Elastohydrodynamic Lubrication: A Gateway to Interfacial Mechanics—Review and Prospect

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Elastohydrodynamic Lubrication (EHL) is commonly known as a mode of fluid-film lubrication in which the mechanism of hydrodynamic film formation is enhanced by surface elastic deformation and lubricant viscosity increase due to high pressure. It has been an active and challenging field of research since the 1950s. Significant breakthroughs achieved in the last 10–15 years are largely in the area of mixed EHL, in which surface asperity contact and hydrodynamic lubricant film coexist. Mixed EHL is of the utmost importance not only because most power-transmitting components operate in this regime, but also due to its theoretical universality that dry contact and full-film lubrication are in fact its special cases under extreme conditions. In principle, mixed EHL has included the basic physical elements for modeling contact, or hydrodynamic lubrication, or both together. The unified mixed lubrication models that have recently been developed are now capable of simulating the entire transition of interfacial status from full-film and mixed lubrication down to dry contact with an integrated mathematic formulation and numerical approach. This has indeed bridged the two branches of engineering science, contact mechanics, and hydrodynamic lubrication theory, which have been traditionally separate since the 1880s mainly due to the lack of powerful analytical and numerical tools. The recent advancement in mixed EHL begins to bring contact and lubrication together, and thus an evolving concept of "Interfacial Mechanics" can be proposed in order to describe interfacial phenomena more precisely and collaborate with research in other related fields, such as interfacial physics and chemistry, more closely. This review paper briefly presents snapshots of the history of EHL research, and also expresses the authors' opinions about its further development as a gateway to interfacial mechanics. [DOI: 10.1115/1.4004457]

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1 Historic Review

1.1 Background and Early Studies (1880s–1940s). Power and motion are transmitted through interfaces at surface contact locations in mechanical components. Contact geometry can be generally categorized as conformal contact and counterformal contact. In a conformal contact, the two surfaces have closely matched curvatures with each other so that the area of surface interaction is large, typically comparable to dimensions of the machine elements. Typical conformal-contact components include journal bearings, piston/ring/cylinder systems, many types of joints and seals, and others. A counterformal contact; on the contrary, is formed by two surfaces whose curvatures do not match. As a result, the contact area is usually small in both principal directions (called point contact), or at least in one direction (line contact), and a localized high-pressure concentration may exist at the interface. Counterformal contact can be found in various gears, rolling element bearings, cam/follower systems, vane pumps, ball screws, metal-rolling tools, traction drives, continuously variable transmissions, and so on.

It is commonly agreed that contact mechanics, as a branch of engineering science, originated from the study by Hertz in 1881 [1], based on an assumption that there is no lubrication at the interface. In the intervening time, the Hertzian theory has been enjoying wide applications due to its simplicity and satisfactory accuracy for frictionless elastic dry contact of smooth surfaces. In engineering practice; however, most power-transmitting components are lubricated with oils or greases, and lubrication is found to be an effective way to improve component performance, efficiency, and durability. Research efforts to understand lubrication mechanisms and predict lubrication performance first became successful for conformal-contact problems. In 1886, Reynolds [2] published his milestone theoretical lubrication analysis based on Tower's journal bearing experiment. The Reynolds equation was derived; and it has been the foundation of hydrodynamic lubrication theory since then. Good agreement was obtained between analyses and experiments for some conformal-contact components, such as fluid-film bearings, in which the hydrodynamic pressure is low, typically on the order of 0.1–10 MPa or less. Because of this success, attempts were made to extend the application of the hydrodynamic lubrication theory to counterformal contact components. One of the remarkable early studies was by Martin, who published his hydrodynamic lubrication analysis for line contacts in spur gears in 1916 [3]. In his study, a pair of gear teeth was simplified to two parallel smooth rigid cylinders lubricated by an incompressible isoviscous Newtonian fluid. Using a simplified Reynolds equation he derived an expression of loading capacity, which can be written in current notation as follows:

$$w/l_e = 4.896\eta_0 UR_x/h_c \quad \text{or} \quad H_c = h_c/R_x = 4.896\eta_0 Ul_e/w \qquad (1)$$

where w/l_e is the load per unit contact length, η_0 the lubricant viscosity under the ambient condition, U rolling velocity, R_x effective radius of curvature of the two cylindrical bodies, and h_c lubricant film thickness at the center of contact. It was found that the predicted lubricant film thickness between gear teeth was extremely small, often on the order of 1–10 nm, far below surface roughness

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of machined gear teeth (which is typically of the order of 100 nm). Observations indicated; however, that in some high speed gears original machining tracks on the functional tooth flanks were clearly visible even after a long duration of operation, demonstrating the existence of a significant hydrodynamic lubrication. This disagreement somewhat discouraged further efforts in the next 10–20 years. But, indeed, Martin's work was a good beginning of the lubrication study for nonconformal-contact components.

Starting in the 1930s, researchers strived to improve the lubrication analyses for counterformal contacts by including the effect of localized elastic deformation of the two surfaces (Peppler, 1936 [4], Meldahl, 1941 [5], etc.) and that of the lubricant viscosity increase in the contact area due to high pressure (Gatcombe, 1945 [6], Blok, 1950 [7], etc.). Both elastic deformation and viscosity increase appeared to have positive influences on lubricant film formation. However, a significant breakthrough in understanding the fundamental elastohydrodynamic lubrication (EHL) mechanism was not seen by the research community until 1949, when Grubin [8] published his paper, "Fundamentals of the Hydrodynamic Theory of Lubrication of Heavily Loaded Cylindrical Surfaces." It is said that Grubin's theory was based on Ertel's preliminary results obtained as early as 1939 [9].

The Grubin solution was the first to take into account both elastic deformation and viscosity increase simultaneously. A simplified approximate solution was developed based mainly on the following two assumptions:

(1) The shape of the elastically deformed cylindrical bodies in a heavily loaded lubricated contact is the same as that in the corresponding dry contact.
(2) The hydrodynamic pressure approaches infinity at the inlet border of the Hertzian contact zone.

The geometric shape of the gap could be calculated; therefore, by an analytical solution from the Hertzian theory for dry contacts. The viscosity also approaches infinity at the inlet border according to the following pressure–viscosity relationship that has been commonly used:

$$\eta = \eta_0 e^{\alpha p} \tag{2}$$

Based on the above-presented discussion, Grubin numerically calculated the integral of a simplified Reynolds equation in the inlet zone and then curve fitted his results in a range of central film thickness values reasonable for certain practical applications. The following expression was then successfully derived for predicting the lubricant film thickness:

$$H_c = h_c/R_x = 1.95(G^* U^*)^{8/11}/W^{*1/11} \tag{3}$$

The Grubin theory well describes the basic characteristics of line-contact EHL, e.g., a nearly constant film thickness in the contact zone, and a pressure distribution close to Hertzian. Although the assumptions were heroic and the theory was approximate, the film thickness results predicted by Eq. (3) were found to be in a reasonably good agreement with experimental data, especially under heavy loading conditions.

1.2 Establishment of Fundamental EHL Theories (1950–1970s). Numerical solutions without the two assumptions previously mentioned for line-contact problems were given much attention in the 1950s and 1960s. The first successful solution was published in 1951 by Petrusevich [10], who presented three cases in detail for different speeds but the same load, as shown in Fig. 1. Impressively, his results demonstrated all the typical EHL characteristics for the first time, including a nearly constant central film thickness and an EHL pressure distribution close to the Hertzian over the majority of the contact zone, a film constriction downstream near the outlet, and, especially, a high-pressure spike at the outlet side right before the film constriction, which was later named the "Petrusevich Spike." Also, the three film thickness values he presented were close to those predicted by currently used formulas developed much later. Based on his limited results, he somehow derived a film thickness formula, which quite correctly reflected the relationship between film thickness and speed, but showed a small film thickness increase with increasing load, that appeared to be difficult for people to understand at that time.

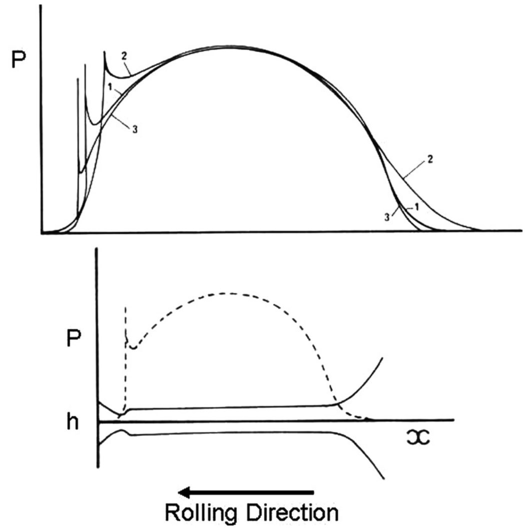

Fig. 1 Line-contact EHL solutions by Petrusevich [10]

Shortly after Petrusevich, Dowson and Higginson presented their milestone paper: "A Numerical Solution to the Elastohydrodynamic Problem" in 1959 [11]. They developed a new solution approach, called the inverse solution, to overcome difficulties associated with slow numerical convergence observed in the early straightforward iterative processes employed in Ref. [5] and other works. The inverse solution procedure appeared to be capable of handling heavily loaded cases and getting a converged solution within a small number of calculation cycles, although it was not fully automatic. A curve-fitting formula for predicting line-contact EHL minimum film thickness, shown in the following, was presented by Dowson and Higginson in 1961 [12]:

$$H_m = h_m/R_x = 1.6G^{*0.6}U^{*0.7}W^{*(-0.13)} \tag{4}$$

This formula was further modified by Dowson in 1965 as follows (see Ref. [13]):

$$H_m = h_m/R_x = 2.65G^{*0.54}U^{*0.7}W^{*(-0.13)} \tag{5}$$

Later, central film thickness formulas were also presented by different researchers based on numerical solutions. The following is from Dowson and Toyoda, 1978 [14]:

$$H_c = h_c/R_x = 3.06G^{*0.56}U^{*0.69}W^{*(-0.10)} \tag{6}$$

In these formulas, four dimensionless parameters are used for line-contact problems: speed parameter U^*, load parameter W^*, materials parameter G^*, and film thickness parameter H. Note that for line contacts there should be only three independent parameters, and these four given in the above-presented text are actually interrelated. However, these parameters are easy to use, making physical sense explicitly, so they have been widely accepted. There have been some other dimensionless parameter groups also in use, which will not be discussed here. Based on their pioneer studies, Dowson and Higginson published the first book on EHL in 1966 [13], which has been considered a classic, laying the foundation of the smooth surface line-contact EHL theory.

Parallel to the above-mentioned theoretical studies, experimental investigations also yielded fruitful results. Early studies were focused mainly on line-contact EHL film thickness measurements using disc/roller machines with the capacitance technique (e.g., 1961–1963 by Crook [15,16] and in 1966 by Dyson et al. [17]) and the X-transmission method (in 1961 by Sibley and Orcutt [18]). The basic trends in EHL were confirmed experimentally, e.g., the film thickness is significantly affected by the rolling speed, but the load effect is nearly negligible. Measured film thickness results were found to be in reasonably good agreement with those predicted by formulas (3)–(5). In addition, EHL pressure distribution was measured with thin-film transducers applied onto disc specimens by Kannel, 1966 [19], Hamilton and Moore, 1971 [20], etc. The Petrusevich spike was observed experimentally.

In 1961, Archard and Kirk [21] were probably the first who experimentally demonstrated a measurable lubricant film in a heavily loaded point contact, formed by two crossed cylinders, although the film thickness appeared to be smaller than that in a line contact under otherwise similar conditions. Note that the capacitance technique was used to measure the average or central film thickness, while the X-ray technique could give approximate results of the minimum film thickness. No detailed information about the shape of the EHL film or the gap between the two surfaces could be provided until optical interferometry, which was originally developed in 1963 and 1966 by Gohar and Cameron [22,23], and further modified in 1969-1970 by Foord et al. [24]. With a superfinished steel ball against a glass disc, one could observe the lubricant film thickness distribution through optical interference fringes under a microscope. Figure 2 shows a sample of such measurements. A remarkable new finding was that in such circular contact the film constriction takes a horseshoe shape and the minimum film thickness is actually located on two sides away from the centerline. Due to its great accuracy and capability to provide detailed mapping of film thickness, the optical interferometry has been a major experimental means in fundamental EHL research since then. Its limitations include the requirement of the use of superfinished transparent optical disks and highly reflective balls.

Simplified inlet analyses of Grubin's type for point-contact problems were developed in 1966 by Archard and Cowking [25], in 1970 by Cheng [26], about 15–20 years later than that by Grubin for line contacts. Full numerical solutions for point contacts did not appear until 1975–1976, more than 10 years behind the successful experimental studies and 20–25 years later than the full solutions of line-contact EHL. This is because additional computing capacity needed by point-contact problems demanded significantly more powerful digital computers, which were not widely available to engineering researchers earlier. In 1975, Ranger et al. [27] presented the first full solution from a straightforward iterative procedure, numerically demonstrating the typical point-contact EHL characteristics and confirming experimental observations from optical interferometry for the first time. It was questionable; however,

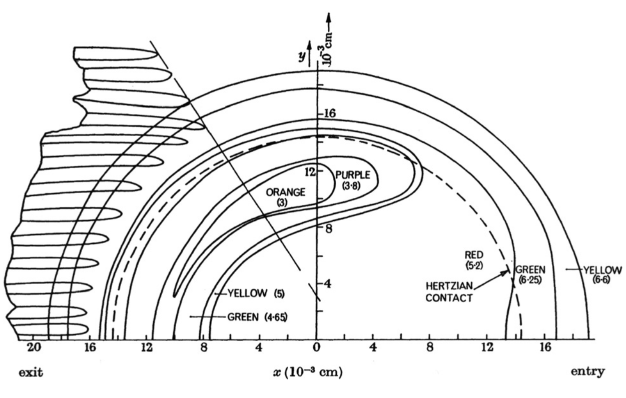

Fig. 2 Optical interference fringes by Gohar and Cameron [23], in an EHL circular contact

that their results showed an increasing film thickness with increasing load, beyond the common understanding at that time. It should be noticed today that Petrusevich and Ranger et al. were the first to present the full numerical solutions in line and point contacts, respectively, but their work did not seem to get full recognition for the same reason: Both studies showed slight film thickness increase with increasing load in the parameter ranges they analyzed. Today, it is understood that the film thickness may gradually increase first and then slightly decrease, if the load is continuously increasing over an extended wide range.

Shortly after Ranger et al., in 1976 and 1977, Hamrock and Dowson [28–31] published a series of papers, systematically investigating the effects of speed, load, materials properties, contact ellipticity, and lubricant starvation on central and minimum film thicknesses in elliptical contacts through full numerical solutions from a straightforward iterative approach similar to that of Ranger et al. The following curve-fitting formulas were derived for point-contact problems [30]:

$$H_c = 2.69G^{*0.53}U^{*0.67}W^{*-0.067}\left(1 - 0.61e^{-0.73k}\right) \tag{7}$$

$$H_m = 3.63G^{*0.49}U^{*0.68}W^{*-0.073}\left(1 - e^{-0.68k}\right) \tag{8}$$

These formulas use dimensionless parameters nearly the same as those in Eqs. (3)–(6), except that the load parameter is slightly different and a parameter of contact ellipticity, $k = b/a$, is added to take into account the effect of point-contact geometry. Comparative studies were conducted later by different researchers. In 1981, it was found by Koye and Winer [32] that the discrepancies in film thickness between formula predictions and optical interferometry results were about 30% as an average under studied testing conditions. In addition to Eqs. (4)–(8) there have been some other formulas published, and their accuracy comparison is a complicated topic. Generally; however, these formulas have been found to be practically acceptable in engineering design and analysis, because the film thickness is dominated by the lubricant entraining action in the inlet zone, where the gap is still large and the effects of thermal and non-Newtonian behaviors and surface roughness are still limited in most cases. Therefore, the isothermal analyses developed in early years based on the Newtonian fluid and smooth surface assumptions are in many cases still acceptable. Continuous efforts have been made with modified models and updated numerical methods to improve the prediction accuracy since then. Recent studies based on more precise characterization of lubricant rheology include those in 2009 by Kumar et al. [33], and others.

1.3 Significant Development on Various Subjects. The development of EHL theory and practice has been prosperous and many significant contributions have been presented since the 1960s. A brief review like this one can only give a snapshot, citing a small portion of published papers and focusing on a limited number of topics. Readers may refer to the 1999 work of Dowson and Ehret [34], the 2001 work of Gohar [35], the 2006 work of Spikes [36], and other works for more historic reviews.

1.3.1 Starvation Effect. Insufficient lubricant supply for various reasons may cause a condition called "starvation," which may significantly affect EHL film formation. Early studies started with optical interferometry experiments, because the lubricant supply could be readily quantified under microscope with the distance from the meniscus inlet boundary of an EHL film to the center of contact (called the inlet distance). Pioneer studies include the 1971 work of Wedeven et al. [37] and the 1974 work of Chiu [38], who defined the starvation problem well, and used the meniscus inlet boundary position as a criterion of starvation severity. Early analytical investigations were conducted in 1971 by Wolveridge et al. [39] for line contacts and in 1977 by Hamrock and Dowson [31] for elliptical contacts, and others, using the inlet distance as an input parameter in predicting the film thickness. Various film thickness reduction formulas have been obtained through curve fitting based on either experimental or numerical results. It was found that the basic trends from those formulas are in good agreement, but quantitative differences still exist. One of the reasons for the differences is probably that in numerical simulations a straight line inlet boundary was assumed, while in the experiments the practical situation was much more complicated. More recently, attempts were made in order to consider realistic conditions, e.g., the analytical study in 1998 by Chevalier et al. [40], and other studies.

1.3.2 Thermal EHL. Thermal behavior is an important subject, as significant temperature increase could negatively affect the EHL performance due to reduced viscosity, and possibly lead to EHL film breakdown, excessive energy loss, and early failures. Pioneer studies on the thermal EHL in line contacts were presented in 1964 by Cheng and Sternlicht [41] and in 1966 by Dowson and Whitaker [42], coupling the energy equation with other EHL equations to solve for temperature variations in the film, in addition to those of pressure and film thickness. Full numerical solutions of the thermal EHL in point contacts were presented in 1984 by Zhu and Wen [43]. It was found that the temperature increase could be significant in the contact zone, but relatively small in the inlet zone, where the entraining action actually dominates the EHL film formation. That is why the effect of temperature rise caused by sliding on film thickness is often limited, except at extremely high speeds, which may result in significant heating due to increased lubricant shear rate and possible reverse flows in the inlet zone. The thermal reduction of film thickness due to inlet heating was studied and prediction formulas derived in 1967 by Cheng [44], in 1975 by Murch and Wilson [45], and others, mainly through simplified inlet solutions of Grubin's type. A more comprehensive prediction was later presented in 1992 by Gupta et al. [46]:

$$C_T = \frac{1 - 13.2\left(\frac{P_h}{E'}\right)L^{0.42}}{1 + 0.213(1 + 2.23S^{0.83})L^{0.64}} \quad (9)$$

where C_T is the thermal reduction factor for film thickness, $L = \eta_0 \beta U^2/k_f$, the thermal loading parameter, and $S = (u_2 - u_1)/U$, the slide-to-roll ratio. Note that Eq. (9) was obtained with the assumption that the temperature–viscosity parameter and the pressure–viscosity coefficient are independent. More detailed studies have recently been conducted, e.g., in 2010 by Kumar et al. [47], employing the temperature-modified Doolittle viscosity equation to consider the lubricant viscosity sensitivity to temperature at high pressure.

Transient flash temperature measurement in a tiny EHL contact is a challenging task. There have mainly been two techniques: (1) measurement with thin-film transducers deposited onto specimens in a rolling–sliding contact (e.g., by Orcutt [48] in 1965 and Kannel and Bell [49] in 1972, among others); (2) detection of infrared radiation on a device similar to that of the optical interferometry (Turchina et al. [50] in 1974, and others).

1.3.3 Friction/Traction in EHL. EHL friction, sometimes called traction, is of great importance as it is directly associated with machine components' performance, efficiency, and energy consumption. For hydrodynamic lubrication, in which pressure is relatively low and lubricant film is thick so that lubricant shear strain rate in the film is low, commonly used industrial lubricants may be considered as Newtonian fluids and modeling friction is relatively simple. For the EHL, however, the frictional mechanism becomes much more complicated. Early experimental studies (e.g. Crook [16], some sample results are shown in Fig. 3) revealed that measured friction is usually much lower than predicted by Newtonian fluid models. A new concept of limiting shear stress was then established in further studies by Plint [51] and Johnson and Cameron [52], both in 1967–1968, and others. Basically, in the inlet zone of an EHL contact, where the entraining action actually dominates the lubricant film formation, the pressure is relatively low and the gap is large, so that the shear strain rate is still low and the New-

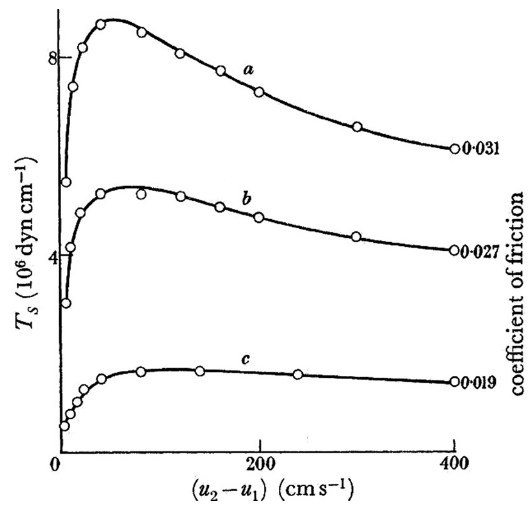

Fig. 3 EHL friction measured by Crook [16] as a function of load and sliding speed. Load (10^7 dyn/cm): (*a*) 20, (*b*) 15, (*c*) 7.5. T_s: measured traction/friction.

tonian models may still be acceptable. That is why the Newtonian models can be successfully used to solve for EHL film thickness. However, a vast majority of sliding friction is generated in the contact zone, where the pressure may reach 1–4 GPa, the lubricant passes through probably in a small fraction of a minisecond, and the film thickness (or gap) tiny, resulting in a lubricant shear rate possibly as high as 10^8 1/s. Under such conditions the lubricant can no longer be considered as Newtonian; the shear stress may increase but cannot go beyond the limit. The limiting shear stress, τ_L, is found to be a property of lubricant, which is also a function of pressure and temperature.

Modeling friction should describe the non-Newtonian viscous–elastic characteristics of lubricants, considering lubricant shear due to both elastic and viscous behaviors under the high-pressure high-shear transient conditions stated earlier. The following Maxwell model is so far widely accepted (see Refs. [53,54], etc.):

$$\dot{\gamma} = \dot{\gamma}_e + \dot{\gamma}_v = \frac{1}{G}\frac{d\tau}{dt} + F(\tau) \quad (10)$$

The viscous term in Eq. (10) can be expressed as follows by Johnson and Tevaarwerk [53] in 1977:

$$F(\tau) = \frac{\tau_L}{\eta}\sinh\left(\frac{\tau}{\tau_L}\right) \quad (11)$$

or by Bair and Winer [54] in 1978:

$$F(\tau) = -\frac{\tau_L}{\eta}\ln\left(1 - \frac{\tau}{\tau_L}\right) \quad (12)$$

In 1996, a modified Carreau model has also been used due to its convenience in application with the shear stress as an independent variable (see Bair and Khonsari [55]):

$$\dot{\gamma} = \frac{\tau}{\eta}\left[1 + \left(\frac{\tau}{G}\right)^2\right]^{(1-n)/(2n)} \quad (13)$$

where n is the power law exponent

More efforts on modeling EHL friction are still ongoing. Fortunately, in engineering practice friction is often relatively easy to evaluate experimentally. Based on friction test data, one can estimate the limiting shear stress of a lubricant under given conditions of contact pressure and temperature.

1.3.4 More Rheology-Related Issues. Defining lubricant rheology under the EHL conditions is a challenging task, because it is difficult, if not impossible, to reproduce such transient high-pressure high-shear strain rate conditions in a laboratory outside the tiny EHL contact zone. Viscosity at the inlet may noticeably affect the EHL film thickness analysis, while lubricant shear characteristics determine the traction and thermal behaviors of an EHL interface. Although transient lubricant properties under high pressure, high shear rate, and varying temperature, as those a fluid may experience in EHL, are difficult to obtain, certain empirical relationships to correlate viscosity with pressure and temperature, as well as to describe other physical phenomena such as "shear thinning," have assisted the advancement of EHL modeling. A thorough review of rheology research is beyond the scope of this paper. Readers may find in-depth coverage of rheology for EHL from the 2007 work of Bair [56] and in general the 1985 work of Tanner [57]. Here, only several commonly used viscosity equations are listed.

In 1893, Barus [58] reported the viscosity data of a marine glue as a function of the average pressure in a linear model. However, the exponential relationship, Eq. (2), more well known in tribology as the Barus equation, is widely used in EHL analyses due to its simplicity and capture of a certain nonlinear pressure–viscosity behavior. Exponential pressure–viscosity relationships were observed on several mineral oils in 1937 by Thomas et al. [59], although the data were given in kinetic viscosity and the density effect was not discussed. Log-scale linear relationships were found in a 1953 ASME report [60]. According to Cameron [61] in 1966, and others, the "Barus" exponential relationship describes the viscosity behavior quite well up to the pressure range of 200–400 MPa at low temperatures. In 1976, Hirst and Moore [62] pointed out that the measured viscosity peaks were much lower than those predicted by Eq. (2).

In order to more accurately describe the pressure–viscosity relation, Cameron [61] presented a power function, employing two constants, θ and n, to gain more flexibility

$$\eta = \eta_0 (1 + \theta p)^n \tag{14}$$

In the meantime, several viscosity models were proposed in 1966 by Roelands [63]. The following Roelands equation is often seen as an improved pressure–viscosity relationship used in EHL analyses with z as the pressure–viscosity index,

$$\eta = \eta_0 \exp^{\left[(\ln \eta_0 + 9.67)\left(\left(1+\frac{p}{p_0}\right)^z - 1\right)\right]} \tag{15}$$

For the pressure in excess of 1 GPa, Eqs. (2), (14), and (15) might considerably overestimate the viscosity. In 1951, Doolittle [64] explored the relationship between viscosity and the fractional free volume using an exponential function and developed the first free-volume viscosity model, showing that the resistance to flow depends on the relative volume of molecules present per unit of free volume. Based on this, improved free-volume viscosity models were developed and the one presented in 1993 by Cook et al. [65] is given in the following:

$$\eta = \eta_0 \exp\left\{ B \frac{V_{\text{occ}}}{V_0} \left[\frac{1}{\frac{V}{V_0} - \frac{V_{\text{occ}}}{V_0}} - \frac{1}{1 - \frac{V_{\text{occ}}}{V_0}} \right] \right\} \tag{16}$$

where V is the volume, V_0 is the volume at ambient pressure, V_{occ} the occupied volume, and B the Doolittle parameter. In Eq. (16), Tait's equation of state can be used for the pressure–density relationship. Note that the density variation is reciprocal of the volume variation. It was found by Liu et al. [66] in 2006, and others that the free-volume model seems to be able to yield EHL simulation results closest to experimental data.

1.3.5 Roughness Effect. In engineering practice, no surface is ideally smooth, and roughness is usually of the same order of magnitude as, or greater than, the film thickness estimated by the smooth surface EHL theory. Effects of surface roughness and topography, therefore, ought to be taken into account in EHL analyses for engineering applications. Great efforts have been made since 1970s, and there have been basically two types of rough surface EHL models: stochastic and deterministic. Early studies employed mainly stochastic models, using a small number of selected statistic parameters to describe the surface and lubrication characteristics. Among various models, the 1978 model by Patir and Cheng [67] for line-contact problems has enjoyed wide recognition. It employed an average Reynolds equation derived by Patir and Cheng [68] in 1978 for hydrodynamics and a load–compliance relation given in 1971 by Greenwood and Tripp [69], based on a simplified stochastic contact model. Obtained solutions showed that transverse roughness, with asperity aspect ratio $\gamma < 1.0$, may lead to a significant increase in film thickness, while longitudinal roughness, $\gamma > 1.0$, may cause a film reduction, as shown in Fig. 4. This influence is rather negligible when the hydrodynamic parameter, $\Lambda = h_{cs}/\sigma$, is large. It becomes more and more significant if Λ continuously decreases. In 1988, this stochastic approach was extended to point contacts by Zhu and Cheng [71].

The stochastic models; however, could only predict basic trends and estimate approximate average values. Parameter variations within the EHL conjunction and localized details, such as maximum and minimum values (which may be critical for studies on lubrication breakdown and failures), were missed. During the last 20 years more attention has been given to deterministic approaches due to advancements in computer technologies and numerical simulation methods. Early deterministic models mainly employed artificial roughness, such as sinusoidal waves and irregularities of simple geometry, e.g., the 1984 model of Goglia et al. [72], the 1987 model of Lubrecht [73], the 1989 model of Kweh et al. [74], the 1975 model of Chang et al. [75], and others. More realistic two-dimensional machined or random roughness was used in 1976 by Venner [76], in 1992 by Kweh, et al. [77], and by others. Full-scale point-contact EHL solutions utilizing digitized three-dimensional machined roughness did not appear until Xu and Sadeghi [78] in 1996 and Zhu and Ai [79] in 1997, etc.

It should be noted that line-contact problems were traditionally solved with simplified two-dimensional models when surfaces were assumed smooth or analyzed stochastically. In reality; however, rough surface topography is usually three dimensional, so that a 3D line-contact deterministic model is needed in order to take into account the roughness effect. In 2009, the first 3D line-contact model was developed by Ren et al. [80], employing an FFT approach with mixed padding that employs periodic extension in the direction perpendicular to motion and zero padding in the other.

Generally, the effects of surface roughness and orientation on the EHL film thickness predicted by the deterministic models are not as great as those predicted by the stochastic models. For line contacts, the basic trends presented by Ren et al. [80] are similar to those by Patir and Cheng [67], but quantitatively the influences appear to be relatively mild. For point contacts, the effects of roughness orientation become more complicated. For example, in a circular contact, the transverse roughness may possibly yield a thinner film than the longitudinal due to significant lateral flows that can be enhanced by the transverse roughness but may have a negative influence on the EHL film formation. So far there seems to be no systematic study found in literature on this topic over a wide range of operating conditions considering various types of contact geometry and roughness orientation.

The importance of surface roughness effect on the lubrication performance and components' life was recognized as early as in the 1960s (Dawson [81] in 1962) and early 1970s (Tallian [82] in 1972), and later by many others. A parameter called the film thickness ratio, or λ ratio, or specific film thickness, defined as the ratio of average film thickness over composite rms roughness,

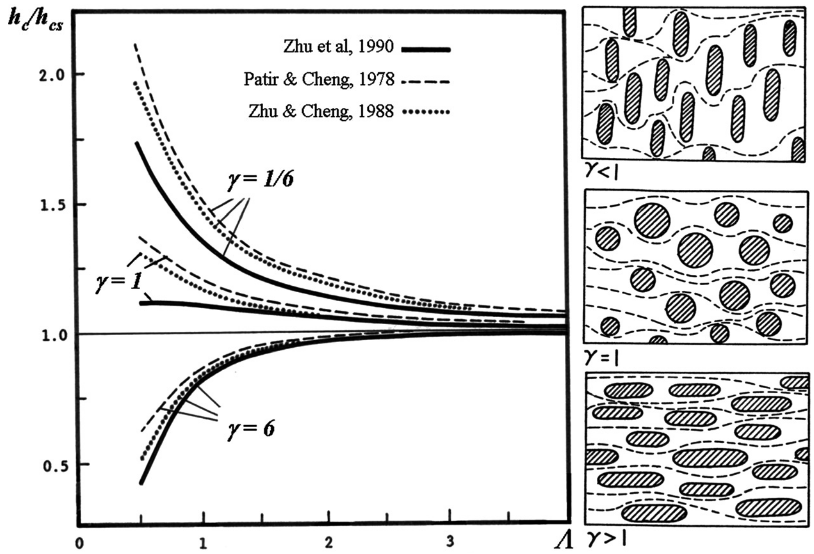

Fig. 4 Roughness orientation effect predicted by stochastic models (from Zhu et al. [70])

$\lambda = h_a/\sigma$, was introduced for evaluation of lubrication effectiveness in a rough surface EHL contact. More discussions will be given in the following for mixed EHL.

1.4 Improvements of Film-Thickness Measurement. As the lubrication theory and practice were fast advancing, new challenges were imposed on EHL experimental technologies. Extremely thin lubricant films and rough asperity contacts may coexist in many engineering applications; they are difficult to be measured with the capacitance, electric resistance, and X-techniques. Efforts; therefore, have been focused more on the optical interferometry since the 1980s. Originally, the resolution of the film thickness measurement with manual calibration methods was limited to about a quarter of the wavelength of the light being used to produce interference fringes, which is usually around 110–160 nm. The great advancement of computer technologies has fueled significant improvements in different ways by different researchers. The main contributions include the following:

(1) Spacer Layer Imaging Method (SLIM), developed at Imperial College, London (Johnston et al. [83] in 1991, Cann et al. [84] in 1996, and others). In order to overcome the above-stated resolution limitation, a combination of a solid spacer layer, having the same reflective index as that of the oil to be measured, with a spectrum analysis technique enables accurate measurement of very thin lubricant films on the nanometer scale.
(2) Relative Optical Interference Intensity Technique, developed at Tsinghua University, China, by Luo et al. [85] in 1996, and others. A monochrome light is used to produce interference fringes and the lubricant film thickness at a certain location is determined by the relative light intensity between the maximum and minimum within the same order of fringe. The intensity is precisely measured through a digitized image analysis, and the resultant film thickness resolution is claimed to be about 0.5 nm.
(3) Thin-Film Colorimetric Interferometry, developed at Brno University of Technology, Czech Republic (Hartl et al. [86] in 1999). This method incorporates a computer-controlled test apparatus with an extensive imaging process software, so that real-time instantaneous evaluation of film thickness distribution can be successfully conducted through colorimetric interferometry and the measurement range is about 1–800 nm.

Based on the much improved resolution and accuracy, ultrathin films with a patterned/textured surface can be measured and detailed mapping obtained (see Fig. 5 for an example with the SLIM). This capability provides useful tools for studying the transition from thick-film and thin-film EHL down to mixed and boundary lubrication. Detailed descriptions and comparison of these techniques is beyond the scope of this brief review.

1.5 Improvements of Numerical Solution Methods

1.5.1 EHL Equations. EHL research has relied heavily on numerical analyses through model-based simulations. The development of the EHL theory and practice; therefore, has been dependent largely upon advancements of computer technologies and numerical solution methods. A typical set of basic equations that formulate rough surface point-contact EHL problems in general is given in the following (equations for relatively simpler problems, such as those of line contacts and/or smooth surfaces, will not be described here).

Reynolds equation:

$$\frac{\partial}{\partial x}\left(\frac{\rho h^3}{12\eta}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\rho h^3}{12\eta}\frac{\partial p}{\partial y}\right)=\frac{u_1+u_2}{2}\frac{\partial(\rho h)}{\partial x}+\frac{\partial(\rho h)}{\partial t} \quad (17)$$

Film-thickness (gap) equation:

$$h=h_0(t)+\frac{x^2}{2R_x}+\frac{y^2}{2R_y}+v(x,y,t)+\delta_1(x,y,t)+\delta_2(x,y,t) \quad (18)$$

Elastic deformation equation:

$$V(x,y,t)=\frac{2}{\pi E'}\iint_\Omega \frac{p(\xi,\varsigma)}{\sqrt{(x-\xi)^2+(y-\varsigma)^2}}d\xi\, d\varsigma \quad (19)$$

Load balance equation:

$$w(t)=\iint_\Omega p(x,y,t)dx\,dy \quad (20)$$

Either a Newtonian or Non-Newtonian lubricant model can be used in association with the Reynolds equation. When a non-

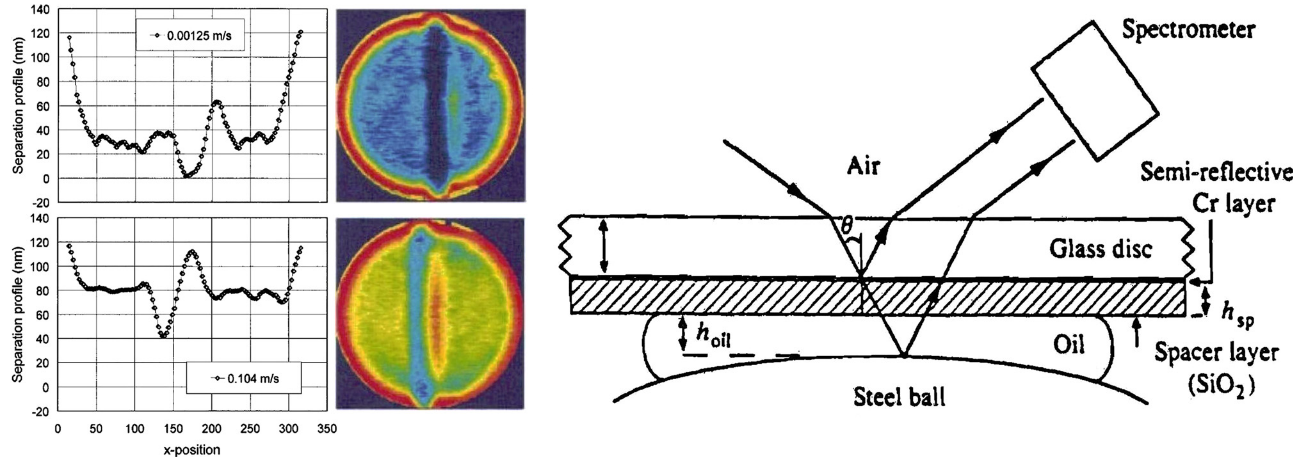

Fig. 5 Improved thin-film measurement with SLIM (sample results shown on the left are from Guantang et al. [87])

Newtonian model is used, η should be the effective viscosity with respect to a certain shear behavior of the lubricant used.

A pressure–viscosity relationship should be included in the model system. The exponential equation, $\eta = \eta_0 e^{\alpha p}$, the Roelands equation, and the free-volume model, etc., can be selected. However, the range of model application deserves attention.

The pressure–density relationship can come from a state equation. The Dowson–Higginsen pressure–density model (see Ref. [13]) and Tait's type of the equation of state (see Ref. [65]) are among the commonly used ones with particularly determined parameters.

1.5.2 Numerical Solution Methods. The above-presented equation system exhibits very strong nonlinear behaviors, resulting mainly from the surface elastic deformation and much increased viscosity due to pressure. Therefore, the numerical solution has long been a great challenge to researchers. With a traditional direct iteration approach, the elastic deformation, $V(x, y)$, can be calculated based on an initial guess of pressure distribution, $p(x, y)$, and then the film thickness, $h(x, y)$, can be readily obtained. Substituting $h(x, y)$ into the discretized Reynolds equation one can solve for all the unknown nodal pressures on the left side of the equation, as everything on the right side is considered as known. Using the newly obtained pressure distribution one can update the deformation and the film thickness and then solve for the pressure again in the next iteration. This procedure can be repeated until a converged solution is achieved that satisfies both the pressure convergence and load balance criteria prescribed.

However, the above-described approach suffers from very slow convergence and undesirable solution instability, as well as numerical overflow, especially under practical and heavy loading conditions. The main problem is that whenever and wherever the pressure is high and/or the film thickness is tiny (or zero), the pressure flow terms on the left side of the Reynolds equation tend to vanish due to the third power of the tiny film thickness, h^3, and exponentially increased viscosity, $\eta = \eta_0 \exp(\alpha p)$. Since in each iteration the coefficient matrix of the discretized Reynolds equation is constructed based only on the pressure flow terms on the left (which may be vanishing), and other terms are all treated as known, the solution process may likely be subject to deterioration due to the ill-conditioned coefficient matrix that violates the leading diagonal Dominance Theorem. Attempts have been made in the past to solve this problem and there have been mainly four types of numerical approaches developed by different researchers:

(1) Inverse Solution. It used the direct iteration only in the low pressure inlet zone and solved for the pressure inversely in the high-pressure contact zone, being able to successfully handle heavy loading cases and obtain solutions within a small number of calculation cycles. Following Dowson and Higginson's first inverse solution in line contacts [11], Evans and Snidle [88] and Hou et al. [89], in 1981 and 1987, respectively, developed similar approaches for point-contact problems. However, the inverse solution procedure requires tricky manual adjustment and domain division from case to case, so it is not fully automatic.

(2) System Analysis through Newton–Raphson Iterative Procedure. The Newton–Raphson method was initially employed in 1975 by Rohde and Oh [90] for line-contact problems and in 1977 by Oh and Rohde [91] for point contacts with rather coarse computational meshes. The number of iterative cycles appeared to be reduced and convergence speed increased, compared with the early direct iterative methods, but the published solutions were still limited to low Hertzian contact pressures, about 0.15–0.35 GPa. Houpert and Hamrock [92] in 1986, and others, improved the numerical method for line contacts based on Okamura's system analysis presented earlier in 1982 [93]. They adopted a nonuniform computational mesh with much increased mesh density around the inlet border of contact zone and also in the vicinity of the pressure spike. Their obtained solutions reached a maximum Hertzian pressure as high as 4.8 GPa. Also, the detailed shape of the secondary pressure spike was revealed to be smooth with a certain height, rather than a sharp singular peak, if a sufficient number of grid nodes were used around the spike. With this system analysis method, all the unknown nodal pressures and the central film thickness are updated simultaneously based on the entire integrated equation system, and a converged solution can be obtained within a small number of iterations. However, it requires a good initial guess of the pressure distribution and, especially, calculation and storage of several large full matrices, which are mainly due to the Jacobian matrix of gradients and its inversion. For point-contact problems with an adequate mesh density, this would demand huge computing time and memory space. Therefore this approach has not been extended to point-contact EHL.

(3) Coupled Differential Deflection Method. This approach was developed in 2000 by Hughes et al. [94] for line-contact problems and modified in 2003 by Holmes et al. [95] for point contacts. A differential form of the deformation equation was derived, with which the effect of pressure appears to be much localized compared to that with the original integral form of deformation equation. As a result, the matrix of pressure-deformation coefficients can be simplified into a bandwidth matrix which enables an elimination solver to be used for line contacts and a coupled iterative technique for point contacts. Based on this, the elastic and hydrodynamic equations can be effectively

coupled and solved simultaneously. In this way, the computation is accelerated and the solution better stabilized under heavy loading conditions.

(4) Improved Methods Based on Direct Iterative Approach. This is also called the "Straightforward Iteration" approach. It was the first used in EHL studies, and its main advantages include its simplicity and small memory requirement. However, its success was limited in early years due to its slow convergence and difficulty to obtain solutions under practical loading conditions. Great efforts were made in 1940s through the mid-1980s, but obtained solutions were still limited to relatively low Hertzian pressures mostly below 0.4–0.5 GPa. In order to increase computational speed and also ensure solution convergence under severe conditions, significant efforts have been made, mainly including the following

(1) Multigrid Method. This method was originally developed in the areas of computational fluid mechanics, then employed in solving EHL problems; first, in 1987, by Lubrecht [73] (a sample numerical solution is shown in Fig. 6), followed by Venner [76] in 1991, Ai [96] in 1993, and many others. It is found that errors whose wavelength is of the order of the mesh size are fast to converge and hence can be reduced quickly, implying that higher frequency errors can be reduced quickly on a finer mesh while lower frequency errors can be removed efficiently on a coarser mesh. Thus in a multigrid process the computational mesh is constantly changed with mesh density increasing and decreasing alternately. Through the repeated transitions between coarse and fine meshes, the total error is minimized quickly. By using this method, together with a much reduced dimensionless ultimate mesh size $\Delta x/a$ from the previous level of about 0.06–0.20 down to 0.0075–0.03, the solution process appears to be significantly

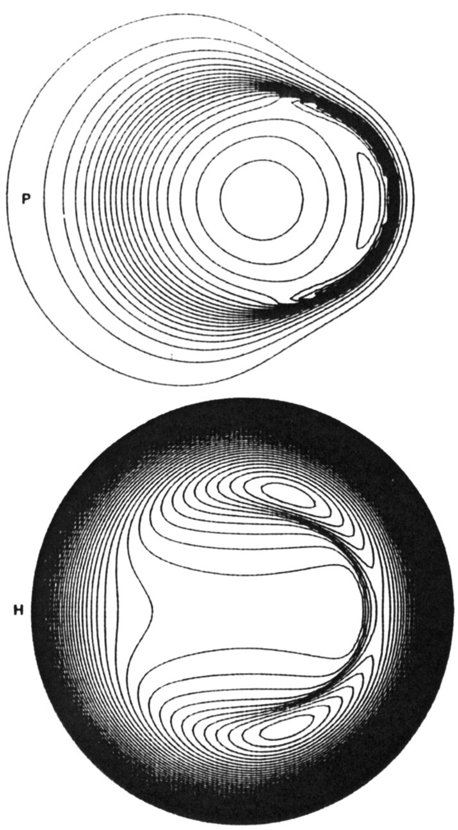

Fig. 6 Multigrid solution of an EHL circular contact using a densified mesh by Lubrecht [73]

accelerated and numerical accuracy improved in both line and point-contact solutions. Therefore, this method has enjoyed wide applications.

(2) Semisystem Approach. As stated earlier, if in each iteration the coefficient matrix of the discretized Reynolds equation is constructed only from the pressure flow terms on the left, and all the other terms on the right are considered as known and calculated by using the available pressure from the initial guess or previous iteration, the solution process may suffer from very slow convergence and numerical instability, especially under heavy loading and/or low speed conditions. The basic idea of the semisystem approach is to consider the entraining flow term as a function of unknown nodal pressures, so that the construction of the coefficient matrix will utilize not only the pressure flow terms but the entraining flow term as well. In this way, the diagonal dominance is guaranteed even when the pressure flow becomes extremely weak. This approach was employed in 1993 by Ai [96], in 1999 by Zhu and Hu [97], in 2000 by Hu and Zhu [98], and by others. It has been proven that with this approach the solution convergence and stability can be ensured even under extremely severe conditions, and cases with ultrathin film, zero-film, and rough surface asperity contacts can be handled.

(3) Progressive Mesh Densification (PMD) Method. Morales-Espejel et al. [99] in 2005, Liu et al. [100] in 2006, and others pointed out that as calculated EHL film thickness is getting smaller, down to nanometer scale or approaching zero, the converged solution becomes dependent more significantly on the mesh density and differential schemes used. In 2007, Zhu [101] further revealed that the converged film thickness is approaching a limit if mesh density continuously increases, and this asymptotic limit can be readily estimated. Based on this, a (PMD) method has been developed (see Ref. [101] for details). It appears to be capable of speeding up the solution process remarkably while ensuring numerical accuracy, beneficial especially to ultrathin film and mixed EHL.

1.5.3 Deformation Calculation Techniques. An important component of the EHL numerical simulation is the calculation of surface elastic deformation, which may demand more than 50%–70% of the total computing time. There have been mainly four types of numerical algorithms for point-contact problems:

(1) Direct summation using influence coefficients, employed earlier by Hamrock and Dowson [28] (influence coefficient calculation was based on zero-order discretization of the pressure distribution), Ranger et al. [27] (bilinear discretization), and Zhu and Wen [43] (biquadratic discretization), and others.
(2) Multilevel multi-integration by Lubrecht and Ioannides [102] in 1991, etc.
(3) Differential deflection method by Evans and Hughes [103] in 2000.
(4) Discrete convolution and FFT, originally developed by Liu et al. [104] in 2000 and Liu and Q. Wang [105] in 2002. It has been employed in the EHL by Wang et al. [106] in 2003, and others.

2 Recent Advancement in Thin-Film and Mixed EHL (Mid-1990s to Present)

As described earlier, the development of the EHL theory and application in early years was either based on smooth surfaces or artificial roughness of simple geometry under full-film conditions with no surface contact, or through stochastic models that did not directly simulate surface contact and could not handle severe conditions with heavy interaction of rough surface asperities (i.e., λ

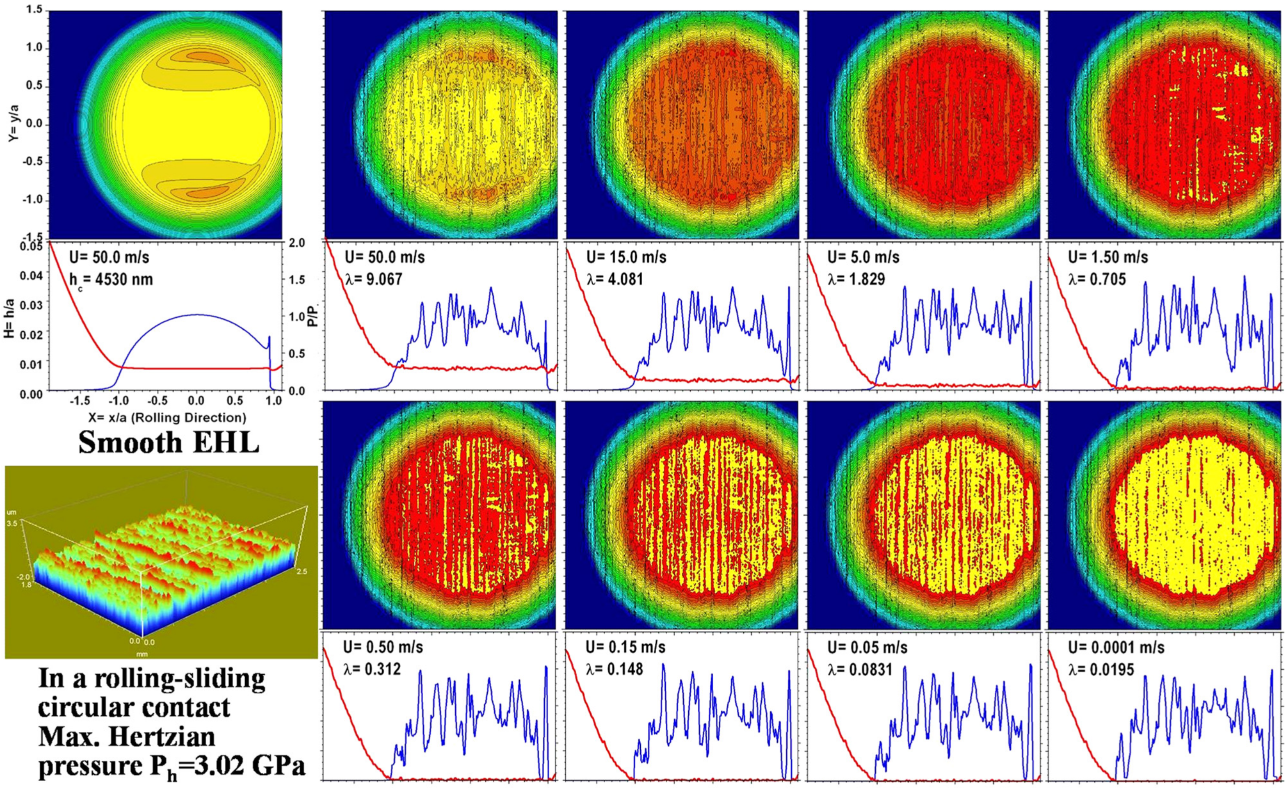

Fig. 7 Unified deterministic model by Zhu and Hu [97], Hu and Zhu [98], and Liu et al. [101] being able to simulate the entire transition from full film and mixed EHL to dry contact with 3D machined roughness

ratio below 0.5). In engineering practice; however, no surface is ideally smooth, and surface roughness is often of the same order of magnitude as, or greater than, the average lubricant film thickness, so that a complete separation between the two surfaces is seldom. Mixed EHL (also called partial EHL) is the mode in which both EHL films and surface asperity contacts coexist and neither can be ignored. Actually, most functional components operate in the mixed lubrication regime, often at λ ratios far below 0.5. Also, in-depth lubrication transition studies and failure analyses require detailed information about distributions and maximum/minimum values of pressure, film thickness, friction, flash temperature, and subsurface stresses resulted from mixed EHL with real machined roughness, which cannot be provided by stochastic models. Powerful deterministic models and experimental methods for mixed EHL studies are certainly in high demand.

The great advancement of computer and information technologies has fueled significant breakthroughs in the thin-film and mixed EHL research and development since mid-1990s. First, improved optical interferometry, described previously, has yielded fruitful results in the areas of ultrathin film and boundary lubrication as well as mixed EHL with textured surfaces. Representative contributions include those by Guangteng and Spikes [107,108] in 1995 and 1997; Luo et al. [85] in 1996; Kaneta and Nishikawa [109] in 1999; Guangteng et al. [87] in 2000; Luo and Liu [110] in 2006; Krupka and Hartl [111,112] in 2007, and others. The thin EHL film measurements can now be done at the nanometer scale, and the transition from full-film EHL to boundary lubrication has been a focus of investigation. In addition, various types of textured surfaces have been used in the experiments showing asperity contact patterns in mixed EHL that can be used for numerical simulation model validation.

Concurrently, deterministic solutions for mixed EHL have achieved significant progress. Basically there have been two approaches for mixed lubrication simulation: the first is to use a unified equation system and solution method for both the lubricated areas and asperity contacts simultaneously, and the second to use separate models for lubrication and contact, respectively. A separate solution with sinusoidal roughness for line contacts was presented in 1995 by Chang [113]. In 1999, Jiang et al. [114] presented the first model with the separate approach for point-contact mixed EHL with machined roughness. Also in 1999, the first unified approach for point contacts with machined 3D roughness was published by Zhu and Hu [97], and then in 2000 by Hu and Zhu [98]. Other unified solutions were obtained by Holmes et al. [115] in 2005, through a coupled differential deflection method, and in 2009 by Li and Kahraman [116] by means of an asymmetric integrated control volume discretization. The separate approach was also employed in 2001 by Zhao et al. [117], in 2002 by Holmes et al. [118], in 2003 by Zhao and Sadeghi [119], in 2004 by Popovici, et al. [120], and others, mainly for start-up and slow-down problems, in which the lubricated and contact areas are clearly separated and the boundary conditions in between can be conveniently handled.

Considering that dry contact is nothing but a special case of lubricated contact under extreme conditions (such as ultralow viscosity, ultralow speed, and/or high pressure concentrated in tiny contact areas), theoretically, there should be no barrier between contact and lubrication, and one should be able to use a unified lubrication equation system to simulate both EHL films and asperity contact simultaneously, if the numerical solver is sufficiently robust. It is important to note that "dry contact" is defined as the state in which there is no effective hydrodynamic fluid film between the two surfaces. Basically, the term of "contact" is still based on the continuum mechanics. It may not be suitable for describing any molecular or atomic level events. Therefore, the effect of possible absorbed lubricant molecules on the surfaces is not considered in contact and mixed lubrication simulations. Newly developed numerical approaches appear to be capable of simulating the entire transition from full-film and mixed EHL down to dry contact under severe operating conditions (see Fig. 7 for an example). This not only bridges contact and lubrication successfully with a unified model, but also provides a powerful tool

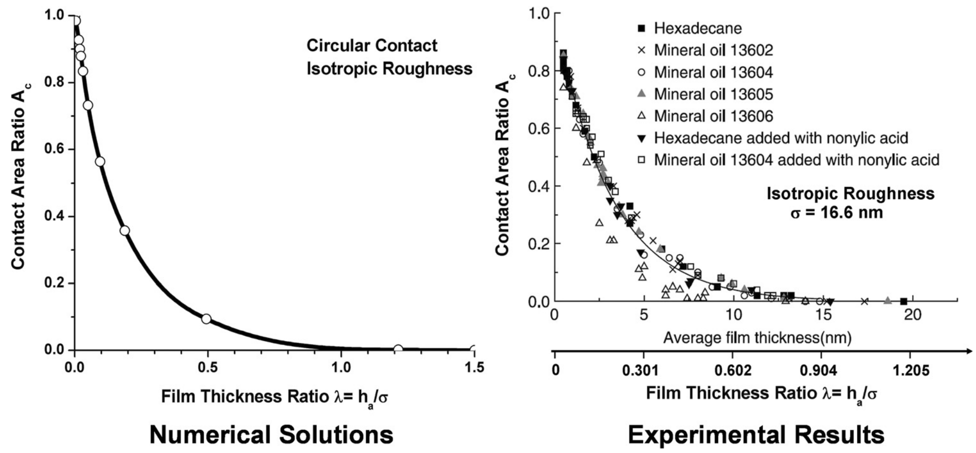

Fig. 8 Comparison of numerical simulation results with experimental data by Luo and Liu [110], for a continuous transition from full film and mixed EHL to boundary lubrication (presented at the STLE Annual Meeting in May, 2010, Las Vegas)

for analyzing engineering interfaces under practical conditions. Based on the mixed EHL model by Zhu and Hu, a virtual texturing approach was developed in 2005 by Wang and Zhu [121] as a design tool for surface optimization in practice.

In order to validate computer simulation models, comparative studies have been conducted in a wide range of lubrication conditions from full-film and mixed EHL down to dry contact. For full-film EHL with various textured surfaces; for example, numerical results were compared with optical interferometry experiments in 2005 by Felix-Quinonez et al. [122], in 2006 and 2009 by Liu et al. [66,123], and others. Good agreement was found. When the entraining velocity is inchmeal decreased, the EHL film thickness reduces and the solution gradually approaches that of dry contact as numerically demonstrated by Zhu and Hu [97], Zhu [101], and in 2010 by Wang et al. [124]. In Fig. 8 this transition is characterized by a continuous change in contact area ratio A_c. Note that $A_c = 0$ indicates a full-film EHL condition with no contact, while $A_c = 1$ represents a complete contact with average film thickness $h_a = 0$. It can be seen that the numerical simulation results agree well with experimental data by Luo and Liu [110]. Friction calculation results also agree with the trend presented by Guangteng and Spikes [108]. If the speed becomes extremely low, the mixed lubrication solutions from the unified model by Zhu and Hu [97,101] are found to be in perfect agreement with those obtained from the classic Hertzian contact theory and the FFT-based dry contact model [104,105], as demonstrated by Refs. [97,101,124], and others (see Fig. 9 for a comparison example from Ref. [124]). This doubtlessly proves that the mixed EHL models can be used to simulate dry contact interfaces.

The developments of numerical and experimental means have supported in-depth rheology research. Encouraging results have been obtained by Bair and co-workers from accurate measurement of high-pressure rheological properties of a number of lubricants [125,126]. The understanding of the sensitivity of EHL results to lubricant rheological, piezo-viscous, and thermal properties of lubricants have been deepened. The use of realistic shear-thinning models with measured rheological properties in EHL analyses may achieve excellent agreement between measured and simulated film thickness (see Refs. [33] and [127,128] for some recent contributions).

3 Interfacial Mechanics

It has been more than 60 years since the first successful EHL solution was published. The EHL theory and practice have progressed from infancy to maturity. Although the focus of tribology research efforts has been dynamically changed in the last 60 years, EHL has constantly remained an active field, attracting significant attention. There has been enormous achievement both in numerical modeling and experimental investigation. However, much still remains to be done in order to solve real interface problems in engineering practice and to meet new challenges continuously imposed by scientific research and technology development. A brief review of the history, such as the one given here, may be helpful for better foreseeing its prospects and looking into the future.

Basically, real engineering problems associated with power-transmitting interfaces of mechanical components are complicated in nature, and few can be solved with an analytical solution. In early years, due to the lack of powerful analytical and numerical tools, problems always had to be greatly simplified in order to utilize available mathematic solution methods and manual calculation tools at that time. That was why possible existence of lubricants was completely ignored when focusing on surface contact aspects of the interfacial phenomena, and; on the other hand, possible surface contact was completely neglected in lubrication studies. Under the circumstances, contact mechanics and hydrodynamic lubrication theory were established in 1880s and they were developed in parallel thereafter. There was almost no cross from one to the other until recently, solely because handling both contact and lubrication simultaneously was very difficult in the past. In addition to the no lubrication assumption in contact analyses or no surface contact assumption in lubrication studies, there have been other assumptions in place for simplification, including those of ideally rigid (or purely elastic) homogeneous isotropic body materials, simple contact geometry, perfectly smooth surfaces or artificial roughness of simple micro-geometry, and isoviscous Newtonian lubricants, and so on, based on which classic contact and lubrication theories were established.

Research progress in both areas was relatively slow until the late 1950s/early 1960s, and they have been greatly accelerated and classic assumptions released one by one, largely due to significant advancement in digital computer technologies in the last 40–50 years. The EHL theory has been commonly considered to be the most important achievement during this time period in the field of lubrication. As soon as solid surface deformation is introduced into lubrication study, elastic dry contact becomes a special case of EHL, if air can be considered as lubricant with an extremely low viscosity and density. The lubrication research development has been broadened with links to more relevant branches of science. Removal of rigid body and isoviscous lubricant assumptions paves the way toward the interaction with the science of materials and lubricants, giving rise to a new research area of greater importance where more practical problems with lubricated interfaces can be tackled.

Although there have been no theoretical barriers between contact mechanics and lubrication theory after the establishment of EHL,

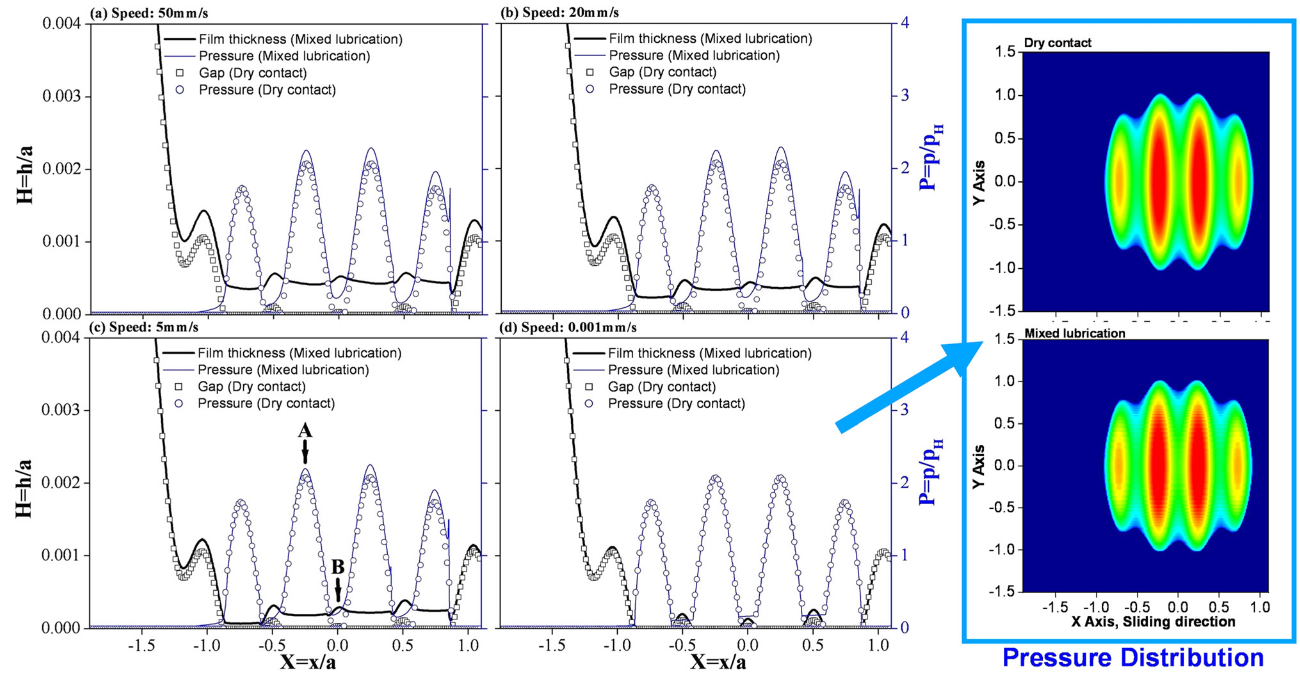

Fig. 9 As the speed decreases, mixed EHL solution gradually approaches that of dry contact (from Wang et al. [124])

they have still been separately developed because of different focuses and different technical treatments that are necessary in many cases. Separate development is also due to the particular difficulty to simulate both surface contact and lubrication with a unified model. This difficulty has been significantly eased after the mid-1990s when the computer and numerical simulation technologies have been explosively advancing. Now it becomes possible to simulate the entire transition from full-film and mixed lubrication down to boundary lubrication or dry contact with a unified mathematic model and numerical approach. These two separate branches of engineering science; therefore, have been realistically bridged.

Once again, interfacial phenomena in engineering practice are always complicated, and contact and lubrication generally coexist in most cases. Our ultimate goal is to develop advanced theories and models to gradually access the complex reality. Deepening and merging of contact and lubrication research efforts suggest a uniform, integrated, and evolving concept of Interfacial Mechanics. We manly have the following considerations:

(1) The classic contact mechanics (well covered by Johnson [129] in 1985, and others) and lubrication theories (described by Pinkus and Sternlicht [130] in 1961, and Hamrock [131] in 1994, and others), as well as conventional EHL reviewed earlier in this paper, have provided a solid foundation for research and engineering applications. As research has been deepening recently, especially in the last 15 years, the classic assumptions are being released one by one. For example, recently the EHL has been extended in 2007 and 2008 by Liu et al. [132,133] to consider coated or layered materials, in 2000 by Kang et al. [134] to take into account the debris effect, and in 2007 by Slack et al. [135] to include the effect of material inclusions. Plastic deformation, which commonly exists and is extremely important in failure analyses, has been included in EHL simulations by Xu et al. [136] in 1996 for line contacts and by Ren et al. [137,138] in 2010 for point contacts. In-depth studies on nanoscale thin-film lubrication, such as those by Luo, et al. [85] and Spikes [139], suggest that the lubricants may no longer behave as continuum media at lubricated interfaces. The above-mentioned studies are only examples but have demonstrated a strong momentum along this direction. Therefore, the conventional terms, such as "contact," "elasto-," and "hydrodynamic," may become insufficient to characterize interfacial phenomena in future research in the course of deepening scientific discovery. Interfacial mechanics is a natural embrace of contact and lubrication theories and a wider advocate for advanced research in much extended integrated scopes.

(2) Contact mechanics and lubrication theories, including EHL, are originally based on continuum mechanics that assumes uniform and continuous materials properties regardless of the scales in space and time. It is often valid when tackling scientific and engineering problems of surface interaction on a macroscale. It has been understood; however, that macroscale phenomena, such as loading capacity, performance, efficiency and durability/reliability of various machines and their components, depend heavily on properties of the interfaces on micro- and nanoscales, where efforts beyond continuum mechanics are needed. With the available mixed EHL models described previously, for instance, the entire transition from full-film and mixed lubrication down to boundary lubrication and surface contact can be numerically simulated, and calculated lubricant film thickness can be only a few nanometers or even zero. Nevertheless, key questions still remain unanswered, such as: Can we convincingly predict interfacial mechanisms with a continuum mechanics based model, if the lubricant film thickness is in the same order of magnitude as lubricant molecules? What is the range of validity of the EHL theory? How can we model nanoscale boundary films that widely exist in mixed and boundary lubrication? How can we characterize frictional behavior at the interfaces under transient thin-film or boundary film conditions? Currently, multiscale models and methods are being developed, linking continuum mechanics with molecule-level event simulations. Recent efforts include those by Luan and Robbins [140] in 2005, Martini et al. [141] in 2006, and Zhu et al. [142] in 2010, and others. This appears to be a new direction of interface research with great potential. Interfacial mechanics can better describe the multiscale nature of the further studies and can be highly engaged with the research vehicles voyaging into the depth of multiscale surface interaction.

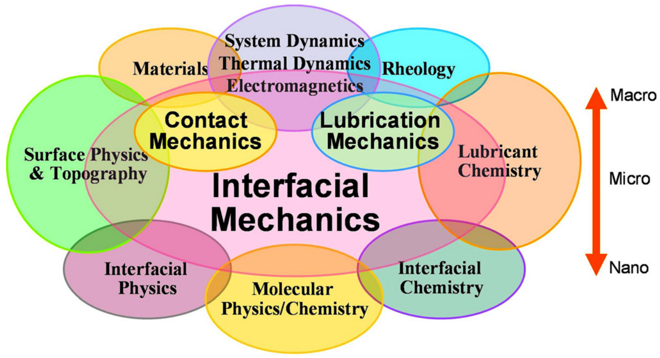

Fig. 10 Interfacial mechanics and its related fields of study

(3) Obviously, in-depth studies of interfacial phenomena are multidisciplinary in nature, involving several branches of continuum and discrete mechanics, materials science, lubricant chemistry and rheology, surface physics and topography/metrology, interfacial physics and chemistry, molecular dynamics, system dynamics, thermal dynamics, and electromagnetics, and possibly more. Design and development of advanced interfaces also require the support of design methodology and manufacturing technologies. Interfacial mechanics is going to cover a broader field with melting borders between individual knowledge branches, where more fruitful interdisciplinary and collaborative efforts are committing toward solutions to more difficult scientific and engineering problems. Its science kernel and technological extensions make interfacial mechanics an evolving field flourishing in the time of rapid developments of science-based simulation of engineering systems.

Figure 10 summarizes the above-presented discussions as a pictorial reference. It does by no means present accurate scientific definitions and precise relations.

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Nomenclature

A_c = contact area ratio (area of surface contact divided by area of Hertzian zone)
a = semiaxis of Hertzian contact ellipse in rolling direction, or radius of Hertzian contact circle, or half-width of Hertzian zone for a line contact
b = semiaxis of Hertzian contact ellipse in the direction perpendicular to rolling
E' = effective elastic modulus
G^* = $\alpha E'$, dimensionless material parameter
h = local film thickness (or gap)
h_a = average film thickness (or average gap)
h_c, H_c = central film thickness, $H_c = h_c/R_x$
h_{cs} = central film thickness predicted by smooth surface EHL theory
h_m, H_m = minimum film thickness, $H_m = h_m/R_x$
k = b/a, contact ellipticity
k_f = lubricant conductivity
l_e = effective length of line contact
p = pressure
p_h = maximum Hertzian pressure
R_q = root mean square (rms) surface roughness
R_x = effective radius of curvature in x (rolling) direction
U = $(u_1 + u_2)/2$, rolling velocity (or entraining velocity)
U^* = $\eta_0 U/(E'R_x)$, dimensionless speed parameter
u_1, u_2 = velocities of surface 1 and surface 2, respectively
W^* = $w/(E'R_x^2)$ for point contact, or $w/(E'R_x l_e)$ for line contact, dimensionless load parameter
w = load
α = pressure–viscosity exponent used in $\eta = \eta_0 \exp(\alpha p)$
β = temperature–viscosity exponent used in $\eta = \eta_0 \exp\{\alpha p + \beta(T_0 - T)\}$
η, η_0 = viscosity and viscosity under ambient condition, respectively
Λ = h_{cs}/σ, hydrodynamic roughness parameter
λ = h_a/σ, film thickness ratio, or specific film thickness, or λ ratio
σ = $(R_{q1}^2 + R_{q2}^2)^{0.5}$, composite rms roughness
τ_L = limiting shear stress

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