construction of the curve is the major challenge. Typically, empirical evidence often shows only the equilibrium price points in which supply and demands are balanced, and there are no such direct and accurate estimates of demand elasticity, defined as the rate of changes in quantity demanded by the customer in response to price changes, at price points far from the equilibrium price. [Perloff, 2008].

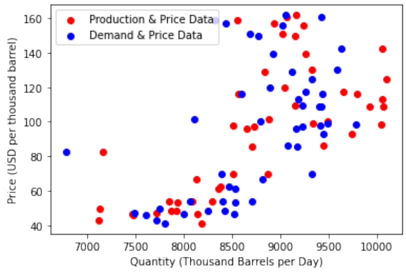
In microeconomics, there are multiple factors whose changes would influence consumer's demand, including consumer's income, tastes or preferences, price of related goods, expectations about future prices,... The Law of Demand given that all other factors remain equal, the higher the price of a good, the lower quantity demanded, which is described by below formula:

$$Q_x = f(P_x, Y)$$

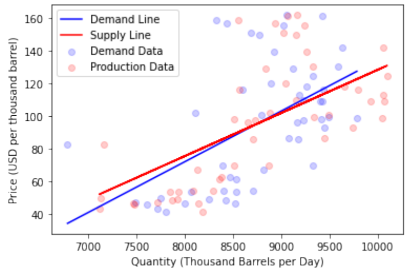
where: Q_x is the quantity demanded of good x ; f: demand function ; P_x: price of the good ; Y: other parameters held constant.

In an experimental attempt to test the viability of the Law of Demand conducted by our group, two data sets (1) New York State Energy Research and Development Authority's Transportation Fuels Production and Demand (1993-2022) and (2) Energy Information Administration's US All Grades All Formulation Retail Gasoline Prices were gathered and processed with these parameters: U.S gasoline quantity of demand and production, and price. The joined data then went through quadratic regression, which resulted demand curves do not satify the Law of Demand (see Figure 2). This denotes the necessity to obtain other variables that impact demand elasticity of a product or service other than pure quantity or value of purchases to successfully estimate demand and hence, consumer surplus.

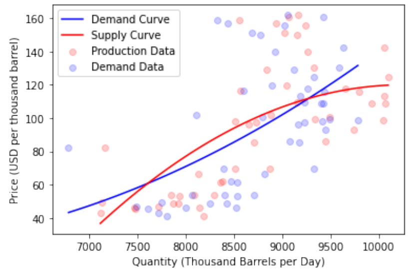
Given such accounts, this part of the document would instead report on proposed methodologies and research attempted in estimating consumer surplus from real-world data from two different sectors: recreation site evaluation and technology-based ride-hailing.



(a) Scatter plot



(b) Linear regression



(c) Quadratic Regression

Figure 2: Illustration 1993-2021 U.S Gasoline Demand and Production data in scatter plot (a) Demand and Supply curves from Linear (b) and Quadratic Regression (c)

3.3 Case reports

3.3.1 Travel Cost Method to estimate Consumer Surplus of Environmental and Historical Recreation sites

Environmental and historical recreation sites, such as national parks or historical heritage, etc., generate an increasing amount of profit for governments and local economies. However, these natural resources's market mechanism, which means calculating the consumer's surplus, is difficult. [Das, 2013].

The travel cost method (TCM) is an indirect method to estimate consumer surplus, from visits to recreational sites [Clawson et al., 1959]. The method includes a survey questionnaire asking a sample group of visitors for variables assumed to affect demand such as demographic information, income, and time of traveling, whose combination forms the function of travel costs.

There are three different types of TCM: the zonal travel cost model, individual travel cost model, and random utility travel cost model, among which the first is also the simplest and widely applied implementation for national parks [Pradeep et al., 2008, Bharali et al., 2012, Tourkolias et al., 2015]. Regression models are then used for fitting and correlating the visit rates from different zones with its travel cost data. Either a linear model that account for socioeconomic variables (income, age, . . .) can be used.

In an attempt to evaluate the Poseidon temple in Greece, a research group implemented the travel cost model with multiple regression to estimate demand function and consumer surplus [Tourkolias et al., 2015]. The two major steps of research include: (1) regression models correlating the visitation rates from the different zones with the corresponding average travel cost from the above-mentioned cost elements, including one linear, two semi-log, and one double-log model, and (2) construct demand curve for a range of hypothetical access prices of the site on the basis of equations generated in step 1.

Consumer surplus is estimated based on two equations:

- Semi-log 2 model: $CS = \dfrac{TC_1 * X_1}{B_1 1}$
- Double-log model : $\mathrm{CS} = \dfrac{X_1}{-B_1}$

  where:

  - TC1: travel cost level
  - X1: visitation rate on the basis of step 1.
  - B1: coefficients of regression models used in step 1.

In step (2), two regression models were implemented: TC model: assuming only one independent variable as the travel cost; Full model: take in multiple independent variables, including the travel cost and other socioeconomic factors.

In summary, the consumer surplus for the monument in question ranges between 1.5 and 24.5 million per year, which

Model	Scenario	Regression function	Adj. R^2	Predicted number of visitors
Semi-log 2 - TC model	1	$\ln(VR) = 10.2^{*} - 0.04\cdot(\text{travel cost})^{*}$	87.2%	168,348
	2	$\ln(VR) = 9.7^{*} - 0.02\cdot(\text{travel cost})^{**}$	75.0%	219,830
	3	$\ln(VR) = 10.4^{*} - 0.04\cdot(\text{travel cost})^{*}$	87.4%	163,600
Semi-log 2 - full model	1	$\ln(VR) = -25.5^{**} - 0.04\cdot(\text{travel cost})^{*} + 15.7\cdot(\text{freelancer})^{**} + 3.7\cdot(\text{visit})^{*\ **}$	99.2%	154,923
	2	$\ln(VR) = 14.8^{**} - 0.02\cdot(\text{travel cost})^{*} \pm 5.5\cdot(\text{gender})^{*} - 2.7\cdot(\text{education})^{**}$	99.4%	174,561
	3	$\ln(VR) = -26.4^{**} - 0.05\cdot(\text{Travel Cost})^{*} + 17.8\cdot(\text{Freelancer})^{**} + 0.6\cdot(\text{Income})^{***}$	99.3%	167,646
Double-log - TC model	1	$\ln(VR) = 15.2^{*} - 2.4\cdot\ln(\text{travel cost})^{*}$	95.4%	158,849
	2	$\ln(VR) = 13.5^{*} - 1.8\cdot\ln(\text{travel cost})^{*}$	90.9%	177,592
	3	$\ln(VR) = 16.9^{*} - 2.7\cdot\ln(\text{travel cost})^{*}$	93.8%	153,265
Double-log - full model	1	$\ln(VR) = -15.7^{**} - 2.6\cdot\ln(\text{travel cost})^{*} + 8.6\cdot\ln(\text{age})^{*}$	99.1%	205,503
	2	$\ln(VR) = 24.0^{**} - 1.8\cdot\ln(\text{travel cost})^{*} - 15.6\cdot\ln(\text{visitation reason})^{***}$	98.0%	238,495
	3	$\ln(VR) = -21.2^{**} - 3.1\cdot\ln(\text{travel cost})^{*} + 3.0\cdot\ln(\text{attractiveness})^{***} + 9.7\cdot\ln(\text{age})^{*}$	99.9%	178,722

Figure 3: Demand functions of Poseidon Park using Travel Cost Method

suggests the total value of the Poseidon temple ranges from 17.2 to 60.8 million per year.

3.3.2 Estimating Uber's consumer surplus using big data

In 2016, Uber claimed their benefiting U.S customers with an estimated **consumer surplus** of 6.76 billion USD for UberX trips in the US [Cohen et al., 2016]. The number came from two major contributing factors: the company's rich pool of individual data and its price surging algorithm.

Uber's price surge system constantly changes according to supply and demand (i.e. prices increases during rush hours). And importantly, the app not only registers successful rides as data points but also records events of ride rejection after knowing the price surge. These notions reflect the potential "local demand and supply conditions" and the opportunity to estimate the price elasticity of demand, demand curve, and ultimately, consumer surplus.

As stated by the principle of demand, one needs to keep all parameters constant except for price to estimate the demand curve. The researchers ran regressions correlating purchase rate with price discontinuities in form of the following equation:

$$\begin{aligned} Outcome =& \alpha + \theta * Window * Post + \beta_2 * Window + + \beta_3 * (1 - Window) * Post + \beta_4 * Wait \\ &+ \beta_5 * (1 - Post) * Generator + \beta_6 * Post * Generator + \epsilon \end{aligned}$$

where:

- θ: key regression coefficient capturing average difference in the outcome variable right after compared to right before a discontinuity.
- $Outcome$: outcome of interest (purchase made or not, ride in rush hour or not, etc.). In the price elasticity estimate step, $Purchase$ is the $Outcome$.
- $Window$: indicator of whether observation made close to a price discontinuity
- $Post$: indicator of observation to the right of price discontinuity (when price increases). Equals 1 if the the observation is to the right of the price discontinuity.
- $Wait$: expected wait time
- $Generator$: measure of surge produced by Uber algorithm.

Estimates of price elasticity at different search levels were then derived from the definition-based equation and regression discontinuity analysis :

$$Elasticity\ (\%\Delta Price) = \frac{\%\Delta Quantity}{\%\Delta Price} = \frac{\theta\%\Delta Purchase\ Rate}{\%\Delta Price}$$

Consumer surplus, then, estimated by summing the surplus estimates yield for sessions of customers who face different levels of price surge from 1.0x to 4.8x. **Demand curve** were generated from three different approaches with different constraints on relative positions of observation to median level of demand, presented as Figure 4).

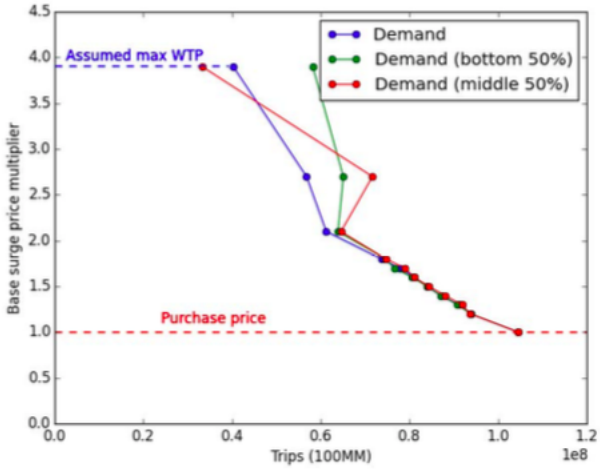


Figure 4: Estimated demand curves generated from different approaches

4 Discussion and conclusion

In theory, intergration is a helpful mathematical method to calculate consumer and producer surplus as the area under known demand and supply curves.

Examination of researches shows that estimating consumer surplus for real-life products is particularly challenging than supply, which motivates a major part of this report dedicated to the former concept. The major challenge lies in the variablity of some important inputs other than price and quantity and methodologies to constructing accurate demand curves for goods and services of different sectors.

Yet considering the importance of consumer surplus as inputs for measurement of consumer benefit in Welfare Economy, which guide public policy toward increasing the total good of society, it is a challenge for economists and researchers alike in measuring consumer's welfare in different segments of the market.

5 Authors' contributions

Ngoc Anh is in charge of collecting data and coding to visualize them to report the real-life application. Hong Ha created the outline for the final synthesis and finds the previous research papers to brainstorm the ideas to be concluded in the paper. Ngoc Ha created the introduction and research about models in economics. Ngoc Ha and Hong Ha worked in formating the LaTex file. All of us researched and contribute to this topic and prepare for the video presentation. All authors read and agree to the final report.

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