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Conformal Contact Problems of Ball-socket and Ball

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Abstract

This paper focuses attention on non-conformal and almost conformal contact of ball and ball-socket. Two-dimensional finite element models are developed to calculate the normal contact stress distribution and contact area. The effects of geometry dimension and external load on the contact pressure distribution and contact region are presented, respectively. Meanwhile, the results of FEM and solutions of Hertz contact theory are compared. The results indicates that contact state of ball and ball-socket changes from point contact to area contact with the increasing of the dimensionless number-curvature radius coefficient f and the number of $f=0.536$ (≈ 0.54) is critical parameter causing the change.

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Keyword: Contact stress; Conformal contact; Finite element method; Hertz contact theory

1. Introduction

Hertz contact theory has been used to calculate the real contact area of rough surfaces, and to investigate the sliding contact and bearing stresses [1]. However, this is only applicable when the contact area is limited to small and narrow region where the contact bodies are considered as half-planes [2]. Especially, applying Hertz contact theory to solve the problems of sliding bearing and spherical plain bearing is very difficult to obtain precise solution where geometry dimensions of contact bodies are almost same radius. Therefore, different approaches are required to solve conformal contact problems.

A closed form solution was developed by Persson [3] to calculate the contact pressure in an infinite plate loaded through a disc passing through a hole with nearly the same radius as the disc. Frictionless surfaces and isotropic elastic materials were assumed. Using similar criteria, Kovalenko [4] developed

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additional closed form solutions for conformal cylinders. Persson's solution was studied further by Ciavarell and Decuzzi [5] to investigate a wider range of loading conditions and material parameters.

In addition, numerical analysis techniques were used for conformal contact cylinders [6, 7]. A combination of numerical and displacement modeling was conducted by Campbell [8] using finite element modeling and displacement methods to calculate the stress distribution on a cylindrical bearing subjected to vertical and horizontal loads. Hsien H. Chen and Kurt M. Marshek [9] presented a numerical procedure for solving the two-dimensional closely conforming elastic body contact problems.

This paper is a part of a study on the interfacial contact behavior of spherical plain bearing. Axisymmetric finite element models (two-dimensional) are developed to simulate conformal contact of ball and ball-socket, as shown Fig.1. The effects of geometry dimensions and external load on the contact pressure distribution and contact region of ball and ball-socket are presented, respectively. Simulation results and Hertz solution are also compared.

2. Hertz contact theory

The classical solution for the local stress and deformation of two elastic bodies apparently contacting at a single point was established by Hertz in 1881. However, Hertz point contact theory is only appropriate to non-conforming contact problem. Essential assumptions of Hertz theory are following three items [10]:

1. The proportional limit of the material is not exceeded, that is, contact bodies are linear elastic material and all deformation occurs in the elastic range.
2. Loading is perpendicular to the surface, that is, the effect of surface shear stresses and friction is neglected.
3. The contact area dimensions are very small compared to the radii of curvature of the bodies under load.

Based on the three assumptions, Hertz solutions are expressed as:

$$\sum \rho = \rho_{11} + \rho_{12} + \rho_{11} + \rho_{22}; F(\rho) = \frac{(\rho_{11} - \rho_{12}) + (\rho_{11} - \rho_{22})}{\sum \rho}$$

$$a = a^* \sqrt[3]{\frac{3Q}{2\sum \rho} \left(\frac{1 - \xi_1^2}{E_1} + \frac{1 - \xi_2^2}{E_2} \right)}; b = b^* \sqrt[3]{\frac{3Q}{2\sum \rho} \left(\frac{1 - \xi_1^2}{E_1} + \frac{1 - \xi_2^2}{E_2} \right)}; \sigma_{\max} = \frac{3Q}{2\pi ab}$$

in which $\sum \rho$ is curvature sum, $F(\rho)$ is curvature difference, a and b are semimajor and semiminor axis of the projected contact ellipse, σ_{\max} is max contact stress, a^* and b^* are dimensionless semimajor and semiminor axis of the projected contact ellipse, Q is normal force between ball and ball-socket, E_1 and E_2 are elastic modulus of ball and ball-socket, ξ_1 and ξ_2 are Poisson's ratio of two bodies.

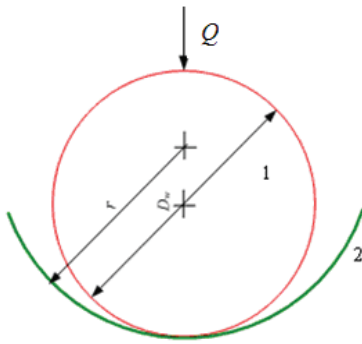


Fig. 1 Contact model of ball and ball-socket

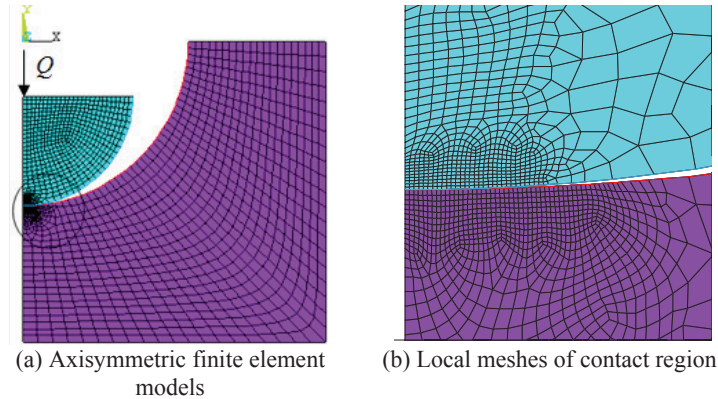


Fig. 2 General description of the 2D finite element model

3. Finite element models

3.1. General

There are three contact models to solve contact problems: point-to-point, point-to-surface and surface-to-surface. The model of point-to-point is suitable to simulate point-to-point contact behavior. Before using point-to-point element, contact location must be known in advance and small sliding distance between two contact areas is only allowed. The model of point-to-surface is applicable to the case that large deforming or distance exists between contact areas, and exact contact region may not be known in advance. However, since it is difficult to quantify the type of contact in most cases, the general contact element-surface-to-surface may be used. The model of surface-to-surface is defined by contact pairs consisting of two contact surfaces. It is considered the most computationally intensive yet conceptually simplest type to use. In the paper, the model is adopted to simulate the contact problem of ball and ball-socket under consideration in axisymmetric problem.

3.2. Two-dimensional model

Using an ANSYS finite element package, a two dimensional model was developed to simulate the two parts: ball and ball-socket, shown in Fig. 2. Four-node quadrilateral elements were used in the model. Fig. 2(a) is whole finite element model and Fig. 2(b) is enlarged local meshes of contact region in the circularity.

Because of symmetry, axisymmetric finite element models were established. The contact surface, namely: ball-ball-socket is presented in Fig. 2. The surface between ball and ball-socket is assumed smooth. Usually, such assumption is allowed because friction between the two bodies is very small.

The ball and ball-socket being as isotropic material simulated in the paper are bearing steel. The elastic modulus of two bodies is $E_1 = E_2 = 2.07 \times 10^5 \text{ MPa}$ and Poisson's ratio is $\xi_1 = \xi_2 = 0.3$. External load applied on the ball is $Q = 1000 \text{ N}$. Inner radius of ball-socket $r = 30 \text{ mm}$ and diameters of ball D_w are 40mm, 48mm, 56mm, 59mm, and 50.6mm, respectively. Therefore, Hertz formulas are expressed as:

$$a = b = \sqrt[3]{\frac{3Q}{4} * \frac{\frac{1-\mu_1^2}{E_1} + \frac{1-\mu_2^2}{E_2}}{\frac{2}{D_w} - \frac{1}{r}}} = 0.1875\sqrt[3]{R}; \sigma_{\max} = \frac{3Q}{2\pi a^2}$$

Where $\frac{1}{R}$ is curvature sum and $\frac{1}{R} = \frac{2}{D_w} - \frac{1}{r}$

4. Results and discussion

The contact pressure distributions on surface under normal applied external load of 1000N were obtained, as shown in Fig.3 (a) and Fig.3 (b). From the contact stress distributions, maximal contact stress occurs on the contact core.

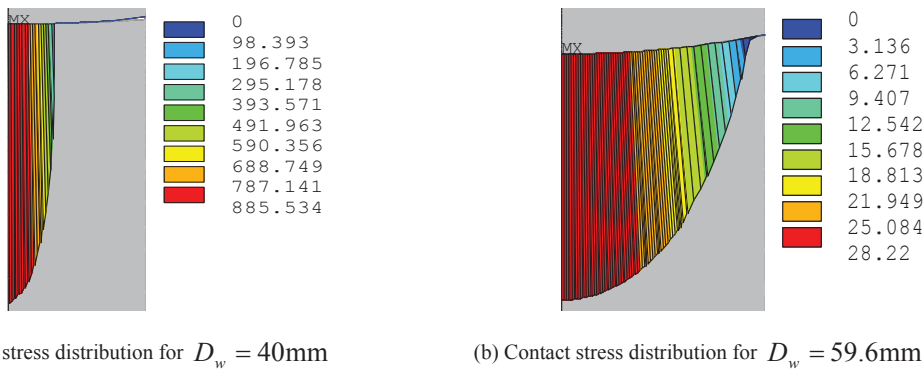


Fig. 3 Contact pressure distribution

The most significant factor affecting the contact pressure distribution and contact region dimension was the radius of ball on the contact surface. When $D_w = 40\text{mm}$ max contact stress was 885.534Mpa and radius of contact circularity was 0.741mm. When $D_w = 59.6\text{mm}$ max contact stress was only 28.22Mpa and radius of contact circularity reached to 4.404mm.

In order to expatiate the effect of radius of ball on contact stress and contact region, the dimensionless number f was introduced to denote osculant degree on contact points between ball and ball-socket. According to the definition of f , it was expressed as the expression of $f = r/D_w$. Where f known as curvature radius coefficient.

The comparing results of ANSYS with Hertz solutions are shown as Table 1.

$\sigma_{A\max}$ is max contact pressure on contact surface for ANSYS result.

$\sigma_{H\max}$ is max contact pressure on contact surface for Hertz solution.

a_A is contact radius on contact surface for ANSYS result

σ_H is contact radius on contact surface for Hertz solution.

Table 1. Comparing results of ANSYS with Hertz solutions

D_w/mm	f	σ_{Hmax}/MPa	σ_{Hmax}/MPa	$\sigma_{Hmax}/\sigma_{Hmax}$	a_A/mm	a_H/mm	a_A/a_H
40	0.750	885.53	885.96	1	0.741	0.734	1.009
48	0.625	556.74	558.12	0.9975	0.930	0.925	1.005
56	0.536	240.07	242.11	0.9916	1.439	1.404	1.025
59	0.508	82.01	92.80	0.8837	2.571	2.268	1.134
59.6	0.503	28.22	50.04	0.5639	4.404	3.089	1.425

Fig.4 describes the change of max contact pressure along with diameter of ball D_w and Fig.5 explains the change of contact radius along with diameter of ball. When $D_w \leq 56$ mm, Hertz solution are consistent with the results of ANSYS. However, when diameter of ball $D_w > 56$, with the increasing diameter of ball, Hertz solutions are much different from the results of ANSYS.

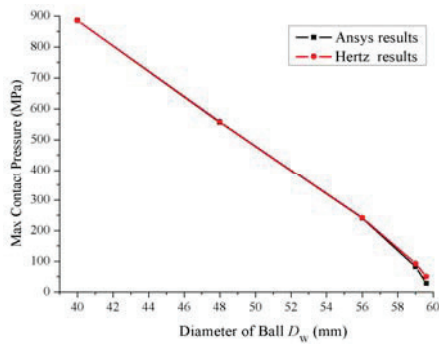


Fig. 4. Max contact pressure along with diameter of ball

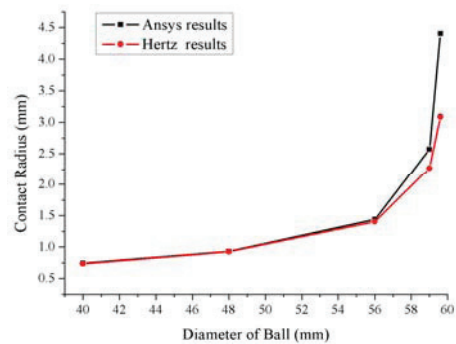


Fig.5. Contact radius along with diameter of ball

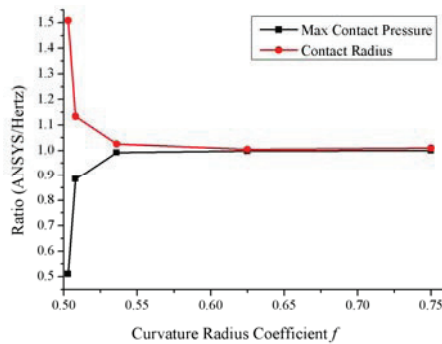


Fig.6. Ratio of result of ANSY and Hertz solution along with curvature radius coefficient

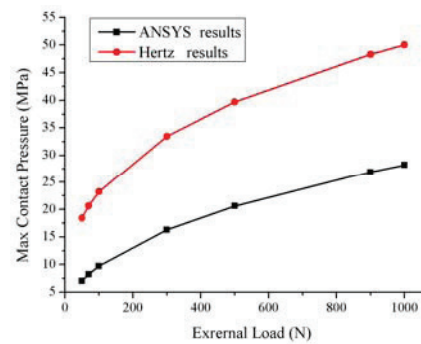


Fig.7. Max contact pressure along with external load under diameter of ball

Fig.6 shows the change of the ratio of result of ANSY and Hertz solution with the dimensionless number f . When curvature radius of coefficient $f < 0.536$ (≈ 0.54), the ratio of result of ANSY and Hertz solution deviates from 1.0 greatly. Hertz solutions are much different from the results of ANSYS. However, when curvature radius coefficient $f > 0.536$ (≈ 0.54), with the increasing of f , the ratio of result of ANSY and Hertz solution is closed to 1.0. A possible explaining of the results is that contact state of

ball and ball-socket changes from point contact (higher pair contact) to area contact (lower pair contact) with the increasing of f . The number of $f=0.536$ (≈ 0.54) is critical number causing the change.

Besides above analysis, the author analyzed the effect of external load on contact properties of ball and ball-socket. Fig.7 describes the change of max contact pressure along with the external load under the diameter of ball $D_s=59.6$ ($f=0.503$). Fig.8 explains the change of contact radius along with the external load under the diameter of ball $D_s=59.6$ ($f=0.503$) and Fig.9 illuminates the ratio of result of ANSYS and Hertz solution along with external load. From the three figures, it is very difficult to obtain precise solution of almost conformal contact problem of ball and ball-socket using Hertz contact theory. Either max contact pressure or contact area deviates from Hertz solution greatly. Finite element method is effective approach to solve such problem.

5. Conclusions

Two-dimensional finite element models of ball and ball-socket was developed to calculate the normal contact pressure distribution and contact area using the ANSYS finite element package. Meanwhile, the results of ANSYS and the solutions of Hertz theory were compared. By analyzing the results of ANSYS and Hertz solutions, FEM is proper to obtain accurate resolution to solve conformal contact problem.

The contact pressure distribution on the contact surface was greatest in the centre of contact zone. No significant difference between the result of ANSYS and the solution of Hertz contact theory was observed when dimensionless number-curvature radius coefficient $f > 0.536$ (≈ 0.54) and significant difference in the contact pressures and contact zone was found when curvature radius coefficient $f \leq 0.536$ (≈ 0.54). A possible explaining of the results is that contact state of ball and ball-socket changes from point contact to area contact with the increasing of f and the number of $f=0.536$ (≈ 0.54) is critical number causing the change.

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