EXERCISES

1. Use the following matrices for problem:
$A = \begin{pmatrix} 1 & 0 & -2 \\ 3 & 2 & 1 \\ 4 & 0 & 3 \end{pmatrix}$; $B = \begin{pmatrix} 1 & 1 & 3 & 0 \\ 4 & 2 & 1 & 1 \\ 3 & 2 & 0 & 1 \end{pmatrix}$; $C = \begin{pmatrix} 5 & 3 \\ 1 & 2 \end{pmatrix}$; $D = \begin{pmatrix} 4 & 2 \\ 3 & 5 \end{pmatrix}$
$E = \begin{pmatrix} 1 & 0 & 4 \\ 5 & 1 & 0 \end{pmatrix}$; $F = \begin{pmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \\ 2 & -3 & -4 \end{pmatrix}$; $G = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 0 & 4 \\ 5 & 1 & 0 \end{pmatrix}$
Perform the operation, if possible
$$C+D;\quad A-F;\quad A+A^T;\quad 8C-3D,\quad 4F+3G;\quad A^2+B;\quad (F+G)A;\quad (A-B)(F-2I_3)$$

2. Give $A = \begin{pmatrix} 1 & -1 & 2 \\ 2 & 3 & 0 \\ 3 & 1 & 4 \end{pmatrix}$ $B = \begin{pmatrix} 1 & -1 & 2 \\ 2 & 3 & 1 \\ 3 & 1 & 5 \end{pmatrix}$ $C = \begin{pmatrix} -2 & 1 & 7 \\ 1 & -5 & 1 \\ 2 & 7 & -3 \end{pmatrix}$. FIND AB, ABC, $2A + BC$

3. Find x, y, z and t

   (a) $\begin{pmatrix} x & 1 & 0 \\ 0 & y & z \\ t & 2 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 1 & 0 \\ 0 & 2 & 3 \\ 4 & -5 & 1 \end{pmatrix}$

   (b) $\begin{pmatrix} 0 & x & 1 \\ 3 & y & y \\ z & 0 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 4 & 1 \\ 3 & 1 & y \\ 1 & 0 & t \end{pmatrix}$

   (c) $\begin{pmatrix} x & 3 & 2x-1 \\ y & 4 & 4y \end{pmatrix} = \begin{pmatrix} 2x-4 & z & 7 \\ 1 & t+1 & 3y+1 \end{pmatrix}$

   (d) $\begin{pmatrix} x & y & x+3 \\ z & 4 & 4y \end{pmatrix} = \begin{pmatrix} 2x-1 & -1 & t \\ x & 5+y & -4 \end{pmatrix}$

4. Solve for x, y and z if
$$3\begin{pmatrix} x & y \\ y & z \end{pmatrix} + 2\begin{pmatrix} 2x & -y \\ 3y & -4z \end{pmatrix} = \begin{pmatrix} 14 & 4-y \\ 18 & 15 \end{pmatrix}$$

5. Solve for x, y, z and w if
$$3\begin{pmatrix} x & 4 \\ 4y & w \end{pmatrix} - 5\begin{pmatrix} 4x & 2z \\ -3 & -2w \end{pmatrix} = \begin{pmatrix} 20 & 30 \\ 5 & -7 \end{pmatrix}$$

6. Solve the Determinant of matrices

   (a) $A = \begin{pmatrix} 1 & 0 & 2 & a \\ 2 & 0 & b & 0 \\ 3 & c & 4 & 5 \\ d & 0 & 0 & 0 \end{pmatrix}$

   (b) $A = \begin{pmatrix} m & 1 & 1 & 1 \\ 1 & m & 1 & 1 \\ 1 & 1 & m & 1 \\ 1 & 1 & 1 & m \end{pmatrix}$

   (c) $C = \begin{pmatrix} x & a & b & 0 & c \\ 0 & y & 0 & 0 & d \\ 0 & e & z & 0 & f \\ g & h & k & u & l \\ 0 & 0 & 0 & 0 & v \end{pmatrix}$

7. Find the conditions for the existence of an inverse matrix. From there, find the inverse of the following matrices:

   (a) $A = \begin{pmatrix} m+1 & -m & 3 \\ 3 & -4m & 2m \\ 1 & -2 & -m \end{pmatrix}$

   (b) $B = \begin{pmatrix} -3m & 2 & 4 \\ 1 & -m & 2 \\ 3 & 1-m & 4 \end{pmatrix}$

(c) $C = \begin{pmatrix} 4m & 3 & -4 \\ 2+2m & 1 & -m \\ 1 & -1 & m-2 \end{pmatrix}$ (d) $D = \begin{pmatrix} -2 & -2 & m \\ -2 & m & -2 \\ m & -2 & -2 \end{pmatrix}$

8. Using elementary row transformations, calculate the following determinants:

(a) $A = \begin{vmatrix} 1 & -1 & 2 \\ -1 & 2 & 1 \\ 2 & -3 & 2 \end{vmatrix}$

(b) $B = \begin{vmatrix} 1 & -2 & 1 & -1 \\ -1 & 4 & -2 & 3 \\ 2 & 0 & 1 & 3 \\ -2 & 6 & 0 & 5 \end{vmatrix}$

(c) $C = \begin{vmatrix} 2 & -1 & 0 & 3 \\ 1 & 1 & 2 & -1 \\ -1 & 2 & 3 & 1 \\ 0 & 1 & 2 & 1 \end{vmatrix}$

(d) $D = \begin{vmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \end{vmatrix}$

(e) $E = \begin{vmatrix} 1 & 2 & 3 & 4 \\ -2 & 1 & 4 & 3 \\ 3 & -4 & -1 & 2 \\ 4 & 3 & -2 & -1 \end{vmatrix}$

9. Using the Gauss - Jordan method, find the inverse of the following matrices (if any)

(a) $A = \begin{pmatrix} 2 & 1 & -1 \\ 0 & 1 & 3 \\ 2 & 1 & 1 \end{pmatrix}$

(b) $B = \begin{pmatrix} 1 & 3 & 2 \\ -1 & 0 & 1 \\ 2 & 2 & 3 \end{pmatrix}$

(c) $C = \begin{pmatrix} 1 & 3 & -2 & 4 \\ -2 & 0 & 1 & 3 \\ 2 & -3 & 0 & 1 \\ 4 & 1 & -2 & 3 \end{pmatrix}$

(d) $D = \begin{pmatrix} 2 & 3 & 0 & 0 \\ 3 & 2 & 0 & 0 \\ 1 & 1 & 3 & 4 \\ 2 & -1 & 2 & 3 \end{pmatrix}$

(e) $E = \begin{pmatrix} 1 & -2 & 1 & -1 \\ -1 & 4 & -2 & 3 \\ 2 & 0 & 1 & 3 \\ -2 & 6 & 0 & 2 \end{pmatrix}$

10. Find the matrix X that

(a) $\begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} X = \begin{pmatrix} 4 & -6 \\ 2 & 1 \end{pmatrix}$

(b) $X \begin{pmatrix} 1 & -1 & 2 \\ 1 & 0 & -1 \\ 1 & 1 & -2 \end{pmatrix} = \begin{pmatrix} 6 & 2 & -7 \\ 15 & 2 & -13 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & -1 & 1 \\ -1 & 2 & 1 \\ -2 & 3 & 1 \end{pmatrix} X = \begin{pmatrix} 1 & 1 & 1 & -1 \\ 1 & 0 & 2 & 2 \\ 1 & -2 & -2 & 0 \end{pmatrix}$

(d) $\begin{pmatrix} 3 & 0 & 1 \\ 8 & 1 & 1 \\ 5 & -3 & -2 \end{pmatrix} \begin{pmatrix} 1 & -1 & 1 \\ 1 & 0 & -1 \\ 1 & 1 & -2 \end{pmatrix} X = \begin{pmatrix} -3 & 2 & 1 \\ 4 & 5 & 6 \\ 5 & 7 & -8 \end{pmatrix}$

11. Let $A = \begin{pmatrix} 3 & 1 & -3 \\ 2 & -5 & 1 \\ 0 & 2 & -1 \end{pmatrix}$ $B = \begin{pmatrix} 2 & -1 & 3 \\ 5 & 0 & 8 \\ 1 & -9 & 0 \end{pmatrix}$. Find the matrix X that

(a) $A + B = X - I_3$

(b) $AXB = B^T$

(c) $XAB = A^T$

(d) $2X - I_3 = 3A - 5B$

12. Find the rank of matrices

(a) $A = \begin{pmatrix} 1 & 2 & -1 & 0 \\ -1 & 2 & 4 & 2 \\ -3 & -2 & 6 & 2 \end{pmatrix}$

(b) $B = \begin{pmatrix} 1 & 2 & 3 & -2 & 6 \\ 2 & -1 & 2 & -1 & 0 \\ 5 & 3 & 8 & 1 & 1 \\ 4 & 9 & 10 & 5 & 2 \end{pmatrix}$

(c) $C = \begin{pmatrix} 1 & 5 & 4 & 3 & 1 \\ 2 & -1 & 2 & -1 & 0 \\ 5 & 3 & 8 & 1 & 1 \\ 4 & 9 & 10 & 5 & 2 \end{pmatrix}$

(d) $D = \begin{pmatrix} 4 & 3 & -5 & 2 & 3 \\ 8 & 6 & -7 & 4 & 2 \\ 4 & 3 & -8 & 2 & 7 \\ 4 & 3 & 1 & 2 & -5 \\ 8 & 6 & -1 & 4 & 6 \end{pmatrix}$

13. Argument accordingly m and λ, the rank of matrices

(a) $A = \begin{pmatrix} 1 & 1 & -3 \\ 2 & 1 & m \\ 1 & m & 3 \end{pmatrix}$

(b) $B = \begin{pmatrix} m & 5m & -m \\ 2m & m & 10m \\ -m & -2m & -3m \end{pmatrix}$

(c) $C = \begin{pmatrix} 3 & 1 & 1 & 4 \\ m & 4 & 10 & 1 \\ 1 & 7 & 17 & 3 \\ 2 & 2 & 4 & 1 \end{pmatrix}$

(d) $D = \begin{pmatrix} -1 & 0 & 2 & 1 & 0 \\ 2 & 0 & -1 & 2 & 2 \\ 1 & 1 & 1 & 3 & 2 \\ -2 & -1 & 1 & m & -2 \end{pmatrix}$

(e) $E = \begin{pmatrix} 3 & \lambda & 1 & 2 \\ 1 & 4 & 7 & 2 \\ 1 & 10 & 17 & 4 \\ 4 & 1 & 3 & 3 \end{pmatrix}$

(f) $F = \begin{pmatrix} -1 & 2 & 1 & -1 & 1 \\ \lambda & -1 & 1 & -1 & -1 \\ 1 & \lambda & 0 & 1 & 1 \\ 1 & 2 & 2 & -1 & 1 \end{pmatrix}$

14. Solve the following systems

(a) $\begin{cases} x - 3y + 3z = 7 \\ x + 2y - z = -2 \\ 3x + 2y + 4z = 5 \end{cases}$

(b) $\begin{cases} x + y + z = 3 \\ x - y + z = 4 \end{cases}$

(c) $\begin{cases} -0.6x_1 + 0.1x_2 + 0.4x_3 = 10 \\ 0.4x_1 - 0.7x_2 + 0.2x_3 = -26 \\ 0.2x_1 + 0.6x_2 - 0.5x_3 = 20 \end{cases}$

(d) $\begin{cases} x_1 + 3x_2 + 2x_3 + 2x_4 = 3 \\ x_1 + x_2 + 3x_3 = 4 \\ 2x_1 + 2x_3 - 3x_4 = 4 \\ x_1 - 3x_2 = 1 \end{cases}$

(e) $\begin{cases} x_1 + x_2 + x_3 + 2x_4 = 1 \\ x_1 - 3x_3 = 2 \\ x\_3x_2 + x_4 = 2 \\ x_2 - 4x_3 + x_4 = 0 \end{cases}$

(f) $\begin{cases} 3x + 2y + z - w = 3 \\ x - y - 2z + 2w = 2 \\ 2x + 3y - z + w = 1 \\ -x + y + 2z - 2w = -2 \end{cases}$

(g) $\begin{cases} 2x_1 + 3x_2 - x_3 + 3x_4 + x_5 = 2 \\ 3x_1 + x_2 - 4x_3 + 3x_4 - x_5 = 0 \\ x_1 + x_2 + 3x_3 - 4x_4 + 2x_5 = 6 \\ x_1 + 2x_2 - 3x_3 + 2x_4 - 2x_5 = -7 \end{cases}$

(h) $\begin{cases} x_1 + 3x_2 + 4x_3 - x_4 + 2x_5 = 1 \\ x_1 - x_2 - 2x_3 + x_4 = 3 \\ x_1 + 2x_2 + 3x_3 + x_4 + 4x_5 = 0 \\ 2x_1 + 2x_2 + 2x_3 + 2x_5 = 4 \end{cases}$

(i) $\begin{cases} 2x_1 - x_2 + x_3 + 3x_4 + 3x_5 = 7 \\ x_1 + x_2 + x_3 + 2x_4 + 2x_5 = 0 \\ 2x_1 + x_2 - x_3 + x_4 + x_5 = -3 \\ 4x_1 - x_2 - 3x_3 + x_4 + x_5 = 1 \end{cases}$

15. Given the system of equation $\begin{cases} x_1 + x_2 - x_3 = 1 \\ 2x_1 + 3x_2 + kx_3 = 3 \\ x_1 + kx_2 + 3x_3 = 2 \end{cases}$ Find k that

(a) The system has a unique solution.

(b) Invalid system.
(c) The system has infinitely many solutions.

16. Given the system of equation $\begin{cases} mx_1 + x_2 + x_3 = 1 \\ x_1 + mx_2 + x_3 = m \\ x_1 + x_2 + mx_3 = m^2 \end{cases}$ Find m that

(a) The system has a unique solution.
(b) Invalid system.
(c) The system has infinitely many solutions.

17. Given the system of equation $\begin{cases} x_1 + 2x_2 + x_3 = 1 \\ 2x_1 + 4x_2 + x_3 = 3 \\ 4x_1 + 8x_2 + 3x_3 = m \end{cases}$ Find m that

(a) The system has a unique solution.
(b) Invalid system.
(c) The system has infinitely many solutions.

18. Given the system of equation $\begin{cases} kx_1 + 2x_2 + x_3 = 1 \\ x_1 + kx_2 + x_3 = 1 \\ x_1 + x_2 + kx_3 = 1 \end{cases}$ Find k that

(a) The system has a unique solution.
(b) Invalid system.
(c) The system has infinitely many solutions.

19. Find the eigenvalues and eigenvectors the matrices:

(a) $\begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 2 \\ 1 & 1 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} 3 & -18 \\ 2 & -9 \end{pmatrix}$

(c) $\begin{pmatrix} 2 & 1 & 6 \\ 0 & 2 & 5 \\ 0 & 0 & 2 \end{pmatrix}$

(d) $\begin{pmatrix} 2 & 3 & 4 \\ 3 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix}$

(e) $\begin{pmatrix} -1 & 1 & 0 \\ 1 & 2 & 3 \\ 0 & 1 & -1 \end{pmatrix}$

(f) $\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}$

(g) $\begin{pmatrix} -1 & 4 & -2 \\ 4 & -1 & 2 \\ 0 & 0 & 6 \end{pmatrix}$

(h) $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 0 \\ 4 & 1 & 1 \end{pmatrix}$

(i) $\begin{pmatrix} 3 & -1 & 1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix}$

(j) $\begin{pmatrix} 0 & 0 & 3 \\ 0 & 3 & 0 \\ 3 & 0 & 0 \end{pmatrix}$

(k) $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{pmatrix}$

(l) $\begin{pmatrix} 1 & 0 & 1 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

(m) $\begin{pmatrix} 2 & 0 & 0 & 0 \\ -4 & 0 & 0 & 0 \\ 3 & 1 & 2 & 0 \\ 0 & -3 & 0 & 6 \end{pmatrix}$

20. In $\mathbf{R}^3$, let $S = \{u_1 = (-9, 7, 0), u_2 = (11, -10, 3), u_3 = (4, 2, m)\}$. Find m that S is a basic of $\mathbf{R}^3$

21. Find the coordinates of the vector $u = (3, -7, 8)$ relative to $B = \{u_1 = (3, 4, 5), u_2 = (-4, 6, -2), u_3 = (4, 5, -1)\}$

22. In $\mathbf{R}^3$,let $S = \{u_1 = (-9, 7, 0), u_2 = (11, -10, 3), u_3 = (4, 2, -1)\}$ and $S' = \{u_1' = (7, 5, 3), u_2' = (-2, -2, 1), u_3' = (6, 5, 4)\}$.

    (a) Roof S and S' are the basics of $\mathbf{R}^3$

    (b) Find the coordinates of the vector $v = (-5, 2, 1)$ relative to S.

    (c) Find the transition matrix from S' to S

23. In $\mathbf{R}^4$, let $S = \{u_1 = (-9, 7, 0, 1), u_2 = (1, -1, 3, -3), u_3 = (4, 2, -1, 0), u_4 = (1, 1, 1, 1)\}$ and $S' = \{u_1' = (1, -1, 3, 1), u_2' = (-2, 0, -2, 1), u_3' = (3, 0, 3, 0), u_4' = (-2, 3, 3, -2)\}$.

    (a) Roof S and S' are the basics $\mathbf{R}^4$

    (b) Find the coordinates of the vector $v = (-5, 2, 1, 0)$ relative to S.

    (c) Find the transition matrix from S' to S

24. Determine the number of dimensions and a basis of the solution space of the system:

    (a) $\begin{cases} 2x_1 + x_2 + 3x_3 = 0 \\ x_1 + 2x_2 = 0 \\ x_2 + x_3 = 0 \end{cases}$

    (b) $\begin{cases} 3x - 1 + x_2 + x_3 + x_4 = 0 \\ 5x_1 - x_2 + x_3 - x_4 = 0 \end{cases}$

    (c) $\begin{cases} 3x_1 + x_2 + 2x_3 = 0 \\ 4x_1 + 5x_3 = 0 \\ x_1 - 3x_2 + 4x_3 = 0 \end{cases}$

    (d) $\begin{cases} x_1 - 2x_2 + x_3 = 0 \\ 2x_1 - 6x_2 + 2x_3 = 0 \\ 3x_1 - 9x_2 + 3x_3 = 0 \end{cases}$

    (e) $\begin{cases} 2x_1 - 4x_2 + x_3 + x_4 = 0 \\ x_1 - 5x_2 + 2x_3 = 0 \\ -2x_2 - 2x_3 - x_4 = 0 \\ x_1 + 3x_2 + x_4 = 0 \\ x_1 - 2x_2 - x_3 + x_4 = 0 \end{cases}$

    (f) $\begin{cases} x + y + z = 0 \\ 3x + 2y - z = 0 \\ 2x - 4y + z = 0 \\ 4x + 8y - 3z = 0 \\ 2x + y - 2z = 0 \end{cases}$

25. Find the basis and dimension of the subspace of $\mathbf{R}^3$ generated by the following vectors:

    (a) $u_1 = (1, -1, 2); u_2 = (2, 1, 3); u_3 = (-1, 5, 0)$

    (b) $a = (2, 4, 1); b = (36, -2); c = (-1, 2, -\frac{1}{2})$

26. Find the basis and dimension of the subspace of $\mathbf{R}^4$ generated by the following vectors:

    (a) $u_1 = (1, 1, -4, -3); u_2 = (2, 0, 2, -2); u_3 = (2, -1, 3, 2)$

    (b) $a = (-1, 1, -2, 0); b = (3, 3, , 6, 0); c = (9, 0, 0, 3)$

    (c) $x = (1, 1, 0, 0); y = (0, 0, 1, 1); z = (-2, 0, 2, 2); t = (0, -3, 0, 3)$

    (d) $v_1 = (1, 0, 1, -2); v_2 = (0, 0, 1, 1); v_3 = (2, 1, 5, -1); v_4 = (1, -1, 1, 4)$

27. Find the coordinate matrix and coordinate vector of w relative to $S = \{u_1; u_2; u_3$, that

    (a) $w = (2, -3, 3); u_1 = (2, 3, -1); u_2 = (-4, 1, 2); u_3 = (1, 2, 3)$

    (b) $w = (5, -12, 3), u_1 = (1, 2, 3), u_2 = (-4, 5, 6); u_3 = (7, -8, 9)$

28. Give the basis for the space Euclide $\mathbf{R^3}$, $S = \{u_1 = (1, -2, 5), u_2 = (3, -4, 1), u_3 = (1, 2, 1)\}$. Find the an orthogonal basis for the space Euclide $\mathbf{R^3}$

29. Give the basis for the space Euclide $\mathbf{R^4}$, $S = \{u_1 = (1, -1, 0, -1), u_2 = (0, 0, 1, 1), u_3 = (1, 2, 1, -2), u_4 = (1, 0, 0, -3)\}$. Find the an orthogonal basis for the space Euclide $\mathbf{R^4}$

30. Given $A = \begin{pmatrix} 3 & 4 & -4 \\ -2 & -1 & 2 \\ -2 & 0 & 1 \end{pmatrix}$. The matrix A Can it be diagonalized? Let's diagonalize A (if any).

31. Let $A = \begin{pmatrix} -2 & 5 & 7 \\ -1 & 6 & 9 \\ 0 & -2 & -3 \end{pmatrix}$. The matrix A Can it be diagonalized? Let's diagonalize A (if any).

32. Let $A = \begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$. The matrix A Can it be diagonalized? Let's diagonalize A (if any).

    (a) $A = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix}$

    (b) $B = \begin{pmatrix} 4 & 2 & -1 \\ -6 & -4 & 3 \\ -6 & -6 & 5 \end{pmatrix}$

    (c) $C = \begin{pmatrix} 5 & 4 & 6 \\ 4 & 5 & 6 \\ -4 & -4 & -5 \end{pmatrix}$

    (d) $D = \begin{pmatrix} 3 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix}$

33. Let $A = \begin{pmatrix} 1 & 1 & 0 \\ -1 & 2 & 1 \\ 1 & 0 & 1 \end{pmatrix}$. The matrix A Can it be diagonalized? Let's diagonalize A (if any).

34. Let $A = \begin{pmatrix} -7 & 24 \\ 24 & 7 \end{pmatrix}$. Find the orthogonal matrix P diagonalize A, Find $P^{-1}AP$ and calculate A^{10}

35. Let $A = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$. Find the orthogonal matrix P diagonalize A, let $P^{-1}AP$ and find A^5

36. Let $A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. Find the orthogonal matrix P diagonalize A, let $P^{-1}AP$ and find A^{20}

37. Let $A = \begin{pmatrix} -4 & 0 & -6 \\ 2 & 1 & 2 \\ 3 & 0 & 5 \end{pmatrix}$. Find A^{10}

38. Let $A = \begin{pmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{pmatrix}$. Find A^{100}

39. Let $A = \begin{pmatrix} -2 & 0 & -36 \\ 0 & -3 & 0 \\ -36 & 0 & -23 \end{pmatrix}$. Find the matrix P is orthonormal diagonalization A and find $P^{-1}AP$

40. Find the canonical holographic form of each of the following holographic forms:

(a) $x_1^2 + x_2^2 + 3x_3^2 + 4x_1x_2 + 2x_1x_3 + 2x_2x_3$
(b) $x_1^2 - x_2^2 + x_3^2 + 2x_1x_2 + 4x_1x_3 + 2x_2x_3$
(c) $x_1^2 - 3x_3^2 - 2x_1x_2 + 2x_1x_3 - 6x_2x_3$
(d) $x_1^2 + 5x_2^2 - 4x_3^2 + 2x_1x_2 - 4x_1x_3$
(e) $4x_1^2 + x_2^2x_3^2 - 4x_1x_2 + 4x_1x_3 - 3x_2x_3$
(f) $x_1x_2 + x_2x_3 + x_1x_3$
(g) $2x_1^2 + 18x_2^2 + 8x_3^2 - 12x_1x_2 + 8x_1x_3 - 27x_2x_3$
(h) $-12x_1^2 - 3x_2^2 - 12x_3^2 + 12x_1x_2 - 24x_1x_3 + 8x_2x_3$

41. Given $T : \mathbf{R}^2 \to \mathbf{R}^3$, is determined by
$$T(x, y) = (x - 3y, y - x; -y)$$
and $S = \{u_1 = (-2, 3), u_2 = (5, -1)\}$ and $S' = \{v_1 = (1, 2, 3), v_2 = (-1, 3, 8), v_3 = (-4, 6, 3)\}$ are the basics of $\mathbf{R}^2$ và $\mathbf{R}^3$. Find the matrix of the transformation T from S to S'

42. Let the transformations $T : \mathbf{R}^3 \to \mathbf{R}^3$, is determined by
$$T(x, y, z) = (2x - 3y, 3y - 4z, x + y + z)$$
and the basics $\mathbf{R}^3$ is $S = \{u_1 = (-1, 2, 3), u_2 = (-1, 3, 8), u_3 = (-4, 6, 3)\}$. Find the matrix of the transformation T relative S.

43. Let the transformation $T : \mathbf{R}^4 \to \mathbf{R}^3$, is determined by
$$T(x_1, x_2, x_3, x_4) = (x_1 - 2x_2; x_2 - 3x_3, x_3 - 4x_4)$$
and $B = \{u_1 = (1, 4, 1, 2), u_2 = (3, 1, 2, 1), u_3 = (-1, 1, 0, 1), u_4 = (2, 3, 1, 0)\}$, $B' = \{v_1 = (1, 2, 3), v_2 = (-1, 3, 2), v_3 = (2, -1, -1)\}$ are the basics $\mathbf{R}^4, \mathbf{R}^3$. Find the matrix of the transformation T from B to B'.

44. Let $A = \begin{pmatrix} 3 & -1 & 2 \\ 2 & -2 & 1 \\ 3 & -4 & -2 \\ 5 & 2 & 1 \end{pmatrix}$ is the matrix of the transformation of $T : \mathbf{R}^3 \to \mathbf{R}^4$. The basic $B = \{u_1 = (2, 3, 3), u_2 = (4, 4, -1), u_3 = (2, 1, 4)\}$ of $\mathbf{R}^3$ và $B' = \{u_1' = (4, 0, 0, 0), u_2' = (-2, 3, 0, 0), u_3' = (3, 2, 1, 0), u_4' = (1, 2, 3, 4)\}$ of $\mathbf{R}^4$. Find $T(9, 7, 8)$

45. Let $A = \begin{pmatrix} 2 & -1 & 2 \\ -1 & -2 & 1 \\ 1 & 4 & 5 \end{pmatrix}$ is the matrix of the transformation of $T : \mathbf{R}^3 \to \mathbf{R}^3$. The basics $B = \{u_1 = (0, -2, 3), u_2 = (4, 4, -1), u_3 = (2, 1, 4)\}$ of $\mathbf{R}^3$ Find $T(9, 7, 8)$

46. Let $A = \begin{pmatrix} 2 & -1 & 2 & -1 \\ -1 & -2 & 1 & 0 \\ 1 & 4 & 5 & 4 \end{pmatrix}$ is the matrix of the transformation of $T : \mathbf{R}^4 \to \mathbf{R}^3$. The basics $u = \{u_1 = (1, 1, , 0, 1), u_2 = (-1, 0, -2, 1), u_3 = (-3, 1, 0, 0), u_4 = (-1, 2, 1, -2)\}$ và $V = \{v_1 = (0, -2, 3), v_2 = (4, 4, -1), v_3 = (2, 1, 4)\}$ in $\mathbf{R}^3$. Find $T((-1, 1, -1, 3))$

47. In, $\mathbf{R}^4$ và $\mathbf{R}^3$ Let $S = \{v_1 = (1, 2, 3, 4), v_2 = (-1, 1, -1, 0), v_3 = (-2, 3, -4, 1), v_4 = (-1, 1, 3, 2)\}$ and $S\{u_1 = (2, 1, 1), u_2 = (3, 5, 1), u_3 = (2, 1, -3)\}$ are the basics of $\mathbf{R}^4$ và $\mathbf{R}^3$. Let $A = \begin{pmatrix} -2 & 0 & 2 & 1 \\ 1 & 1 & -1 & 1 \\ 3 & 2 & 1 & -2 \end{pmatrix}$ is the matrix of the transformation of $T : \mathbf{R}^4 \to \mathbf{R}^3$. Tìm $T(-3, 0, -2; 1)$