Final Report

Discrete Structures

Regulations

You should solve and submit this report to your elearning account within 14 days, from the beginning of May 12th, 2025, to the end of May 25th, 2025. Late submission is not accepted.

This is a group final report. The number of students in a group should be 2 or 3.

Students need to submit a compressed file named with your Student IDs. For example, in a group of 3 students, said 52300123, 52300136 and 52301001, then the submission file name is 52300123_52300136_52301001.zip/rar, including this structure:

- The document file is in Word format (.doc/docx), named by your Student IDs, eg. 52300123_52300136_52301001.docx, using our faculty's format, from 25 to 35 pages.
    - The tasks of each member and self-evaluation of your group should be declared at the end of this report.
    - English is required for high-quality classes. Format violations will cost from 10% to 50% of your total scores.
    - Any case of plagiarism will get 0.

In the test, there are questions that need to be customized according to a student ID. Only the smallest student ID in your group will be used for this purpose. For example, in a group of 3 students, said 52300123, 52300136 and 52301001, then only 52300123 are used. Let $\overline{abcd}$ be the 4-digit number combined by the last 4 digits in this student. In our example, student 52300123 represents the group and has $\overline{abcd}$ = 123.

Problem 1: Password

A hacker is trying to hack a password. He knows that this password has 3 characters, each of which is a distinct number from 1 to 9. He also learns from his trials that:

a. 472: one number is correct but in an incorrect position.

b. 581: one number is correct but in an incorrect position.

c. 483: one number is correct and in the correct position.

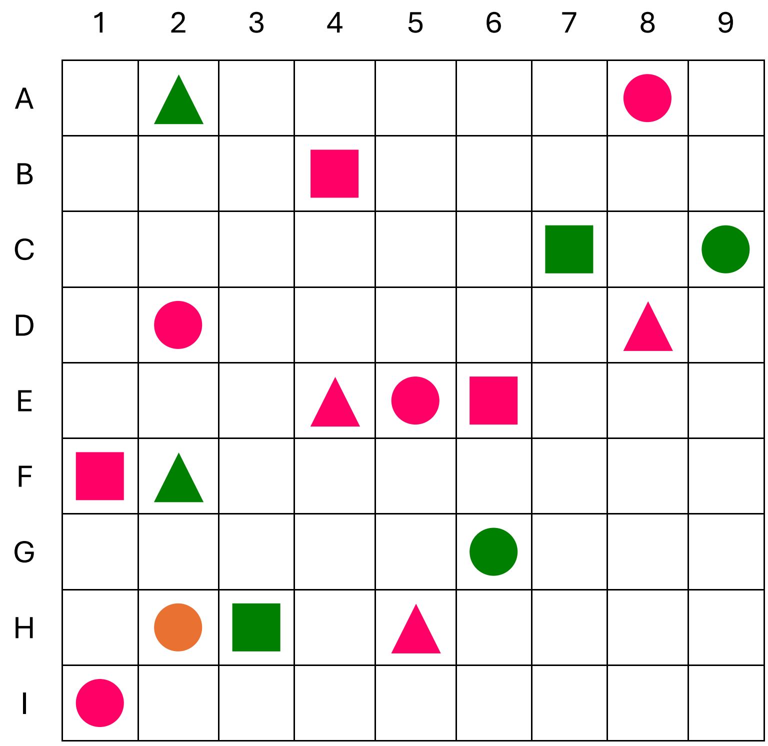
d. 317: two numbers are correct but in incorrect positions.

e. 956: all numbers are incorrect.

Please help him to find the password with good reasoning.

Problem 2: Tarski's world

Giving the following Tarski's world.



Items are named by their positions. These notations are defined:

- Triangle(x), meaning “x is a triangle,”
- Circle(x), meaning “x is a circle,”
- Square(x), meaning “x is a square,”
- Red(x), meaning “x is red,”
- Green(x), meaning “x is green,”
- Orange(x), meaning “x is orange,”
- RightOf(x, y), meaning “x is to the right of y (but possibly in a different row),”
- LeftOf(x, y), meaning “x is to the left of y (but possibly in a different row),”
- AboveOf(x, y), meaning “x is to the above of y (but possibly in a different column),”
- BelowOf(x, y), meaning “x is to the below of y (but possibly in a different column).”

The domain for all variables is the set of objects in Tarski’s world shown in the picture.

a. Modify the above Tarski's world as follows:

- If $\overline{abcd}$ % 7 = 0 then add a red square in A5.
- If $\overline{abcd}$ % 7 = 1 then add a green triangle in F3.
- If $\overline{abcd}$ % 7 = 2 then delete the item at G6.
- If $\overline{abcd}$ % 7 = 3 then delete the item at H5.
- If $\overline{abcd}$ % 7 = 4 then change the item at E4 into an orange square.

- If $\overline{abcd}$ % 7 = 5 then change the item at E5 into a green circle.
- If $\overline{abcd}$ % 7 = 6 then change the item at E6 into a red triangle.

Re-draw your new Tarski's world.

b. Determine the truth or falsity of all the following statements, based on the modified Tarski's world. Give the reasons for your justification.

i. ∀x, Circle(x) → Green(x)
ii. ∀x, Triangle(x) → ~Orange(x)
iii. ∃x such that Red(x) ∧ Triangle(x)
iv. ∃x such that ~Green(x) ∧ BelowOf(x, E4)
v. ∀x, Square(x) → RightOf(E5, x).
vi. ∃x such that AboveOf(E5, x) ∧ LeftOf(x, E5).
vii. There is a triangle x such that for all squares y, x is above y.
viii. For all circles x, there is a square y such that y is to the right of x.
ix. There is a circle x and there is a square y such that y is below x.
x. For all circles x and for all triangles y, x and y have the same color.

Problem 3: Euclid's algorithm and Bezout's identity

a. Using Euclid's algorithm to calculate *gcd*(2025, 1000 + *m*) and *lcm*(2025, 1000 + m), where *m* is the last 3 digits of your student ID. For example, if your student ID is 52200**123** then you need to calculate *gcd*(2025, 1123) and *lcm*(2025, 1123).

b. Apply above result(s) in to find 5 integer solution pairs (x,y) of this equation:

$$2025x + (1000 + m)y = gcd(2025, 1000 + m)$$

For example, if your student ID is 52000**123** then your equation is:

$$2025x + 1123y = gcd(2025, 1123)$$

Problem 4: Equation with interger solutions

Solve the equation ($\overline{ab}$ + 1) x + 101y = ($\overline{cd}$ + 2) for integers x, y.

Where *a, b, c, d* are the last digits of your StudentID*.* For example, Student ID 52301234 has $\overline{ab}$ = 12 and $\overline{cd}$ = 34.

Problem 5: Recurrence relation

Solve this recurrence relation.

$$a_n = 8.a_{n-1} - 15.a_{n-2}$$

with $a_0 = 5$ and $a_1 = m$,

where *m* is the last 2 digits of your student ID. For example, if your student ID is 52300**23** then $a_1 = 23$.

Problem 6: Self-study on planar graphs

Part A – Theoretical Review (Written Report)

Please prepare a concise written report (2–3 pages) addressing the following points:

1. **Definition and Basic Concepts**
    - What is a *planar graph*?
    - Definitions of *plane embedding*, *face*, *outer face*, and *crossing*.
2. **Euler's Formula**
    - State and explain Euler's formula for connected planar graphs.
    - Apply Euler's formula to simple examples (e.g., trees, K_4).
3. **Non-planar Graphs**
    - Introduce Kuratowski's Theorem.
    - Explain why K_5 and $K_{3,3}$ are non-planar.
    - Discuss the idea of *graph minor* and how it relates to planarity.
4. **Applications of Planar Graphs**
    - Give one real-world application of planar graphs (e.g., map coloring, VLSI design).

Please ensure that your explanations are in your own words and supported by diagrams where appropriate.

Part B – Problem Solving

- If $\overline{abcd}$ % 4 = 0 then solve question 1.
- If $\overline{abcd}$ % 4 = 1 then solve question 2.
- If $\overline{abcd}$ % 4 = 2 then solve question 3.
- If $\overline{abcd}$ % 4 = 3 then solve question 4.

Show your reasoning clearly.

1. **(Planarity Check)**
   Determine whether the following graphs are planar. Justify your answers:
    - (a) K_5 minus one edge
    - (b) $K_{3,3}$ minus one edge
    - (c) A graph consisting of a cycle C_6 with one diagonal (i.e., an edge between two non-adjacent vertices)
2. **(Face Counting with Euler's Formula)**
   Given a connected planar graph with 10 vertices and 15 edges, how many faces

does any planar embedding of this graph have? Illustrate with a possible drawing.

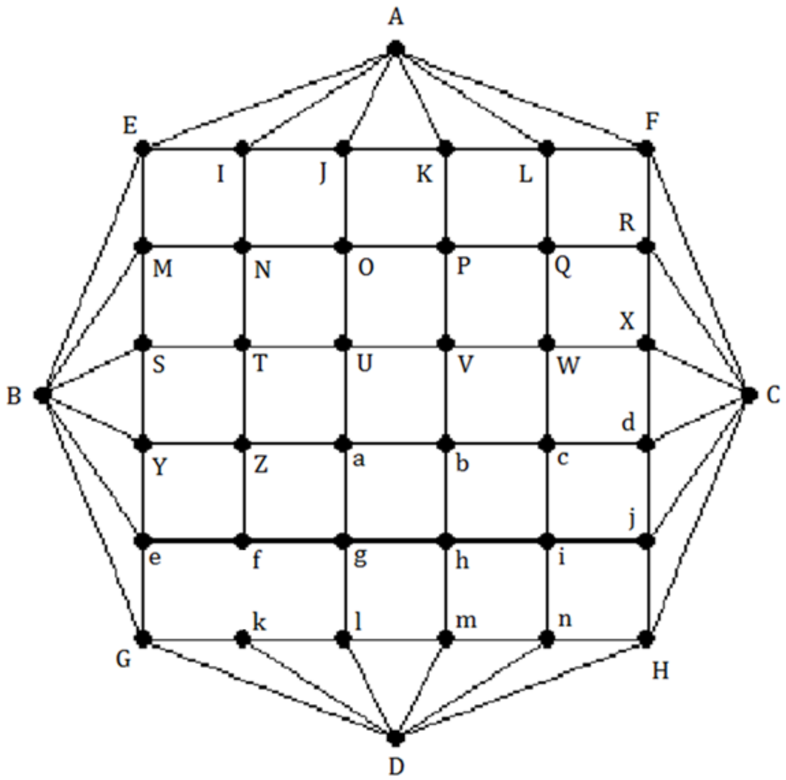
3. **(Drawing Planar Embeddings)**
   Provide a planar embedding (no edge crossings) for the graph defined by:
   - Vertices: {A, B, C, D, E}
   - Edges: {AB, AC, AD, AE, BC, BD, CD}

   If it is not possible to draw it without crossings, explain why.

4. **(Application Problem)**
   Consider a map divided into 6 regions such that no more than three regions meet at a point. Show how to model this as a planar graph and explain how the Four Color Theorem applies.

Problem 7: Eulerian circuit

a. Does the following graph have an Eulerian circuit or Eulerian trail? Why?



c. If the graph has an Eulerian trail, based on the idea of Hierholzer's algorithm, find an Eulerian trail of the graph. Present your solution step by step.

Problem 8: Map coloring

Given this map of the United States of Africa:

Source: https://en.wikipedia.org/wiki/United_States_of_Africa

a. Modeling this map by a graph.

b. Color the map with a minimum number of colors. Present your solution step by step.

i. If $\overline{abcd}$ % 4 = 0 then start from Ethiopia.

ii. If $\overline{abcd}$ % 4 = 1 then start from Morocco.

iii. If $\overline{abcd}$ % 4 = 2 then start from Chad.

iv. If $\overline{abcd}$ % 4 = 3 then start from Angola.

Rubric

Criteria	Scale	1	2	3	Self-evalutaion	Reason
	Score /10	0 score	1/2 score	Full score		
Problem 1	1	Do nothing or wrongly.	Right password but the reasoning is not good enough.	Right password and good reasoning.		
Problem 2	1	Do nothing or wrongly. Wrong a)	Right result but wrong explaination.	Right result and right explaination.		
Problem 3	1	Do nothing or wrongly.	Correct gcd and lcm, but incorrect solutions of the Bezout's identity.	Correct calculation, detailed explanation.		
Problem 4	1	Do nothing or wrongly.	Right method but wrong result.	All are correct.		
Problem 5	1	Do nothing or wrongly.	Correct calculation but wrong result or conclusion.	Correct calculation, detailed explanation.		
Problem 6A	1	Do nothing or wrongly	Not enough details, no example, no comment	Correct calculations, detailed explanations		
Problem 6B	0.5	Do nothing or wrongly.	Right method but wrong result.	Correct answer, detailed explanation.		
Problem 7	2	Do nothing or wrongly.	a-Correct recognition, right explanation.	a-Correct recognition, right explanation.		

			b-Good idea but incorrect applications.	b-Good idea, right calculation, detailed explanation.		
Problem 8	1.5	Do nothing or wrongly.	Correct modeling but wrong coloring.	Correct modeling but right coloring.		
Total	10			Result	0	