



## Signal Processing I

## Laboratory 4

## Analysis of Continuous-Time Systems Using Laplace.

## Part I

We will continue using Matlab's symbolic math processor in this lab.

We will analyse Problem 4.4.13 as shown in the next figure:

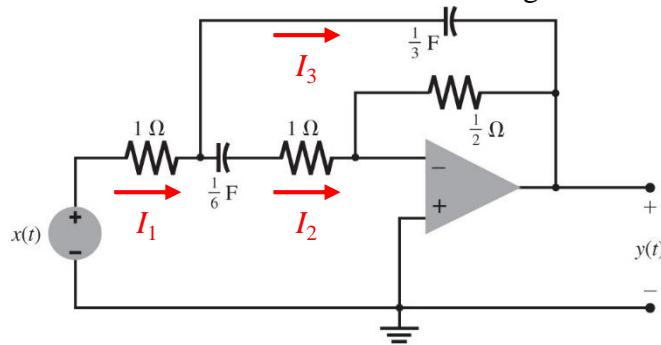


Figure 1.

Using basic circuit theory

$$\begin{aligned} -X(s) + I_1(s) + \frac{6}{s}I_3(s) + I_2(s) &= 0 \\ I_3(s) + I_2(s) &= I_1(s) \\ I_2(s) &= -2Y(s) \\ -X(s) + I_1(s) + \frac{3}{s}I_3(s) + Y(s) &= 0 \end{aligned}$$

Using the symbolic tool in Matlab we can form the matrix problem

$$\begin{bmatrix} -1 & 1 & \frac{6}{s} + 1 & 0 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & -1/2 & 0 \\ -1 & 1 & 0 & 3/s \end{bmatrix} \begin{bmatrix} X \\ I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ Y \\ -Y \end{bmatrix}$$

as

```
>> syms Y s
>> A = sym([-1 1 (1+6/s) 0; 0 -1 1 1 ; 0 0 -1/2 0; -1 1 0 3/s])
>> B = sym([0; 0; Y; -Y])
>> inv(A)*B
```

ans =

$$\begin{array}{ll}
 - (Y^*s)/3 - (2*Y^*(s^2 + 12*s + 18))/(3*s) & X(s) \\
 - (Y^*s)/3 - Y^*((2*s)/3 + 6) & I_1 \\
 - 2*Y & I_2 \\
 - (Y^*s)/3 - Y^*((2*s)/3 + 4) & I_3
 \end{array}$$

From the matrix inversion we have

$$-\frac{s}{3}Y(s) - \frac{2Y(s)[s^2 + 12s + 18]}{3s} = X(s)$$

Using again symbolic math expressions

```
>> ff = - (Y^*s)/3 - (2*Y^*(s^2 + 12*s + 18))/(3*s)
>> simplify(ff)
```

ans =

$$-(Y^*(s^2 + 8*s + 12))/s$$

We then have

$$\begin{aligned}
 -\frac{Y(s)[s^2 + 8s + 12]}{s} &= X(s) \\
 \Rightarrow \frac{Y(s)}{X(s)} &= \frac{-s}{[s^2 + 8s + 12]} = H(s)
 \end{aligned} \tag{1}$$

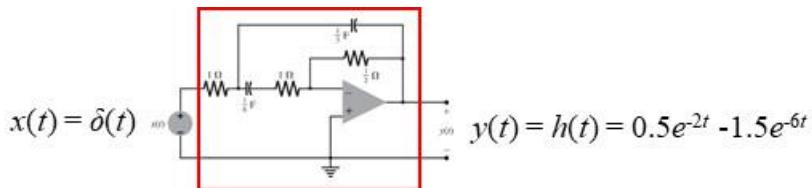
And that is how you can easily calculate the transfer function of a system. The impulse response of the system is just the inverse that can also easily be calculated as

```
>> H = sym(-s/(s^2 + 8*s + 12))
>> imp_response = ilaplace(H)
```

imp\_response =

$$\exp(-2*t)/2 - (3*\exp(-6*t))/2$$

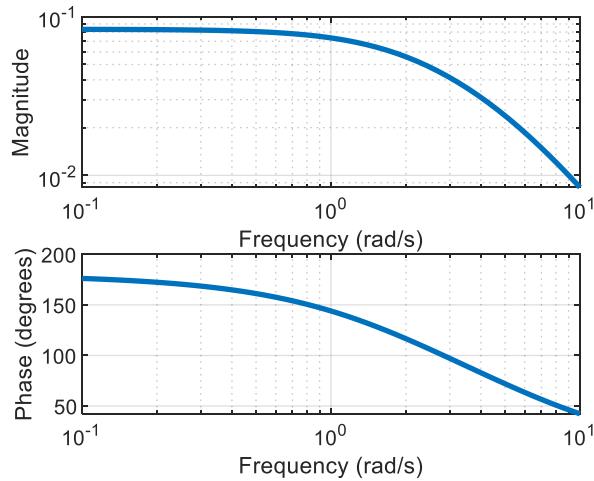
And literally that is what you obtained, if your circuit was in a box (red rectangle) you try to identify it by doing



Once you have  $h(t)$  you can compute any output for any input signal  $x(t)$  by **convolutions**. Most importantly, if  $h(t)$  models degradation of a signal, like an undesired echo in audio, motion blurring in an image etc., knowing  $h(t)$  may be the first step to counteract these unwanted degradations.

If you want to have an idea of the frequency response of a system, you can easily do this in Matlab as identifying the coefficients of the polynomials on  $H(s)$  (equation 1 above) and doing the following:

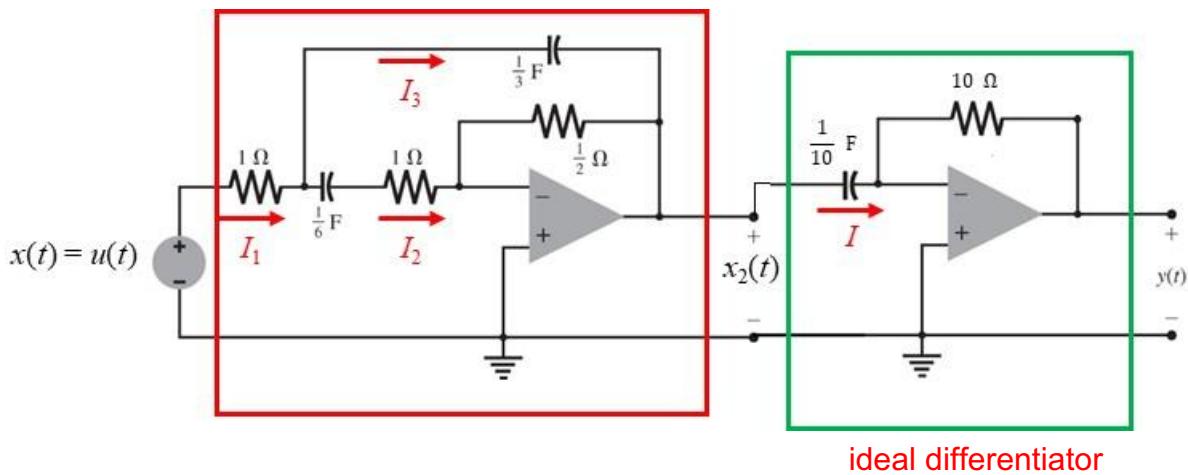
```
b = [0 -1];
a = [1 8 12];
w = logspace(-1,1);
freqs(b,a,w)
```



**Figure 2.**

There is no lab equipment in the world that can generate an impulse though or an oscilloscope that can record it. But it is very easy to have  $x(t)$  be a unit step. It is just a constant voltage connected to a switch that when you close it and feed the system you can easily check for the step response. Let us introduce a second system in the green square in the figure.

Question: What is the transfer function  $H_2(s) = Y(s)/X_2(s)$



Question: What is the system within the green box doing?

Question: What is the final  $Y(s)$  and  $y(t)$  for the unit step input?  
 $H(s) \cdot X(s)$   $\text{ilaplace}(H(s))$

Question: From what you know about Linear Time-Invariant systems, can you explain in the time domain the results you obtained in the last question?

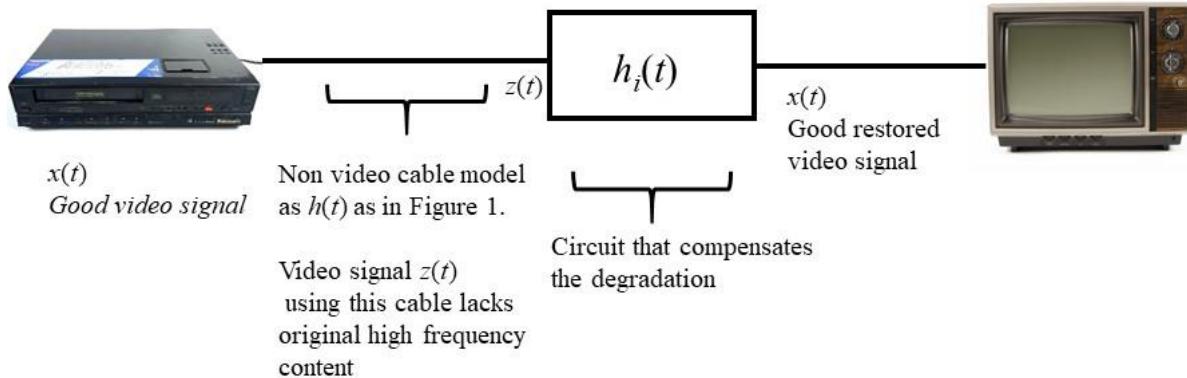
Question: Is the system in Figure 1 stable?

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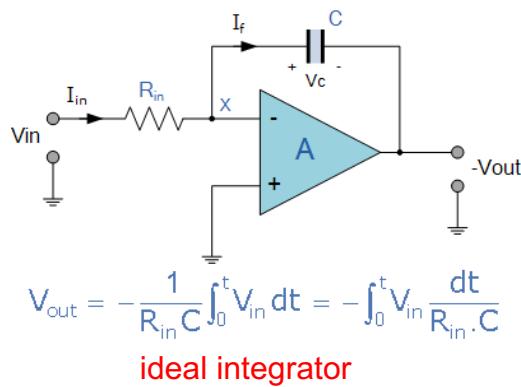
You found an analog video system that you would like to connect to an analog TV. You do not have video coaxial cables and the cable you have available will attenuate higher frequencies and is modeled by the system in Figure 1.

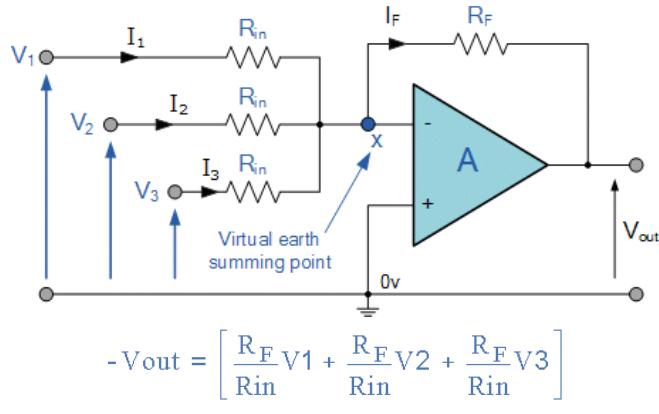
Question: Can you *describe* an analog circuit that can be connected to your available cable and compensate for the higher frequencies attenuation?

Hint 1: your nice input video signal is  $x(t)$  that enters  $h(t)$  and that output needs to go to the system you designed  $h_i(t)$  that will be connected to your TV. Work in the Laplace domain.



Hint 2:





Question: You found that you have some spare general purpose 741 OpAmps. Can you use them for this project? You have been provided the data sheet to make such decision.

## Part II

$$H(s)$$

Following the same approach as before, calculate i) the transfer function, ii) the impulse response of the following circuit and iii) determine what type of filter is this.

$$h(t)$$

$$\begin{aligned} R_1 &= 200; \\ C_1 &= 2e-6; \\ R_2 &= 200; \\ C_2 &= 1e-6; \\ R_3 &= 100; \\ C_3 &= 1e-6; \\ R_{\text{load}} &= 1000; \end{aligned}$$

