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A new focus on the Walther equation for lubricant viscosity determination

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ABSTRACT

Lubricants are widely used in industrial machinery in order to separate solid tribological surfaces and support high loads under severe conditions. In tribological contacts, viscosity plays an important role in the film-forming abilities of the lubricant, but this property is strongly dependent on temperature. Consequently, small variations in temperature cause appreciable variations in the viscosity of lubricating oils. For this reason it is of practical value to be able to predict viscosity changes with temperature. This paper presents a new focus on the Walther equation to determine the viscosity of commercial lubricants at different temperatures. This new approach provides very good correlation with experimental measurements. Copyright © 2006 John Wiley & Sons, Ltd.

KEY WORDS: Viscosity; lubricants; Walther equation; Newtonian

INTRODUCTION

The main function of a lubricant in a mechanical system is to control friction and wear. The viscosity of lubricating oils plays a fundamental role in this process. It is well known that oil viscosity changes with pressure, shear rate and temperature and that lubricants' film-forming properties are proportional to it. Thus, in practical applications, lubricant viscosity is chosen to provide optimum performance at the operating temperature. Since the viscosity of a lubricant is extremely sensitive to variations in temperature, one of the most important questions is how viscosity varies with temperature.

Several viscosity–temperature equations exist; some of them are derived entirely from experimental research but others come completely from theoretical models.[1–5] The usual approach to obtaining a viscosity–temperature relationship is to find an equation for $d\eta/dt$ as a function of temperature or viscosity. Thus, upon integration, an expression for viscosity variation with temperature is found.

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In recent years, computational simulations of molecular behavior of synthetic lubricants have been proposed to predict and tailor lubricant viscosity and rheological characteristics.[6,7] However such approaches are highly specialized, complex and cumbersome. In practical applications maintenance technicians require quick and easy viscosity predictions. Traditionally, the ASTM viscosity–temperature chart (ASTM D 341) is the method most widely used to determine the viscosity of a lubricant. Although based on Walther's equation this chart is old, fairly accurate over a limited range of temperatures and completely empirical.

Among the existing mathematical models for the influence of temperature on lubricant performance, Vogel's equation has proven to be the most accurate. Three values of viscosity at different lubricant temperatures are needed in order to determine the constants in this equation. Cameron has also indicated that allowing the constants to take their real values usually makes the equation as accurate as experimental data.[4] In Walther's equation a constant value of $C = 0.7$ is generally specified for all the lubricants;[8] however, some researchers such as Nissan *et al*. have shown over the years that this value must be $C = 0.6$,[9] when viscosity is expressed in centistokes.

The present paper describes a procedure for prediction of commercial lubricant viscosity based on the general Walther equation. Viscosity measurements for commercial oils were carried out at a wide range of temperatures up to 100°C. The flexibility and accuracy of the method were also evaluated comparing the experimental viscosity values using a new focus on the general Walther equation (NFGW), Vogel's equation, Eyring's model and the traditional procedure used for Walther's equation. The criteria used to validate the model were the correlation, R^2, and the standard deviation of the predicted viscosity from experimental values.

Experimental viscosity data of crude oils from a published reference were also compared with predicted values using the proposed procedure.

NEW FOCUS ON THE GENERAL WALTHER EQUATION

A number of mathematical models to describe the viscosity–temperature relationship can be found in the scientific literature.[1,4,9–11] Table 1 lists those most commonly employed to describe the relationship for lubricants, along with some advantages and limitations in their use.[8]

Walther's equation forms the basis for the ASTM viscosity–temperature chart, one of the most popularly used methods to determine the viscosity of oil:

$$\log\log(\eta + 0.7) = A + B \cdot \log(T) \tag{1}$$

In this equation, viscosity values of the lubricant under study at temperatures of 40°C and 100°C must be known. The constant C is taken to be 0.6 or 0.7. This information allows the constants A and B to be evaluated. Traditionally, Walther's equation has a constant value of $C = 0.7$ for all lubricants. However, using a different value of C for each lubricant might lead to a better viscosity–temperature correlation, similar to Vogel's.[4]

Equation 1 suggests that a graph of log log(viscosity) versus log(temperature) will give a straight line $y = mx + b$. As is well known, to be able to mathematically construct a line two points are needed (for lubricants commonly the viscosity values at 40°C and 100°C). A more accurate result requires an intermediate point between the previously chosen points. For Walther's equation, this means solving three equations in three unknows (A, B and C) at three known arbitrary temperatures for which the

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Table 1. Viscosity–temperature models (adapted from ref. 8).

Model	Equation	Observations
Reynolds	$\eta = B \cdot e^{-AT}$	Accurate only for a limited temperature range
Slotte	$\eta = A/(B + T)^C$	Reasonable; useful in numerical analysis
Sun	$\log \eta = A + B/T + C \cdot T + D \cdot T^2$	A system of 4 equations and 4 unknowns has to be solved
Vasquez	$\eta = A \cdot e^{B/T^C}$	Useful for petroleum products
Eyring	$\eta = A \cdot e^{B/T}$	Gives good correlation
Walther	$(\eta + C) = A \cdot D^{1/T^B}$	Works well for mineral and synthetic oils under normal conditions. Forms the bases of ASTM viscosity–temperature chart
Vogel	$\eta = A \cdot e^{B/(T-C)}$	Highly accurate, very useful in engineering

viscosity is known. Following this approach, the constant C is determined by an iterative procedure and takes a different value for each lubricant (see Appendix):

$$\log \log(\eta + C) = A + B \cdot \log(T) \tag{2}$$

However, this leaves unanswered the question of what intermediate point in the temperature range is best suited to determining the unknown constants most accurately.

In this study the kinematic viscosity of 10 commercial lubricating oils was determined experimentally. These oils were monograde and multigrade, mineral and synthetic oils. Experimental kinematic viscosity measurements were made at temperatures of 40, 50, 60, 63, 65, 70, 75, 80, 90 and 100°C. Tests were carried out with calibrated Cannon-Fenske capillary viscometers according to standards ASTM D 445 and D 2515 in a temperature-controlled bath.[12] Viscosities were measured with a precision of ±0.2%, while the temperature of the bath was controlled to ±0.05°C. The estimated total error in viscosity determination was ±0.4%.

Some calculations were carried out initially using the mean arithmetic temperature (70°C) as the third point in the temperature interval between 100 and 40°C. Although this procedure was better and more precise than the ordinary Walther equation, it was not sufficiently good since Vogel's equation was giving more accurate predictions. Then calculations were performed taking temperatures of 60, 63, 65, 70 and 75°C as the intermediate point in the temperature range.

It turned out that taking the mean logarithmic temperature (63.25°C ≅ 63°C) was the most accurate value; however, no mathematical explanation can justify this result. For the sake of brevity only a few results at different temperatures are presented in this paper.

After the experimental viscosity measurements were carried out, viscosity determination using a lubricant-specific C value with Walther's equation was performed taking the viscosity values at the two extremes of the temperature range (40 and 100°C) and a third viscosity value at the mean logarithmic temperature of the experimental range (63°C).

RESULTS

Commercial Lubricating Oils

Kinematic viscosity in centistokes was predicted for a set of commercially available lubricating oils, both mineral and synthetic. Table 2 shows experimental values against mathematically obtained vis-

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Table 2. Experimental versus predicted lubricating oil viscosity, NFGW(63).

LUBRICANT	ν (cSt)	TEMPERATURE °C 40	50	60	63	65	70	75	80	90	100	R^2
PENNZOIL SAE 5W50	EXP.	105.639	68.887	47.250	43.125	40.527	34.979	30.425	26.457	20.925	17.159	0.9999
	CALC.	105.639	68.883	47.665	43.125	40.441	34.727	30.155	26.458	20.954	17.159	
PENNZOIL SAE 15W40	EXP.	103.323	64.269	42.559	38.257	35.412	30.288	25.983	22.334	17.289	13.872	0.9999
	CALC.	103.323	64.224	42.727	38.257	35.639	30.129	25.791	22.334	17.281	13.872	
PENNZOIL SAE 20W50	EXP.	139.225	84.685	55.356	49.239	45.380	38.529	32.714	27.903	21.612	17.194	0.9998
	CALC.	139.225	84.636	55.264	49.239	45.727	38.380	32.644	28.111	21.555	17.194	
MOBIL SAE 20W50	EXP.	121.368	75.700	50.844	45.865	43.030	36.574	30.894	27.175	20.923	17.109	0.9998
	CALC.	121.368	76.177	51.104	45.865	42.792	36.315	31.204	27.123	21.148	17.109	
MOBIL SAE 40	EXP.	133.184	76.923	48.023	42.423	39.317	32.198	27.021	22.929	17.009	13.197	0.9999
	CALC.	133.184	76.968	48.167	42.423	39.105	32.243	26.968	22.855	17.009	13.197	
MEXLUB SAE 15W40	EXP.	96.313	59.751	39.794	35.564	33.078	28.079	24.194	20.916	16.023	12.938	0.9999
	CALC.	96.313	59.750	39.721	35.564	33.131	28.013	23.987	20.780	16.096	12.938	
ESSO SAE 15W40	EXP.	88.127	54.745	37.276	33.328	31.317	26.843	23.340	20.078	15.548	12.476	0.9999
	CALC.	88.127	55.282	37.119	33.328	31.104	26.416	22.714	19.757	15.418	12.476	
QUAKER SAE 15W40	EXP.	94.343	58.593	38.737	34.781	32.508	27.497	23.917	20.609	15.478	12.318	0.9999
	CALC.	94.343	58.587	38.885	34.781	32.376	27.312	23.319	20.134	15.471	2.318	
QUAKER SAE 50	EXP.	155.572	91.592	58.108	51.428	48.464	39.461	33.974	28.657	21.592	16.630	0.9999
	CALC.	155.572	91.592	58.166	51.428	47.523	39.415	33.148	28.240	21.229	16.630	
CASTROL SAE 5W50	EXP.	117.290	75.913	52.333	47.325	44.411	38.121	33.526	29.169	23.591	19.097	0.9998
	CALC.	117.290	75.913	52.331	47.325	44.374	38.110	33.123	29.108	23.165	19.097	

cosity values determined using the NFGW equation over a temperature range from 40°C to 100°C. The NFGW uses three known viscosity values at their respective temperatures. Two temperatures are the extremes of the temperature range (40°C and 100°C) under analysis and the third value is the logarithmic mean value (63°C). Viscosity values were also calculated taking the mid value as 60°C, 65°C, 70°C and 75°C; however, the best correlation (R^2) was obtained with the logarithmic mean.

The viscosity values calculated using the NFGW equation deviate from the experimental measurements by less than 0.1%. In all the calculated values the correlation coefficient was greater than 0.999. On the other hand, the ordinary Walther and Eyring relations were less suitable for description of the temperature dependence of lubricant viscosity, with correlation coefficients less than 0.992 and standard deviations of calculated viscosity more than 2%.

Figure 1(a) compares a number of theoretical models against the experimental viscosity values. This shows that the NFGW63 equation represented by the dashed line with asterisk is the closest to the experimental data (solid line), while the second best option is Vogel's equation represented by a solid diamond symbol. The area of interest is shown magnified in Figure 1(b) to this to be seen more crearly.

Table 3 shows the constants A, B, and C for the Eyring, ordinary Walther, Vogel and NFGW models, in the latter case with mid temperatures 60, 63, 65, 70, 75°C.

Estimates of viscosity values at different temperatures for the set of commercially available lubricating oils are shown in Tables 4–13. The calculated standard deviation for each model used indicates how far the predicted value is from the experimental measurements; the smaller this figure, the better the model. In all cases the NFGW equation proved to perform better than the rest of the models.

Vogel's equation used to be the most accurate of all viscosity relations. This equation has three constants A, B and C with distinct physical meaning. A has units of viscosity, represents the oil 'thickness', and gives the inherent viscosity of the oil. It could conceivably be used as the basis of a logical means of classifying oils into grades. The exponential term B has units of temperature and is the 'viscosity variation' term. C appears to be related in some way to the molecular composition of the oil; however, this has not been properly verified. By way of comparison, for the general Walther equation B represents the slope of a linear behavior typically found in Newtonian lubricants on a log log(viscosity) versus log(temperature) plot, taking a negative value for liquids and positive for gases. A change in A shifts the whole viscosity level of the oil (A represents the intercept of the line). The term

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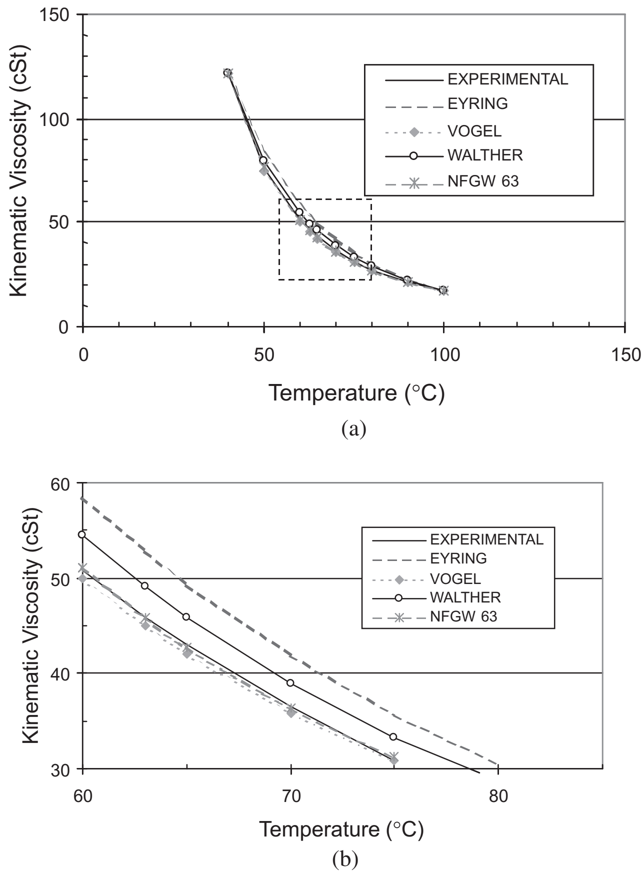

Figure 1. (a) Comparison of viscosity models. (b) Comparison for temperature of 60–80°C.

C has to do with molecular polarity of the oil. The present authors have analysed several fluids using the general Walther equation, observing a gradual change in *C* as polarity was increased from highly non-polar fluids to a highly polar fluid.

Crude Oils

Table 14 shows experimental viscosity measurements of US petroleum crudes at temperatures from 0 to 100°C taken from Amin and Maddox.[13] For crude oils from California, Wyoming and Oklahoma, constants for Vogel equation were evaluated at 40, 70 and 100°C, while for the ordinary Walther equation and Eyring, constants were evaluated at 40 and 100°C. However, for crude oil from Pennsylvania, Vogel constants were determined at temperatures of 20, 50 and 80°C, while for the Eyring and ordinary Walther equation, constants were determined at 20 and 80°C. Thus in the first case the logarithmic temperature for NFGW equation was 46°C and for the later 37°C.

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Table 3. Constants for viscosity models of lubricating oils.

LUBRICANT	CONSTANTS									
	EYRING		WALTHER		VOGEL			NFGW(60)		
	A	B	A	B	A	B	C	A	B	c
PENNZOIL SAE 5W50	0.639060	554.805989	7.165936	–2.748329	0.639060	554.805989	–204.530313	8.489993	–3.280423	–3.665429
PENNZOIL SAE 15W40	0.000390	3910.620159	8.138406	–3.138802	0.412399	580.253832	–208.100494	9.223487	–3.574590	–2.077955
PENNZOIL SAE 20W50	0.000312	4073.323334	7.994769	–3.070481	0.552074	545.520345	–214.505495	9.019580	–3.481985	–2.751825
MOBIL SAE 20W50	0.000620	3815.648813	7.607195	–2.920063	0.854695	454.794753	–221.380494	8.857200	–3.422157	–3.396000
MOBIL SAE 40	0.000076	4502.191811	9.168970	–3.542522	0.262157	633.710281	–211.439699	9.942896	–3.853127	–1.196443
MEXLUB SAE 15W40	0.000365	3909.543179	8.272798	–3.195284	0.397735	571.339788	–209.072729	9.202247	–3.568580	–1.522700
ESSO SAE 15W40	0.000462	3807.370917	8.176363	–3.160030	0.280427	669.784881	–169.670136	9.229281	–3.582968	–1.674440
QUAKER SAE 15W40	0.000390	3910.620159	8.138406	–3.138802	0.412399	580.253832	–208.100494	9.223487	–3.574590	–2.077955
QUAKER SAE 50	0.000142	4354.487581	8.478194	–3.260332	0.228116	751.012878	–198.052443	9.356687	–3.612966	–2.166558
CASTROL SAE 5W50	0.000462	3807.370917	8.176363	–3.160030	0.280427	669.784881	–169.670136	9.229281	–3.582968	–1.674440

LUBRICANT	CONSTANTS											
	NFGW(63)			NFGW(65)			NFGW(70)			NFGW(75)		
	A	B	C	A	B	C	A	B	C	A	B	c
PENNZOIL SAE 5W50	8.314945	–3.210108	–3.185515	8.275156	–3.194124	–3.072902	8.178513	–3.155299	–2.793683	8.136393	–3.138377	–2.669416
PENNZOIL SAE 15W40	9.148561	–3.544507	–1.912587	9.263050	–3.590474	–2.163901	9.054670	–3.506808	–1.700439	9.008176	–3.488139	–1.593298
PENNZOIL SAE 20W50	9.048243	–3.493491	–2.834010	9.172789	–3.543485	–3.183330	8.984897	–3.468062	–2.651462	9.011200	–3.478621	–2.727667
MOBIL SAE 20W50	8.762397	–3.384091	–3.136537	8.665733	–3.345276	–2.8645	8.637857	–3.334082	–2.78459	8.950526	–3.459628	–3.644566
MOBIL SAE 40	9.893165	–3.833171	–1.086325	9.809.146	–3.799455	–0.896891	9.915020	–3.841941	–1.134902	9.860951	–3.820244	–1.014202
MEXLUB SAE 15W40	9.324520	–3.617674	–1.774160	9.353529	–3.629321	–1.832599	9.281472	–3.600390	–1.686586	9.157949	–3.550793	–1.429499
ESSO SAE 15W40	9.313540	–3.616802	–1.835940	9.184670	–3.565054	–1.587429	9.010266	–3.495016	–1.236883	8.772531	–3.399534	–0.730900
QUAKER SAE 15W40	9.148561	–3.544507	–1.912587	9.263050	–3.590474	–2.163901	9.054670	–3.506808	–1.700439	9.008176	–3.488139	–1.593298
QUAKER SAE 50	9.340434	–3.606443	–2.119933	9.042814	–3.486992	–1.226526	9.322503	–3.599247	–2.068251	8.949479	–3.449528	–0.930011
CASTROL SAE 5W50	9.313540	–3.616802	–1.835940	9.184670	–3.565054	–1.587429	9.010266	–3.495016	–1.236883	8.772531	–3.399534	–0.730900

Table 4. Experimental versus predicted viscosity for Pennzoil SAE 15W40 API SL.

TEMP. (ºC)	VISCOSITY (cSt)									Δν = EXPERIMENTAL VISCOSITY – PREDICTED VISCOSITY							
	EXP.	EYRING	WALTHER	VOGEL	NFGW(60)	NFGW(63)	NFGW(65)	NFGW(70)	NFGW(75)	EYRING	WALTHER	VOGEL	NFGW(60)	NFGW(63)	NFGW(65)	NFGW(70)	NFGW(75)
40	103.323	103.323000	103.323000	103.323000	103.323000	103.323000	103.323000	103.323000	103.323000	0	0	0	0	0	0	0	0
50	64.269	70.204597	66.537330	63.927631	64.056196	64.224335	63.967901	64.435513	64.540505	–5.935597	–2.268330	0.341369	0.212804	0.044665	0.301099	–0.166513	–0.271505
60	42.559	48.821320	45.102045	42.709879	42.559000	42.727430	42.470677	42.939838	43.045713	–6.262320	–2.543045	–0.15087	0	–0.168430	0.088323	–0.380838	–0.486713
63	38.2571	43.965598	40.490954	38.310262	38.099641	38.257070	38.017104	38.455840	38.554985	–5.708528	–2.233884	–0.05319	0.157429	0	0.239966	–0.198770	–0.297915
65	35.412	41.042183	37.760326	35.731378	35.490140	35.639051	35.412000	35.827162	35.921039	–5.630183	–2.348326	–0.31937	–0.078140	–0.227051	0	–0.415162	–0.509039
70	30.288	34.677523	31.932136	30.290625	30.004024	30.129419	29.938363	30.288000	30.367298	–4.389523	–1.644136	–0.00262	0.283976	0.158581	0.349637	0	–0.079298
75	25.983	29.442028	27.254722	25.983000	25.690249	25.791221	25.637421	25.919110	25.983000	–3.459028	–1.271722	0	0.292751	0.191779	0.345579	0.063890	0
80	22.334	25.113088	23.462544	22.524917	22.257007	22.334285	22.216610	22.432268	22.481314	–2.779088	–1.128544	–0.19091	0.076993	–0.000285	0.117390	–0.098268	–0.147314
90	17.289	18.512737	17.795993	17.402522	17.246475	17.281301	17.228308	17.325528	17.347706	–1.223737	–0.506993	–0.11352	0.042525	0.007699	0.060692	–0.036528	–0.058706
100	13.872	13.872000	13.872	13.872000	13.872000	13.872000	13.872000	13.872000	13.872000	0	0	0	0	0	0	0	0
							STANDARD DEVIATION (σ)			**2.448509**	**0.973090**	**0.173060**	**0.130203**	**0.126446**	**0.145057**	**0.165957**	**0.196492**

Table 15 shows the proper constants *A*, *B* and *C* to be used in each mathematical model to predict the viscosity of the crude oil analysed in the specific temperature range.

Table 16 reports experimental and predicted viscosity values at different temperatures. Standard deviation values (σ) for Wyoming, Pennsylvania and Oklahoma crude oils indicate that the NFGW equation with three constants correlated better than ordinary Walther, Eyring or Vogel and clearly show an excellent agreement with experimental values.

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Table 5. Experimental versus predicted viscosity for Pennzoil SAE 20W50 API SL.

TEMP. (ºC)	VISCOSITY (cSt)									Δν = EXPERIMENTAL VISCOSITY – PREDICTED VISCOSITY							
	EXP.	EYRING	WALTHER	VOGEL	NFGW(60)	NFGW(63)	NFGW(65)	NFGW(70)	NFGW(75)	EYRING	WALTHER	VOGEL	NFGW(60)	NFGW(63)	NFGW(65)	NFGW(70)	NFGW(75)
40	139.225	139.225000	139.225000	139.225000	139.225000	139.22500	139.22500	139.22500	139.22500	0	0	0	0	0	0	0	0
50	84.685	93.090007	88.109266	83.686416	84.729082	84.636656	84.236006	84.841517	84.756210	–8.405007	–3.424266	0.998584	–0.044082	0.048344	0.448994	–0.156517	–0.071210
60	55.356	63.765170	58.783837	54.809691	55.356000	55.264560	54.869681	55.467241	55.382841	–8.409170	–3.427837	0.546309	0	0.091440	0.486319	–0.111241	–0.026841
63	49.239	57.173409	52.538195	48.934002	49.324248	49.239000	48.871523	49.427891	49.349245	–7.934409	–3.299195	0.304998	–0.085248	0	0.367477	–0.188891	–0.110245
65	45.38	53.219192	48.852531	45.510049	45.807090	45.726686	45.380000	45.904918	45.830682	–7.839192	–3.472531	–0.130049	–0.427090	–0.346686	0	–0.524918	–0.450682
70	38.529	44.652019	41.022202	38.338422	38.447144	38.379793	38.089805	38.529000	38.466910	–6.123019	–2.493202	0.190578	0.081856	0.149207	0.439195	0	0.062090
75	32.714	37.653333	34.77894	32.714000	32.698173	32.644205	32.412111	32.763895	32.714000	–4.939333	–2.064940	0	0.015827	0.069795	0.301889	–0.049895	0
80	27.903	31.905256	29.748912	28.235966	28.151698	28.110583	27.933942	28.201795	28.163771	–4.002256	–1.845912	–0.332966	–0.248698	–0.207583	–0.030942	–0.298795	–0.260771
90	21.612	23.223248	22.297181	21.669228	21.574138	21.555766	21.476953	21.596562	21.579539	–1.611248	–0.685181	–0.057228	0.037862	0.056234	0.135047	0.015438	0.032461
100	17.194	17.194000	17.194000	17.194000	17.194000	17.194000	17.194000	17.194000	17.194000	0	0	0	0	0	0	0	0
									STANDARD DEVIATION (σ)	**3.392443**	**1.409201**	**0.383683**	**0.155284**	**0.149940**	**0.214731**	**0.172312**	**0.158421**

Table 6. Experimental versus predicted viscosity for Mobil SAE 20W50 API SL.

TEMP. (ºC)	VISCOSITY (cSt)									Δν = EXPERIMENTAL VISCOSITY – PREDICTED VISCOSITY							
	EXP.	EYRING	WALTHER	VOGEL	NFGW(60)	NFGW(63)	NFGW(65)	NFGW(70)	NFGW(75)	EYRING	WALTHER	VOGEL	NFGW(60)	NFGW(63)	NFGW(65)	NFGW(70)	NFGW(75)
40	121.368	121.3680	121.6380	121.3680	121.3680	121.3680	121.3680	121.3680	121.3680	0	0	0	0	0	0	0	0
50	75.700	83.243169	79.526543	74.579108	75.919695	76.177553	76.440897	76.517226	75.666866	–7.543169	–3.826543	1.120892	–0.219695	–0.477553	–0.740897	–0.817226	0.033134
60	50.844	58.401467	54.511654	50.000413	50.844000	51.104146	51.371229	51.448738	50.590007	–7.557467	–3.667654	0.843587	0	–0.260146	–0.527229	–0.604738	0.253993
63	45.865	52.726888	49.071509	44.955323	45.621520	45.865000	46.115548	46.188260	45.383920	–6.861888	–3.206509	0.909677	0.243480	0	–0.250548	–0.323260	0.481080
65	43.030	49.303223	45.838538	42.005365	42.561856	42.792520	43.030000	43.098734	42.337038	–6.273223	–2.808538	1.024635	0.468144	0.237480	0	–0.068734	0.692962
70	36.574	41.828299	38.907505	35.797281	36.120353	36.315062	36.515706	36.574000	35.930879	–5.254299	–2.333505	0.776719	0.453647	0.258938	0.058294	0	0.643121
75	30.894	35.654641	33.312095	30.894000	31.046611	31.203723	31.365859	31.413039	30.894000	–4.760641	–2.418095	0	–0.152611	–0.309723	–0.471859	–0.519039	0
80	27.175	30.529931	28.751874	26.962079	27.003019	27.123476	27.247941	27.284190	26.886116	–3.354931	–1.576874	0.212921	0.171981	0.051524	–0.072941	–0.109190	0.288884
90	20.923	22.673181	21.893316	21.135983	21.093462	21.147902	21.204247	21.220689	21.040746	–1.750181	–0.970316	–0.212983	–0.170462	–0.224902	–0.281247	–0.297689	–0.117746
100	17.109	17.109000	17.109	17.109000	17.109000	17.109000	17.109000	17.109000	17.109000	0	0	0	0	0	0	0	0
									STANDARD DEVIATION (σ)	**2.926338**	**1.4018539**	**0.511364**	**0.24750019**	**0.239413**	**0.274341**	**0.290983**	**0.292577**

Table 7. Experimental versus predicted viscosity for Mobil SAE 40 API SL.

TEMP. (ºC)	VISCOSITY (cSt)									Δν = EXPERIMENTAL VISCOSITY – PREDICTED VISCOSITY							
	EXP.	EYRING	WALTHER	VOGEL	NFGW(60)	NFGW(63)	NFGW(65)	NFGW(70)	NFGW(75)	EYRING	WALTHER	VOGEL	NFGW(60)	NFGW(63)	NFGW(65)	NFGW(70)	NFGW(75)
40	133.184	133.18400	133.184	133.184000	133.184000	133.18400	133.184000	133.18400	133.18400	0	0	0	0	0	0	0	0
50	76.923	85.355637	79.212202	76.247955	76.816654	76.968205	77.225205	76.901556	77.066561	–8.432637	–2.289202	0.675045	0.106346	–0.045205	–0.302205	0.021444	–0.143561
60	48.023	56.183907	50.338948	47.841820	48.023000	48.167045	48.412020	48.103651	48.260715	–8.160907	–2.315948	0.181180	0	–0.144045	–0.389020	–0.080651	–0.237715
63	42.423	49.800420	44.43519	42.209648	42.290138	42.423000	42.649098	42.364529	42.509431	–7.377420	–2.012190	0.213352	0.132862	0	–0.226098	0.058471	–0.086431
65	39.317	46.007650	40.997413	38.956393	38.980389	39.104990	39.317000	39.050148	39.186066	–6.690650	–1.680413	0.360607	0.336611	0.212010	0	0.266852	0.130934
70	32.198	37.894583	33.815986	32.219917	32.140412	32.243286	32.418618	32.198000	32.310270	–5.696583	–1.617986	–0.02191	0.057588	–0.045286	–0.220618	0	–0.112270
75	27.021	31.386598	28.219413	27.021000	26.886599	26.967986	27.106832	26.932150	27.021000	–4.365598	–1.198413	0	0.134401	0.053014	–0.085832	0.088850	0
80	22.929	26.135365	23.802562	22.944115	22.793699	22.855010	22.959698	22.828009	22.894976	–3.206365	–0.873562	–0.01511	0.135301	0.073990	–0.030698	0.100991	0.034024
90	17.009	18.397808	17.42889	17.086653	16.982832	17.009725	17.055719	16.997879	17.027271	–1.388808	–0.41989	–0.07765	0.026168	–0.000725	–0.046719	0.011121	–0.018271
100	13.197	13.197000	13.197	13.197000	13.197000	13.197000	13.197000	13.197000	13.197000	0	0	0	0	0	0	0	0
									STANDARD DEVIATION (σ)	**3.253017**	**0.8866531**	**0.2351721**	**0.103432**	**0.092497**	**0.142844**	**0.093092**	**0.103845**

Table 8. Experimental versus predicted viscosity for Mexlub SAE 15W40 API SJ.

TEMP. (ºC)	VISCOSITY (cSt)									Δν = EXPERIMENTAL VISCOSITY – PREDICTED VISCOSITY							
	EXP.	EYRING	WALTHER	VOGEL	NFGW(60)	NFGW(63)	NFGW(65)	NFGW(70)	NFGW(75)	EYRING	WALTHER	VOGEL	NFGW(60)	NFGW(63)	NFGW(65)	NFGW(70)	NFGW(75)
40	96.313	96.3130	96.3130	96.3130	96.3130	96.3130	96.3130	96.3130	96.3130	0	0	0	0	0	0	0	0
50	59.751	65.448496	61.956265	59.524499	60.002230	59.750351	59.690769	59.839010	60.093897	–5.697496	–2.205265	0.226501	–0.251230	0.000649	0.060231	–0.088010	–0.342897
60	39.794	45.518411	41.980157	39.755162	39.7940	39.721347	39.661770	39.810089	40.066284	–5.724411	–2.186157	0.038838	0	0.072653	0.132230	–0.016089	–0.272284
63	35.564	40.992376	37.687631	35.660127	35.800336	35.5640	35.508360	35.646976	35.886678	–5.428376	–2.123631	–0.096127	–0.236336	0	0.055640	–0.082976	–0.322678
65	33.078	38.267385	35.146453	33.260464	33.354230	33.130667	33.078000	33.209117	33.435960	–5.189385	–2.068453	–0.182464	–0.276230	–0.052667	0	–0.131117	–0.357960
70	28.079	32.334530	29.724600	28.199456	28.201365	28.012999	27.968703	28.079000	28.270317	–4.255530	–1.645600	–0.120456	–0.122365	0.066001	0.110297	0	–0.191317
75	24.194	27.454010	25.375179	24.1940	24.138416	23.986660	23.951011	24.039846	24.1940	–3.260010	–1.181179	0	0.055584	0.207340	0.242989	0.154154	0
80	20.916	23.418399	21.850055	20.979373	20.896459	20.780260	20.752990	20.820965	20.939093	–2.502399	–0.934055	–0.063373	0.019541	0.135740	0.163010	0.095035	–0.023093
90	16.023	17.264905	16.584074	16.218754	16.148916	16.096516	16.084239	16.114863	16.168184	–1.241905	–0.561074	–0.195754	–0.125916	–0.073516	–0.061239	–0.091863	–0.145184
100	12.938	12.9380	12.9380	12.9380	12.9380	12.9380	12.9380	12.9380	12.9380	0	0	0	0	0	0	0	0
									STANDARD DEVIATION (σ)	**2.292090**	**0.886081**	**0.123269**	**0.125515**	**0.085985**	**0.092073**	**0.088649**	**0.152078**

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Table 9. Experimental versus predicted viscosity for Esso SAE 15W40 API SL.

TEMP. (ºC)	VISCOSITY (cSt)									Δν = EXPERIMENTAL VISCOSITY – PREDICTED VISCOSITY							
	EXP.	EYRING	WALTHER	VOGEL	NFGW(60)	NFGW(63)	NFGW(65)	NFGW(70)	NFGW(75)	EYRING	WALTHER	VOGEL	NFGW(60)	NFGW(63)	NFGW(65)	NFGW(70)	NFGW(75)
40	88.127	88.127000	88.127	88.127000	88.127000	88.127000	88.127000	88.127000	88.127000	0	0	0	0	0	0	0	0
50	54.745	60.493492	57.415531	55.932394	55.436815	55.281738	55.519062	55.842077	56.285753	–5.748492	–2.670531	–1.187394	–0.691815	–0.536738	–0.774062	–1.097077	–1.540753
60	37.276	42.473470	39.322659	37.944764	37.276000	37.119583	37.359278	37.687653	38.142723	–5.197470	–2.046659	–0.668764	0	0.156417	–0.083278	–0.411653	–0.866723
63	33.328	38.355039	35.403709	34.143666	33.474511	33.328000	33.552528	33.860683	34.288704	–5.027039	–2.075709	–0.815666	–0.146511	0	–0.224528	–0.532683	–0.960704
65	31.317	35.869792	33.077425	31.902819	31.243104	31.104327	31.317000	31.609194	32.015602	–4.552792	–1.760425	–0.585819	0.073896	0.212673	0	–0.292194	–0.698602
70	26.843	30.442384	28.09673	27.142413	26.533109	26.415907	26.595598	26.843000	27.188350	–3.599384	–1.253730	–0.299413	0.309891	0.427093	0.247402	0	–0.345350
75	23.34	25.958225	24.08193	23.340000	22.808952	22.714326	22.859450	23.059718	23.340000	–2.618225	–0.741930	0	0.531048	0.625674	0.480550	0.280282	0
80	20.078	22.234683	20.813408	20.264791	19.829470	19.756871	19.868244	20.022229	20.238327	–2.156683	–0.735408	–0.186791	0.248530	0.321129	0.209756	0.055771	–0.160327
90	15.548	16.523342	15.90171	15.670457	15.450768	15.417912	15.468337	15.538283	15.636876	–0.975342	–0.353710	–0.122457	0.097232	0.130088	0.079663	0.009717	–0.088876
100	12.476	12.476000	12.476	12.476000	12.476000	12.476000	12.476000	12.476000	12.476000	0	0	0	0	0	0	0	0
									STANDARD DEVIATION (σ)	**2.165821**	**0.941152**	**0.409654**	**0.322657**	**0.311471**	**0.333097**	**0.398438**	**0.528750**

Table 10. Experimental versus predicted viscosity for Quaker SAE 15W40 API SI.

TEMP. (ºC)	VISCOSITY (cSt)									Δν = EXPERIMENTAL VISCOSITY – PREDICTED VISCOSITY							
	EXP.	EYRING	WALTHER	VOGEL	NFGW(60)	NFGW(63)	NFGW(65)	NFGW(70)	NFGW(75)	EYRING	WALTHER	VOGEL	NFGW(60)	NFGW(63)	NFGW(65)	NFGW(70)	NFGW(75)
40	94.343	94.343000	94.343000	94.343000	94.343000	94.343000	94.343000	94.343000	94.343000	0	0	0	0	0	0	0	0
50	58.593	63.759852	60.196099	59.299520	58.439781	58.587116	58.734472	58.832735	59.553656	–5.166852	–1.603099	–0.706520	0.153219	0.005884	–0.141472	–0.239735	–0.960656
60	38.737	44.116422	40.529825	39.725929	38.737000	38.884750	39.033155	39.132278	39.866204	–5.379422	–1.792825	–0.988929	0	–0.147750	–0.296155	–0.395278	–1.129204
63	34.781	39.670884	36.327100	35.599498	34.642954	34.781000	34.920035	35.012870	35.701818	–4.889884	–1.546100	–0.818498	0.138046	0	–0.139035	–0.231870	–0.920818
65	32.508	36.997665	33.843576	33.169884	32.245748	32.376464	32.508000	32.595873	33.249215	–4.489665	–1.335576	–0.661884	0.262252	0.131536	0	–0.087873	–0.741215
70	27.497	31.187139	28.556858	28.018489	27.201737	27.311880	27.422822	27.497000	28.050573	–3.690139	–1.059858	–0.521489	0.295263	0.185120	0.074178	0	–0.553573
75	23.917	26.418490	24.328927	23.917000	23.230751	23.319498	23.408981	23.468907	23.917000	–2.501490	–0.411927	0	0.686249	0.597502	0.508019	0.448093	0
80	20.609	22.484392	20.911811	20.611854	20.066506	20.134470	20.203056	20.249024	20.593673	–1.875392	–0.302811	–0.002854	0.542494	0.474530	0.405944	0.359976	0.015327
90	15.478	16.504882	15.825249	15.702404	15.440613	15.471282	15.502258	15.523056	15.679647	–1.026882	–0.347249	–0.224404	0.037387	0.006718	–0.024258	–0.045056	–0.201647
100	12.318	12.318000	12.318	12.318000	12.318000	12.318000	12.318000	12.318000	12.318000	0	0	0	0	0	0	0	0
									STANDARD DEVIATION (σ)	**2.106453**	**0.699883**	**0.389767**	**0.239870**	**0.235356**	**0.245181**	**0.259042**	**0.462481**

Table 11. Experimental versus predicted viscosity for Quaker SAE 50 API SL.

TEMP. (ºC)	VISCOSITY (cSt)									Δν = EXPERIMENTAL VISCOSITY – PREDICTED VISCOSITY							
	EXP.	EYRING	WALTHER	VOGEL	NFGW(60)	NFGW(63)	NFGW(65)	NFGW(70)	NFGW(75)	EYRING	WALTHER	VOGEL	NFGW(60)	NFGW(63)	NFGW(65)	NFGW(70)	NFGW(75)
40	155.572	155.5720	155.5720	155.5720	155.5720	155.5720	155.5720	155.5720	155.5720	0	0	0	0	0	0	0	0
50	91.562	101.16973 0	94.838062	92.343453	91.532061	91.592609	92.698867	91.658299	93.049384	–9.607730	–3.276062	–0.781453	0.0299389	–0.030609	–1.136867	–0.096299	–1.487384
60	58.108	67.513211	61.369769	59.212835	58.108000	58.166228	59.243560	58.229999	59.587833	–9.405211	–3.261769	–1.104835	0	–0.058228	–1.135560	–0.121999	–1.479833
63	51.428	60.079767	54.410798	52.477000	51.374218	51.428000	52.427722	51.487271	52.747864	–8.651767	–2.982798	–1.049000	0.0537819	0	–0.999722	–0.059271	–1.319864
65	48.464	55.648562	50.336832	48.557050	47.472394	47.523120	48.464000	47.578691	48.765769	–7.184562	–1.872832	–0.093050	0.9916058	0.940880	0	0.885309	–0.301769
70	39.461	46.128054	41.769488	40.367054	39.372657	39.414765	40.198164	39.461000	40.450281	–6.667054	–2.308488	–0.906054	0.0883428	0.046235	–0.737164	0	–0.989281
75	33.974	38.442971	35.033403	33.974000	33.114755	33.148231	33.772578	33.184947	33.974000	–4.468971	–1.059403	0	0.8592446	0.825769	0.201422	0.789053	0
80	28.657	32.204011	29.675516	28.913074	28.215135	28.240465	28.713818	28.268243	28.867028	–3.547011	–1.018516	–0.256074	0.4418647	0.416535	–0.056818	0.388757	–0.210028
90	21.592	22.932391	21.869309	21.563473	21.217697	21.228906	21.438694	21.241151	21.506927	–1.340391	–0.277309	0.028527	0.3743026	0.363094	0.153306	0.350849	0.085073
100	16.63	16.63	16.63	16.63	16.63	16.63	16.63	16.63	16.63	0	0	0	0	0	0	0	0
									STANDARD DEVIATION (σ)	**3.759929**	**1.3148188**	**0.482532**	**0.3745346**	**0.372993**	**0.559232**	**0.372954**	**0.667735**

Table 12. Experimental versus predicted viscosity for Castrol SAE 5W50 API SJ.

TEMP. (ºC)	VISCOSITY (cSt)									Δν = EXPERIMENTAL VISCOSITY – PREDICTED VISCOSITY							
	EXP.	EYRING	WALTHER	VOGEL	NFGW(60)	NFGW(63)	NFGW(65)	NFGW(70)	NFGW(75)	EYRING	WALTHER	VOGEL	NFGW(60)	NFGW(63)	NFGW(65)	NFGW(70)	NFGW(75)
40	117.290	117.2900	117.2900	117.2900	117.2900	117.2900	117.2900	117.2900	117.2900	0	0	0	0	0	0	0	0
50	75.913	82.708457	79.621506	75.703530	75.914468	75.912616	75.952880	75.926039	76.535263	–6.795457	–3.708506	0.209470	–0.001468	0.000384	–0.039880	–0.013039	–0.622263
60	52.333	59.558880	56.286690	52.504419	52.333000	52.331043	52.372646	52.344942	52.977393	–7.225880	–3.953690	–0.171419	0	0.001957	–0.039646	–0.011942	–0.644393
63	47.325	54.177627	51.083336	47.586785	47.327019	47.325000	47.364332	47.338234	47.934868	–6.852627	–3.758336	–0.261785	–0.002019	0	–0.039332	–0.013234	–0.609868
65	44.411	50.910539	47.964958	44.682369	44.375630	44.373836	44.411000	44.386287	44.953837	–6.499539	–3.553958	–0.271369	0.035370	0.037164	0	0.024713	–0.542837
70	38.121	43.717423	41.206786	38.493948	38.111991	38.110460	38.142184	38.121000	38.605310	–5.596423	–3.085786	–0.372948	0.009009	0.010540	–0.021184	0	–0.484310
75	33.526	37.705226	35.669477	33.526000	33.124159	33.122915	33.148713	33.131537	33.526000	–4.179226	–2.143477	0	0.401841	0.403085	0.377287	0.394463	0
80	29.169	32.656375	31.094583	29.485150	29.109263	29.108306	29.128222	29.114961	29.420032	–3.487375	–1.925583	–0.316150	0.059737	0.060694	0.040778	0.054039	–0.251032
90	23.591	24.788970	24.089925	23.392171	23.165983	23.165558	23.174660	23.168595	23.308473	–1.197970	–0.498925	0.198829	0.425017	0.425442	0.416340	0.422405	0.282527
100	19.097	19.097	19.097	19.097	19.097	19.097	19.097	19.097	19.097	0	0	0	0	0	0	0	0
									STANDARD DEVIATION (σ)	**2.886066**	**1.598956**	**0.210003**	**0.170294**	**0.170110**	**0.174576**	**0.171442**	**0.336901**

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Table 13. Experimental versus predicted viscosity for Pennzoil SAE 5W50 API SL.

TEMP. (ºC)	VISCOSITY (cSt)								
	EXP.	EYRING	WALTHER	VOGEL	NFGW(60)	NFGW(63)	NFGW(65)	NFGW(70)	NFGW(75)
40	105.639	105.639000	105.639000	105.639000	105.639000	105.639000	105.639000	105.639000	105.639000
50	68.887	74.477150	71.568706	68.677808	68.483828	68.883356	68.974558	69.196331	69.293290
60	47.25	53.608315	50.537681	47.740577	47.250000	47.664781	47.759894	47.992010	48.093783
63	43.125	48.758635	45.856848	43.270169	42.733884	43.125000	43.214957	43.434390	43.530678
65	40.527	45.814583	43.053251	40.624473	40.069428	40.441602	40.527000	40.736150	40.827912
70	34.979	39.333593	36.981579	34.973960	34.409422	34.727008	34.800127	34.979000	35.057651
75	30.425	33.917680	32.011220	30.425000	29.896097	30.154870	30.214545	30.360698	30.425000
80	26.457	29.370441	27.907790	26.717041	26.258220	26.458371	26.504597	26.617906	26.667810
90	20.925	22.286588	21.630183	21.114397	20.862623	20.954443	20.975706	21.027881	21.050902
100	17.159	17.159000	17.159000	17.159000	17.159000	17.159000	17.159000	17.159000	17.159000

TEMP. (ºC)	Δν = EXPERIMENTAL VISCOSITY – PREDICTED VISCOSITY							
	EYRING	WALTHER	VOGEL	NFGW(60)	NFGW(63)	NFGW(65)	NFGW(70)	NFGW(75)
40	0	0	0	0	0	0	0	0
50	–5.590150	–2.681706	0.209192	0.403172	0.003644	–0.087558	–0.309331	–0.406290
60	–6.358315	–3.287681	–0.490577	0	–0.414781	–0.509894	–0.742010	–0.843783
63	–5.633635	–2.731848	–0.145169	0.391116	0	–0.089957	–0.309390	–0.405678
65	–5.287583	–2.526251	–0.097473	0.457572	0.085398	0	–0.209150	–0.300912
70	–4.354593	–2.002579	0.005040	0.569578	0.251992	0.178873	0	–0.078651
75	–3.492680	–1.586220	0	0.528903	0.270130	0.210455	0.064302	0
80	–2.913441	–1.450790	–0.260041	0.198780	–0.001371	–0.047597	–0.160906	–0.210810
90	–1.361588	–0.705183	–0.189397	0.062377	–0.029443	–0.050706	–0.102881	–0.125902
100	0	0	0	0	0	0	0	0
STANDARD DEVIATION (σ)	**2.367225**	**1.162759**	**0.189952**	**0.233443**	**0.186770**	**0.194697**	**0.238811**	**0.265830**

Table 14. Experimental viscosities for US petroleum crude.

TEMPERATURE (ºC)	KINEMATIC VISCOSITY (cSt)			
	PENNSYLVANIA	CALIFORNIA	WYOMING	OKLAHOMA
0	0.894	1.250	1.530	2.050
10	0.787	1.090	1.310	1.700
20	0.706	0.959	1.140	1.440
30	0.643	0.858	1.000	1.240
40	0.590	0.768	0.890	1.090
50	0.536	0.701	0.798	0.966
60	0.499	0.649	0.739	0.877
80	0.443	0.554	0.611	0.719
100	–	0.485	0.526	0.611

Table 15. Constants for viscosity prediction for crude oils.

CRUDE OIL	CONSTANTS									
	EYRING		WALTHER		VOGEL			NFGW(46)		
	A	B	A	B	A	B	C	A	B	C
WYOMING	0.033799	1024.246538	11.012484	–4.691343	0.092241	450.215115	–114.538706	8.008023	–3.464610	0.807999
OKLAHOMA	0.029787	1127.301688	10.301383	–4.366825	0.101087	443.465192	–126.659905	8.884325	–3.789218	0.762057
CALIFORNIA	0.044045	895.165268	10.840974	–4.655498	0.097362	432.909274	–103.545014	8.057289	–3.517003	0.782050
PENNSYLVANIA	0.045447	804.129782	11.569674	–5.025944	0.155664	203.576196	–158.501981	9.328353	–4.103107	0.744909

CONCLUSIONS

This paper presents a new focus on the procedure to determine viscosity–temperature variations using the general Walther equation. This NFGW equation uses three viscosity values at their respective temperatures. Given two kinematic viscosity values of the lubricant, at their particular temperature, the third value is taken viscosity at the logarithmic mean of the two previously chosen values. For lubricant viscosities at 40°C and 100°C, the third temperature is approximately 63°C.

Viscosity predictions for both formulated commercial automotive oils and crude oils make evident that NFGW equation offers a better viscosity–temperature correlation than mathematical models from the past. Although no mathematical justification for using the mean logarithmic temperature could be found, certainly this intermediate temperature works best for this approach. It is important to stress the fact that even though the use of Walther's equation with $C = 0.7$ is currently widespread even for

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Table 16. Experimental versus predicted viscosities for US petroleum crude at different temperatures.

WYOMING									
KINEMATIC VISCOSITY (cSt)						$\Delta\nu = \nu_{exp} - \nu_{calc}$			
T (ºC)	EXP.	EYRING	WALTHER	VOGEL	NFGW(46)	EYRING	WALTHER	VOGEL	NFGW(46)
0	1.530	1.436823	1.711991	1.576379	1.530	0.093177	–0.181991	–0.046379	0
10	1.310	1.258605	1.403889	1.332139	1.308622	0.051395	–0.093889	–0.022139	–0.001378
20	1.140	1.112499	1.181418	1.147162	1.136232	0.027501	–0.041418	–0.007162	–0.003768
30	1.000	0.991392	1.016005	1.003656	0.999456	0.008608	–0.016005	–0.003656	–0.000544
40	0.890	0.890	0.890	0.890	0.889182	0	0	0	0.0008180
46	0.833	0.836918	0.828413	0.832762	0.833	–0.003918	0.004587	0.000238	0
50	0.798	0.804323	0.792050	0.798359	0.799028	–0.006323	0.005950	–0.000359	–0.001028
60	0.739	0.731328	0.714592	0.723309	0.724424	0.007672	0.024408	0.015691	0.014576
70	0.661	0.668654	0.652437	0.661	0.662026	–0.007654	0.008563	0	0.001026
80	0.611	0.614462	0.601927	0.608635	0.609341	–0.003462	0.009073	0.002365	0.001659
100	0.526	0.526	0.526	0.526	0.526	0	0	0	0
STANDARD DEVIATION (σ)						**0.030565**	**0.059294**	**0.015521**	**0.004273**

OKLAHOMA									
VISCOSITY (cSt)						$\Delta\nu = \nu_{exp} - \nu_{calc}$			
T (ºC)	EXP.	EYRING	WALTHER	VOGEL	NFGW(46)	EYRING	WALTHER	VOGEL	NFGW(46)
0	2.050	1.846625	2.179028	2.086519	2.050	0.203375	–0.129028	–0.036519	0
10	1.700	1.596166	1.768963	1.719527	1.703028	0.103834	–0.068963	–0.019527	–0.003028
20	1.440	1.393466	1.474268	1.450400	1.443635	0.046534	–0.034268	–0.010400	–0.003635
30	1.240	1.227456	1.255926	1.247221	1.244911	0.012544	–0.015926	–0.007221	–0.004911
40	1.090	1.090	1.090	1.090	1.089476	0	0	0	0.000524
46	1.012	1.018689	1.009002	1.012133	1.012	–0.006689	0.002998	–0.000133	0
50	0.966	0.975110	0.961199	0.965760	0.965730	–0.009110	0.004801	0.000240	0.000270
60	0.877	0.878168	0.859391	0.865767	0.865697	–0.001168	0.017609	0.011233	0.011303
70	0.784	0.795706	0.777670	0.784003	0.783754	–0.011706	0.006330	0	0.000242
80	0.719	0.725024	0.711192	0.716208	0.715857	–0.006024	0.009073	0.002365	0.001659
100	0.611	0.611	0.611	0.611	0.611	0	0	0	0
STANDARD DEVIATION (σ)						**0.064950**	**0.042571**	**0.012499**	**0.004081**

fluids other than automotive lubricants,[14] making use of the general Walther equation employing a fluid-specific C value lends more accuracy to the process of viscosity determination.

APPENDIX

In order to calculate A, B and C constants for the NFGW equation, three viscosity values (ν_1, ν_2, ν_3) at their corresponding temperature (T_1, T_2, T_3) must be given. We have

$$\log\log(\nu + C) = A + B \cdot \log(T)$$

Substituting (ν_i, T_i), $i = 1, 2, 3$, in turn,

$$\log\log(\nu_1 + C) = A + B \cdot \log(T_1) \quad \text{(a)}$$

$$\log\log(\nu_2 + C) = A + B \cdot \log(T_2) \quad \text{(b)}$$

$$\log\log(\nu_3 + C) = A + B \cdot \log(T_3) \quad \text{(c)}$$

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Table 16. *Continued*.

CALIFORNIA									
VISCOSITY (cSt)						$\Delta\nu = \nu_{exp} - \nu_{calc}$			
T (ºC)	EXP.	EYRING	WALTHER	VOGEL	NFGW(46)	EYRING	WALTHER	VOGEL	NFGW(46)
0	1.250	1.167249	1.365389	1.249992	1.250	0.082751	–0.115389	0.000008	0
10	1.090	1.039676	1.146928	1.084395	1.085860	0.050324	–0.056928	0.005605	0.004140
20	0.959	0.933387	0.985369	0.954944	0.956448	0.025613	–0.026369	0.004056	0.002552
30	0.858	0.843948	0.862859	0.851727	0.852687	0.014052	–0.004859	0.006273	0.005313
40	0.768	0.768	0.768	0.768	0.768277	0	0	0	–0.000277
46	0.725	0.727820	0.721107	0.725106	0.725	–0.002820	0.003893	–0.000106	0
50	0.701	0.702985	0.693248	0.699066	0.698739	–0.001985	0.007752	0.001934	0.002261
60	0.649	0.646896	0.633450	0.641551	0.640817	0.002104	0.015550	0.007449	0.008183
70	0.593	0.598174	0.584995	0.593	0.592098	–0.005174	0.008005	0	0.000902
80	0.554	0.555581	0.545288	0.551595	0.550764	–0.001581	0.008712	0.002405	0.003236
100	0.485	0.485	0.485	0.485	0.485	0	0	0	0
STANDARD DEVIATION (σ)						**0.027365**	**0.038002**	**0.002849**	**0.002613**

PENNSYLVANIA									
VISCOSITY (cSt)						$\Delta\nu = \nu_{exp} - \nu_{calc}$			
T (ºC)	EXP.	EYRING	WALTHER	VOGEL	NFGW(37)	EYRING	WALTHER	VOGEL	NFGW(37)
0	0.894	0.863039	0.925868	0.919070	0.894	0.030961	–0.031878	–0.025070	0
10	0.787	0.777816	0.800334	0.797043	0.786622	0.009184	–0.013334	–0.010043	0.000378
20	0.706	0.706	0.706	0.706	0.702373	0	0	0	0.003627
30	0.643	0.644921	0.633603	0.635931	0.635187	–0.001921	0.009397	0.007069	0.007813
37	0.596	0.607444	0.592629	0.595931	0.596	–0.011444	0.003371	0.000069	0
40	0.59	0.592542	0.577054	0.580613	0.580856	–0.002542	0.012946	0.009387	0.009147
50	0.536	0.547278	0.532219	0.536	0.536378	–0.011278	0.003781	0	–0.000378
60	0.499	0.507889	0.496213	0.499365	0.499586	–0.008889	0.002787	–0.000365	–0.000586
80	0.443	0.443	0.443	0.443	0.443	0	0	0	0
STANDARD DEVIATION (σ)						**0.013174**	**0.013528**	**0.010172**	**0.003776**

Rearranging equations (a), (b) and (c) gives

$$A = \log\log(\nu_1 + C) - B\cdot\log(T_1) \quad \text{(d)}$$

$$A = \log\log(\nu_2 + C) - B\cdot\log(T_2) \quad \text{(e)}$$

$$A = \log\log(\nu_3 + C) - B\cdot\log(T_3) \quad \text{(f)}$$

Combining equations (d) and (e) to eliminate A gives:

$$\log\log(\nu_1 + C) - B\cdot\log(T_1) = \log\log(\nu_2 + C) - B\cdot\log(T_2)$$

Rearranging this equation gives:

$$B\log(T_2) - B\log(T_1) = \log\log(\nu_2 + C) - \log\log(\nu_1 + C)$$

or

$$B\cdot\log\left(\frac{T_2}{T_1}\right) = \log\left(\frac{\log(\nu_2 + C)}{\log(\nu_1 + C)}\right)$$

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where

$$B \cdot \log\left(\frac{T_2}{T_1}\right) = \frac{1}{X}$$

$$\frac{1}{X} = \log\left(\frac{\log(v_2 + C)}{\log(v_1 + C)}\right) \tag{g}$$

Combining equations (e) and (f) gives:

$$\frac{1}{Y} = \log\left(\frac{\log(v_3 + C)}{\log(V_2 + C)}\right) \tag{h}$$

where:

$$\frac{1}{Y} = B \cdot \log\left(\frac{T_3}{T_2}\right)$$

Combining (g) and (h) gives:

$$\frac{1}{X} \cdot \log\left(\frac{\log(v_2 + C)}{\log(v_1 + C)}\right) = \frac{1}{Y} \cdot \log\left(\frac{\log(v_3 + C)}{\log(v_2 + C)}\right)$$

$$Y \cdot \log\left(\frac{\log(v_2 + C)}{\log(v_1 + C)}\right) = X \cdot \log\left(\frac{\log(v_3 + C)}{\log(v_2 + C)}\right)$$

$$\left[\frac{\log(v_2 + C)}{\log(v_1 + C)}\right]^Y = \left[\frac{\log(v_3 + C)}{\log(v_2 + C)}\right]^X$$

$$\frac{[\log(v_2 + C)]^Y}{[\log(v_1 + C)]^Y} = \frac{[\log(v_3 + C)]^X}{[\log(v_2 + C)]^X}$$

If $(X + Y) = Z$, then

$$[\log(v_2 + C)]^Z = [\log(v_3 + C)]^X \cdot [\log(v_1 + C)]^Y \tag{i}$$

Iterating equation (i), C constant can be found. Substituting C in (g) and (h) gives two similar values for B (the average is taken). B and C are substituted in (d), (e) and (f), obtaining three similar values for A (again the average is taken).

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