

(Mt. Whitney), and the smallest possible value is  $-282$  (Death Valley). The set of all possible values of  $Y$  is the set of all numbers in the interval between  $-282$  and  $14,494$ —that is,

$$\{y: y \text{ is a number, } -282 \leq y \leq 14,494\}$$

and there are an infinite number of numbers in this interval. ■

## Two Types of Random Variables

In Section 1.2, we distinguished between data resulting from observations on a counting variable and data obtained by observing values of a measurement variable. A slightly more formal distinction characterizes two different types of random variables.

### DEFINITION

A **discrete** random variable is an rv whose possible values either constitute a finite set or else can be listed in an infinite sequence in which there is a first element, a second element, and so on (“countably” infinite).

A random variable is **continuous** if *both* of the following apply:

1. Its set of possible values consists either of all numbers in a single interval on the number line (possibly infinite in extent, e.g., from  $-\infty$  to  $\infty$ ) or all numbers in a disjoint union of such intervals (e.g.,  $[0, 10] \cup [20, 30]$ ).
2. No possible value of the variable has positive probability, that is,  $P(X = c) = 0$  for any possible value  $c$ .

Although any interval on the number line contains an infinite number of numbers, it can be shown that there is no way to create an infinite listing of all these values—there are just too many of them. The second condition describing a continuous random variable is perhaps counterintuitive, since it would seem to imply a total probability of zero for all possible values. But we shall see in Chapter 4 that *intervals* of values have positive probability; the probability of an interval will decrease to zero as the width of the interval shrinks to zero.

### Example 3.6

All random variables in Examples 3.1–3.4 are discrete. As another example, suppose we select married couples at random and do a blood test on each person until we find a husband and wife who both have the same Rh factor. With  $X$  = the number of blood tests to be performed, possible values of  $X$  are  $D = \{2, 4, 6, 8, \dots\}$ . Since the possible values have been listed in sequence,  $X$  is a discrete rv. ■

To study basic properties of discrete rv’s, only the tools of discrete mathematics—summation and differences—are required. The study of continuous variables requires the continuous mathematics of the calculus—integrals and derivatives.

## EXERCISES Section 3.1 (1–10)

1. A concrete beam may fail either by shear ( $S$ ) or flexure ( $F$ ). Suppose that three failed beams are randomly selected and the type of failure is determined for each one. Let  $X$  = the number of beams among the three selected that failed by shear. List each outcome in the sample space along with the associated value of  $X$ .
2. Give three examples of Bernoulli rv’s (other than those in the text).
3. Using the experiment in Example 3.3, define two more random variables and list the possible values of each.