MACHINE LEARNING

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HW week1

1 Problem 1

y1	0.01	0.02	0.03	0.1	0.1
y2	0.05	0.1	0.05	0.07	0.2
y3	0.1	0.05	0.03	0.05	0.04
	x1	x2	x3	x4	x5

a) Marginal distribution:

- p(x)

$$p(x_1) = p(X = x_1) = p(X = x_1, Y = y_1) + p(X = x_1, Y = y_2) + p(X = x_1, Y = y_3) = 0.01 + 0.05 + 0.01 = 0.16 \tag{1}$$

$$p(x_2) = p(X = x_2) = p(X = x_2, Y = y_1) + p(X = x_2, Y = y_2) + p(X = x_2, Y = y_3) = 0.02 + 0.1 + 0.05 = 0.17 \tag{2}$$

$$p(x_3) = p(X = x_3) = p(X = x_3, Y = y_1) + p(X = x_3, Y = y_2) + p(X = x_3, Y = y_3) = 0.03 + 0.05 + 0.03 = 0.11 \tag{3}$$

$$p(x_4) = p(X = x_4) = p(X = x_4, Y = y_1) + p(X = x_4, Y = y_2) + p(x = x_4, Y = y_3) = 0.1 + 0.07 + 0.05 = 0.22 \tag{4}$$

$$p(x_5) = p(X = x_5) = p(X = x_5, Y = y_1) + p(X = x_5, Y = y_2) + p(x = x_5, Y = y_3) = 0.1 + 0.2 + 0.04 = 0.34 \tag{5}$$

- p(y)

$$p(y_1) = p(Y = y_1) = 0.01 + 0.02 + 0.03 + 0.1 + 0.1 = 0.26 \tag{6}$$

$$p(y_2) = p(Y = y_2) = 0.05 + 0.1 + 0.05 + 0.07 + 0.2 = 0.47 \tag{7}$$

$$p(y_3) = p(Y = y_3) == 0.1 + 0.05 + 0.03 + 0.05 + 0.04 = 0.27 \tag{8}$$

b) Condition distribution:

- $p(x|Y=y_1)$

$$\begin{aligned}
p(x_1|Y=y_1) &= \frac{p(X=x_1,Y=y_1)}{p(y_1)} = \frac{0.01}{0.26} \approx 0.038 && (9)\\
p(x_2|Y=y_1) &= \frac{p(X=x_2,Y=y_1)}{p(y_1)} = \frac{0.02}{0.26} \approx 0.077 && (10)\\
p(x_3|Y=y_1) &= \frac{p(X=x_3,Y=y_1)}{p(y_1)} = \frac{0.03}{0.26} \approx 0.115 && (11)\\
p(x_4|Y=y_1) &= \frac{p(X=x_4,Y=y_1)}{p(y_1)} = \frac{0.1}{0.26} \approx 0.385 && (12)\\
p(x_5|Y=y_1) &= \frac{p(X=x_5,Y=y_1)}{p(y_1)} = \frac{0.1}{0.26} \approx 0.385 && (13)\\
& && (14)
\end{aligned}$$

- $p(x|Y=y_3)$

$$\begin{aligned}
p(x_1|Y=y_3) &= \frac{p(X=x_1,Y=y_3)}{p(y_3)} = \frac{0.1}{0.27} \approx 0.37 && (15)\\
p(x_2|Y=y_3) &= \frac{p(X=x_2,Y=y_3)}{p(y_3)} = \frac{0.05}{0.27} \approx 0.185 && (16)\\
p(x_3|Y=y_3) &= \frac{p(X=x_3,Y=y_3)}{p(y_3)} = \frac{0.03}{0.27} \approx 0.11 && (17)\\
p(x_4|Y=y_3) &= \frac{p(X=x_4,Y=y_3)}{p(y_3)} = \frac{0.05}{0.27} \approx 0.185 && (18)\\
p(x_5|Y=y_3) &= \frac{p(X=x_5,Y=y_3)}{p(y_3)} = \frac{0.04}{0.27} \approx 0.148 && (19)\\
& && (20)
\end{aligned}$$

2 Problem 2

We have:

$$\begin{aligned}
E_Y[E_X[x|y]] &= \sum E(X|Y=y)\cdot P(Y=y)\\
&= \sum_y \sum_x xP(X=x|Y=y)\cdot P(Y=y)\\
&= \sum_y \sum_x xP(Y=y|X=x)\cdot P(X=x)\\
&= \sum_x P(X=x) \sum_y P(Y=y|X=x)\\
&= E_X(X)
\end{aligned}$$

3 Problem 3

Got:

- X: people who use product X

- Y: people who use product Y

We have:
P(X)=0.207 , P(Y)=0.5 , P(A—B)=0.365

a) $P(AB) = P(A|B) \cdot P(B) = 0.5 \cdot 0.365 = 0.1825$

b) $P(B|\overline{A}) = \frac{P(\overline{A}|B) \cdot P(B)}{P(\overline{A})} = \frac{(1-0.365) \cdot 0.5}{1-0.207} = 0.4004$

4 Problem 4

We have the standard deviation: $V_X[x] = E_X[(x-\mu)^2]$
Using the properties of avarage we have:

$$\begin{aligned} V_X[x] &= E_X[(x-\mu)^2] \\ &= E_X[x^2 - 2x\mu + \mu^2] \\ &= E_X[x^2] - E_X[2x\mu] + E_X[\mu^2] \\ &= E_X[x^2] - 2\mu E_X[x] + \mu^2 \\ &= E_X[x^2] - 2\mu^2 + \mu^2 \\ &= E_X[x^2] - \mu^2 \end{aligned}$$

We have: $\mu = E_X[x]$
=> $V_X[x] = E_X[(x-\mu)^2] = E_X[x^2] - (E_X[x])^2$

5 Problem 5

We have:

- A: door 1 has a car
- B: Monty open the door 2
- C: door 3 has a car

$P(A) = \frac{1}{3}$
$P(B) = \frac{1}{2}$
We have: $P(B|A) = \frac{1}{2}$ is the probability Monty open the door 2 given that the door 1 has the car
$P(A|B) = \frac{1}{3}$ is the probability that the car is in the door 1 after Monty open the door 2
Event A and event C are 2 mutually exclusive events so:
=> $P(C) = 1 - P(A) = \frac{2}{3}$
=> The chances of winning the car are indeed 2 times higher (2/3) when you switch than when you stick (1/3)