7b)

$$B = \begin{bmatrix} -3m & 2 & 4 \\ 1 & -m & 2 \\ 3 & 1-m & 4 \end{bmatrix}$$

$$\det(B) = \begin{vmatrix} -3m & 2 & 4 \\ 1 & -m & 2 \\ 3 & 1-m & 4 \end{vmatrix} = 12m^2 + 12 + 4 - 4m + 12m - 8 \quad 6m^2 +$$

$$= 6m^2 + 14m + 8$$

Để B có nghịch đảo thì $\det(B) \neq 0 \Leftrightarrow 6m^2 + 14m + 8 \neq 0$

$$\Rightarrow \begin{cases} x \neq -1 \\ x \neq \dfrac{-4}{3} \end{cases}$$

$$A_{11} = (-1)^{1+1} \begin{vmatrix} -m & 2 \\ 1-m & 4 \end{vmatrix} = -4m - (2-2m) = -2m - 2$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = -(4-6) = 2$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 1 & -m \\ 3 & 1-m \end{vmatrix} = (1-m) - (-3m) = 1 + 2m$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} 2 & 4 \\ 1-m & 4 \end{vmatrix} = (-1) \cdot \left(8 - (4-4m)\right) = -4 - 4m$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} -3m & 4 \\ 3 & 4 \end{vmatrix} = -12m - 12$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} -3m & 2 \\ 3 & 1-m \end{vmatrix} = \left(\left(-3m \cdot (1-m)\right) - 6\right) = (-3m + 3m^2 - 6)(-1)$$

$$= -3m^2 + 3m + 6$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} 2 & 4 \\ -m & 2 \end{vmatrix} = 4 + 4m$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} -3m & 4 \\ 1 & 2 \end{vmatrix} = (-1)\left(-6m - 4 \right) = 6m + 4$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} -3m & 2 \\ 1 & 2-m \end{vmatrix} = 3m^2 - 2$$

$$\Rightarrow A^{-1} = \frac{1}{6m^2 + 14m + 8} \begin{bmatrix} -2m-2 & -4-4m & 4+4m \\ 2 & -12m-12 & 6m+4 \\ 1+2m & -3m^2+3m+6 & 3m^2+2 \end{bmatrix}$$

9c)

$$E = \begin{bmatrix} 1 & -2 & 1 & -1 \\ -1 & 4 & -2 & 3 \\ 2 & 0 & 1 & 3 \\ -2 & 6 & 0 & 2 \end{bmatrix}$$

Ta có MTMR:

$$\check{E} = (E|I) = \left[\begin{array}{cccc|cccc} 1 & -2 & 1 & -1 & 1 & 0 & 0 & 0 \\ -1 & 4 & -2 & 3 & 0 & 1 & 0 & 0 \\ 2 & 0 & 1 & 3 & 0 & 0 & 1 & 0 \\ -2 & 6 & 0 & 2 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{array}{l} R_2 + R_1 \to R_2 \\ R_3 - 2R_1 \to R_3 \\ \xrightarrow{\hspace{2cm}} \\ R_4 + 2R_1 \to R_4 \end{array} \left[\begin{array}{cccc|cccc} 1 & -2 & 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 2 & -1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 4 & -1 & 5 & -2 & 0 & 1 & 0 \\ 0 & 2 & 2 & 0 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{array}{l} R_3 - 2R_2 \to R_3 \\ \xrightarrow{\hspace{2cm}} \\ R_4 - R_2 \to R_4 \end{array} \left[\begin{array}{cccc|cccc} 1 & -2 & 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 2 & -1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & -4 & -2 & 1 & 0 \\ 0 & 0 & 3 & -2 & 1 & -1 & 0 & 1 \end{array}\right] \xrightarrow{R_4 - 3R_3 \to R_4}$$

$$\left[\begin{array}{cccc|cccc} 1 & -2 & 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 2 & -1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & -4 & -2 & 1 & 0 \\ 0 & 0 & 0 & -5 & 13 & 5 & -3 & 1 \end{array}\right] \xrightarrow{R_4 \to R_4} \left[\begin{array}{cccc|cccc} 1 & -2 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 2 & -1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & \frac{7}{5} & -\frac{2}{5} & & \\ 0 & 0 & 0 & -5 & \frac{13}{5} & & & \end{array}\right]$$

$$\xrightarrow{-\frac{1}{5}R_4 \to R_4} \left[\begin{array}{cccc|cccc} 1 & -2 & 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 2 & -1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & -4 & -2 & 1 & 0 \\ 0 & 0 & 0 & 1 & -\frac{13}{5} & -1 & \frac{3}{5} & -\frac{1}{5} \end{array}\right] \xrightarrow{R_3 - R_4 \to R_3} \left[\begin{array}{cccc|cccc} 1 & -2 & 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 2 & -1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & \frac{-7}{5} & -1 & \frac{2}{5} & \frac{1}{5} \\ 0 & 0 & 0 & 1 & \frac{-13}{5} & -1 & \frac{3}{5} & \frac{-1}{5} \end{array}\right]$$

$$\frac{1}{2}R_2 \to R_2 \Rightarrow \left[\begin{array}{cccc|cccc} 1 & -2 & 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -\frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 & -\frac{7}{5} & -1 & \frac{2}{5} & \frac{1}{5} \\ 0 & 0 & 0 & 1 & -\frac{13}{5} & -1 & \frac{3}{5} & -\frac{1}{5} \end{array}\right] \quad \begin{array}{c} R_1 + R_4 \to R_1 \\ \hline R_2 - R_4 \to R_2 \end{array} \Rightarrow$$

$$\left[\begin{array}{cccc|cccc} 1 & -2 & 1 & 0 & -\frac{8}{5} & -1 & \frac{3}{5} & -\frac{1}{5} \\ 0 & 1 & -\frac{1}{2} & 0 & \frac{31}{10} & \frac{3}{2} & -\frac{3}{5} & \frac{1}{5} \\ 0 & 0 & 1 & 0 & -\frac{7}{5} & -1 & \frac{2}{5} & \frac{1}{5} \\ 0 & 0 & 0 & 1 & -\frac{13}{5} & -1 & \frac{3}{5} & -\frac{1}{5} \end{array}\right]$$

$$\begin{array}{c} R_1 - R_3 \to R_1 \\ R_2 + \frac{1}{2}R_3 \to R_2 \end{array} \Rightarrow \left[\begin{array}{cccc|cccc} 1 & -2 & 0 & 0 & -\frac{1}{5} & 0 & \frac{1}{5} & -\frac{2}{5} \\ 0 & 1 & 0 & 0 & \frac{12}{5} & 1 & -\frac{2}{5} & \frac{3}{10} \\ 0 & 0 & 1 & 0 & -\frac{7}{5} & -1 & \frac{2}{5} & \frac{1}{5} \\ 0 & 0 & 0 & 1 & -\frac{13}{5} & -1 & \frac{3}{5} & -\frac{1}{5} \end{array}\right]$$

$$R_1 + 2R_2 \to R_1 \Rightarrow \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \frac{23}{5} & 2 & -\frac{3}{5} & \frac{1}{5} \\ 0 & 1 & 0 & 0 & \frac{12}{5} & 1 & -\frac{2}{5} & \frac{3}{10} \\ 0 & 0 & 1 & 0 & -\frac{7}{5} & -1 & \frac{2}{5} & \frac{1}{5} \\ 0 & 0 & 0 & 1 & -\frac{13}{5} & -1 & \frac{3}{5} & -\frac{1}{5} \end{array}\right]$$

\# Vậy ta có

$$(I \mid E^{-1}) = \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \frac{23}{5} & 2 & -\frac{3}{5} & \frac{1}{5} \\ 0 & 1 & 0 & 0 & \frac{12}{5} & 1 & -\frac{2}{5} & \frac{3}{10} \\ 0 & 0 & 1 & 0 & -\frac{7}{5} & -1 & \frac{2}{5} & \frac{1}{5} \\ 0 & 0 & 0 & 1 & -\frac{13}{5} & -1 & \frac{3}{5} & -\frac{1}{5} \end{array}\right] \Rightarrow E^{-1} = \left[\begin{array}{cccc} \frac{23}{5} & 2 & -\frac{3}{5} & \frac{1}{5} \\ \frac{12}{5} & 1 & -\frac{2}{5} & \frac{3}{10} \\ -\frac{7}{5} & -1 & \frac{2}{5} & \frac{1}{5} \\ -\frac{13}{5} & -1 & \frac{3}{5} & -\frac{1}{5} \end{array}\right]$$

14/f)

$$\begin{cases} 3x + 2y + z - w = 3 \\ x - y - 2z + 2w = 2 \\ 2x + 3y - z + w = 1 \\ -x + y + 2z - 2w = -2 \end{cases} \qquad \left[\begin{array}{cccc|c} 3 & 2 & 1 & -1 & 3 \\ 1 & -1 & -2 & 2 & 2 \\ 2 & 3 & -1 & 1 & 1 \\ -1 & 1 & 2 & -2 & -2 \end{array}\right]$$

$$\det(A) = \begin{vmatrix} 3 & 2 & 1 & -1 \\ 1 & -1 & -2 & 2 \\ 2 & 3 & -1 & 1 \\ -1 & 1 & 2 & -2 \end{vmatrix} = 3 \cdot (-1)^{1+1}\begin{vmatrix} -1 & -2 & 2 \\ 3 & -1 & 1 \\ 1 & 2 & -2 \end{vmatrix} + 2(-1)^{2+1}\begin{vmatrix} 2 & 1 & -1 \\ 3 & -1 & 1 \\ 1 & 2 & -2 \end{vmatrix}$$

$$+ 2(-1)^{3+1}\begin{vmatrix} 2 & 1 & -1 \\ -1 & -2 & 2 \\ 1 & 2 & -2 \end{vmatrix} + (-1)(-1)^{4+1}\begin{vmatrix} 2 & 1 & -1 \\ -1 & -2 & 2 \\ 3 & -1 & 1 \end{vmatrix}$$

$$= 3(0) + (-1)(0) + 2 \cdot (0) + 1(0) = 0$$

$$\Leftrightarrow \det(A) = 0$$

$\rightarrow$ Ma trận hệ số không có tính khả nghịch

$\leadsto$ Không thể giải theo phương pháp Crames