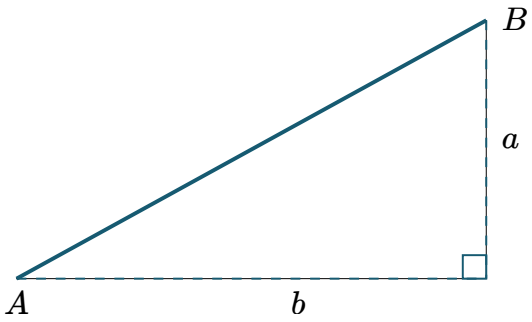
Gradient (or slope) of a Line, and Inclination

The **gradient** (also known as **slope**) of a line is defined as

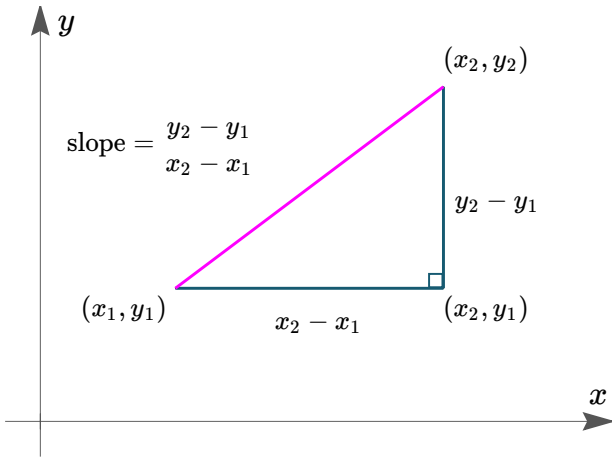
$$\text{gradient} = \frac{\text{vertical rise}}{\text{horizontal run}}$$

In the following diagram, the gradient of the line AB is given by: $\frac{a}{b}$



Application: Road sign, indicating a steep gradient
A 15% road gradient is equivalent to $m = 0.15$.

In general, for the line joining the points (x_1, y_1) and (x_2, y_2), we have:



Slope of the line joining the points (x_1, y_1) and (x_2, y_2).

We can now write the formula for the slope of a line.

Gradient of a Line Formula

We see from the diagram above, that the **gradient** (usually written m) is given by:

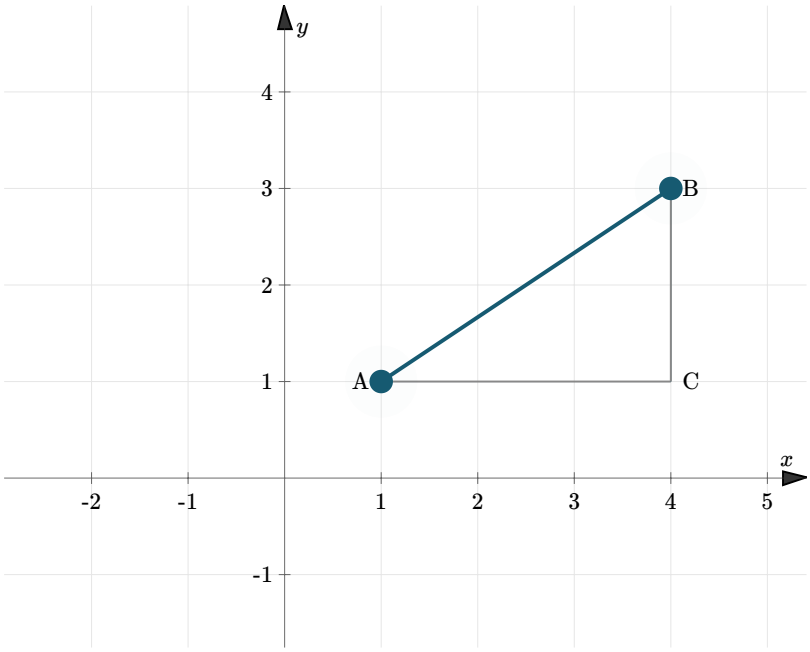
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Interactive graph - slope of a line

You can explore the concept of slope of a line in the following interactive graph (it's not a fixed image).

Drag either point A (x_1, y_1) or point B (x_2, y_2) to investigate how the gradient formula works. The numbers will update as you interact with the graph.

Notice what happens to the sign (plus or minus) of the slope when point B is above or below A.



Slope $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{BC}{AC}$

$= \dfrac{3.00 - 1.00}{4.00 - 1.00}$

$= 0.67$

You can move the graph up-down, left-right if you hold down the "Shift" key and then drag the graph.

If you get lost, you can always refresh the page.

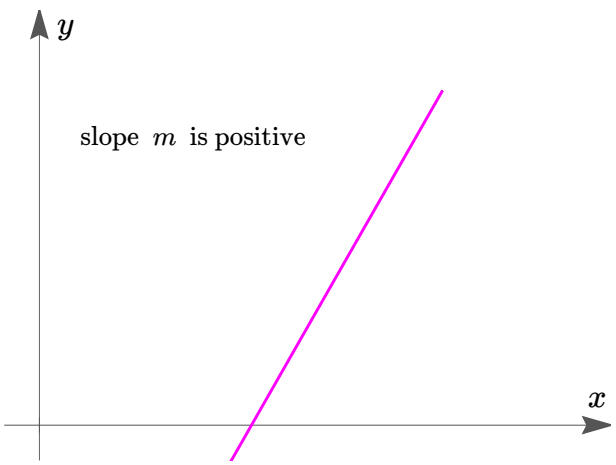
Example

Find the slope of the line joining the points $(-4, -1)$ and $(2, -5)$.

Answer

Positive and Negative Slopes

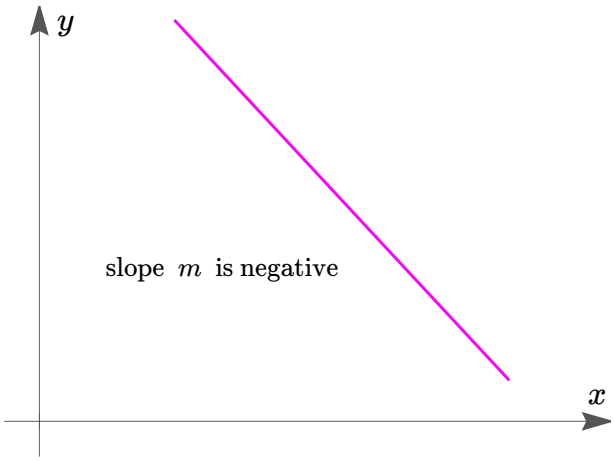
In general, a **positive slope** indicates the value of the dependent variable (usually y) **increases** as we go left to right:



The line has **positive slope**.

The **dependent variable** in the above graph is the y-value, while the **independent** variable is x.

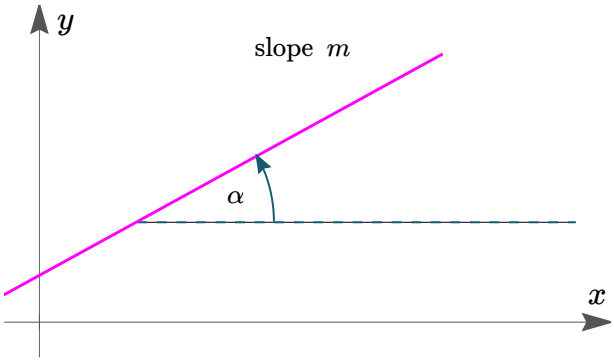
A **negative slope** means that the value of the dependent variable (usually y) is **decreasing** as we go left to right:



The line has **negative slope**.

Inclination

We have a line with slope m and the angle that the line makes with the x-axis is α.



Angle α is the **inclination** of the line with slope m.

From trigonometry, we recall that the tan of angle α is given by:

$$\tan \alpha = \frac{\text{opposite}}{\text{adjacent}}$$

Now, since slope is also defined as opposite/adjacent, we have:

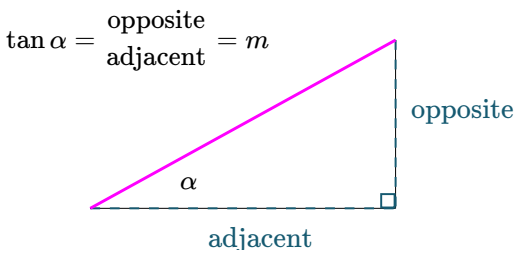


Diagram illustrating $\tan \alpha = m$.

This gives us the result:

$$\boxed{\tan \alpha = m}$$

Then we can find angle α using

$$\boxed{\alpha = \arctan m}$$

(That is, $\alpha = \tan^{-1} m$)

This angle α is called the **inclination** of the line.

Exercise 1

Find the inclination of the line with slope 2.

Answer

NOTE: The size of angle α is (by definition) only between 0° and 180°.

Exercise 2

Find the slope of the line with inclination $\alpha = 137^\circ$.

Answer