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p) $\sum_{n=1}^{\infty} \frac{2^n (n!)^2}{n^{2n}}$

$a_n = \frac{2^n (n!)^2}{n^{2n}} \Rightarrow a_{n+1} = \frac{2^{n+1}[(n+1)!]^2}{(n+1)^{2(n+1)}}$

Áp dụng TCTS ta có:

$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{2^{n+1}[(n+1)!]^2}{(n+1)^{2(n+1)}} \cdot \frac{n^{2n}}{2^n (n!)^2}\right|$

$= \lim_{n\to\infty} \frac{2^n . 2 . (n!)^2 (n+1)^2}{(n+1)^{2(n+1)}} . \frac{n^{2n}}{2^n (n!)^2}$

$= \lim_{n\to\infty} \frac{2 . n^{2n} . (n+1)^2}{(n+1)^{2n} . (n+1)^2}$

$= 2\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^{2n} \quad (1^\infty)$

$= 2\lim_{n\to\infty}\left[\left(1 + \frac{(-1)}{n+1}\right)^{\frac{n+1}{-1}}\right]^{2n . \frac{-1}{n+1}}$

$= 2 . e^{\lim_{n\to\infty}\frac{-2n}{n+1}} = 2e^{\lim_{n\to\infty} -2} = 2.e^{-2} < 1$

$\Rightarrow$ Chuỗi hội tụ

q) $\sum_{n=1}^{\infty} \frac{1}{2^n} . \left(1 + \frac{1}{n}\right)^{n^2}$

$a_n = \frac{1}{2^n}\left(1 + \frac{1}{n}\right)^{n^2} \Rightarrow a_{n+1} = \frac{1}{2^{n+1}} . \left(1 + \frac{1}{n+1}\right)^{(n+1)^2}$

Áp dụng TCTS ta có:

$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \frac{1}{2^{n+1}} . \left(1 + \frac{1}{n+1}\right)^{(n+1)^2}$

$= \lim_{n\to\infty}\left|\frac{\frac{1}{2^{n+1}} . \left(1 + \frac{1}{n+1}\right)^{(n+1)^2}}{\frac{1}{2^n} . \left(1 + \frac{1}{n}\right)^{n^2}}\right|$

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$$= \lim_{n\to\infty} \frac{1}{2} \cdot \frac{\left(1 + \frac{1}{n+1}\right)^{(n+1)^2}}{\left(1 + \frac{1}{n}\right)^{n^2}}$$

$$= \frac{1}{2} \lim_{n\to\infty} \frac{\left[\left(1 + \frac{1}{n+1}\right)^{(n+1)}\right]^{(n+1)}}{\left[\left(1 + \frac{1}{n}\right)^{n}\right]^{n}}$$

$$= \frac{1}{2} \lim_{n\to\infty} \frac{e^{n+1}}{e^{n}} = \frac{1}{2} \lim_{n\to\infty} e = \frac{1}{2} e > 1$$

$\Rightarrow$ Chuỗi phân kỳ.

r) $\sum_{n=1}^{\infty} \frac{n^2+5}{2^n}$

$a_n = \frac{n^2+5}{2^n} \Rightarrow a_{n+1} = \frac{(n+1)^2+5}{2^{(n+1)}}$

Áp dụng TCTS ta có:

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{(n+1)^2+5}{2^{(n+1)}} \cdot \frac{2^n}{n^2+5}\right|$$

$= \cancel{n^2+2n+1+}$

$$= \lim_{n\to\infty} \left|\frac{n^2+2n+1+5}{2^n \cdot 2} \cdot \frac{2^n}{n^2+5}\right|$$

$$= \lim_{n\to\infty} \frac{1}{2} \cdot \left(1 + \frac{2n+1}{n^2+5}\right)$$

$$= \frac{1}{2} \lim_{n\to\infty} \left[\left(1 + \frac{2n+1}{n^2+5}\right)^{\frac{n^2+5}{2n+1}}\right]^{\frac{2n+1}{n^2+5}}$$

$$= \frac{1}{2} \cdot e^{\lim_{n\to\infty} \frac{2n+1}{n^2+5}} = \frac{1}{2} e^0 = \frac{1}{2} < 1$$

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$\Rightarrow$ Chuỗi hội tụ

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8) $\sum_{n=1}^{\infty} \frac{(3n+1)!}{8^n n^2}$

$a_n = \frac{(3n+1)!}{8^n n^2} \Rightarrow a_{n+1} = \frac{[3(n+1)+1]!}{8^{(n+1)}(n+1)^2}$

Áp dụng TCTS ta có:

$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{[3(n+1)+1]!}{8^{(n+1)}(n+1)^2}\cdot\frac{8^n.n^2}{(3n+1)!}\right|$

$= \lim_{n\to\infty} \frac{(3n+4)!}{8^{(n+1)}(n+1)^2}\cdot\frac{8^n.n^2}{(3n+1)!}$

$= \lim_{n\to\infty} \frac{(3n+1)!(3n+2)(3n+3)(3n+4).8^n.n^2}{8.8^n(n+1)^2.(3n+1)!}$

$= \lim_{n\to\infty} \frac{3n^2(3n+2)(3n+4)(n+1)}{8^n(n+1)^2}$

$= \lim_{n\to\infty} \frac{3n^2(3n+2)(3n+4)}{8(n+1)}$

$= \lim_{n\to\infty} \frac{27n^4+54n^3+24n^2}{8n+8}$

$= \lim_{n\to\infty} \frac{108n^3+162n^2+48n}{8} = +\infty > 1$

$\Rightarrow$ Chuỗi phân kỳ.

HẢI TIẾN