Q1. The radius of the disk S_x whose diameter run from the parabola $y = x^2$ to the parabola $y = 2 - x^2$ is

$$r_x = \frac{2 - x^2 - x^2}{2} = 1 - x^2$$

$$V = \int_{-1}^{1} \text{Area}(S_x)\,dx = \int_{-1}^{1} \pi (1-x^2)^2\,dx = \frac{16\pi}{15}.$$

2.

a) $$V = \int_0^{\pi} \frac{(2\sqrt{\sin x})^2 \sin(\pi/3)}{2}\,dx = \int_0^{\pi} \sqrt{3} \sin x\,dx$$

$$= 2\sqrt{3}.$$

b)

$$V = \int_0^{\pi} (2\sqrt{\sin x})^2\,dx = \int_0^{\pi} 4 \sin x\,dx = 8$$

c)

3)

$$V = \int_0^{\sqrt{2}} 2\pi y \, y^2 \, dy = 2\pi$$

4)

$$V = \int_0^3 2\pi x \frac{9x}{\sqrt{x^3+9}} dx = 2\pi \int_0^{27} \frac{3dt}{\sqrt{t+9}} = 9\pi .$$

5)

a)

$$V = \int_0^1 2\pi y \, 12(y^2 - y^3) \, dy .$$

$$= 24\pi \int_0^1 (y^3 - y^4) \, dy = \frac{6\pi}{5} .$$

b) $$V = \int_0^1 2\pi(1-y)\, 12(y^2-y^3)\, dy = 24\pi \int_0^1 (y^2 - 2y^3 + y^4)\, dy .$$

$$= \frac{4\pi}{5} .$$

c) $$V = \int_0^1 2\pi \left(\frac{8}{5} - y\right) 12 (y^2 - y^3)\, dy .$$

$$\cancel{= 24\pi \int_0^1 (\text{[illegible]})\, dy =}$$

$$= 24\pi \int_0^1 \left(y^4 - \frac{13}{5} y^3 + \frac{8}{5} y^2\right) dy = 2\pi .$$

d) $$V = \int_0^1 2\pi \left(\frac{2}{5} + y\right) 12 (y^2 - y^3)\, dy = 24\pi \int_0^1 \left(\frac{2y^2}{5} + \frac{3y^3}{5} - y^4\right) dy$$

$$= 2\pi$$

6)

$$D = \left\{ (x;y) \in \mathbb{R}^2 : 0 \leq x \leq 4, x \leq y \leq \frac{x+4}{2} \right\}.$$

a) The volume is

$$V = \int_0^4 \pi \left(\frac{(x+4)^2}{4} - x^2 \right) dx = \int_0^4 \frac{\pi(16 + 8x - 3x^2)}{4} dx = 16\pi$$

b)

$$V = \int_0^4 2\pi x \left(\frac{x+4}{2} - x \right) dx = \pi \int_0^4 x(4-x) dx = \frac{32\pi}{3}$$

c) $$V = \int_0^4 2\pi (4-x) \left(\frac{x+4}{2} - x \right) dx = \pi \int_0^4 (4-x)^2 dx = \frac{512\pi}{3}.$$

d) $$V = \int_0^4 \pi \left((8-x)^2 - \frac{(12-x)^2}{4} \right) dx.$$

$$= \frac{\pi}{4} \int_0^4 (112 - 40x + 3x^2) dx = 48\pi$$

7) We have $y^2 + 2y = 2x + 1 \Leftrightarrow x = \frac{y^2 + 2y - 1}{2} =: g(y)$

Leng of the curve is

$$\int_{-1}^3 \sqrt{1 + (g'y)^2}\, dy = \int_{-1}^3 \sqrt{1 + (y+1)^2}\, dy \approx 9.29$$

8.) We have:

$$\left((1 - x^{2/3})^{3/2} \right)' = -x^{-1/3} (1 - x^{2/3})^{1/2}.$$

The the length of the astroid is.

$$8 \int_{\sqrt{2}/4}^1 \sqrt{1 + x^{-2/3}(1 - x^{2/3})}\, dx = 8 \int_{\sqrt{2}/4}^1 x^{-1/3} dx = 12 x^{2/3} \Big|_{\sqrt{2}/4}^1$$