$S = x^2 + 4xh$

$$S(x) = x^2 + 4x \cdot \frac{500}{x^2}$$

$$S(x) = x^2 + \frac{2000}{x}$$

$$S'(x) = 2x - \frac{2000}{x^2}$$

$$2x = \frac{2000}{x^2} \Rightarrow x = 10.$$

$$\Rightarrow h = \frac{500}{10^2} = 5$$

6a.

a. $$\lim_{x \to 1} \frac{2x^2 - (3x+1)\sqrt{x} + 2}{x - 1}.$$

L'hopital rule.

$$= \lim_{x \to 1} \frac{(2x^2 - (3x+1)\sqrt{x} + 2)'}{(x+1)'}$$

$$= \lim_{x \to 1} \frac{4x - \left(\frac{3}{2\sqrt{x}} + \frac{3x+1}{2\sqrt{x}}\right).}{1.}$$

$$= 4.1 - \frac{3}{2\sqrt{1}} + \frac{3 \times 1 + 1}{2\sqrt{1}}$$

$$= -1.$$

9a:

$f(x) = x^4 + x - 5$.

Use newton's method.

$$x_{n+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

x_i: the initial value.

x_{i+1}: the value to be computed

So : $f'(x) = 4x^3 + 1$

$x_0 = -1$

$\Rightarrow x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{f(-1)}{f'(1)} = -2.$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = -2 \frac{f(-2)}{f'(-2)} = -1{,}645$

$x_0 = 1.$

9b: $f(x) = x^4 - 2$ $\quad f'(x) = 4x^3$

$x_0 = 1 \Rightarrow f(1) = 1 - 2 = -1$

$f'(1) = 4.$

$x_1 = 1 - \frac{-1}{4} = 1 + \frac{1}{4} = 1{,}25$

$x_1 = 1{,}25 \Rightarrow f(1{,}25) = (1{,}25)^4 - 2 = 0{,}4414$

$f'(1{,}25) = 4(1{,}25)^3$

$= 7{,}8125.$

$x_2 = 1{,}25 - \frac{0{,}4414}{7{,}8125} = 1{,}1935$

KLONG

(A)

1) Let the dimensions of the rectangle be x, y.
Given the area $A = x.y = 16$
The perimeter $P = 2(x+y)$.

$$y = \frac{16}{x}$$

Substitute y into the perimeter formula.

$$P = 2\left(x + \frac{16}{y}\right)$$

$$\frac{dP}{dx} = 2\left(1 - \frac{16}{x^2}\right) = 0$$

$$\Leftrightarrow 1 = \frac{16}{x^2} \Rightarrow x = 4.$$

$$\Rightarrow y = \frac{16}{4} = 4$$

$$P = 2(4+4) = 16\text{cm}$$

2)

1. Let the triangle have vertices at $(-1, 0)$, $(0, 1)$ and $(1, 0)$.

2. +) slope of line AB: -1 (since it's a 45°)

+) AB: $y = -x + 1$.

b)

$$A = x.y = x(-x+1) = -x^2 + x$$

4)

let the dimensions x (parallel to the river) and y (perpendicular)

Perimeter constraint: $x + 2y = 800$

Area $A = x \cdot y$.

$$y = \frac{800 - x}{2}.$$

$$A = x \, \frac{800 - x}{2} = 400x - \frac{x^2}{2}.$$

Differentiate A with respect to x.

$$\frac{dA}{dx} = 400 - x.$$

set $\frac{dA}{dx} = 0$. | $400 - x = 0$, $\Rightarrow x = 400$

$$y = \frac{800 - 400}{2} = 200.$$

Maximum Area: $400 \times 200 = 80\,000 \, m^2$

5)

let the side length of the box x, and the height be h volume constraint

$$x^2 \cdot h = 500.$$

a) $h = \frac{500}{x^2}$

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a) AB : $y = -x + 1$.

c) $\frac{dA}{dx} = -2x + 1$.

$\frac{dA}{dx} = 0$:

$\Rightarrow -2x + 1 = 0 \Rightarrow x = \frac{1}{2}$

+) $x = \frac{1}{2}$ into $y = -x + 1 \Rightarrow y = \frac{1}{2}$.

Maximum area is: $A = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$.

3) , let x be the side length of each square cut from the corners.

+ The new dimensions of the box are $(8-2x)$, $(15-2x)$, and height x.

+ $V = x(8-2x)(15-2x)$

~~To maximize V differentiate V with respect to x and set it to zero. Again solving for x,~~

$V = x(120 - 30x - 16x + 4x^2)$

$= 4x^3 - 46x^2 + 120x$

$\frac{dV}{dx} = 12x^2 - 92x + 120 = 0$

6b.

$$\lim_{x\to 0} \frac{x(\cos x - 1)}{\sin x - x}$$

$$= \lim_{x\to 0} \frac{\cos x - 1 - x\sin x}{\cos x - 1}$$

$$= \lim_{x\to 0} \frac{-\sin x - x\cos x - \sin x}{-\sin x}$$

$$= \lim_{x\to 0} \frac{-\cos x - \cos x + x\sin x - \cos x}{-\cos x}$$

$$= 3.$$

7)

$$f(x) = \begin{cases} \frac{9x - 3\sin 3x}{5x^3} & \text{if } x \neq 0 \\ c & \text{if } x = 0. \end{cases}$$

The function is continuous at $x = 0$ if $\lim_{x\to 0} f(x) = 0$.

$$\lim_{x\to 0} f(x) = \frac{9x - 3\sin 3x}{5x^3}$$

$$\lim_{x\to 0} f(x) = \frac{9 - 9\cos 3x}{15x^2}.$$

$$\lim_{x\to 0} f(x) = \frac{27\sin 3x}{30x}.$$

$$\lim_{x\to 0} f(x) = \frac{81\cos 3x}{30} = \frac{27}{10}.$$

Thus $c = \frac{27}{10}$.

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