Problem 2:

1. $f(x) = \sqrt{x(1-x)}$ is continuos on $[0,1]$ because $x(1-x)$ always is positive within this interval.

2.

$$f'(x) = \frac{d}{dx}\left(\sqrt{x(1-x)}\right) = \frac{1-2x}{2\sqrt{x(1-x}}.$$

This deriviative exists for all $x \in (0,1)$, so $f(x)$ is differentiable on $(0,1)$

3. Since $f(x)$ satisfies the conditions of the MVT, there exists a point $c \in (0,1)$

$$f'(c) = \frac{f(1) - f(0)}{1-0}.$$

$f(1) = 0$, $f(0) = 0$

$$\Rightarrow f'(c) = 0$$

$\dfrac{1-2c}{2\sqrt{c(1-c)}} = 0$, we get $c = \dfrac{1}{2}$.

Thus, $f(x) = \sqrt{x(1-x)}$ satisfies the Mean Value Theorem on $[0, 1]$ with $c = \dfrac{1}{2}$.

Problem 3:

a) $y = x^2 - 4$.

+) $y = 0 \Rightarrow x = \pm 2$.

+) $y' = 2x$

$y' = 0 \Rightarrow x = 0$.

b) $y = x^2 + 8x + 15$.

$y = 0 \Rightarrow x^2 + 8x + 15 = 0$

$\Leftrightarrow \left[\begin{array}{l} x = -3 \\ x = -5 \end{array}\right.$

$y' = 2x + 8$

$y' = 0 \Leftrightarrow 2x + 8 = 0 \Leftrightarrow x = -4$.

Problem 4.

a) $y = x^3 - 3x^2 + 4 = (x+1)(x-2)^2$.

$y = 0 \Leftrightarrow \left[\begin{array}{l} x = -1 \\ x = 2 \end{array}\right.$

$y' = 3x^2 - 6x$

$y' = 0 \Rightarrow \left[\begin{array}{l} x = 0 \\ x = 2 \end{array}\right.$

b) $y = x^3 - 33x^2 + 216x = x(x-9)(x-24)$

$y = 0 \Leftrightarrow \left[\begin{array}{l} x = 0 \\ x = 9 \\ x = 24 \end{array}\right.$

$y' = 0 \Leftrightarrow \left[\begin{array}{l} x = 6 \\ x = 12 \end{array}\right.$

KLONG

6)

a) $f(x) = x^4 + 3x + 1$, $x \in [-2; -1]$.

$f'(x) = 4x^3 + 3$.

$f'(x) = 0$.

$\Leftrightarrow$ $4x^3 + 3 = 0$

$\Leftrightarrow$ $x = -\sqrt[3]{\frac{3}{4}}$.

$f(-2) = 11$, $f(-1) = -1$.

One zero in $[-2; -1]$.

b) $r(\theta) = \theta + \sin^2\left(\frac{\theta}{3}\right) - 8$.

1) $r'(\theta) = 1 + \frac{2}{3}\sin\left(\frac{\theta}{3}\right)\cos\left(\frac{\theta}{3}\right)$.

2. One zero since $r(\theta)$ is continue and $r'(\theta)$ doesn't change sign

)

$T(0) = -19°C$, $T(14) = 100°C$.

MVT. $T'(c) = \frac{100-(-19)}{14} = 8,5$ °C/sec

Problem 9:

1, $y''(1) = 0$

2.

$y' = 3x^2 + 2bx + c$

$y'' = 6x + 2b$

3. $y''(1) = 0.$

$x = 1$ into y''

$y''(1) = 6.1 + 2b = 6 + 2b.$

for $x = 1$ to be an inflection point, we need $y''(1) = 0$

$6 + 2b = 0 \Rightarrow b = -3.$

To make $x = 1$ a point of inflection, b must be -3

8,

a) for $j(t)$

$j'(t) = 13 - 3t^2$.

$j'(t) = 0$

$\Leftrightarrow$ $12 - 3t^2 = 0$

$\Leftrightarrow$ $t = \pm 2$.

+) $j(-2) = 12(-2) - (-2)^3 = -24 + 8 = -16$.

$j(2) = 12(2) - 2^3 = 24 - 8 = 16$

local extrema: $(-2, -16)$ (minimum), $(2, 16)$ maximum.

for $h(x)$.

$h'(x) = x^2 - 4x + 4$.

$h'(x) = 0$

$\Leftrightarrow$ $x^2 - 4x + 4 = 0 \Leftrightarrow (x - 2)^2 = 0$

$\Leftrightarrow$ $x = 2$.

Critical $x = 2$.

Evaluate $h(x)$ at $x = 2$

$h(2) = \frac{2^3}{3} - 2(2)^2 + 4(2) = \frac{8}{3} - 8 + 8 = \frac{2}{3}$.

Local extremum: $\left(2, \frac{8}{3}\right)$ minimum.

Caculus 1 : Tutorial 5.

Problem 1.

$$AP = \sqrt{x^2+4}.$$

$$PB = \sqrt{(10-x)^2+25}.$$

$$AP + PB = \sqrt{x^2+4} + \sqrt{(10-x)^2+25}.$$

$$= f(x).$$

$$f'(x) = \frac{d}{dx}\sqrt{x^2+4} + \frac{d}{dx}\sqrt{(10-x)^2+25}.$$

$$= \frac{(x^2+4)'}{2\sqrt{x^2+4}} + \frac{((10-x)^2)'}{2\sqrt{(10-x)^2+25}}.$$

$$= \frac{2x}{2\sqrt{x^2+4}} + \frac{-2.(10-x)}{2\sqrt{(10-x)^2+25}}.$$

$$= \frac{x}{\sqrt{x^2+4}} + \frac{-(10-x)}{\sqrt{(10-x)^2+25}}$$

let fx min if and only if : $f'(x) = 0$

$$\Rightarrow \frac{x}{\sqrt{x^2+4}} + \frac{-(10-x)}{(10-x)^2+25} = 0$$

$$\Leftrightarrow \frac{x}{\sqrt{x^2+4}} = \frac{10-x}{(10-x)^2+25}.$$

$$\Leftrightarrow x = 20/7.$$