Problem 2.

$f(x) = x^2 + 1$.

We divide the interval $[0, 3]$ into n equal subinterval

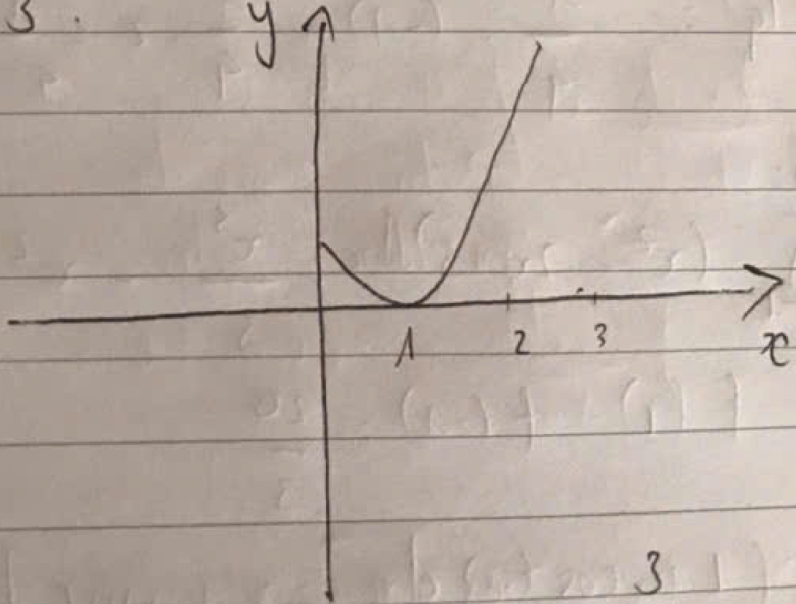
Then we have the grid point given by $x_1 = 0$, $x_{n+1} = 3$ and $x_k = x_3 + (k-1) \cdot \frac{3}{n}$ $(k = 2, \dots n)$

$$S_n = \sum_{k=1}^{n} f(x_n + 1) \frac{3}{n} = \frac{3}{n} \left[\sum^{n} \left(\frac{3k}{n}\right)^2 + 1 \right].$$

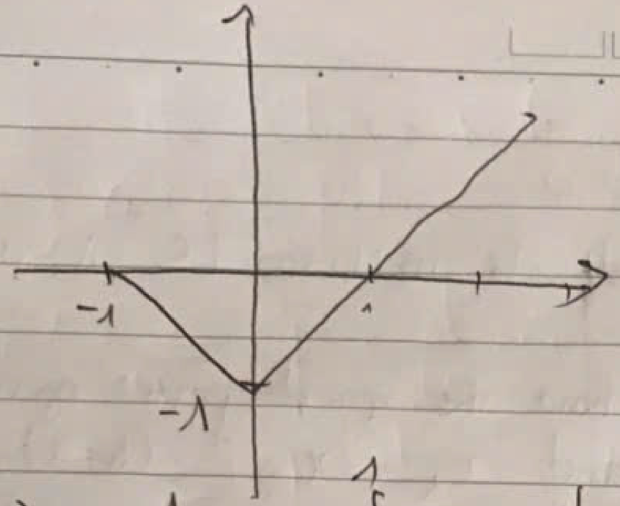
$$= 3 + \frac{9(n+1)(2n+1)}{2n^2}.$$

$$\rightarrow \lim_{x \to \infty} S_n = 12.$$

Problem 3.



The average value: $\frac{1}{3-0} \cdot \int_0^3 (x-1)^2 dx = 1.$



i) $av(g) = \frac{1}{1-(-1)} \cdot \int_{-1}^{1} (|x|-1)\,dx.$

$= \frac{1}{2} \int_{-1}^{0} (-x-1)\,dx + \frac{1}{2} \cdot \int_{0}^{1} (x-1)\,dx = \frac{1}{2}.$

ii) $av(g) = \frac{1}{3-1} \cdot \int_{1}^{3} (|x|-1)\,dx = \frac{1}{2} \int_{1}^{3} (x-1)\,dx = 1$

iii) $av(g) = \frac{1}{4} \cdot (-1) + \frac{1}{4} \cdot 2 = \frac{1}{4}.$

Problem 4.

a) $\int_{-1}^{1} (x^2 - 2x + 3)\,dx = \frac{x^3}{3} - x^2 + 3x \Big|_{-1}^{1}.$

$f(1) - f(-1) = \frac{20}{3}.$

b) $\int_{0}^{\pi} (1 + \cos(x))\,dx = x + \sin x \Big|_{0}^{\pi}$

$f(\pi) - f(0) = \pi$

5.

a) $\int_0^{\sqrt{\pi/2}} x \cos(x^2)\,dx$.

Let $u = x^2 \to du = 2x\,dx \to \frac{du}{2} = x\,dx$

x	0	$\sqrt{\pi/2}$
u	0	$\pi/2$

We have $\int_0^{\pi/2} \frac{1}{2}\cos u\,du = \frac{1}{2}\sin u \Big|_0^{\pi/2} = \frac{1}{2}$.

b) $\int_2^5 \frac{x\,dx}{\sqrt{1+x^2}}$

Let $u = 1+x^2 \to du = 2x\,dx \to \frac{1}{2}du = x\,dx$

x	2	5
u	5	26

We have: $\int_5^{26} \frac{1}{2} \cdot \frac{1}{\sqrt{u}}\,du = \frac{1}{2} 2\sqrt{u} \Big|_5^{26}$

$= \sqrt{26} - \sqrt{5}$.

Problem 6.

a) $\int_1^{\sqrt{t}} \left(x^4 + \frac{3}{x^3}\right)dx = \frac{x^5}{5} - \frac{3}{2x^2}\Big|_1^{\sqrt{t}}$.

$= \left(\frac{\sqrt{t}^5}{5} - \frac{3}{2t}\right) - \left(\frac{1}{5} - \frac{3}{2}\right)$

$= \frac{t^2\sqrt{t}}{5} - \frac{3}{2t} + \frac{13}{10}$

7 $\frac{d}{dt}\int_1^{\sqrt{t}} \left(x^4 + \frac{3}{x^3}\right)dx = \frac{d}{dt}\left(\frac{t^2\sqrt{t}}{5} - \frac{3}{2t} + \frac{13}{10}\right)$

$= \frac{t\sqrt{t}}{2} + \frac{3}{2t^2}$

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b) We use the Chain Rule of differentiation

$$\frac{d}{dt}\int_1^{\sqrt{t}} \left(x^4 + \frac{3}{t^2}\right) dx$$

$$= \left[\sqrt{t}^{\,4} + \frac{3}{(\sqrt{t})^3}\right] \cdot \frac{d}{dt}(\sqrt{t}\,))$$

$$= t^2 + \frac{3}{t\sqrt{t}}\,) \cdot \frac{1}{2\sqrt{t}} = \frac{t\sqrt{t}}{2}.$$

Problem 7.

a) $$\int \frac{(1+\sqrt{x})^{1/3}}{\sqrt{x}}\,dx = \Big|\; u = 1+\sqrt{x}$$

$$du = \frac{dx}{2\sqrt{x}} \rightarrow \frac{dx}{\sqrt{x}} = 2du.$$

Then: $$\int 2u^{1/3}\,du = \frac{3}{2}u^{4/3} + C = \frac{3}{2}(1+\sqrt{x})^{4/3} + C$$

b) $$\int \frac{9r^2\,dr}{\sqrt{1-r^3}}\; x \;\Big|\; u = 1-r^3$$

$$\rightarrow du = -3r^2\,dr.$$

$$\rightarrow 9r^2\,dr = -3du$$

Then: $$\int \frac{-3\,du}{\sqrt{u}} = -6\sqrt{u} + C = -6\sqrt{1-r^3} + C.$$

Problem 8:

a) $x = 2y^2 \Rightarrow y = \sqrt{\frac{x}{2}}$

$x = 0$

$y = 3 \rightarrow x = 18$

$$\rightarrow S = \int_0^{18} \sqrt{\frac{x}{2}}\,dx$$

$$= 36.$$

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b) $y^2 = 4x \to y = 2\sqrt{x}$.

$y = 4x - 2$.

$\to 2\sqrt{x} = 4x - 2$.

$\to \left[\begin{array}{l} x = 1 \\ x = \frac{1}{2} \end{array}\right. \to S = \int_0^1 (4x - 2\sqrt{x} - 2)\,dx$.

$= \frac{4}{3}$.

c) $y = \sin x$

$y = x \longrightarrow S = \int_0^{\pi/4} (\sin x - x)\,dx$

$\to \sin x = 0 \qquad = \left(-\cos\left(\frac{\pi}{4}\right) + 1\right) - \frac{\pi^2}{32}$

Problem 9:

$S = -\underbrace{\int_{-2}^{0} (x\sqrt{4-x^2})\,dx}_{I_1} + \underbrace{\int_0^2 x\sqrt{4-x^2}\,dx}_{I_2}$.

$I_1 = \frac{1}{2}\int_0^4 \sqrt{u}\,.du = \frac{8}{3}$.

$I_2 = \int_0^2 (x\sqrt{4-x^2})\,dx$.

x	0	2
u	4	0

$\to I_2 = -\frac{1}{2}\int_4^0 \sqrt{u}\,.du = \frac{1}{2}\int_0^4 \sqrt{u}\,du = \frac{8}{3}$

$\to S = \frac{16}{3}$.

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Problem 10:

$2x^2 \geq x^4 - 2x$ for x from -2 to 2

$\to S = \int_{-2}^{2} (2x^2 - x^4 + 2x^2)dx = \frac{128}{15}$