Homework #1

Math 222, Spring 2024

1. Find the components and the length of $\overrightarrow{PQ}$.

   (a) $P = (0, 3),\ Q = (4, 0).$

   (b) $P = (1, 5),\ Q = (-7, 3).$

   (c) $P = (2, -1, 5),\ Q = (-1, 0, 3).$

2. Calculate $P + Q$ and $P - Q$ for the following:

   (a) $P = (2, 7),\ Q = (4, 3).$

   (b) $P = (-8, 1),\ Q = (-3, 5).$

   (c) $P = (3, -1, 2),\ Q = (2, -3, 1).$

3. Sketch $\vec{v},\ 3\vec{v},\ \vec{w},\ 2\vec{w},\ \vec{v} + \vec{w}$, and $\vec{v} - \vec{w}$ for the following:

   (a) $\vec{v} = \langle 1, 0 \rangle\ ,\ \vec{w} = \langle 0, 1 \rangle\ .$

   (b) $\vec{v} = \langle -1, 1 \rangle\ ,\ \vec{w} = \langle 1, 2 \rangle\ .$

   (c) $\vec{v} = \langle 1, -2 \rangle\ ,\ \vec{w} = \langle 3, 1 \rangle\ .$

4. Which of the following vectors are parallel to $\langle 2, 4 \rangle$ and which point in the same direction?

   (a) $\langle 1, 1 \rangle$

   (b) $\langle 1, 2 \rangle$

   (c) $\langle -1, 2 \rangle$

   (d) $\langle -1, -2 \rangle$

   (e) $\langle 10, 20 \rangle$

   (f) $\langle 3, 4 \rangle$

   (g) $\langle -5, -10 \rangle$

5. Determine whether $\overrightarrow{AB}$ and $\overrightarrow{PQ}$ are equivalent.

   (a) $A = (3, 8),\ B = (5, 4),\ P = (2, 7),\ Q = (4, 3).$

   (b) $A = (-3, 5),\ B = (-8, 1),\ P = (-8, 1),\ Q = (-3, 5).$

   (c) $A = (1, 3, 5),\ B = (4, 5, 4),\ P = (4, 6, 8),\ Q = (7, 8, 7).$

6. Find the given vector.
   (a) The unit vector in the direction of $\langle -6, 8 \rangle$.
   (b) The unit vector in the direction of $\langle 1, 2 \rangle$.
   (c) The vector of length 7 in the direction of $\langle 2, 1 \rangle$.
   (d) The vector of length 5 in the direction of $\langle 2, 1 \rangle$.
   (e) The unit vector in the direction opposite to $\langle 2, -3 \rangle$.
   (f) The unit vector in the direction of $\langle 2, 4, 7 \rangle$.
   (g) The vector of length 40 in the direction of $\langle -3, 2, 1 \rangle$.
7. Find the components and length of the following vectors:
   (a) $3\hat{\imath} + 2\hat{\jmath}$.
   (b) $\hat{\imath} - 4\hat{\jmath}$.
   (c) $2\hat{\imath} + 5\hat{\jmath} - \hat{k}$.
   (d) $3\hat{\imath} + 2\hat{\jmath} - 5\hat{k}$.
8. For the following, let $R = (3, 2, 1)$.
   (a) Find a point Q such that $\overrightarrow{RQ} = \langle 40, 38, 23 \rangle$.
   (b) Find a point Q such that $\overrightarrow{QR} = \langle 7, -4, 1 \rangle$.
   (c) Assuming $Q = (4, -8, 12)$, find $\overrightarrow{QR}$.
9. Find a vector parametrization for the line with the given description.
   (a) Passes through $(2, 5)$ in the direction of the vector: $\langle 4, -3 \rangle$.
   (b) Passes through $(7, -2, 5)$ in the direction of the vector: $\langle 1, 0, 6 \rangle$.
   (c) Passes through both $(2, 6)$ and $(-4, -1)$.
   (d) Passes through both $(8, -3, 5)$ and $(-1, 0, 9)$.
10. Determine if the lines intersect, and if so, find their point of intersection.
   (a) $\vec{r}_1(t) = \langle 2, 0, 2 \rangle + t \langle 0, -1, 0 \rangle$ and $\vec{r}_2(t) = \langle 2, 1, 0 \rangle + t \langle 0, 3, 1 \rangle$.
   (b) $\vec{r}_1(t) = \langle 2, 0, 2 \rangle + t \langle 0, -1, 0 \rangle$ and $\vec{r}_2(t) = \langle 2, 1, 1 \rangle + t \langle 0, 3, 1 \rangle$.

(c) $\vec{r}_1(t) = \langle 1, 9, 2\rangle + t\langle -2, 4, -5\rangle$ and $\vec{r}_2(t) = \langle 7, -1, -5\rangle + t\langle -2, 3, 6\rangle$.

11. Compute the dot product.

    (a) $\langle 2, 3\rangle \cdot \langle -5, 6\rangle$
    (b) $\langle 2, 3, 4\rangle \cdot \langle -5, 6, -7\rangle$
    (c) $\langle -1, -8\rangle \cdot \langle -4, 3\rangle$
    (d) $(2\hat{\imath} - 3\hat{\jmath}) \cdot (4\hat{\jmath} - 7\hat{k})$
    (e) $(3\hat{\imath} - 2\hat{\jmath} + 4\hat{k}) \cdot (\hat{\imath} + 5\hat{\jmath} - 6\hat{k})$

12. Determine if the two given vectors are orthogonal. If they are not orthogonal, then determine if the angle between them is acute or obtuse.

    (a) $\langle 2, 3\rangle$ and $\langle 3, -2\rangle$.
    (b) $\langle 2, 3\rangle$ and $\langle -3, -2\rangle$.
    (c) $\langle 2, 3\rangle$ and $\langle 3, 2\rangle$.
    (d) $\langle 4, 4, -2\rangle$ and $\langle 3, -2, 2\rangle$.
    (e) $\langle 1, 0, 2\rangle$ and $\langle -4, -1, 3\rangle$.
    (f) $\langle 1, 1, 2\rangle$ and $\langle 4, 4, -2\rangle$.

13. Find the angle between the vectors. Express your answer in terms of an inverse cosine.

    (a) $\langle 3, 4\rangle$ and $\langle 5, 12\rangle$.
    (b) $\langle 7, 24\rangle$ and $\langle -9, 40\rangle$.
    (c) $\langle 1, -2, 2\rangle$ and $\langle 6, 8, 0\rangle$.

14. Find the angle between the vectors. Use a calculator to express your answer to four decimal places. (Use radians obviously!)

    (a) $\langle 4, 5\rangle$ and $\langle 6, 7\rangle$.
    (b) $\langle 2, -9\rangle$ and $\langle -3, 8\rangle$.
    (c) $\langle 3, -2, 1\rangle$ and $\langle 2, 3, 4\rangle$.
    (d) $\langle 0, 1, 2\rangle$ and $\langle 6, -3, 0\rangle$.

15. Find a vector orthogonal to the given vector.

    (a) $\langle 7, 6 \rangle$.

    (b) $\langle 1, 2, 3 \rangle$.

    (c) $\langle -2, 3, -4 \rangle$.

16. Sketch $\vec{u}$ and $\vec{v}$, and find and sketch the projection of $\vec{u}$ along $\vec{v}$.

    (a) $\vec{u} := \langle 3, 4 \rangle$ and $\vec{v} := \langle 1, 0 \rangle$.

    (b) $\vec{u} := \langle 3, 4 \rangle$ and $\vec{v} := \langle 2, 0 \rangle$.

    (c) $\vec{u} := \langle 3, 4 \rangle$ and $\vec{v} := \langle 0, 1 \rangle$.

    (d) $\vec{u} := \langle 3, 4 \rangle$ and $\vec{v} := \langle 0, 3 \rangle$.

    (e) $\vec{u} := \langle 3, 4 \rangle$ and $\vec{v} := \langle 3, 4 \rangle$.

    (f) $\vec{u} := \langle 3, 4 \rangle$ and $\vec{v} := \langle 4, -3 \rangle$.

    (g) $\vec{u} := \langle 3, 4 \rangle$ and $\vec{v} := \langle -3, -4 \rangle$.

17. Find the projection of $\vec{u}$ along $\vec{v}$.

    (a) $\vec{u} := \langle 1, 2, 3 \rangle$ and $\vec{v} := \langle 2, 5, 0 \rangle$.

    (b) $\vec{u} := \langle 0, 1, -2 \rangle$ and $\vec{v} := \langle 1, -4, 5 \rangle$.

18. Find the decomposition: $\vec{\mathbf{a}} = \vec{\mathbf{a}}_{\parallel} + \vec{\mathbf{a}}_{\perp}$ with respect to $\vec{\mathbf{b}}$ for the following:

    (a) $\vec{a} := \langle 2, 7 \rangle$ and $\vec{b} := \langle 3, -5 \rangle$.

    (b) $\vec{a} := \langle 1, 2, -2 \rangle$ and $\vec{b} := \langle 3, -2, 1 \rangle$.