523YC0018 – Lê Nguyễn Cao Duy

7/b)

b) $B = \begin{pmatrix} -3m & 2 & 4 \\ 1 & -m & 2 \\ 3 & 1-m & 4 \end{pmatrix}$

$\det B \quad \begin{vmatrix} -3m & 2 & 4 \\ 1 & -m & 2 \\ 3 & 1-m & 4 \end{vmatrix} = 12m^2 + 12 + 4 - 4m + 12m - 8 + 6m + 6m^2$

$= 18m^2 + 14m + 8$

Để B có nghịch đảo thì $\det B \neq 0 \Leftrightarrow 18m^2 + 14m + 8 \neq 0$

Đế B^2 B có nghịch đảo thì $\det B \neq 0 \Leftrightarrow 18m^2 + 14m + 8 \neq 0$

Vậy B không có nghịch đảo

9/e)

$\begin{pmatrix} 1 & -2 & 1 & -1 \end{pmatrix}$

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$R_4 - 3R_3 \to R_4$

$$\begin{pmatrix} 1 & -2 & 1 & -1 \\ 0 & 2 & -1 & 2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & -5 \end{pmatrix}$$

g/e/

$$E = \begin{pmatrix} 1 & -2 & 1 & -1 \\ -1 & 4 & -2 & 3 \\ 2 & 0 & 1 & -2 \\ -2 & 6 & 0 & 2 \end{pmatrix}$$

Ta có: Ma trận mở rộng

$$\det E = \begin{pmatrix} 1 & -2 & 1 & -1 & 1 & 0 & 0 & 0 \\ -1 & 4 & -2 & 3 & 0 & 1 & 0 & 0 \\ 2 & 0 & 1 & -2 & 0 & 0 & 1 & 0 \\ -2 & 6 & 0 & 2 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$\xrightarrow{\begin{array}{c} R_2 + R_1 \to R_2 \\ R_3 - 2R_1 \to R_3 \\ R_4 + 2R_1 \to R_4 \end{array}}$
$$\begin{pmatrix} 1 & -2 & 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 2 & -1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 4 & -1 & 0 & -2 & 0 & 1 & 0 \\ 0 & 2 & 2 & 0 & 2 & 0 & 0 & 1 \end{pmatrix}$$

$\xrightarrow{\begin{array}{c} R_3 - 2R_2 \to R_3 \\ R_4 - R_2 \to R_4 \end{array}}$
$$\begin{pmatrix} 1 & -2 & 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 2 & -1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -4 & -4 & -2 & 1 & 0 \\ 0 & 0 & 3 & -2 & 1 & -1 & 0 & 1 \end{pmatrix}$$

$\xrightarrow{R_4 - 3R_3 \to R_4}$
$$\begin{pmatrix} 1 & -2 & 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 2 & -1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -4 & -4 & -2 & 1 & 0 \\ 0 & 0 & 0 & -5 & 13 & 5 & -3 & 1 \end{pmatrix}$$

$\xrightarrow{\frac{1}{-5} R_4 \to R_4}$
$$\begin{pmatrix} 1 & -2 & 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 2 & -1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -4 & -4 & -2 & 1 & 0 \\ 0 & 0 & 0 & 1 & -13/5 & -1 & 3/5 & -1/5 \end{pmatrix}$$

$\xrightarrow{\begin{array}{c} R_1 + R_4 \to R_1 \\ R_3 + 4R_4 \to R_3 \end{array}}$
$$\begin{pmatrix} 1 & -2 & 1 & 0 & ? & ? & ? & ? \\ 0 & 2 & -1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -7/5 & -1 & 2/5 & -1/5 \\ 0 & 0 & 0 & 1 & -13/5 & -1 & 3/5 & -1/5 \end{pmatrix}$$

$\xrightarrow{R_2 + \frac{1}{2} R_3 \to R_2}$
$$\begin{pmatrix} 1 & -2 & 1 & -1 & ? & ? & ? & ? \\ 0 & 1 & -1/2 & 1 & ? & ? & ? & ? \\ 0 & 0 & 1 & 0 & -7/5 & -1 & 2/5 & -1/5 \\ 0 & 0 & 0 & 1 & -13/5 & -1 & 3/5 & -1/5 \end{pmatrix}$$

$\xrightarrow{R_1 + R_4 \to R_1}$
$$\begin{pmatrix} 1 & -2 & 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1/2 & 1 & ? & ? & ? & ? \\ 0 & 0 & 1 & 0 & -7/5 & -1 & 2/5 & -1/5 \\ 0 & 0 & 0 & 1 & -13/5 & -1 & 3/5 & -1/5 \end{pmatrix}$$

$$\left(\begin{array}{cccc|cccc} 1 & -2 & 1 & 0 & -8/5 & -1 & 3/5 & -1/5 \\ 0 & 1 & 0 & 0 & 12/5 & 1 & -2/5 & 3/10 \\ 0 & 0 & 1 & 0 & -7/5 & -1 & 2/5 & 1/5 \\ 0 & 0 & 0 & 1 & -13/5 & -1 & 3/5 & -1/5 \end{array}\right) \xrightarrow{R_1 - R_3 \to R_1} \left(\begin{array}{cccc|cccc} 1 & -2 & 0 & 0 & -1/5 & 0 & 1/5 & -2/5 \\ 0 & 1 & 0 & 0 & 12/5 & 1 & -2/5 & 3/10 \\ 0 & 0 & 1 & 0 & -7/5 & -1 & 2/5 & 1/5 \\ 0 & 0 & 0 & 1 & -13/5 & -1 & 3/5 & -1/5 \end{array}\right)$$

$$\xrightarrow{R_1 + 2R_2 \to R_1} \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 23/5 & 2 & -3/5 & 1/5 \\ 0 & 1 & 0 & 0 & 12/5 & 1 & -2/5 & 3/10 \\ 0 & 0 & 1 & 0 & -7/5 & -1 & 2/5 & 1/3 \\ 0 & 0 & 0 & 1 & -13/5 & -1 & 3/5 & -1/5 \end{array}\right)$$

Vậy

$$E = \left(\begin{array}{cccc} 1 & -2 & 1 & -1 \\ -1 & 4 & -2 & 3 \\ 2 & 0 & 1 & 3 \\ -2 & 6 & 0 & 2 \end{array}\right)^{(-1)} = \left(\begin{array}{cccc} 23/5 & 2 & -3/5 & 1/5 \\ 12/5 & 1 & -2/5 & 3/10 \\ -7/5 & -1 & 2/5 & 1/5 \\ -13/5 & -1 & 3/5 & -1/5 \end{array}\right)$$

14/ f)

$$\begin{cases} 3x + 2y + z - w = 3 \\ x - 2y - 2z + 2w = 2 \\ 2x + 3y - z + w = 1 \\ -x + y + 2z - 2w = -2 \end{cases}$$

$$\det A = \begin{vmatrix} 3 & 2 & 1 & -1 \\ 1 & -1 & -2 & 2 \\ 2 & 3 & -1 & 1 \\ -1 & 1 & 2 & -2 \end{vmatrix} \Rightarrow \det A = 3(-1)^{1+1}\begin{vmatrix} -1 & -2 & 2 \\ 3 & -1 & 1 \\ 1 & 2 & -2 \end{vmatrix} + 1(-1)^{1+2}\begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 1 \\ -1 & 2 & -2 \end{vmatrix}$$

$$+ 2(-1)^{1+3}\begin{vmatrix} 1 & -1 & 2 \\ 2 & 3 & 1 \\ -1 & 1 & -2 \end{vmatrix} + (-1)(-1)^{1+4}\begin{vmatrix} 1 & -1 & -2 \\ 2 & 3 & -1 \\ -1 & 1 & 2 \end{vmatrix} = 3(\cancel{0})\cancel{1}\cancel{8} - 12 \neq 0 \Rightarrow A \text{ có nghiệm}$$

$$A_1 = \begin{vmatrix} 3 & 2 & 1 & -1 \\ 2 & -1 & -2 & 2 \\ 1 & 3 & -1 & 1 \\ 2 & 1 & 2 & -2 \end{vmatrix} \Rightarrow \det A_1 = 0 \quad ; \quad A_2 = \begin{vmatrix} 3 & 3 & 1 & -1 \\ 1 & 2 & -2 & 2 \\ 2 & 1 & -1 & 1 \\ -1 & 2 & 2 & -2 \end{vmatrix} \Rightarrow \det A_2 = 0$$

$$A_3 = \begin{vmatrix} 3 & 2 & 3 & -1 \\ 1 & -1 & 2 & 2 \\ 2 & 3 & 1 & 1 \\ -1 & 1 & 2 & -2 \end{vmatrix} \Rightarrow \det A_3 = \cancel{100}\,80 \quad ; \quad A_4 = \begin{vmatrix} 3 & 2 & 1 & 3 \\ 1 & -1 & -2 & 2 \\ 2 & 3 & -1 & 1 \\ -1 & 1 & 2 & 2 \end{vmatrix} \Rightarrow \det A_4 = 80$$

$$\Rightarrow \begin{cases} x = \dfrac{\det A_1}{\det} = 0 \\ y = 0 \\ z = 80/-12 = -20/3 \\ w = 80/-12 = -20/3 \end{cases}$$