Lubrication at point contacts

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Measurements have been made of the friction, electrical resistance, and electrical capacity between rotating steel cylinders with their axes mutually at right angles. The lubricant was a plain hydrocarbon mineral oil. Nominally the surfaces come together at a single point and the apparatus is designed to ensure that this condition is maintained even if the cylinders wear. It is shown that hydrodynamic lubrication exists over a wide range of conditions. At loads of a few kilograms it persists even when the speed falls below 1 cm/s and at higher speeds ($\sim$ 100 cm/s) it is maintained even when the load becomes large enough to cause bulk plastic flow of hardened steel. Hitherto it has been considered that only boundary lubrication could occur under these extreme conditions.

At very light loads classical hydrodynamic theory applies but as the load is increased a departure from classical theory occurs because the viscosity of the oil increases under the applied pressure. At heavier loads the pressures become large enough to cause appreciable elastic deformation of the surfaces and a state of elasto-hydrodynamic lubrication is achieved. Under elasto-hydrodynamic conditions the film thickness can be deduced from the measurements of electrical capacity. A simplified theory of elasto-hydrodynamic lubrication at point contacts is developed, and the measured values of film thickness are in fairly good agreement with those derived from the theory. However, the variations of film thickness with viscosity, speed and radius of curvature forecast by the theory differ significantly from those obtained experimentally. The values of the film thickness range from 2×10^{-6} cm to more than 1×10^{-4} cm. The results, over the whole range, conform to a regular pattern and there is no evidence of any disturbing influence of the surface molecular fields, even with the thinnest films.

1. Introduction

The shape of rubbing surfaces has a large influence upon the mechanism of their lubrication. Three main geometrical arrangements can be distinguished, conforming surfaces, surfaces in nominal line contact and surfaces which come together, nominally, at a single point. The classical hydrodynamic theory of Reynolds (1886) was developed for conforming surfaces between which relatively thick (10^{-2} to 10^{-4} cm) films of lubricant are formed, even at heavy loads, by reason of the favourable geometry. In contrast, the present paper is concerned with the lubrication of bodies in nominal point contact, the arrangement least favourable to hydrodynamic lubrication. The high local pressures, the rapid convergence of the film in the direction of motion, and the ease with which the lubricant can leak away sideways would all seem to act against the establishment of a hydrodynamic film.

The classical theory of hydrodynamics assumed that the surfaces were rigid and that the viscosity of the lubricant was invariant. When similar assumptions are made in the hydrodynamic calculations for a point contact (Howlett 1946; Kapitza 1955) one obtains what may conveniently be called a classical theory for this geometrical arrangement. According to this theory the minimum thickness h_0 of the film between spherical surfaces of radius R_1 and R_2 is

$$h_0 = 28{\cdot}4\eta_0^2 V^2 R^3 W^{-2}, \tag{1}$$

where R is the relative radius of curvature ($1/R = 1/R_1 + 1/R_2$), η_0 is the viscosity of the lubricant, W is the load, and V is the sum of the surface velocities. Putting $\eta_0 = 0{\cdot}2$ poise, $V = 30$ cm/s, $R = 0{\cdot}3$ cm (conditions typical of the widely used four-ball lubricant tester), and taking the relatively low load of 1 Kg (10^6 dyne), one obtains $h_0 = 0{\cdot}3$ Å which is far less than the carbon-carbon spacing in a single molecule of oil. Classical theory therefore predicts that hydrodynamic lubrication at point contacts can occur only at very light loads. At the heavier loads commonly of practical interest, it is usually assumed that boundary lubrication predominates, i.e. the surfaces are separated by films of molecular dimensions only.

Bodies which touch, nominally, at a line provide a geometrical arrangement intermediate, in the severity of its hydrodynamic conditions, between conforming surfaces and the point contact. Recent experiments (Lewicki 1955; Crook 1957; 1958*a*, *b*; Cameron & MacConochie 1960) with disks or gears in nominal line contact have shown that relatively thick films of oil can exist at loads greater than those forecast by the relevant classical theory (Martin 1916), and it has been possible to go some way towards explaining these experimental results by the development of the elasto-hydrodynamic theory of lubrication (Gatcombe 1945; Poritsky 1952; Grubin 1949; Petrusevich 1951; Blok 1958; Dowson & Higginson 1959). This theory allows for the increase of viscosity with pressure (Bridgman 1949) and takes account of the elastic deformation of the surfaces in the load-bearing region. These two effects combine to oppose the fall in film thickness with increasing load which the classical theory predicts.

Theory provides no clear guide to indicate whether bodies in nominal point contact behave in a similar manner. The theories of Kapitza (1955) and Korovchinskii (1958) include the effect of a pressure-dependent viscosity but no elasto-hydrodynamic theory, which must include also the effect of deformation, yet exists. Korovchinskii, however, argues on more general grounds that hydrodynamic effects would be negligible even if deformation were included in the analysis.

Similarly, the experimental evidence from earlier studies of point contacts (see, for example, Clayton 1937) is not decisive. It was usually assumed that the value of the coefficient of friction, μ, provided a reliable guide to the state of lubrication, values of the order of 10^{-3} to 10^{-2} being typical of hydrodynamic lubrication and values of about 0·1 indicating boundary conditions. Intermediate values of μ were taken to indicate a mixed régime. However, it is now clear from the line contact studies that these intermediate values can also occur under purely hydrodynamic conditions when the viscosity of the lubricant is raised by the pressure. More direct and unambiguous investigations are required.

Beeck, Givens & Smith (1941) showed, by means of electrical resistance measurements, that hydrodynamic lubrication could occur at point contacts at speeds as low as 1 cm/s. More recently Cameron (1954) and Cameron, Siropongse & Rogers (1958) have attempted to measure the film thickness in the four-ball machine using a method based upon the discharge voltage of the film. The results thus obtained must be dominated by the behaviour of the oil in the regions where the surfaces approach one another most closely and the method must therefore be very sensitive to surface irregularities. Moreover, it has been shown that changes in the solid particle

content of the oil can change the calibration by a factor of three. Because of these and other difficulties, recalibration has changed the values of the film thickness obtained by Cameron by an order of magnitude.

In the present work the state of lubrication at point contacts is investigated using measurements of the electrical resistance and electrical capacity between the surfaces. The resistance measurements show quite clearly that fluid film lubrication can occur over a very wide range of conditions. The main problems are then the determination of the shape and thickness of the lubricant film and of the variations of the thickness with the viscosity of the oil, the velocity, and the radius of the surfaces; it is shown that this information can be deduced from the measurements of the capacity. In this work it is essential to ensure that wear of the surfaces does not change the contact conditions during an experiment. In the four-ball machine and other forms of apparatus, wear can produce local conformity and consequent improved lubrication, but with the special apparatus to be described below, a constant point contact geometry is achieved.

2. Experimental

2·1. *The crossed-cylinders machine*

Figure 1 shows the essential parts of the apparatus, the rotating cylinders being at A and B. As well as rotating, the lower specimen, A, traverses slowly to and fro in a direction at 45° to the two specimen axes so that the point of contact traces out a helical path on both specimens. Because the two rotations and the traverse are not synchronous, all parts of the specimens are subject to equal wear and they then retain their cylindrical shape.

To achieve this, specimen A is mounted on a carriage, C, which is moved to and fro by a hydraulic ram, D, whose speed is continuously variable up to a maximum of 6 in. (15 cm) per second. At each speed of rotation the speed of traverse was adjusted to ensure that there was no overlapping of the helical track traced out by the area of contact. The length of the traverse is controlled by electrical limit switches and an experiment can be performed upon any selected portion of the specimens. The rotation of the lower specimen is effected through a telescopic shaft, E, and Hardy Spicer flexible couplings, F. The lower specimen assembly is insulated, G, from the body of the machine so that the electrical capacity and electrical resistance between the specimens can be measured. Electrical connexions to the specimens themselves and to thermocouples embedded in them are made by mercury contacts, H, of a type described by Kenyon (1954).

The upper specimen, B, is mounted on an arm, J, which is pivoted at one end in a gimbal mounting, K. The rotating drive is flexibly coupled to this specimen by an Oldham coupling situated at the intersection of the gimbal axes. A load W' is applied to the end of the upper specimen arm and the load W between the specimens varies from 10 W' to 10 $W'/3$ over the usable portions of the specimens. In most of the experiments described in this paper only a short length of specimen was used and the load was effectively constant. The friction between the specimens tends to rotate the upper specimen arm about the vertical gimbal axis and this movement is

restrained by the vertical rod, L. The friction is recorded by measuring the deflexion of this rod by means of a 'Talymin' sensing head. The measured force is a component of the frictional force and it depends upon the velocities V_1 and V_2 of the upper and lower specimens. Calibration of the friction records is by horizontal loads applied at M.

The specimens are cylinders 4·75 in. (12·1 cm) long, mounted between running centres which transmit the rotation, and diameters up to 3 in. (7·6 cm) can be accommodated in the machine. The rotations of the two specimens are independently controlled and can be varied from 2·5 rev/min to more than 3000 rev/min. In the experiments described below both specimens were rotated at the same speed.

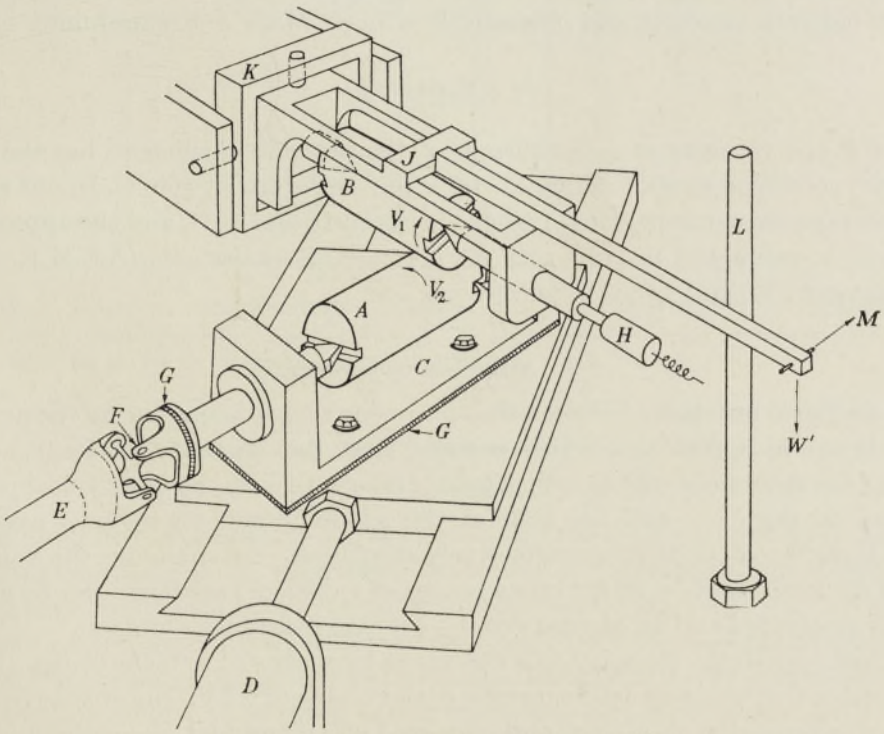

FIGURE 1. The crossed cylinders machine.

In the main experiments the specimen surfaces were very smooth, being ground or polished to a finish of better than 2 μin. (5×10^{-6} cm) c.l.a.; this finish was further improved by running-in. The electrical capacity measurements were made with specimens of hardened high speed tool steel. The temperature of the specimens was measured by thermocouples embedded in them, and disposed about 0·1 in. (2·5 mm) beneath the surface. The temperature of the specimens has a large influence upon their lubrication and in early experiments it was found that at high speeds the temperature of the specimens varied with the load; these variations could not be suppressed even when a large flow of lubricating oil at a controlled temperature was used. In later experiments the variations in temperature were almost eliminated by using hollow specimens through the body of which flowed a stream of oil at a controlled temperature.

2·2. *The oil*

The specimens were lubricated by a jet of a plain mineral oil (turbine oil to Admiralty specification OM100). The variations of the viscosity and electrical resistivity of this oil with temperature have been given by Crook (1958*a*). He also showed that the Clausius–Mosotti relation is applicable; therefore changes in the dielectric constant of the oil (2·3 at n.t.p.) with temperature or pressure should be small and correlate closely with changes in the density.

The variation of the viscosity of the oil with pressure has been measured in an apparatus of the type described by Bridgman (1949)† up to pressures of about 1000 atm. To a good approximation the results are consistent with the exponential relation between viscosity and pressure (Bradbury, Mark & Kleinschmidt 1951),

$$\eta = \eta_0 \exp(\alpha p), \tag{2}$$

where η is the viscosity at a pressure p. α, the pressure coefficient, has the value $2{\cdot}2 \times 10^{-9}$ cm^2/dyne at 60 °C falling to $1{\cdot}7 \times 10^{-9}$ cm^2/dyne at 100 °C. In our experiments the specimen temperatures ranged between 15 and 50 °C and the appropriate values of α, estimated from results for a range of similar oils (A.S.M.E. 1953), range between 3·0 and $2{\cdot}3 \times 10^{-9}$ cm^2/dyne.

2·3. *Electrical apparatus*

The electrical resistance between the specimens was measured using the arrangement shown in figure 2(*a*) which ensured that the maximum voltage applied between the specimens was 95 mV. Figure 2(*b*) shows a sketch of a typical pattern observed on the c.r.o. and the scale at the left-hand side shows that resistance values from 0·1 R_2 to 10 R_2 are effectively displayed. By changing the values of R_1 and R_2 (keeping $R_1 = 20\,R_2$) other ranges of values of resistance can be successively investigated and an estimate of the average resistance obtained.

The inter-specimen capacity was measured by means of a Wein bridge (Hague 1945) with a frequency of 1000 c/s and a peak voltage of 0·2 V. The output from the bridge was presented on a c.r.o. and consisted of a sinusoidal trace interrupted by peaks due to low resistance pulses of short duration. These peaks could make the bridge difficult to balance but their effect was reduced by introducing an amplitude limiting circuit and a frequency selective amplifier between the bridge and the c.r.o. Capacity measurements were found to be consistent to better than ± 1 pF under the most favourable conditions.

The capacities recorded below are the measured capacities, minus the capacity measured with the specimens rotating but separated by a fixed distance (0·25 in.), plus the calculated inter-specimen capacity (approximately 1 or 2 pF) for this separation. In each experiment the inter-specimen capacity was measured as a function of the load, the speeds and temperature of the specimens being maintained constant.

† We are greatly indebted to Dr H. Naylor of the Thornton Research Centre (Shell Research Limited) for these measurements.

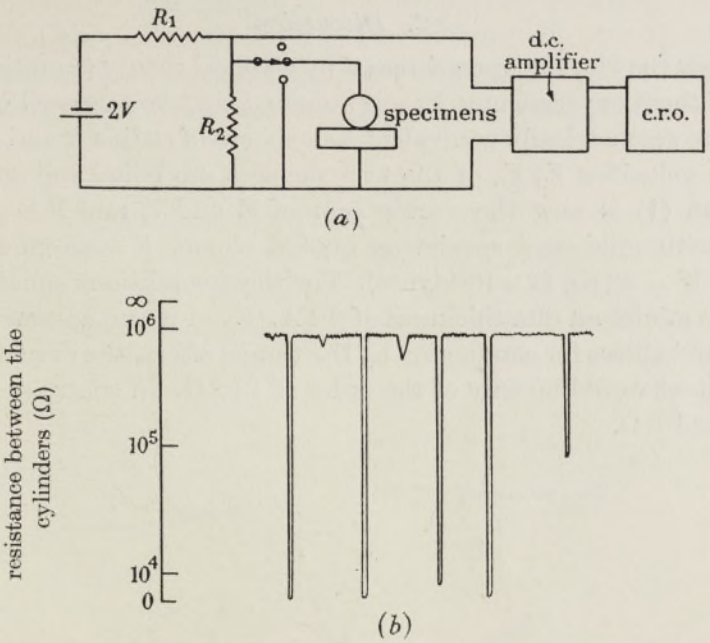

FIGURE 2. Electrical resistance measurements. (*a*) Circuit diagram. $R_1 = 20R_2$ and R_2 has many values between 0·1 and $10^5\ \Omega$. (*b*) Typical pattern observed on the c.r.o. at light loads. $R_2 = 10^5\ \Omega$. Resistance scale at the left.

3. MEASUREMENTS OF THE ELECTRICAL RESISTANCE

3·1. *Results*

The specimens were run at a given speed and the load was gradually increased, being maintained at each value until the electrical resistance stopped rising (indicating that running-in had been substantially completed). The pattern observed on the c.r.o. (figure 2(*b*)) showed a generally high level ($> 10^6\ \Omega$), demonstrating clearly the presence of a hydrodynamic film. Variations in the resistance took the form of brief excursions to lower values indicating that intermittent contacts took place through the film due either to local protuberances on the surfaces or to the presence of solid particles in the oil. Changes in the average resistance were due primarily to changes in the frequency of these low-resistance excursions.

High levels of electrical resistance persisted to very heavy loads. With mild-steel specimens (V.p.n. 180) of diameter 3·8 cm running at surface speeds of 60 cm/s the average value of the resistance at a load of 40 Kg was $10^4\ \Omega$. This load is greater than that required to cause the onset of bulk plastic flow. In similar conditions with hardened tool steel specimens (V.p.n. 620), a load of 300 Kg was reached before the average resistance had fallen to $10^4\ \Omega$. In another experiment with hardened tool steel specimens a single traversal of the cylinders was performed, the load between them ranging up to a maximum of 500 Kg. A Talysurf profilometer record of the surface is shown in figure 3. Apart from the bulk plastic flow shown by this record the specimens were quite unmarked and the electrical resistance exceeded $1000\ \Omega$.

3·2. *Discussion*

Consider now the film thickness forecast by classical theory (equation (1)) for the conditions of these experiments. It will be noted that two crossed cylinders each of radius R are geometrically equivalent to a sphere of radius R and a flat surface, and that the velocities V_1, V_2, of the two surfaces are equal and at right angles; V in equation (1) is now the vector sum of V_1 and V_2 and $V = \sqrt{2}.V_1$. In the experiment with mild-steel specimens quoted above, $V = 85$ cm/s, $R = 1{\cdot}9$ cm, $\eta_0 = 1$ P and $W = 40$ Kg (4×10^7 dynes). For these conditions equation (1) yields a value of the minimum film thickness of 0·1 Å. Even if one assumes a somewhat thicker film and allows for conduction by the tunnel effect, the electrical resistance of such a contact would be only of the order of $10^{-3}\,\Omega$. In contrast, the measured resistance was $10^4\,\Omega$.

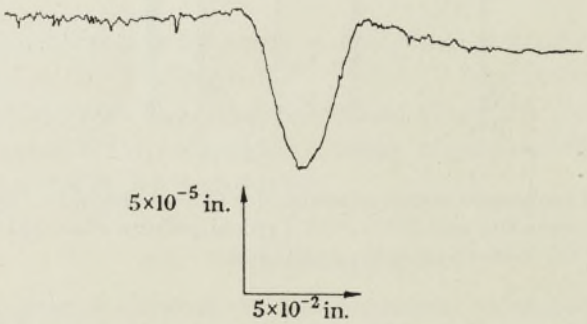

FIGURE 3. Plastic flow of hardened steel under hydrodynamic conditions. $R = 1{\cdot}52$ cm; $V = 140$ cm/s; $W = 420$ Kg; temperature 17 °C; $\mu \simeq 0{\cdot}04$; electrical resistance = 10^3 to $10^4\,\Omega$.

In the experiment of figure 3 the conditions were even more severe. Nevertheless the unmarked appearance of the specimens, the low value of the coefficient of friction ($\sim 0{\cdot}04$), and the high electrical resistance (10^3 to $10^4\,\Omega$), all combine to show that effective fluid lubrication existed even when the pressures were sufficient to cause plastic deformation of the hardened steel. Clearly these results cannot be reconciled with the classical theory.

4. MEASUREMENTS OF THE ELECTRICAL CAPACITY

4·1. *Results at light loads*

Figure 4 shows the results of a single experiment in which the inter-specimen capacity, C, and the coefficient of friction, μ, were measured as a function of the load, W. Three main régimes can be distinguished:

(a) At loads below W_1 ($\simeq 0{\cdot}5$ Kg) C increases steadily and μ falls with increasing load.

(b) At loads between W_1 and W_2 ($\simeq 3$ Kg) μ rises with increasing load but C rises less rapidly and approaches a constant value.

(c) At loads greater than W_2, μ tends towards a constant value and C increases steadily.

Results similar to those of figure 4 have been obtained for a range of experimental conditions. However, when V, R, or η_0 were reduced W_1 and W_2 became smaller and were soon less than the smallest load (100 g) which could be applied to the specimens. Under these conditions only the behaviour of (*c*) above was observed.

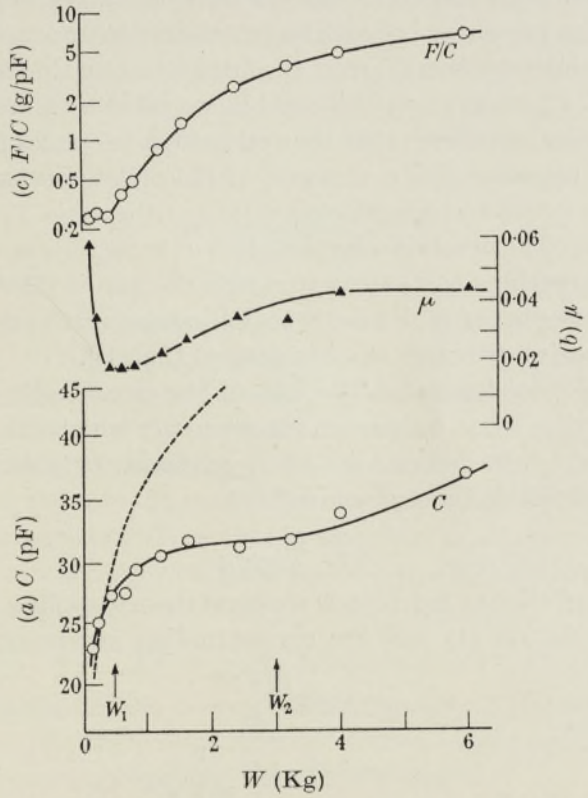

FIGURE 4. Results of a single experiment at light loads. $R = 3{\cdot}77$ cm. $V = 90$ cm/s. $\eta_0 = 1{\cdot}75$ P. The variation with load (W) of (*a*) the electrical capacity (C), (*b*) the coefficient of friction (μ) and (*c*) the ratio F/C, where F is the frictional force. - - - - -, Theoretical capacity (C').

4·2. *Discussion*

When the specimens are undeformed and the gap between them is completely filled with oil,

$$C = 2{\cdot}14[\ln(R/h_0) - 1]\,\text{pF}. \tag{3}$$

Values of C have been calculated from values of h_0 derived from equation (1), but these values of C need to be adjusted to take account of the incomplete filling of the gap with oil. Such corrections, based upon observations using transparent glass specimens, reduce C by amounts varying between 25 and 35 % and the adjusted values, C', are indicated by the broken line of figure 4. It is clear that there is reasonable agreement between theory and experiment only at the lightest loads and at loads greater than a few hundred grams the experimental values of C fall well below the theoretical values. Thus at a load of 2 Kg the theoretical value of h_0 is 1×10^{-5} cm which gives a calculated capacity of 42 pF; the measured inter-specimen capacity was approximately 32 pF which corresponds to a minimum

film thickness of 1×10^{-4} cm. At light loads also, the coefficient of friction μ falls with increasing load, as is predicted by the classical theory. At loads greater than W_1 this fall is arrested and μ rises with increasing load, which is the opposite trend to that forecast by classical theory.

These departures from classical theory would be simply explained if at loads greater than W_1 the pressures became large enough to cause an increase in the viscosity. Such an increase would arrest the fall in the film thickness and also cause the friction to rise. Changes in viscosity can be revealed by a method due to Crook (to be published) who has shown that the ratio of the frictional force due to sliding and the electrical capacity C is a measure of the effective viscosity in the load-bearing region. In figure 4(*c*) are plotted values of the ratio F/C, where F is the total frictional force. The detailed interpretation of these results is complicated by a number of factors (such as the extent to which the gap is filled with oil) but the increase of F/C between 0·4 and 4 Kg is tenfold and must be due primarily to an increase in the effective viscosity at loads greater than W_1.

The increase in C at loads greater than W_2 could be caused either by a reduction in thickness of the film or by an increase in the area over which the surfaces approach one another closely, i.e. by deformation of the surfaces. It is shown below that the capacity increases because the surfaces deform.

4·3. *Theory*

It will be assumed that at light loads classical theory applies, the film thickness being given by equation (1) and the corresponding maximum pressure by the equation

$$p'_{\max.} = 0{\cdot}55\frac{\eta_0 V R^{\frac{1}{2}}}{h_0^{\frac{3}{2}}} = 3{\cdot}6 \times 10^3 \frac{W^3}{\eta_0^2 V^2 R^4}. \tag{4}$$

It will also be assumed that as the load is increased classical theory becomes inadequate because the pressure reaches a value sufficient either to cause an increase in the viscosity or to cause significant deformation of the specimens.

Consider first the increase of viscosity with pressure. For any assumed geometrical configuration of the surfaces the pressure p which exists at any point within the film is greater than the pressure p' which would have existed when using an oil of constant viscosity η_0; p' will be called the reduced pressure. If the exponential relation between η and p (equation (2)) is assumed,

$$p = -(1/\alpha)\ln(1 - p'\alpha) \tag{5}$$

and this expression shows that the effect of the pressure coefficient of viscosity increases in magnitude as p' approaches $1/\alpha$. Thus when $p' = 0{\cdot}2/\alpha$, $p = 1{\cdot}12p'$, but when $p' = 0{\cdot}8/\alpha$, $p = 2p'$, and as p' approaches $1/\alpha$, p/p' approaches infinity. (In practice the approach of p towards infinitely large values must be arrested by deformation of the surfaces.) It seems reasonable to assume that the effect of the dependence of viscosity upon pressure becomes important when $p'_{\max.}$ exceeds $0{\cdot}8/\alpha$. Using equation (4) it will be seen that this occurs at a load

$$W_1 = 6{\cdot}05\eta_0^{\frac{2}{3}} V^{\frac{2}{3}} R^{\frac{4}{3}} \alpha^{-\frac{1}{3}}. \tag{6}$$

Consider now the deformation of the specimens. Under a static load W the maximum pressure in the contact (Hertz 1896) is

$$q_{\max.} = 0{\cdot}39E^{\frac{8}{9}}W^{\frac{1}{9}}R^{-\frac{8}{9}}, \tag{7}$$

where E is the Young's modulus of the material of the specimens. At very light loads the maximum hydrodynamic pressure $p'_{\max.}$ is very much smaller than the corresponding value of $q_{\max.}$ for the same load; therefore the local deformation is negligibly small. As the load is increased $p_{\max.}$ approaches $q_{\max.}$. It will be assumed that local deformation becomes important at a load W_2 such that $p'_{\max.} = q_{\max.}$. Thus, from equations (4) and (7),

$$W_2 = 5{\cdot}8E^{\frac{1}{4}}\eta_0^{\frac{3}{4}}R^{\frac{5}{4}}V^{\frac{3}{4}}. \tag{8}$$

Equations (6) and (8) can now be compared with the experimental results of figure 4. The equations forecast that the increase of viscosity with pressure should occur at a load of 0·7 Kg and that deformation should become important at loads greater than 1·8 Kg. The experimental results suggest corresponding loads of 0·5 Kg and 3 Kg. In view of the approximate nature of the analysis, the agreement is satisfactory. The equations also correctly forecast the reduction in W_1 and W_2 when V, η_0 or R is reduced. It should also be noted that W_2 is greater than W_1 for the full range of experimental conditions described in this paper, i.e. as the load is increased the rise in viscosity with pressure is always manifest before local deformation becomes appreciable.

4·4. *Results at heavier loads*

At loads greater than W_2 the inter-specimen capacity, C, increases with increasing load. Results of more than two hundred experiments covering a wide range of conditions have been found to conform to the relation

$$C = C_0 + kW^{\frac{2}{3}}, \tag{9}$$

where C_0 and k are constants for the given experiment. This is illustrated in figure 5 which gives a selection of the experimental results.

4·5. *Discussion*

If the oil is assumed to be incompressible the basic hydrodynamic equation (Kapitza 1955) is

$$\frac{\partial}{\partial x}\left[\frac{h^3}{\eta}\frac{\partial p}{\partial x}\right] + \frac{\partial}{\partial y}\left[\frac{h^3}{\eta}\frac{\partial p}{\partial y}\right] = 6V\frac{\partial h}{\partial x}, \tag{10}$$

where the x direction is taken as the direction of V, and the second term represents the side leakage in a direction perpendicular to V. For bodies in line contact this term is absent and equation (10) can then be integrated, giving

$$\frac{\partial p}{\partial x} = 6\eta V\left(\frac{h-h^*}{h^3}\right), \tag{11}$$

where h^* is the value of h at the pressure maximum. This equation has been used (Grubin 1949; Cameron *et al.* 1958; Crook 1958*b*) to show that $(h-h^*)$ must be small

when η becomes high, because of the pressure, and therefore the film must be very nearly parallel in the main load-bearing region. However, it is uncertain to what extent this is also true in the conditions of point contact where the second term of equation (10) is present. (Examination of equation (10) suggests that $\partial h/\partial x$ should be small but $\partial h/\partial y$ may not be.)

Consider, therefore, the evidence of the experiments depicted in figure 5. If it is assumed that the surfaces in the pressure zone are nearly parallel the pressures in

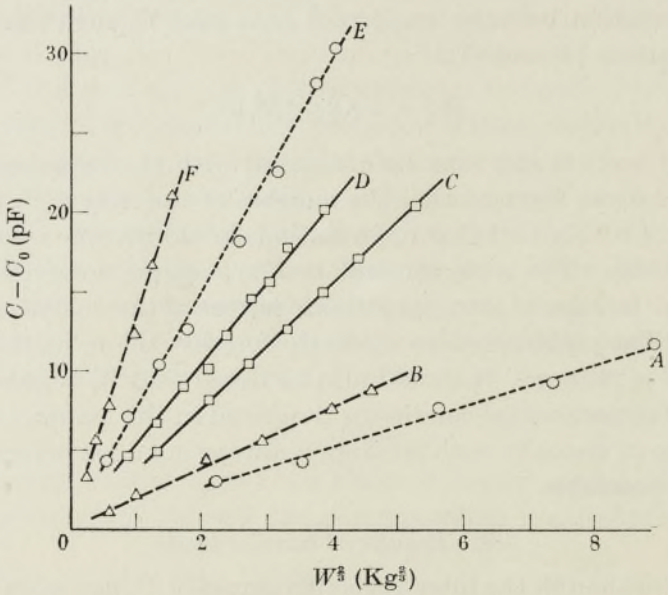

FIGURE 5. Results for a range of conditions at heavier loads. The variation of the electrical capacity (C) with the load (W) always conforms to the relation $C = C_0 + kW^{\frac{2}{3}}$.

graph ...	A	B	C	D	E	F
diam (in.)	3	0·5	1·55	1·55	3	0·5
speed (rev/min)	320	1280	160	80	20	40
temp. (°C)	19	20	26	26	21	23

this region must be close to the Hertzian values and the shape of the specimens is then as shown in figure 6. The inter-specimen capacity is then

$$C = C^* + 0{\cdot}2A/h \text{ pF}, \tag{12}$$

where h (cm) is the thickness of the parallel film, A (cm^2) is the area of the parallel region and the dielectric constant of the oil is taken as 2·3. Calculation shows that C^*, the capacity between the curved portion of the specimens outside the parallel region, is dependent upon h and R but is nearly independent of the size of the deformed region. Since the deformation is assumed to be Hertzian, A is proportional to $W^{\frac{2}{3}}$ and equation (12) becomes

$$C = C^* + KW^{\frac{2}{3}}/h, \tag{13}$$

where K is a constant which can be calculated from the known experimental conditions.

The outstanding feature of the experiments is that, at the heavier loads, all the results conform to equation (9). Comparison with equation (13), and its method of derivation, strongly suggests two main conclusions. First, in the main pressure zone the surfaces are separated by a nearly parallel film and the pressure distribution is approximately Hertzian. Secondly, the thickness of the parallel film is, to a first approximation, independent of the load (cf. equations (9) and (13)). These two conclusions are the principal results forecast by the elasto-hydrodynamic theory for line contacts. Therefore, although no elasto-hydrodynamic theory for point contacts yet exists, the experiments show quite clearly that it must contain features in common with the current theory for line contacts.

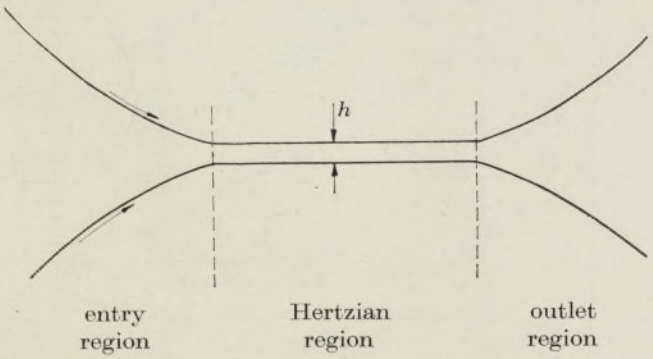

FIGURE 6. The elasto-hydrodynamic régime.

5. THE THICKNESS OF THE OIL FILM

Since K in equation (13) is known it is now possible to calculate values of the film thickness h from the slopes of graphs such as those of figure 5. A small correction to these values of h is however necessary because, if the deformation is assumed to be Hertzian, C^* varies a little with the load. At the most this variation amounted to only 2 to 3 pF and in a typical experiment the correction increased the value of h by about 20 %. A second correction was made to allow for the change in dielectric constant with pressure; this can be calculated with sufficient accuracy from density–pressure curves for a range of similar oils (A.S.M.E. 1953) and it increased the value of h by about 10 to 20 %. The effect of temperature upon the dielectric constant was neglected since, for the conditions of these experiments, the theoretical rise in temperature of the oil (Archard 1959) has been shown to have only a minor influence compared with that of the pressure.

5·1. *The controlling viscosity*

The influence of the viscosity upon the film thickness was investigated by changing the temperature of the system. However, the temperature (T_s) of the specimens often differed significantly from the temperature (T_L) of the oil flowing to them from the supply. It was therefore necessary to establish whether the controlling vicosity (η_0 in the theory of this paper) is to be taken as η_s, the viscosity of the oil at the surface temperature T_s, or as η_L its viscosity at the temperature T_L.

A special series of experiments was therefore performed and the results were analyzed statistically. Using the specimens with internal temperature control the values of η_L and η_s were each varied over a wide range while other experimental variables were maintained constant. A set of results (figure 7 (*a*)) was then chosen which showed no significant correlation between the values of η_L and η_s. The values of h deduced from these experiments show no significant correlation with η_L (figure 7 (*b*)) but, as can be clearly seen (figure 7 (*c*)), the correlation with η_s is highly significant. This demonstrates unambiguously that the controlling viscosity is the viscosity of the oil at the measured temperature of the specimens.

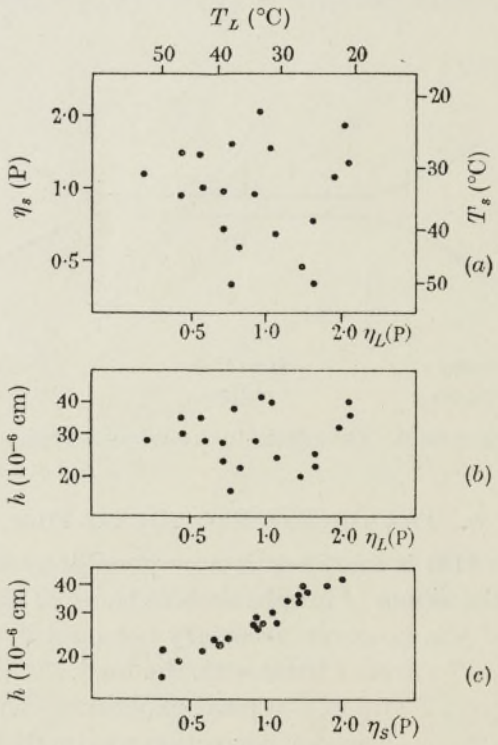

FIGURE 7. Experiments to determine the controlling viscosity. R = 1·97 cm; V = 93 cm/s; h, measured film thickness; η_L, oil viscosity at temperature T_L of oil supply; η_s, oil viscosity at temperature T_s of the specimens.

5·2. *Experimental values of the film thickness*

In all experiments reported here two identical specimens were used and were rotated at equal speeds. The most extensive series of experiments was performed with specimens of 1·55 in. (3·94 cm) diameter running at values of V between 0·72 and 370 cm/s and with specimen temperatures between 15 and 55 °C. The primary effect of a change in temperature is to change the value of η_0, but there is a small effect upon the value of α, the pressure coefficient of viscosity. It will be assumed (see § 5·3 below) that h is a function of $(\alpha\eta_0)$ and in figure 8, h is plotted as a function of $(\alpha\eta_0)$ for five of the ten speeds used in these experiments. By interpolating on graphs such as those of figure 8 one can also obtain the film thickness as a function of speed for a given temperature; the results thus obtained for temperatures of 20,

30, 40 and 50 °C are shown in figure 9. In a similar way the values of the film thickness at 20 °C have been obtained with specimens having diameters ranging from 0·25 in. (6·35 mm) to 2·97 in. (7·54 cm). From these results figure 10 has been con-

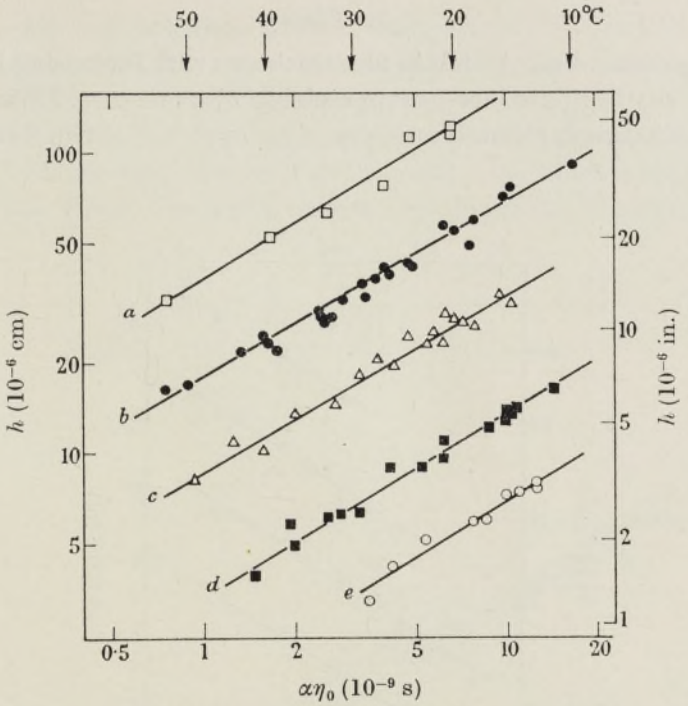

FIGURE 8. The variation of film thickness (h) with temperature. $R = 1{\cdot}97$ cm. h is plotted against ($\alpha\eta_0$) where η_0 is the viscosity and α is the pressure coefficient of viscosity, both measured at the bulk temperature of the specimens. (*a*) 370 cm/s; (*b*) 93 cm/s; (*c*) 23 cm/s; (*d*) 5·8 cm/s; (*e*) 1·5 cm/s.

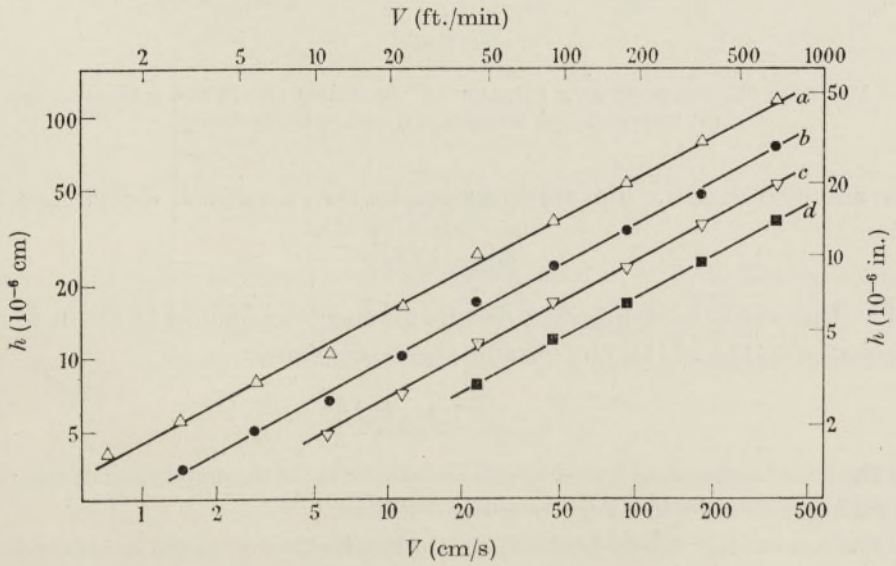

FIGURE 9. Film thickness (h) as a function of speed (V). $R = 1{\cdot}97$ cm. (*a*) 20 °C; (*b*) 30 °C; (*c*) 40 °C; (*d*) 50 °C.

structed which shows the film thickness as a function of the radius of the specimens for a number of different speeds. It will be noted that in figures 8, 9 and 10 double logarithmic scales have been used and the plotted results then lie close to straight lines.

5·3. *Theory*

In §4 it was shown that the fall in film thickness with increasing load is arrested when the viscosity begins to increase appreciably with pressure. This rise in viscosity occurs as the maximum reduced pressure $p'_{\max.}$ approaches the value $1/\alpha$. It will

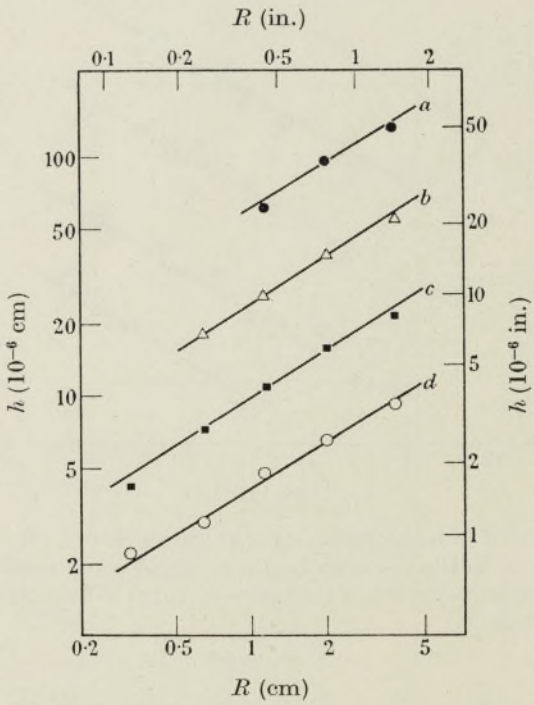

FIGURE 10. Film thickness (h) as a function of the radius (R) of the specimens (at 20 °C). (a) 250 cm/s; (b) 50 cm/s; (c) 10 cm/s; (d) 2 cm/s.

now be assumed that the film thickness approaches a value h_1 determined by the criterion

$$p'_{\max.} = 1/\alpha \tag{14}$$

and that this value is maintained despite further increments in the load. Then from equations (4) and (14) one obtains the expression

$$h_1 = 0{\cdot}67(\alpha\eta_0 V)^{\frac{8}{3}}R^{\frac{1}{3}} \tag{15}$$

which therefore represents a first approximate value of the thickness of the parallel film existing under elasto-hydrodynamic conditions.

A fuller analysis of elasto-hydrodynamic lubrication at point contacts has also been made (Archard, unpublished). This employs methods similar to those used by Grubin (1949) in his analysis of line contacts and makes an allowance for the effect

of side leakage in the entry region (figure 6). The film thickness h_2 obtained in this way is given by the equation

$$h_2 = M(\alpha\eta_0 V)^{0\cdot741} R^{0\cdot407}(E/W)^{0\cdot074}, \tag{16}$$

where M is a numerical constant $\simeq 0\cdot84$. Comparison of equations (15) and (16) shows that equation (15) is a much closer approximation than might have been expected. In addition, equation (16) shows

(*a*) that the film thickness decreases very slowly with increasing load, and

(*b*) that the relationship between h and the other variables is slightly modified; h varies with η_0, α, V and R raised to powers slightly greater than those of equation (15).

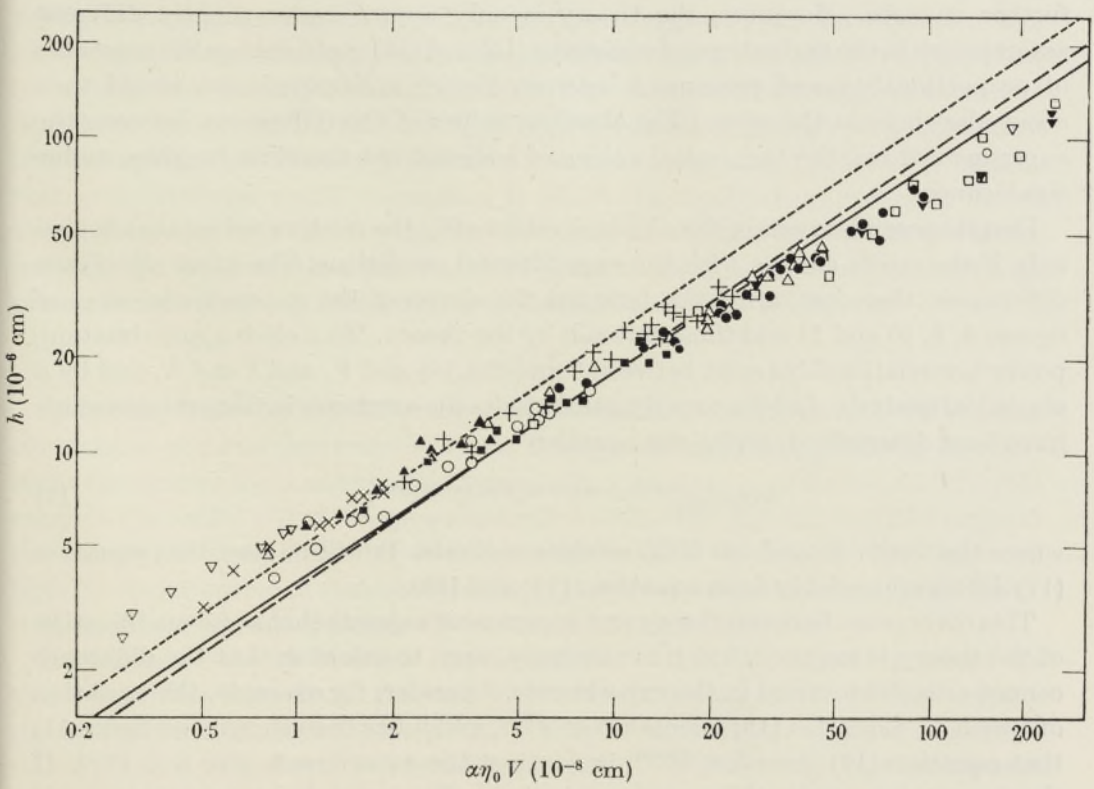

FIGURE 11. Film thickness (h) as a function of ($\alpha\eta_0 V$) for $R = 1\cdot97$ cm. Experimental values (h_e) at different speeds in cm/s; ▼, 370; □, 185; ●, 93; △, 46·5; +, 23·2; ■, 11·6; ○, 5·8; ▲, 2·9; ×, 1·5; ▽, 0·73. ———, h_1 (theoretical, equation (15)); - - - - -, h_2 (theoretical, equation (16)) for min. loads; — — —, h_2 (theoretical, equation (16)) for max. loads.

5·4. *Discussion*

In figure 11 the experimental values of the film thickness (h_e) for specimens of radius $R = 1\cdot97$ cm are plotted as a function of ($\alpha\eta_0 V$). Theoretical values of the film thickness for the same experiments have been calculated from equations (15) and (16). Equation (15) predicts a single value of h_1 for each experiment but a range of values of h_2 (equation (16)) is possible because h_2 is not independent of the load.

Moreover, at the highest values of $(\alpha\eta_0 V)$ loads of up to 40 Kg were used but at the lowest values the maximum load which could be applied was only 1 Kg; the effect of this trend towards increasing loads with increasing $(\alpha\eta_0 V)$ upon the graph of h_2 has been to depress its slope from 0·74, for a fixed load, to 0·70. Values of h_1, and values of h_2 corresponding to the maximum and minimum loads used at each value of $(\alpha\eta_0 V)$, are plotted in figure 11.

Figure 11 shows quite clearly the consistency of the experiments; all the results lie within 25 % of the best line. The overall agreement between the values of h_e, h_1 and h_2 is also very satisfactory but, and this should be emphasized, the fact that agreement between theory and experiment is best at the centre of the range may be fortuitous. The values of h_e include corrections which increased them by some 30 to 40 % and it is conceivable that other factors, not yet included, might make further changes. Moreover, the theory is only approximate; slightly different assumptions in the derivations of equations (15) and (16) could change the constants of proportionality and agreement between theory and experiment would then occur elsewhere in the range. The absolute values of the differences between the experimental and the theoretical values of h should not therefore be given undue significance.

Despite possible errors in the absolute values of h_e, the relative values are affected only if the errors change with the experimental conditions. The more significant differences, therefore, are those between the slopes of the experimental lines of figures 8, 9, 10 and 11 and those forecast by the theory. To a close approximation, power law relationships exist between h and $(\alpha\eta_0)$, h and V, and h and R, and by a statistical analysis of all the experimental results the constants in these relationships have been determined, giving the equation

$$h \propto (\alpha\eta_0)^{0\cdot57\pm0\cdot02} V^{0\cdot55\pm0\cdot01} R^{0\cdot62\pm0\cdot03}, \tag{17}$$

where the limits quoted are 90 % confidence limits. It will be seen that equation (17) differs appreciably from equations (15) and (16).

This divergence between theory and experiment suggests that some modification of the theory is required, but it is necessary, first, to establish that the difference cannot arise from errors in the experiments. Consider, for example, the variation of h with V. Equation (15) forecasts $h \propto V^{0\cdot67}$, and it has been shown (see figure 11) that equation (16) gives $h \propto V^{0\cdot70}$; in contrast the experiments give $h \propto V^{0\cdot55}$. If the theory were correct this would require large changes in the systematic errors of the experiments; for example, if the values of h_e are assumed correct at one end of the velocity range they would need to be in error by a factor of two at the other. Alternatively, if the discrepancy arose through errors in the measurement of temperature this would require the recorded temperature, if correct at the lowest speed, to be too low by as much as 20 °C at the highest speed. Likewise the divergence cannot be due to the heating of the oil by frictional losses. The temperature rise of the oil as it passes through the main load bearing region can be calculated with sufficient accuracy from existing theory (Archard 1959). For the six lowest speeds of figure 9 this rise in temperature is negligible (about 1 °C or less) but the values of h_e for these speeds still conform to the relationship $h \propto V^{0\cdot55}$.

Finally one can note the character of the changes required to adjust the experiments to agree with the theory. When considering the dependence of h upon V it is necessary to raise the high values of h_e and to reduce the low values. When considering the dependence of h upon R the reverse is true; the high values of h_e must be reduced and the low values increased. It is difficult to imagine systematic errors which will meet both requirements and, in view of the arguments in the preceding paragraph, it is concluded that the imperfections lie in the theory.

An outstanding feature of the results is that, at the lower speeds, values of h_e of a few hundred ångströms have been deduced. (The lowest value is about 200 Å.) Figures 8, 9 and 10 show that these values form part of a logical and consistent pattern of results extending up to values greater than 1 μ. One may also note that these results have been interpreted in terms of a theory which is concerned solely with the bulk properties of the oil. It has been suggested (see, for example, Henniker 1949; Deryaguin 1958) that the properties of a liquid close to an interface are modified by the surface molecular fields but estimates of the range of this effect vary between zero and 1000 Å. If such effects upon the viscosity were significant in the experiments described above one would expect that as the film became thinner a thickness would be reached at which the modified properties of the oil would intervene and cause a marked deviation of the results from their regular pattern. No such deviation has been noted and there appears to be no compelling reason for invoking any effect due to the surface molecular fields.

6. Conclusions

It is clear from the experimental results that elasto-hydrodynamic lubrication can exist at nominal point contacts over a very wide range of conditions. The present experiments cover a twelvefold range in the viscosity (η_0) of the oil, a twelvefold range in the radius (R) of the specimens and a more than five hundredfold range in the velocity V. There is a sixtyfold range in the deduced values of the film thickness (h_e). No major source of systematic error in the experiments has been found which is likely to modify the deduced relationship $h_e \propto (\alpha\eta_0)^{0.57} V^{0.55} R^{0.62}$.

No published theory of elasto-hydrodynamic lubrication at point contacts yet exists but it has been shown that application of the principles used in the current theory for line contacts allows a simple theory to be outlined. This yields a result which must lie close to the relationship $h \propto (\alpha\eta_0 V)^{\frac{2}{3}} R^{\frac{1}{3}}$. Although this theory provides estimates of the film thickness which are close to those found in the experiments it needs to be modified so as to forecast the correct dependence of h upon $(\alpha\eta_0)$, V and R.

The experiments of this paper cover a range of conditions, particularly with heavy loads or with very low speeds, where hitherto it has been assumed that only boundary lubrication could occur. However, it has been found that the surfaces are not protected solely by boundary films having molecular dimensions and possessing properties derived from molecular behaviour; instead it is clear that the thicknesses of the films are determined by the bulk properties of the oil. It appears that hydrodynamic lubrication occurs much more frequently and boundary lubrication much less frequently than is usually supposed.

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