$$h_1^{sum} = x_1 * w_1 + x_2 * w_2 + b_1$$
$$h_2^{sum} = x_1 * w_3 + x_2 * w_4 + b_2$$

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

$$h_1 = \sigma(h_1^{sum}) = \frac{1}{1 + e^{-h_1^{sum}}}$$
$$h_2 = \sigma(h_2^{sum}) = \frac{1}{1 + e^{-h_2^{sum}}}$$

$$o_1^{sum} = h_1 * w_5 + h_2 * w_6 + b_3$$
$$o_1 = \sigma(o_1^{sum}) = \frac{1}{1 + e^{-o_1^{sum}}}$$

$$\begin{aligned} L &= \frac{1}{1} \sum_{i=1}^{1} (y_i - \hat{y}_i)^2 \\ &= (y_{true} - y_{pred})^2 \\ &= (1 - 0.9824945598864127)^2 \\ &= 0.0003064404335703929 \end{aligned}$$

$$L = (y_{true} - y_{pred})^2$$

$$L = L(y_{pred})$$

$$
\begin{aligned}
y_{pred} &= o_1 \\
&= \sigma(o_1^{sum}) \\
&= \frac{1}{1+e^{-o_1^{sum}}} \\
&= \frac{1}{1+e^{-(h_1*w_5+h_2*w_6+b_3)}} \\
&= \frac{1}{1+e^{-\left(\frac{1}{1+e^{-h_1^{sum}}}*w_5+\frac{1}{1+e^{-h_2^{sum}}}*w_6+b_3\right)}} \\
&= \frac{1}{1+e^{-\left(\frac{1}{1+e^{-(x_1*w_1+x_2*w_2+b_1)}}*w_5+\frac{1}{1+e^{-(x_1*w_3+x_2*w_4+b_2)}}*w_6+b_3\right)}}
\end{aligned}
$$

$$
L = L(y_{pred}) = L(w_1, w_2, w_3, w_4, w_5, w_6, b_1, b_2, b_3)
$$

$$
\frac{\partial L}{\partial w_i}, \quad (i = 1, 2, 3, 4, 5, 6)
$$

$$
\frac{\partial L}{\partial b_j}, \quad (j = 1, 2, 3)
$$

$$
w_i = w_i - \eta * \frac{\partial L}{\partial w_i}, \quad (i = 1, 2, 3, 4, 5, 6)
$$

$$
b_j = b_j - \eta * \frac{\partial L}{\partial b_j}, \quad (j = 1, 2, 3)
$$

where, η is a constant and it is called **learning rate**.

$$
y_{pred} = o_1
$$

$$
y_{pred} = \frac{1}{1+e^{-(h_1*w_5+h_2*w_6+b_3)}}
$$

$$
\begin{aligned}
\frac{\partial L}{\partial w_1} &= \frac{\partial L}{\partial y_{pred}} \frac{\partial y_{pred}}{\partial w_1} \\
&= -2(y_{true} - y_{pred}) \frac{\partial y_{pred}}{\partial w_1} \\
\frac{\partial y_{pred}}{\partial w_1} &= \frac{\partial y_{pred}}{\partial h_1} \frac{\partial h_1}{\partial w_1} \\
&= \frac{\partial\left(\frac{1}{1+e^{-(h_1*w_5+h_2*w_6+b_3)}}\right)}{\partial h_1} \frac{\partial h_1}{\partial w_1} \\
&= \frac{\partial\left(\frac{1}{1+e^{-(h_1*w_5+h_2*w_6+b_3)}}\right)}{\partial(h_1*w_5+h_2*w_6+b_3)} \frac{\partial(h_1*w_5+h_2*w_6+b_3)}{\partial h_1} \frac{\partial h_1}{\partial w_1} \\
&= w_5 \frac{\partial\left(\frac{1}{1+e^{-(h_1*w_5+h_2*w_6+b_3)}}\right)}{\partial(h_1*w_5+h_2*w_6+b_3)} \frac{\partial h_1}{\partial w_1} \\
&= w_5 \frac{\partial \sigma(o_1^{sum})}{\partial o_1^{sum}} \frac{\partial h_1}{\partial w_1}
\end{aligned}
$$

$$\frac{\partial \sigma(o_1^{sum})}{\partial o_1^{sum}} = \sigma'(o_1^{sum}) = \sigma(o_1^{sum})(1 - \sigma(o_1^{sum}))$$

$$\begin{aligned}
\frac{\partial y_{pred}}{\partial w_1} &= w_5 \sigma'(o_1^{sum}) \frac{\partial h_1}{\partial w_1} \\
&= w_5 \sigma'(o_1^{sum}) \frac{\partial h_1}{\partial h_1^{sum}} \frac{\partial h_1^{sum}}{\partial w_1} \\
&= w_5 \sigma'(o_1^{sum}) \frac{\partial \sigma(h_1^{sum})}{\partial h_1^{sum}} \frac{\partial h_1^{sum}}{\partial w_1} \\
&= w_5 \sigma'(o_1^{sum}) \sigma'(h_1^{sum}) \frac{\partial h_1^{sum}}{\partial w_1} \\
&= w_5 \sigma'(o_1^{sum}) \sigma'(h_1^{sum}) x_1
\end{aligned}$$

$$\frac{\partial L}{\partial w_1} = -2(y_{true} - y_{pred}) w_5 \sigma'(o_1^{sum}) \sigma'(h_1^{sum}) x_1$$

$$\frac{\partial L}{\partial w_2} = -2(y_{true} - y_{pred}) w_5 \sigma'(o_1^{sum}) \sigma'(h_1^{sum}) x_2$$
$$\frac{\partial L}{\partial w_3} = -2(y_{true} - y_{pred}) w_6 \sigma'(o_1^{sum}) \sigma'(h_2^{sum}) x_1$$
$$\frac{\partial L}{\partial w_4} = -2(y_{true} - y_{pred}) w_6 \sigma'(o_1^{sum}) \sigma'(h_2^{sum}) x_2$$

$$\frac{\partial L}{\partial w_5} = -2(y_{true} - y_{pred}) \sigma'(o_1^{sum}) h_1$$
$$\frac{\partial L}{\partial w_6} = -2(y_{true} - y_{pred}) \sigma'(o_1^{sum}) h_2$$

$$\frac{\partial L}{\partial b_1} = -2(y_{true} - y_{pred}) w_5 \sigma'(o_1^{sum}) \sigma'(h_1^{sum})$$
$$\frac{\partial L}{\partial b_2} = -2(y_{true} - y_{pred}) w_6 \sigma'(o_1^{sum}) \sigma'(h_2^{sum})$$
$$\frac{\partial L}{\partial b_3} = -2(y_{true} - y_{pred}) \sigma'(o_1^{sum})$$

$$g_1 \circledast g_2 \circledast \cdots \circledast g_n = \overset{n}{\underset{i=1}{\circledast}} g_i$$

$$\underbrace{g \circledast g \circledast \cdots \circledast g}_{m} = g^{\circledast m}$$

$\circledast$
$\bigoplus$

σ

$\boldsymbol{b}^1$

$\boldsymbol{b}^2$

$\boldsymbol{k}^1$

$\boldsymbol{k}^2$

$\boldsymbol{w}$

$\boldsymbol{b}$

$\odot$

$\boldsymbol{o}$

$$\begin{bmatrix} o_0 \\ o_1 \\ o_2 \\ o_3 \\ o_4 \\ o_5 \\ o_6 \\ o_7 \\ o_8 \\ o_9 \end{bmatrix}$$

$\boldsymbol{C}^1$

$\boldsymbol{C}^2$

$\boldsymbol{P}^1$

$\boldsymbol{P}^2$

$k^1_{m,n,1}$

$k^1_{m,n,2}$

$k^1_{m,n,6}$

$k^2_{m,n,1}$

$k^2_{m,n,2}$

$k^2_{m,n,3}$

b^1_1

b^1_2

b^1_6

b^2_1

b^2_2

$C^1_{g,h,1}$

$C^1_{g,h,2}$

C^1_6

$C^2_{p,q,1}$

$P^1_{r,s,1}$

$P^1_{r,s,2}$

$P^2_{u,v,1}$


V