TECHNICAL NOTE

COMBINED SENSITIVITIES OF T2K-II AND NOνA EXPERIMENTS TO CP-VIOLATION IN NEUTRINO SECTOR

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0.1 PHYSICS MOTIVATION

In order to explain the solar neutrino anomaly [1] and atmospheric neutrino anomaly [2], neutrino oscillation phenomenon, in which one type of neutrino can change into another, has been proposed. In 1957, B. Pontecovo [3] firstly suggested a neutrino-antineutrino transition to explain these anomalies. The neutrino flavor oscillation was introduced later in 1962 by Z. Maki, M. Nakagawa and S. Sakata [4]. The neutrino oscillation phenomenon was observed by Super-Kamiokande experiment [21], SNO experiment [22] and later conclusively confirmed by number of neutrino experiments with different detection techniques at different energy range and different baselines. Discovery of neutrino oscillation which indicates that neutrinos have mass and mix among states, matter a lot since this is the only experimental evidence for the incompleteness of the Standard Model of fundamental particles.

Except for some anomalies, the up-to-date (anti-)neutrino data from various experiments can be well described by a 3×3 unitary mixing matrix, so-called PMNS matrix. This unitary matrix, as discussed more details in the next section, are parameterized by three mixing angles (θ_{12}, θ_{13} and θ_{23}) and one single Dirac phase $\delta_{\rm CP}$[1] which represents CP violation in the lepton sector. The three mixing angles are determined to be non-zero [23] and this allows neutrino experiments to make measurement on the CP violation in the lepton sector, which is one of the most central objective in the present and at near future of neutrino physics. Besides these four parameters, the oscillation probabilities depend on the mass-squared differences among the mass eigenstate, neutrino energy and the distance neutrino travels. At the current landscape of neutrino oscillation physics, two scales of mass-squared differences are determined. However their mass ordering is still unknown and also one of the most important question need to be addressed in the future.

T2K and NOvA are two among the world leading neutrino experiments in searching the CP violation in the lepton sector. The combined sensitivity of these two experiments was performed and shows that this sensitivity can be up to 2σ or higher if the true value δ_{CP} is about $-\pi/2$ [11]. We are revising this analysis with three main updates including (i) possible T2K run extension up to 2026, so-called T2K-II (ii) improvement in selection performance and systematic uncertainties in both experiment and (iii) the ultimate precision on mixing angle θ_{13} can be achieved by the reactor measurements. The first twos are crucial since the measurement is dominated by the statistical errors. The thirds one is needed to break down the $\delta_{CP}-\theta_{13}$ degeneracy with accelerator-based long baseline experiment.These combined critical factors enhances capability to search CP violation to unprecedented level of sensitivity.

The paper is organised as follows, the PMNS formalism of neutrino oscillation is in-

[1] If neutrino is Majorana particle, there are two phases added into the PMNS matrix. However the oscillation amplitudes are not sensitive to these two phases.

troduced with an intense focus on how CP violation can be measured. T2K(-II) and NOvA experiments are overviewed and their inputs for this analysis are presented in section 0.3. The outcomes of combined sensitivity between the T2K-II and NOvA experiment with constraint from reactor are presented in section 0.4.

0.2 NEUTRINO OSCILLATION FRAMEWORK

In three-flavor neutrino oscillation framework, the flavor definitive eigenstates are related to the mass definitive eigenstates by a 3×3 unitary PMNS matrix, shown in Eq. 1,

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{\text{PMNS}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}. \tag{1}$$

The unitarity of PMNS matrix $UU^\dagger = I$ yields nine independent parameters. If the PMNS matrix were real, it could be described by three rotation angles θ_{12}, θ_{13} and θ_{23} via orthogonal rotation matrix R

$$R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2}$$

where $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$. Since PMNS matrix is unitary and not real, it must contain six more additional degrees of freedom in term of complex phase $e^{i\delta}$. Five among these six phases can be absorbed into the definition of the particles and leaves only one single phase δ. This can be seen as follow.

The charged currents for leptonic weak interaction

$$-i\frac{g_W}{\sqrt{2}}(\bar{e}, \bar{\mu}, \bar{\tau})\gamma^\mu \frac{1}{2}(1-\gamma^5) \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

The four-vector currents are unchanged by transformation

$$l_\alpha \to l_\alpha e^{i\theta_\alpha}, \quad \nu_k \to \nu_k e^{i\theta_k} \quad \text{and} \quad U_{\alpha k} \to U_{\alpha k} e^{i(\theta_\alpha - \theta_k)} \tag{3}$$

where l_α is the charged lepton of the type $\alpha = e, \mu, \tau$. Since the phases are arbitrary, all other phases can be defined in term of θ_e:

$$\theta_k = \theta_e + \theta'_k$$

The transformation (3) therefore becomes

$$l_\alpha \to l_\alpha e^{i(\theta_e + \theta'_\alpha)}, \quad \nu_k \to \nu_k e^{i(\theta_e + \theta'_k)} \quad \text{and} \quad U_{\alpha k} \to U_{\alpha k} e^{i(\theta'_\alpha - \theta'_k)}$$

For electron $\theta_e = \theta_e + \theta'_e \quad \Rightarrow \theta'_e = 0$. It is can be seen now that only five phases are independent and can be absorbed into the particle definitions. The PMNS matrix thus can be parameterized by three mixing angles $(\theta_{12}, \theta_{13}, \theta_{23})$ and a single Dirac phase δ_{CP}, expressed in Eq. 4.

$$
\begin{aligned}
U_{PMNS} &= \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \\
&= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}
\end{aligned} \tag{4}
$$

where $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$ and δ_{CP} Dirac phase represents the CP violation in lepton sector [2]. As mentioned before, CP is violated if $U^*_{\alpha i} U_{\beta i} U_{\alpha_j} U^*_{\beta_j}$ contains an imaginary component. Therefore, δ is also called the CP-violating phase δ_{CP}.

The oscillation probability from muon neutrino to electron neutrino is

$$
\begin{aligned}
P(\nu_\mu \to \nu_e) &= |\langle \nu_e | \Psi(\vec{x}, t) \rangle|^2 = c_e c_e^* \\
&= |U^*_{\mu 1} U_{e1} e^{-i\phi_1} + U^*_{\mu 2} U_{e2} e^{-i\phi_2} + U^*_{\mu 3} U_{e3} e^{-i\phi_3}|^2
\end{aligned} \tag{5}
$$

In compact form

$$
P(\nu_\alpha \to \nu_\beta) = |\sum_i U^*_{\alpha i} U_{\beta i} e^{-i\phi_i}|^2 \tag{6}
$$

If $\phi_1 = \phi_2 = \phi_3 (\approx \frac{m^2}{2E})$, from unitary condition we have $P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta}$. This means that the oscillations occur if the neutrinos have mass and the masses are not the same.

Using the identity properties of complex number:

$$
|z_1 + z_2 + z_3|^2 = |z_1|^2 + |z_2|^2 + |z_3|^2 + 2Re[z_1 z_2^* + z_1 z_3^* + z_2 z_3^*] \tag{7}
$$

Then equation (5) becomes

$$
\begin{aligned}
P(\nu_\mu \to \nu_e) &= |U^*_{\mu 1} U_{e1} e^{-i\phi_1} + U^*_{\mu 2} U_{e2} e^{-i\phi_2} + U^*_{\mu 3} U_{e3} e^{-i\phi_3}|^2 \\
&= |U^*_{\mu 1} U_{e1}|^2 + |U^*_{\mu 2} U_{e2}|^2 + |U^*_{\mu 3} U_{e3}|^2 \\
&+ 2Re[U^*_{\mu 1} U_{e1} U_{\mu_2} U^*_{e2} e^{i(\phi_2 - \phi_1)}] \\
&+ 2Re[U^*_{\mu 1} U_{e1} U_{\mu_3} U^*_{e3} e^{i(\phi_3 - \phi_1)}] \\
&+ 2Re[U^*_{\mu 2} U_{e2} U_{\mu_3} U^*_{e3} e^{i(\phi_3 - \phi_2)}]
\end{aligned} \tag{8}
$$

[2] If neutrino is Majorana particle, the mixing matrix includes two additional phases which do not appear in the expression of oscillation probabilities.

In compact form

$$P(\nu_\alpha \to \nu_\beta) = \sum_i |U^*_{\alpha i} U_{\beta i}|^2 + 2\sum_{j>i} Re[U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta_j} e^{i(\phi_j - \phi_i)}] \tag{9}$$

From the unitary condition we derive

$$\begin{aligned}
& |U^*_{\mu 1} U_{e1} + U^*_{\mu 2} U_{e2} + U^*_{\mu 3} U_{e3}|^2 = 0 \\
\Rightarrow \;& |U^*_{\mu 1} U_{e1}|^2 + |U^*_{\mu 2} U_{e2}|^2 + |U^*_{\mu 3} U_{e3}|^2 \\
+ \;& 2Re[U^*_{\mu 1} U_{e1} U_{\mu_2} U^*_{e2} + U^*_{\mu 1} U_{e1} U_{\mu_3} U^*_{e3} + U^*_{\mu 2} U_{e2} U_{\mu_3} U^*_{e3}] \\
= \;& 0
\end{aligned} \tag{10}$$

In compact form

$$\sum_i |U^*_{\alpha i} U_{\beta i}|^2 + 2\sum_{j>i} Re[U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta_j}] = \delta_{\alpha\beta} \tag{11}$$

It is followed from (8) and (10):

$$\begin{aligned}
P(\nu_\mu \to \nu_e) &= 2Re\left[U^*_{\mu 1} U_{e1} U_{\mu_2} U^*_{e2}\left(e^{i(\phi_2 - \phi_1)} - 1\right)\right] \\
&+ 2Re\left[U^*_{\mu 1} U_{e1} U_{\mu_3} U^*_{e3}\left(e^{i(\phi_3 - \phi_1)} - 1\right)\right] \\
&+ 2Re\left[U^*_{\mu 2} U_{e2} U_{\mu_3} U^*_{e3}\left(e^{i(\phi_3 - \phi_2)} - 1\right)\right]
\end{aligned} \tag{12}$$

In compact form

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} + 2\sum_{j>i} Re[U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta_j}(e^{i(\phi_j - \phi_i)}) - 1)] \tag{13}$$

We have

$$\begin{aligned}
& Re[U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta_j}(e^{i(\phi_j - \phi_i)}) - 1)] \\
= \;& Re[U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta_j}(\cos(\phi_j - \phi_i) - 1 + i\sin(\phi_j - \phi_i))] \\
= \;& Re\left\{(Re[U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta_j}] + iIm[U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta_j}])(-2\sin^2(\frac{\phi_j - \phi_i}{2}) + i\sin(\phi_j - \phi_i))\right\} \\
= \;& -2Re[U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta_j}]\sin^2(\frac{\phi_j - \phi_i}{2}) - Im[U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta_j}]\sin(\phi_j - \phi_i)
\end{aligned} \tag{14}$$

From (14), we can write the oscillation pobability in a normal form

$$\begin{aligned}
& P(\nu_\mu \to \nu_e) = \\
- \;& 4Re\left[U^*_{\mu 1} U_{e1} U_{\mu_2} U^*_{e2}\right]\sin^2(\frac{\phi_2 - \phi_1}{2}) - 2Im\left[U^*_{\mu 1} U_{e1} U_{\mu_2} U^*_{e2}\right]\sin(\phi_2 - \phi_1) \\
- \;& 4Re\left[U^*_{\mu 1} U_{e1} U_{\mu_3} U^*_{e3}\right]\sin^2(\frac{\phi_3 - \phi_1}{2}) - 2Im\left[U^*_{\mu 1} U_{e1} U_{\mu_3} U^*_{e3}\right]\sin(\phi_3 - \phi_1) \\
- \;& 4Re\left[U^*_{\mu 2} U_{e2} U_{\mu_3} U^*_{e3}\right]\sin^2(\frac{\phi_3 - \phi_2}{2}) - 2Im\left[U^*_{\mu 2} U_{e2} U_{\mu_3} U^*_{e3}\right]\sin(\phi_3 - \phi_2)
\end{aligned} \tag{15}$$

In compact form

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{j>i} Re\left[U^*_{\alpha i}U_{\beta i}U_{\alpha_j}U^*_{\beta_j}\right]\sin^2(\frac{\phi_j-\phi_i}{2}) - 2\sum_{j>i} Im\left[U^*_{\alpha i}U_{\beta i}U_{\alpha_j}U^*_{\beta_j}\right]\sin(\phi_j-\phi_i) \tag{16}$$

If the neutrino interacts at a time T at a distance L along its direction of flight, the difference in phase of the three mass eigenstates are written as

$$\phi_j - \phi_i = p_j.x_j - p_i.x_i = (E_j - E_i)T - (p_j - p_i)L$$

With assuming that $p_j = p_i = p$ for neutrinos of the same source, then

$$\begin{aligned}\phi_j - \phi_i &= (E_j - E_i)T \approx \left[p_j(1+\frac{m_j^2}{2p_j^2}) - p_i(1+\frac{m_i^2}{2p_i^2})\right]T \\ &= \frac{m_j^2-m_i^2}{2p}T = \frac{\Delta m^2_{ji}L}{2E}\end{aligned} \tag{17}$$

In the above calculation, we used the approximation $T \approx L$ and $p \approx E$ for $v_\nu \approx c$ and $m_\nu \ll E_\nu$

We finally get the most common form of the oscillation probability:

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{j>i} Re\left[U^*_{\alpha i}U_{\beta i}U_{\alpha_j}U^*_{\beta_j}\right]\sin^2(\frac{\Delta m^2_{ji}}{4E}L) - 2\sum_{j>i} Im\left[U^*_{\alpha i}U_{\beta i}U_{\alpha_j}U^*_{\beta_j}\right]\sin(\frac{\Delta m^2_{ji}}{2E}L) \tag{18}$$

The probability for a α-flavour neutrino with energy E to change to β-flavour after traveling a distance of L can be calculated as follows

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{i>j}\Re\left[U^*_{\alpha i}U_{\beta i}U_{\alpha_j}U^*_{\beta_j}\right]\sin^2\left(\frac{\Delta m^2_{ij}}{4E}L\right) + 2\sum_{i>j}\Im\left[U^*_{\alpha i}U_{\beta i}U_{\alpha_j}U^*_{\beta_j}\right]\sin\left(\frac{\Delta m^2_{ij}}{2E}L\right), \tag{19}$$

where $\Delta m^2_{ij} = m_i^2 - m_j^2$. For antineutrinos, the oscillation probabilities can be obtained by replace the mixing matrix elements with their complex conjugate.

Equation (19) is completely the same as eq. (18).

For antineutrinos, we just take the complex conjugate of the product matrix and get

$$P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) = \delta_{\alpha\beta} - 4\sum_{j>i} Re\left[U^*_{\alpha i}U_{\beta i}U_{\alpha_j}U^*_{\beta_j}\right]\sin^2(\frac{\Delta m^2_{ji}}{4E}L) + 2\sum_{j>i} Im\left[U^*_{\alpha i}U_{\beta i}U_{\alpha_j}U^*_{\beta_j}\right]\sin(\frac{\Delta m^2_{ji}}{2E}L) \tag{20}$$

The probabilities (18) and (20) are called *transition probabilities*, and the *survival probability* for a flavor is

$$P(\nu_\alpha \to \nu_\alpha) = P(\bar{\nu}_\alpha \to \bar{\nu}_\alpha) = 1 - 4\sum_{j>i} |U_{\alpha i}|^2 |U_{\alpha_j}|^2 \sin^2(\frac{\Delta m^2_{ji}}{4E} L) \tag{21}$$

By using the natural unit conversion for $1eV^{-1}$of length $= 1.97 \times 10^{-7} m$, we can practically express the phase in (18), (20) and (21) as

$$\begin{aligned} \frac{\Delta m^2_{ji}[eV^2]L[eV]}{4E[eV]} &= \frac{\Delta m^2_{ji}[eV^2]L[m]}{4 \times 1.97 \times 10^{-7} E[eV]} \\ &= 1.269 \frac{\Delta m^2_{ji}[eV^2]L[m]}{E[MeV]} = 1.269 \frac{\Delta m^2_{ji}[eV^2]L[km]}{E[GeV]} \end{aligned} \tag{22}$$

From (18) and (20), the difference between the neutrino and antineutrino oscillation probability indicates CP violation in neutrino sector

$$\begin{aligned} \mathscr{A}_{\text{CP}} &= P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) \\ &= 4\sum_{j>i} Im\left[U^*_{\alpha i} U_{\beta i} U_{\alpha_j} U^*_{\beta_j}\right] \sin(\frac{\Delta m^2_{ij}}{2E} L) \end{aligned} \tag{23}$$

If CP is violated, $U^*_{\alpha i} U_{\beta i} U_{\alpha_j} U^*_{\beta_j}$ has to contain an imaginary component.

For $\alpha = \mu$ and $\beta = e$, then

$$\begin{aligned} \mathscr{A}_{\text{CP}} &= P(\nu_\mu \to \nu_e) - P(\bar{\nu}_\mu \to \bar{\nu}_e) \\ &= 4\sum_{j>i} Im\left[U^*_{\mu i} U_{ei} U_{\mu_j} U^*_{e_j}\right] \sin(\frac{\Delta m^2_{ij}}{2E} L) \end{aligned} \tag{24}$$

$$\begin{aligned} &= 4Im\left[U^*_{\mu 1} U_{e1} U_{\mu_2} U^*_{e_2}\right] \sin(\frac{\Delta m^2_{12}}{2E} L) \\ &+ 4Im\left[U^*_{\mu 1} U_{e1} U_{\mu_3} U^*_{e_3}\right] \sin(\frac{\Delta m^2_{13}}{2E} L) \\ &+ 4Im\left[U^*_{\mu 2} U_{e2} U_{\mu_3} U^*_{e_3}\right] \sin(\frac{\Delta m^2_{23}}{2E} L) \end{aligned} \tag{25}$$

From the unitary condition we have

$$U_{\mu 1} U^*_{e1} + U_{\mu_2} U^*_{e_2} + U_{\mu 3} U^*_{e3} = 0 \tag{26}$$

Multiply two sides of the equation (26) with $U^*_{\mu 1} U_{e1}$ and $U^*_{\mu 2} U_{e2}$ respectively and then add them up, we have

$$
\begin{aligned}
& U^*_{\mu 1}U_{e1}U_{\mu 1}U^*_{e1} + U^*_{\mu 1}U_{e1}U_{\mu_2}U^*_{e_2} + U^*_{\mu 1}U_{e1}U_{\mu 3}U^*_{e3} \\
+ \quad & U^*_{\mu 2}U_{e2}U_{\mu 1}U^*_{e1} + U^*_{\mu 2}U_{e2}U_{\mu_2}U^*_{e_2} + U^*_{\mu 2}U_{e2}U_{\mu 3}U^*_{e3} = 0 \\
\Leftrightarrow \quad & 0 = |U_{\mu 1}|^2|U_{e1}|^2 + |U_{\mu 2}|^2|U_{e2}|^2 \\
+ \quad & Re[U^*_{\mu 1}U_{e1}U_{\mu_2}U^*_{e_2}] + Re[U^*_{\mu 2}U_{e2}U_{\mu 1}U^*_{e1}] + Re[U^*_{\mu 1}U_{e1}U_{\mu 3}U^*_{e3}] + Re[U^*_{\mu 2}U_{e2}U_{\mu 3}U^*_{e3}] \\
+ \quad & i\left\{Im[U^*_{\mu 1}U_{e1}U_{\mu_2}U^*_{e_2}] + Im[U^*_{\mu 2}U_{e2}U_{\mu 1}U^*_{e1}] + Im[U^*_{\mu 1}U_{e1}U_{\mu 3}U^*_{e3}] + Im[U^*_{\mu 2}U_{e2}U_{\mu 3}U^*_{e3}]\right\} \\
\Rightarrow \quad & Im[U^*_{\mu 1}U_{e1}U_{\mu_2}U^*_{e_2}] + Im[U^*_{\mu 2}U_{e2}U_{\mu 1}U^*_{e1}] + Im[U^*_{\mu 1}U_{e1}U_{\mu 3}U^*_{e3}] + Im[U^*_{\mu 2}U_{e2}U_{\mu 3}U^*_{e3}] = 0
\end{aligned}
\tag{27}
$$

Note that

$$[U^*_{\mu 1}U_{e1}U_{\mu_2}U^*_{e_2}]^* = U^*_{\mu 2}U_{e2}U_{\mu 1}U^*_{e1} \Rightarrow Im[U^*_{\mu 1}U_{e1}U_{\mu_2}U^*_{e_2}] = -Im[U^*_{\mu 2}U_{e2}U_{\mu 1}U^*_{e1}]$$

Therefore, from (27) we get

$$Im[U^*_{\mu 1}U_{e1}U_{\mu 3}U^*_{e3}] = -Im[U^*_{\mu 2}U_{e2}U_{\mu 3}U^*_{e3}] \tag{28}$$

Multiply two sides of the equation (26) with $U^*_{\mu 1}U_{e1}$ and $U^*_{\mu 3}U_{e3}$ respectively and then add them up, we have

$$
\begin{aligned}
& U^*_{\mu 1}U_{e1}U_{\mu 1}U^*_{e1} + U^*_{\mu 1}U_{e1}U_{\mu_2}U^*_{e_2} + U^*_{\mu 1}U_{e1}U_{\mu 3}U^*_{e3} \\
+ \quad & U^*_{\mu 3}U_{e3}U_{\mu 1}U^*_{e1} + U^*_{\mu 3}U_{e3}U_{\mu_2}U^*_{e_2} + U^*_{\mu 3}U_{e3}U_{\mu 3}U^*_{e3} = 0 \\
\Leftrightarrow \quad & 0 = |U_{\mu 1}|^2|U_{e1}|^2 + |U_{\mu 3}|^2|U_{e3}|^2 \\
+ \quad & Re[U^*_{\mu 1}U_{e1}U_{\mu_3}U^*_{e_3}] + Re[U^*_{\mu 3}U_{e3}U_{\mu 1}U^*_{e1}] + Re[U^*_{\mu 1}U_{e1}U_{\mu 2}U^*_{e2}] + Re[U^*_{\mu 3}U_{e3}U_{\mu 2}U^*_{e2}] \\
+ \quad & i\left\{Im[U^*_{\mu 1}U_{e1}U_{\mu_3}U^*_{e_3}] + Im[U^*_{\mu 3}U_{e3}U_{\mu 1}U^*_{e1}] + Im[U^*_{\mu 1}U_{e1}U_{\mu 2}U^*_{e2}] + Im[U^*_{\mu 3}U_{e3}U_{\mu 2}U^*_{e2}]\right\} \\
\Rightarrow \quad & Im[U^*_{\mu 1}U_{e1}U_{\mu_3}U^*_{e_3}] + Im[U^*_{\mu 3}U_{e3}U_{\mu 1}U^*_{e1}] + Im[U^*_{\mu 1}U_{e1}U_{\mu 2}U^*_{e2}] + Im[U^*_{\mu 3}U_{e3}U_{\mu 2}U^*_{e2}] = 0
\end{aligned}
\tag{29}
$$

Note that

$$[U^*_{\mu 1}U_{e1}U_{\mu_3}U^*_{e_3}]^* = U^*_{\mu 3}U_{e3}U_{\mu 1}U^*_{e1} \Rightarrow Im[U^*_{\mu 1}U_{e1}U_{\mu_3}U^*_{e_3}] = -Im[U^*_{\mu 3}U_{e3}U_{\mu 1}U^*_{e1}]$$

and

$$[U^*_{\mu 3}U_{e3}U_{\mu 2}U^*_{e2}]^* = U^*_{\mu 2}U_{e2}U_{\mu 3}U^*_{e3} \Rightarrow Im[U^*_{\mu 3}U_{e3}U_{\mu 2}U^*_{e2}] = -[U^*_{\mu 2}U_{e2}U_{\mu 3}U^*_{e3}]$$

Therefore, from (29) we get

$$Im[U^*_{\mu 1}U_{e1}U_{\mu 2}U^*_{e2}] = Im[U^*_{\mu 2}U_{e2}U_{\mu 3}U^*_{e3}] \tag{30}$$

By using (28) and (30), we can rewrite (29) as

$$\begin{aligned}\mathscr{A}_{\mathrm{CP}} &= P(\nu_\mu \to \nu_e) - P(\bar{\nu}_\mu \to \bar{\nu}_e) \\ &= 4Im\left[U^*_{\mu 1}U_{e1}U_{\mu_3}U^*_{e_3}\right](\sin\Delta_{13} - \sin\Delta_{12} - \sin\Delta_{23}) \end{aligned} \tag{31}$$

Where $\Delta_{13} = \frac{\Delta m^2_{13}}{2E}L$, $\Delta_{12} = \frac{\Delta m^2_{12}}{2E}L$ and $\Delta_{23} = \frac{\Delta m^2_{23}}{2E}L = \Delta_{13} - \Delta_{23}$
By a simple trigonometry calculation, we have

$$\begin{aligned}\sin\Delta_{13} - \sin\Delta_{12} - \sin(\Delta_{13} - \Delta_{12}) &= -4\sin\frac{\Delta_{12}}{2}\sin\frac{\Delta_{13}}{2}\sin\frac{\Delta_{23}}{2} \\ &= 4\sin\frac{\Delta_{21}}{2}\sin\frac{\Delta_{31}}{2}\sin\frac{\Delta_{32}}{2}\end{aligned}$$

Then we can rewrite (31) as

$$\begin{aligned}&\mathscr{A}_{\mathrm{CP}} = P(\nu_\mu \to \nu_e) - P(\bar{\nu}_\mu \to \bar{\nu}_e) \\ &= 16Im\left[U^*_{\mu 1}U_{e1}U_{\mu_3}U^*_{e_3}\right]\left(\sin\frac{\Delta_{21}}{2}\sin\frac{\Delta_{31}}{2}\sin\frac{\Delta_{32}}{2}\right) \\ &= 16Im\left[U^*_{\mu 1}U_{e1}U_{\mu_3}U^*_{e_3}\right]\sin\left(\frac{\Delta m^2_{21}L}{4E}\right)\sin\left(\frac{\Delta m^2_{31}L}{4E}\right)\sin\left(\frac{\Delta m^2_{32}L}{4E}\right)\end{aligned} \tag{32}$$

The general form in term of oscillation parameters can be obtained as shown in Eq. 33. The coefficients in the denominator should be 4 instead of 2!

$$\begin{aligned}\mathscr{A}_{\mathrm{CP}} =& P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) = 4\sum_{j>i}\Im\left[U^*_{\alpha i}U_{\beta i}U_{\alpha_j}U^*_{\beta_j}\right]\sin\left(\frac{\Delta m^2_{ji}}{2E}L\right), \\ &= \pm 2\delta_{\alpha\beta}\cos\theta_{13}\sin 2\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}\sin\delta_{CP}\sin\frac{\Delta m^2_{21}L}{4E}\sin\frac{\Delta m^2_{32}L}{4E}\sin\frac{\Delta m^2_{13}L}{4E}\end{aligned} \tag{33}$$

in which $\{\alpha,\beta\} = \{e,\mu,\tau\}$; $\{i,j\} = \{1,2,3\}$, $j > i$ and $\Delta m^2_{ji} = m^2_j - m^2_i$; the positive (negative) sign is applied based on (anti-) cyclic permutation of ordered flavor (e,μ,τ). Apparently CP violation can be measured via the neutrino oscillation phenomenon if only three mixing angles are non-zero. The up-to-date neutrino data shows that Nature supports this scenario and it opens the door to search CP violation in the lepton sector with neutrino oscillation measurements. This CP violation source might be a promising explanation for the matter asymmetry in the Universe.

In practical, CP violation can be measured by comparing the rate of electron neutrinos appearance from muon neutrinos, $P(\nu_\mu \to \nu_e)$, with its of electron antineutrinos appearance from muon anti-neutrinos, $P(\overline{\nu}_\mu \to \overline{\nu}_e)$ in accelerator-based experiments or comparing the first with electron antineutrino disappearance in the reactor-based experiments[3]

[3] Accelerator-based measurements leads to an intrinsic $\delta_{CP} - \theta_{13}$ degeneracy while reactor-based measurement can precisely measure θ_{13}. Their combined information thus can provide constraint on δ_{CP}.

$*$ **Evolution of neutrino flavors in matter**

The relation between mass eigenstates and flavor eigenstates

$$|\nu_\alpha\rangle = \sum_k U^*_{\alpha k} |\nu_k\rangle$$

The total Hamiltonian in matter is

$$H = H_0 + H_1$$

Where

$$\begin{aligned} H_0|\nu_k\rangle &= E_k|\nu_k\rangle; \quad \text{with} \quad E_k = \sqrt{\vec{p}_k^{\,2} + m_k^2} \approx p_k + \frac{m_k^2}{2p_k} \\ H_1|\nu_\alpha\rangle &= V_\alpha|\nu_\alpha\rangle \end{aligned}$$

The Schrodinger equation for neutrino in matter is

$$\begin{aligned} i\frac{d}{dt}|\nu_\alpha(t)\rangle &= H|\nu_\alpha(t)\rangle \\ &= (H_0 + H_1)|\nu_\alpha(t)\rangle \\ &= (E_k + V_\alpha)|\nu_\alpha(t)\rangle \\ &= \left[\left(p_k + \frac{m_k^2}{2p_k}\right) + V_\alpha\right]|\nu_\alpha(t)\rangle \end{aligned}$$

For $v \approx c$ (means $t \approx x$) we have $p_k \approx E$. We can see that $E + V_{NC}$ is the same for all neutrinos. They generate a phase common to all flavors and will cancel out in transition. Hence we can ignore them here for simplicity. So we rewrite the above equation as

$$i\frac{d}{dt}|\nu_\alpha(t)\rangle = \left(\frac{m_k^2}{2E} + V_{CC}\delta_{\alpha e}\right)|\nu_\alpha(t)\rangle$$

Or in explicit form

$$i\frac{d}{dt}\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \left[\frac{1}{2E}U\begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix}U^\dagger + \begin{pmatrix} V_{CC} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right]\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \tag{34}$$

Where U is an unitary matrix.

$*$ **Complete oscillation probability in matter**

Since $v \approx c$ so $x \approx ct = t$ for $c = 1$. We can rewrite the Schrodinger equation in matter as

$$i\frac{d\nu}{dx} = H\nu$$

Where

$$
\begin{aligned}
H &= H_0 + H_1 \\
&= \frac{1}{2E} U \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \Delta m^2_{31} \end{pmatrix} U^\dagger + \frac{1}{2E} \left[U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m^2_{21} & 0 \\ 0 & 0 & 0 \end{pmatrix} U^\dagger + \begin{pmatrix} a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right]
\end{aligned}
$$

and $a = 2EV_{CC} = 2\sqrt{2}G_F E N_e$.

Since Δm^2_{21} and $a \ll \Delta m^2_{31}$, we can treat H_1 as a pertubation.

The Schrodinger equation has a solution of Dyson series form

$$\nu(x) = S(x)\nu(0) \tag{35}$$

With

$$S(x) \equiv T e^{\int_0^x H(s)ds}$$

T is the symbol of time ordering. The oscillation probability at distance L then can be calculate through $S(x)$

$$P(\nu_\alpha \to \nu_\beta) = |S_{\beta\alpha}(L)|^2 \tag{36}$$

We can calculate the pertubation to the first order in a and Δm^2_{21}. We have

$$S_0(x) = e^{-iH_0 x}$$

and

$$S_1(x) = e^{-iH_0 x}(-i)\int_0^x ds H_1(s) = e^{-iH_0 x}(-i)\int_0^x ds e^{iH_0 s} H_1 e^{-iH_0 s}$$

We now calculate $S_0(x)$ and $S_1(x)$ as the following

$$
\begin{aligned}
(S_0(x))_{\beta\alpha} &= \left[U e^{-i\frac{x}{2E} diag(0,0,\Delta m^2_{31})} U^\dagger \right]_{\beta\alpha} \\
&= \sum_{i,j} \left[U_{\beta i} \left(e^{-i\frac{x}{2E} diag(0,0,\Delta m^2_{31})} \right)_{ij} U^*_{\alpha j} \right]
\end{aligned}
$$

Note that

$$e^{-i\frac{x}{2E} diag(0,0,\Delta m^2_{31})} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\frac{\Delta m^2_{31} x}{2E}} \end{pmatrix}$$

Hence

$$
\begin{aligned}
(S_0(x))_{\beta\alpha} &= U_{\beta 1}U^*_{\alpha 1}.1 + U_{\beta 1}U^*_{\alpha 2}.0 + U_{\beta 1}U^*_{\alpha 3}.0 \\
&\quad + U_{\beta 2}U^*_{\alpha 1}.0 + U_{\beta 2}U^*_{\alpha 2}.1 + U_{\beta 2}U^*_{\alpha 3}.0 \\
&\quad + U_{\beta 3}U^*_{\alpha 1}.0 + U_{\beta 3}U^*_{\alpha 2}.0 + U_{\beta 2}U^*_{\alpha 3}.e^{-i\frac{\Delta m^2_{31} x}{2E}} \\
&= U_{\beta 1}U^*_{\alpha 1} + U_{\beta 2}U^*_{\alpha 2} + U_{\beta 3}U^*_{\alpha 3}.e^{-i\frac{\Delta m^2_{31} x}{2E}}
\end{aligned}
$$

By using $U_{\beta 1}U^*_{\alpha 1}+U_{\beta 2}U^*_{\alpha 2}+U_{\beta 3}U^*_{\alpha 3}=\delta_{\alpha\beta} \quad \Rightarrow$

$$(S_0(x))_{\beta\alpha}=\delta_{\alpha\beta}+U_{\beta 3}U^*_{\alpha 3}\left(e^{-i\frac{\Delta m^2_{31}x}{2E}}-1\right) \tag{37}$$

Similarity for calculating $(S_1(x))_{\beta\alpha}$. We have

$$\begin{aligned}
(S_1(x))_{\beta\alpha} &= \left(e^{-iH_0x}(-i)\int_0^x ds e^{iH_0s}H_1e^{-iH_0s}\right)_{\beta\alpha} \\
&= -i\int_0^x ds\left(e^{-iH_0(x-s)}H_1e^{-iH_0s}\right)_{\beta\alpha} \\
&= -i\int_0^x ds\left(Ue^{-i\frac{x-s}{2E}diag(0,0,\Delta m^2_{31})}U^\dagger H_1 Ue^{-i\frac{s}{2E}diag(0,0,\Delta m^2_{31})}U^\dagger\right)_{\beta\alpha} \\
&= -i\int_0^x ds\sum_{i,j',i',j}\left[U_{\beta i}\left(e^{-i\frac{x-s}{2E}diag(0,0,\Delta m^2_{31})}\right)_{ij'}U^*_{\gamma j'}(H_1)_{\gamma\sigma}U_{\sigma i'}\left(e^{-i\frac{s}{2E}diag(0,0,\Delta m^2_{31})}\right)_{i'j}U^*_{\beta j}\right]
\end{aligned}$$

Since $\left(e^{-i\frac{x-s}{2E}diag(0,0,\Delta m^2_{31})}\right)_{ij'}=0$ for $j'\neq i$

and $\left(e^{-i\frac{s}{2E}diag(0,0,\Delta m^2_{31})}\right)_{i'j}=0$ for $i'\neq j \quad \Rightarrow$

$$\begin{aligned}
(S_1(x))_{\beta\alpha} &= -i\int_0^x ds\sum_{i,j}\left[U_{\beta i}\left(e^{-i\frac{x-s}{2E}diag(0,0,\Delta m^2_{31})}\right)_{ii}U^*_{\gamma i}(H_1)_{\gamma\sigma}U_{\sigma j}\left(e^{-i\frac{s}{2E}diag(0,0,\Delta m^2_{31})}\right)_{jj}U^*_{\beta j}\right] \\
&= -i\sum_{i,j}U_{\beta i}U^*_{\gamma i}(H_1)_{\gamma\sigma}U_{\sigma j}U^*_{\beta j}\int_0^x ds\left(e^{-i\frac{\Delta m^2_{31}}{2E}[(x-s)\delta_{i3}+s\delta_{j3}}\right)
\end{aligned} \tag{38}$$

Let

$$X_{ij}=U_{\beta i}U^*_{\gamma i}(H_1)_{\gamma\sigma}U_{\sigma j}U^*_{\beta j}$$

and

$$Y_{ij}=\int_0^x ds\left(e^{-i\frac{\Delta m^2_{31}}{2E}[(x-s)\delta_{i3}+s\delta_{j3}}\right)=\int_0^x ds\left(e^{-i\frac{\Delta m^2_{31}x}{2E}\delta_{i3}}.e^{-i\frac{\Delta m^2_{31}s}{2E}(\delta_{j3}-\delta_{i3}}\right)$$

We first calculate the X_{ij} term

We see that

$$U^*_{\gamma i}(H_1)_{\gamma\sigma}U_{\sigma j}=\frac{1}{2E}[U^*_{\gamma i}(V_{12})_{\gamma\sigma}U_{\sigma j}+U^*_{\gamma i}(V_a)_{\gamma\sigma}U_{\sigma j}]$$

Since $(V_{12})_{22}=\Delta m^2_{21}$ and $(V_{12})_{\gamma\sigma}=0$ for $\gamma\neq 2$ or $\sigma\neq 2$ then

$$\frac{1}{2E}U^*_{\gamma i}(V_{12})_{\gamma\sigma}U_{\sigma j}=\frac{\Delta m^2_{21}}{2E}\delta_{2i}\delta_{2j}$$

Since $(V_a)_{11}=a$ and $(V_a)_{\gamma\sigma}=0$ for $\gamma\neq 1$ or $\sigma\neq 1$ then

$$\frac{1}{2E}U^*_{\gamma i}(V_a)_{\gamma\sigma}U_{\sigma j}=\frac{a}{2E}U^*_{1i}U_{1j}$$

Therefore

$$U^*_{\gamma i}(H_1)_{\gamma\sigma}U_{\sigma j} = \frac{\Delta m^2_{21}}{2E}\delta_{2i}\delta_{2j} + \frac{a}{2E}U^*_{1i}U_{1j}$$

and

$$\begin{aligned}
X_{ij} &= U_{\beta i}U^*_{\gamma i}(H_1)_{\gamma\sigma}U_{\sigma j}U^*_{\beta j} \\
&= U_{\beta i}\left[\frac{\Delta m^2_{21}}{2E}\delta_{2i}\delta_{2j} + \frac{a}{2E}U^*_{1i}U_{1j}\right]U^*_{\alpha j} \\
&= \frac{\Delta m^2_{21}}{2E}U_{\beta i}U^*_{\alpha j}\delta_{2i}\delta_{2j} + \frac{a}{2E}U_{\beta i}U^*_{1i}U_{1j}U^*_{\alpha j} \qquad (39)
\end{aligned}$$

and the Y_{ij} integral is

$$\begin{aligned}
Y_{11} &= Y_{12} = Y_{21} = Y_{22} = x \\
Y_{13} &= Y_{23} = Y_{31} = Y_{32} = \left(-i\frac{\Delta m^2_{31}}{2E}\right)^{-1}\left(e^{-i\frac{\Delta m^2_{31}x}{2E}} - 1\right) \\
Y_{33} &= xe^{-i\frac{\Delta m^2_{31}x}{2E}}
\end{aligned}$$

In general

$$\begin{aligned}
Y_{ij} &= (1-\delta_{i3})(1-\delta_{j3})x + \delta_{i3}\delta_{j3}xe^{-i\frac{\Delta m^2_{31}x}{2E}} \qquad (40) \\
&\quad +[(1-\delta_{i3})\delta_{j3} + \delta_{i3}(1-\delta_{j3})]\left(-i\frac{\Delta m^2_{31}}{2E}\right)^{-1}\left(e^{-i\frac{\Delta m^2_{31}x}{2E}} - 1\right)
\end{aligned}$$

Insert (40) and (39) into (37) we get

$$\begin{aligned}
(S_1(x))_{\beta\alpha} &= -i\frac{ax}{2E}e^{-i\frac{\Delta m^2_{31}x}{2E}}U_{\beta 3}U^*_{\alpha 3}|U_{13}|^2 \\
&\quad -ix\left[\frac{\Delta m^2_{21}}{2E}U_{\beta 2}U^*_{\alpha 2} + \frac{a}{2E}(U_{\beta 1}U^*_{11}U_{11}U^*_{\alpha 1} + U_{\beta 1}U^*_{11}U_{12}U^*_{\alpha 2}\right. \\
&\quad \left. +U_{\beta 2}U^*_{12}U_{11}U^*_{\alpha 1} + U_{\beta 2}U^*_{12}U_{12}U^*_{\alpha 2})\right] \\
&\quad -i\left(-i\frac{\Delta m^2_{31}}{2E}\right)^{-1}\left(e^{-i\frac{\Delta m^2_{31}x}{2E}} - 1\right)\frac{a}{2E}(U_{\beta 1}U^*_{11}U_{13}U^*_{\alpha 3} + U_{\beta 2}U^*_{12}U_{13}U^*_{\alpha 3} \\
&\quad +U_{\beta 3}U^*_{13}U_{11}U^*_{\alpha 1} + U_{\beta 3}U^*_{13}U_{12}U^*_{\alpha 2})
\end{aligned}$$

Note that $\sum_{k=1}^{2}U^*_{\alpha k}U_{1k} = \delta_{\alpha 1} - U^*_{\alpha 3}U_{13}$. We now can calculate the factors that are relevant to matrix elements as the following

$$\begin{aligned}
& U_{\beta 1}U^*_{11}U_{11}U^*_{\alpha 1} + U_{\beta 1}U^*_{11}U_{12}U^*_{\alpha 2} + U_{\beta 2}U^*_{12}U_{11}U^*_{\alpha 1} + U_{\beta 2}U^*_{12}U_{12}U^*_{\alpha 2} \\
={}& (U_{\beta 1}U^*_{11} + U_{\beta 2}U^*_{12})(U_{11}U^*_{\alpha 1} + U_{12}U^*_{\alpha 2}) \\
={}& (\delta_{\beta 1} - U_{\beta 3}U^*_{13})(\delta_{\alpha 1} - U_{13}U^*_{\alpha 3}) \\
={}& \delta_{\alpha 1}\delta_{\beta 1} - \delta_{\alpha 1}U_{\beta 3}U^*_{13} - \delta_{\beta 1}U_{13}U^*_{\alpha 3} + U_{\beta 3}U^*_{\alpha 3}|U_{13}|^2 \\
={}& \delta_{\alpha 1}\delta_{\beta 1} + U_{\beta 3}U^*_{\alpha 3}(|U_{13}|^2 - \delta_{\alpha 1} - \delta_{\beta 1})
\end{aligned}$$

$$
\begin{aligned}
& U_{\beta 1}U^*_{11}U_{13}U^*_{\alpha 3}+U_{\beta 2}U^*_{12}U_{13}U^*_{\alpha 3}+U_{\beta 3}U^*_{13}U_{11}U^*_{\alpha 1}+U_{\beta 3}U^*_{13}U_{12}U^*_{\alpha 2}\\
= & U_{13}U^*_{\alpha 3}(\delta_{\beta 1}-U_{\beta 3}U^*_{13})+U_{\beta 3}U^*_{13}(\delta_{\alpha 1}-U_{13}U^*_{\alpha 3})\\
= & \delta_{\alpha 1}U_{\beta 3}U^*_{13}+\delta_{\beta 1}U_{13}U^*_{\alpha 3}-2U_{\beta 3}U^*_{\alpha 3}|U_{13}|^2\\
= & U_{\beta 3}U^*_{\alpha 3}(\delta_{\alpha 1}+\delta_{\beta 1}-2|U_{13}|^2)
\end{aligned}
$$

Therefore

$$
\begin{aligned}
(S_1(x))_{\beta\alpha} = & -i\frac{ax}{2E}e^{-i\frac{\Delta m^2_{31}x}{2E}}U_{\beta 3}U^*_{\alpha 3}|U_{13}|^2\\
& -i\frac{x}{2E}\left[\Delta m^2_{21}U_{\beta 2}U^*_{\alpha 2}+a(\delta_{\alpha 1}\delta_{\beta 1}+U_{\beta 3}U^*_{\alpha 3}(|U_{13}|^2-\delta_{\alpha 1}-\delta_{\beta 1}))\right]\\
& -\frac{a}{\Delta m^2_{31}}\left(e^{-i\frac{\Delta m^2_{31}x}{2E}}-1\right)(2|U_{13}|^2-\delta_{\alpha 1}-\delta_{\beta 1})U_{\beta 3}U^*_{\alpha 3}
\end{aligned} \tag{41}
$$

From (41) and (37) and note that $\sin X/2=\frac{e^{iX/2}-e^{-iX/2}}{2i}$ with $X=\frac{\Delta m^2_{31}}{2E}$ we get

$$
\begin{aligned}
(S(x))_{\beta\alpha} = & (S_0(x))_{\beta\alpha}+(S_1(x))_{\beta\alpha}\\
= & \delta_{\alpha\beta}+U_{\beta 3}U^*_{\alpha 3}\left(e^{-i\frac{\Delta m^2_{31}x}{2E}}-1\right)-\frac{a}{\Delta m^2_{31}}\left(e^{-i\frac{\Delta m^2_{31}x}{2E}}-1\right)(2|U_{13}|^2-\delta_{\alpha 1}-\delta_{\beta 1})U_{\beta 3}U^*_{\alpha 3}\\
& -i\frac{ax}{2E}e^{-i\frac{\Delta m^2_{31}x}{2E}}U_{\beta 3}U^*_{\alpha 3}|U_{13}|^2+\left(i\frac{ax}{2E}U_{\beta 3}U^*_{\alpha 3}|U_{13}|^2-i\frac{ax}{2E}U_{\beta 3}U^*_{\alpha 3}|U_{13}|^2\right)\\
& -i\frac{\Delta m^2_{31}x}{2E}\left[\frac{\Delta m^2_{21}}{\Delta m^2_{31}}U_{\beta 2}U^*_{\alpha 2}+\frac{a}{\Delta m^2_{31}}(\delta_{\alpha 1}\delta_{\beta 1}+U_{\beta 3}U^*_{\alpha 3}(|U_{13}|^2-\delta_{\alpha 1}-\delta_{\beta 1}))\right]
\end{aligned}
$$

By rearranging the common terms the above equation becomes

$$
\begin{aligned}
(S(x))_{\beta\alpha} = & \delta_{\alpha\beta}-i2e^{-i\Delta_{31}}\sin\Delta_{31}U_{\beta 3}U^*_{\alpha 3}\left[(1-C)-\frac{iax}{2E}|U_{13}|^2\right]\\
& -i2\Delta_{31}\left[\varepsilon U_{\beta 2}U^*_{\alpha 2}+\frac{a}{\Delta m^2_{31}}\delta_{\alpha 1}\delta_{\beta 1}+CU_{\beta 3}U^*_{\alpha 3}\right]\\
= & \delta_{\alpha\beta}+A+B
\end{aligned} \tag{42}
$$

Where $\Delta_{31}=\frac{\Delta m^2_{31}x}{4E}$; $\varepsilon=\frac{\Delta m^2_{21}}{\Delta m^2_{31}}$ and $C=\frac{a}{\Delta m^2_{31}}(2|U_{13}|^2-\delta_{\alpha 1}-\delta_{\beta 1})$

The oscillation probability now can be calculated

$$
\begin{aligned}
P(\nu_\alpha\to\nu_\beta) = & |(S(x))_{\beta\alpha}|^2\\
= & \delta_{\alpha\beta}(1+A+A^*+B+B^*)+AA^*+BB^*+A^*B+AB^*
\end{aligned} \tag{43}
$$

• The tearm which is relevant to $\delta_{\alpha\beta}$

$$
\begin{aligned}
& \delta_{\alpha\beta}(1+A+A^*+B+B^*) \\
= & \ \delta_{\alpha\beta}\left\{1-i2e^{-i\Delta_{31}}\sin\Delta_{31}U_{\beta 3}U^*_{\alpha 3}\left[(1-C)-\frac{iax}{2E}|U_{13}|^2\right]\right. \\
& +i2e^{i\Delta_{31}}\sin\Delta_{31}U^*_{\beta 3}U_{\alpha 3}\left[(1-C)+\frac{iax}{2E}|U_{13}|^2\right] \\
& -i2\Delta_{31}\left[\varepsilon U_{\beta 2}U^*_{\alpha 2}+\frac{a}{\Delta m^2_{31}}\delta_{\alpha 1}\delta_{\beta 1}+CU_{\beta 3}U^*_{\alpha 3}\right] \\
& \left.+i2\Delta_{31}\left[\varepsilon U^*_{\beta 2}U_{\alpha 2}+\frac{a}{\Delta m^2_{31}}\delta_{\alpha 1}\delta_{\beta 1}+CU^*_{\beta 3}U_{\alpha 3}\right]\right\} \\
= & \ \delta_{\alpha\beta}\left[1-4(1-C)|U_{\alpha 3}|^2\sin^2\Delta_{31}-\frac{ax}{E}|U_{\alpha 3}|^2|U_{13}|^2\sin 2\Delta_{31}\right] \\
= & \ \delta_{\alpha\beta}\left[1-4|U_{\alpha 3}|^2\sin^2\Delta_{31}\left(1-\frac{2a}{\Delta m^2_{31}}(|U_{13}|^2-\delta_{\alpha 1})\right)-\frac{ax}{E}|U_{\alpha 3}|^2|U_{13}|^2\sin 2\Delta_{31}\right]
\end{aligned}
$$

• In order to calculate the tearm which is irrelevant to $\delta_{\alpha\beta}$, we first calculate its components

$$
\begin{aligned}
AA^* &= 4\sin^2\Delta_{31}|U_{\beta 3}|^2|U_{\alpha 3}|^2\left[(1-2C+C^2)+\left(\frac{ax}{2E}\right)^2|U_{13}|^4\right] \\
BB^* &= 4(\Delta_{31})^2\left[\varepsilon^2|U_{\beta 2}|^2|U_{\alpha 2}|^2+\varepsilon\frac{2a}{\Delta m^2_{31}}|U_{13}|^2\delta_{\alpha 1}\delta_{\beta 1}\right. \\
& \quad +2\varepsilon C.Re(U^*_{\beta 3}U_{\alpha 3}U_{\beta 2}U^*_{\alpha 2})+C^2|U_{\beta 3}|^2|U_{\alpha 3}|^2 \\
& \quad \left.+\frac{2aC}{\Delta m^2_{31}}|U_{13}|^2\delta_{\alpha 1}\delta_{\beta 1}+\left(\frac{a}{\Delta m^2_{31}}\right)^2\right] \\
A^*B &+ AB^* = 2Re(AB^*) \\
&= 4\varepsilon(1-C)\Delta_{31}\sin 2\Delta_{31}Re(U^*_{\beta 3}U_{\alpha 3}U_{\beta 2}U^*_{\alpha 2})-8\varepsilon(1-C)\Delta_{31}\sin^2\Delta_{31}Im(U^*_{\beta 3}U_{\alpha 3}U_{\beta 2}U^*_{\alpha 2}) \\
& \quad -8\varepsilon\left(\frac{ax}{2E}\right)\Delta_{31}\sin^2\Delta_{31}|U_{13}|^2Re(U^*_{\beta 3}U_{\alpha 3}U_{\beta 2}U^*_{\alpha 2}) \\
& \quad -4\varepsilon\left(\frac{ax}{2E}\right)\Delta_{31}\sin^2\Delta_{31}|U_{13}|^2Im(U^*_{\beta 3}U_{\alpha 3}U_{\beta 2}U^*_{\alpha 2}) \\
& \quad +4(1-C)\frac{a}{\Delta m^2_{31}}\Delta_{31}\sin 2\Delta_{31}|U_{13}|^2\delta_{\alpha 1}\delta_{\beta 1}-8\frac{a^2x}{2E\Delta m^2_{31}}\Delta_{31}\sin^2\Delta_{31}|U_{13}|^2\delta_{\alpha 1}\delta_{\beta 1} \\
& \quad +4(1-C)C\Delta_{31}\sin 2\Delta_{31}|U_{\beta 3}|^2|U_{\alpha 3}|^2-8\left(\frac{axC}{2E}\right)\Delta_{31}\sin^2\Delta_{31}|U_{13}|^2|U_{\beta 3}|^2|U_{\alpha 3}|^2
\end{aligned}
$$

Since $C \propto a$, $\varepsilon\Delta_{31}=\Delta_{21}=\frac{\Delta m^2_{21}}{4E}$ and we have made the approximations:

$$
\frac{ax}{2E} \ll 1; \quad \frac{\Delta m^2_{21}}{2E} \ll 1
$$

we can neglect all the terms that contain $a^2, C^2, aC, \varepsilon a, \varepsilon C, \varepsilon C\Delta_{31}, \varepsilon a\Delta_{31}$ and leave

$$
\begin{aligned}
AA^* &= 4\sin^2\Delta_{31}|U_{\beta 3}|^2|U_{\alpha 3}|^2\left[1-2\frac{a}{\Delta m^2_{31}}(2|U_{13}|^2-\delta_{\alpha 1}-\delta_{\beta 1})\right] \\
BB^* &= 4\Delta^2_{21}|U_{\beta 2}|^2|U_{\alpha 2}|^2 \\
A^*B &+ AB^* = 2Re(AB^*) \\
&= 4\Delta_{21}\sin 2\Delta_{31}Re(U^*_{\beta 3}U_{\alpha 3}U_{\beta 2}U^*_{\alpha 2}) - 8\Delta_{21}\sin^2\Delta_{31}Im(U^*_{\beta 3}U_{\alpha 3}U_{\beta 2}U^*_{\alpha 2}) \\
&\quad +4\frac{ax}{4E}\sin 2\Delta_{31}|U_{13}|^2\delta_{\alpha 1}\delta_{\beta 1} + 4\frac{ax}{4E}\sin 2\Delta_{31}|U_{\beta 3}|^2|U_{\alpha 3}|^2(2|U_{13}|^2-\delta_{\alpha 1}-\delta_{\beta 1})
\end{aligned}
$$

The general form of oscillation probability is therefore

$$
\begin{aligned}
P(\nu_\alpha \to \nu_\beta) &= \delta_{\alpha\beta}(1+A+A^*+B+B^*)+AA^*+BB^*+A^*B+AB^* \\
&= \delta_{\alpha\beta}\left[1-4|U_{\alpha 3}|^2\sin^2\Delta_{31}\left(1-\frac{2a}{\Delta m^2_{31}}(|U_{13}|^2-\delta_{\alpha 1})\right)-\frac{ax}{E}|U_{\alpha 3}|^2|U_{13}|^2\sin 2\Delta_{31}\right] \\
&\quad +4\sin^2\Delta_{31}|U_{\beta 3}|^2|U_{\alpha 3}|^2\left[1-2\frac{a}{\Delta m^2_{31}}(2|U_{13}|^2-\delta_{\alpha 1}-\delta_{\beta 1})\right] \\
&\quad -8\Delta_{21}\sin^2\Delta_{31}Im(U^*_{\beta 3}U_{\alpha 3}U_{\beta 2}U^*_{\alpha 2}) \\
&\quad +4\sin 2\Delta_{31}\Big[\Delta_{21}Re(U^*_{\beta 3}U_{\alpha 3}U_{\beta 2}U^*_{\alpha 2}) \\
&\quad +\frac{ax}{4E}\left(|U_{13}|^2\delta_{\alpha 1}\delta_{\beta 1}+|U_{\beta 3}|^2|U_{\alpha 3}|^2(2|U_{13}|^2-\delta_{\alpha 1}-\delta_{\beta 1})\right)\Big] \\
&\quad +4\Delta^2_{21}|U_{\beta 2}|^2|U_{\alpha 2}|^2
\end{aligned}
\tag{44}
$$

$*$ **Survival probability $P(\nu_\mu \to \nu_\mu)$ in matter**

For $\alpha = \beta = \mu$ we have

$$
\begin{aligned}
P(\nu_\mu \to \nu_\mu) &= 1+4\sin^2\Delta_{31}|U_{\mu 3}|^2\left[(|U_{\mu 3}|^2-1)-\frac{2a}{\Delta m^2_{31}}|U_{e3}|^2\left(2|U_{\mu 3}|^2-1\right)\right] \\
&\quad +4\Delta_{31}\sin 2\Delta_{31}|U_{\mu 3}|^2\left[\frac{\Delta m^2_{21}}{\Delta m^2_{31}}|U_{\mu 2}|^2+\frac{a}{\Delta m^2_{31}}|U_{e3}|^2\left(2|U_{\mu 3}|^2-1\right)\right] \\
&\quad +4\Delta^2_{21}|U_{\mu 2}|^4
\end{aligned}
\tag{45}
$$

From the PMNS matrix we see that

$$U_{e2} = s_{12}c_{13}; \quad U_{e3} = s_{13}e^{-i\delta}$$

and

$$U_{\mu 2} = c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta}; \quad U_{\mu 3} = s_{23}c_{13}$$

Therefore

$$\begin{aligned}
P(\nu_\mu \to \nu_\mu) &= 1+4s_{23}^2c_{13}^2(s_{23}^2c_{13}^2-1)\sin^2\Delta_{31} \\
&+4s_{23}^2c_{13}^2s_{13}^2\left(2s_{23}^2c_{13}^2-1\right)\frac{2a}{\Delta m_{31}^2}\sin^2\Delta_{31} \\
&+4s_{23}^2c_{13}^2s_{13}^2\left(2s_{23}^2c_{13}^2-1\right)\frac{a}{\Delta m_{31}^2}\Delta_{31}\sin 2\Delta_{31} \\
&+4s_{23}^2c_{13}^2(c_{12}^2c_{23}^2+s_{12}^2s_{13}^2s_{23}^2-2s_{12}s_{13}s_{23}c_{12}c_{23}\cos\delta)\Delta_{21}\sin 2\Delta_{31} \\
&+4(c_{12}^2c_{23}^2+s_{12}^2s_{13}^2s_{23}^2-2s_{12}s_{13}s_{23}c_{12}c_{23}\cos\delta)^2\Delta_{21}^2 \qquad (46)
\end{aligned}$$

As you can see in the equation (46), the second term dominates. The third and the forth are related to matter effect.

$*$ **Transition probability $P(\nu_\mu \to \nu_e)$ in matter**

For $\alpha = \mu$ and $\beta = e$ we have

$$\begin{aligned}
P(\nu_\mu \to \nu_e) &= 4\sin^2\Delta_{31}|U_{e3}|^2|U_{\mu 3}|^2 \\
&-8\sin^2\Delta_{31}|U_{e3}|^2|U_{\mu 3}|^2\frac{a}{\Delta m_{31}^2}(2|U_{e3}|^2-1) \\
&+4\sin 2\Delta_{31}\frac{ax}{4E}|U_{e3}|^2|U_{\mu 3}|^2(2|U_{e3}|^2-1) \\
&-8\Delta_{21}\sin^2\Delta_{31}Im(U_{e3}^*U_{\mu 3}U_{e2}U_{\mu 2}^*) \\
&+4\Delta_{21}\sin 2\Delta_{31}Re(U_{e3}^*U_{\mu 3}U_{e2}U_{\mu 2}^*) \\
&+4\Delta_{21}^2|U_{e2}|^2|U_{\mu 2}|^2 \qquad (47)
\end{aligned}$$

From the PMNS matrix we see that

$$U_{e2} = s_{12}c_{13}; \quad U_{e3} = s_{13}e^{-i\delta}$$

and

$$U_{\mu 2} = c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta}; \quad U_{\mu 3} = s_{23}c_{13}$$

Therefore

$$\begin{aligned}
P(\nu_\mu \to \nu_e) &= 4s_{13}^2s_{23}^2c_{13}^2\sin^2\Delta_{31} \\
&-8s_{13}^2s_{23}^2c_{13}^2\frac{a}{\Delta m_{31}^2}(2s_{13}^2-1)\sin^2\Delta_{31} \\
&+4s_{13}^2s_{23}^2c_{13}^2\frac{ax}{4E}(2s_{13}^2-1)\sin 2\Delta_{31} \\
&-8s_{12}s_{13}s_{23}c_{12}c_{13}^2c_{23}\sin\delta\Delta_{21}\sin^2\Delta_{31} \\
&+4s_{12}s_{13}s_{23}c_{13}^2(c_{12}c_{23}\cos\delta - s_{12}s_{13}s_{23})\Delta_{21}\sin 2\Delta_{31} \\
&+4s_{12}^2c_{13}^2(c_{12}^2c_{23}^2+s_{12}^2s_{13}^2s_{23}^2-2s_{12}s_{13}s_{23}c_{12}c_{23}\cos\delta)\Delta_{21}^2 \qquad (48)
\end{aligned}$$

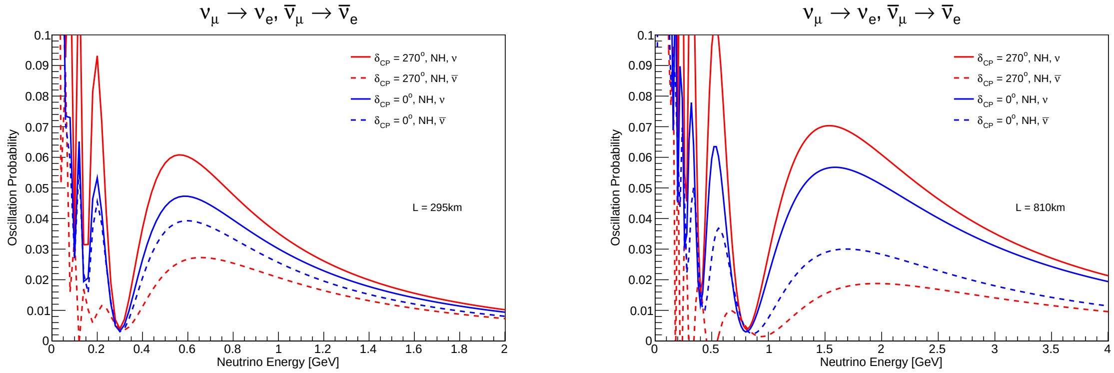
For $\frac{\Delta m^2_{21}x}{4E} \ll 1$ and $\Delta m^2_{31} \approx \Delta m^2_{32}$, you can make a replacement with: $\Delta_{21} = \sin\Delta_{21}$; $\cos\Delta_{31} = \cos\Delta_{32}$; $\sin\Delta_{31} = \sin\Delta_{32}$ and the probability of $\nu_\mu \to \nu_e$ oscillation can be written as follows

$$\begin{aligned} P(\nu_\mu \to \nu_e) &\approx 4s^2_{13}s^2_{23}c^2_{13}\sin^2\Delta_{31} \\ &-8s^2_{13}s^2_{23}c^2_{13}\frac{a}{\Delta m^2_{31}}(2s^2_{13}-1)\sin^2\Delta_{31} \\ &+8s^2_{13}s^2_{23}c^2_{13}\frac{aL}{4E}(2s^2_{13}-1)\sin\Delta_{31}\cos\Delta_{32} \\ &-8s_{12}s_{13}s_{23}c_{12}c^2_{13}c_{23}\sin\delta_{CP}\sin\Delta_{21}\sin\Delta_{31}\sin\Delta_{32} \\ &+8s_{12}s_{13}s_{23}c^2_{13}(c_{12}c_{23}\cos\delta_{CP} - s_{12}s_{13}s_{23})\sin\Delta_{21}\sin\Delta_{31}\cos\Delta_{32} \\ &+4s^2_{12}c^2_{13}(c^2_{12}c^2_{23} + s^2_{12}s^2_{13}s^2_{23} - 2s_{12}s_{13}s_{23}c_{12}c_{23}\cos\delta_{CP})\sin^2\Delta_{21}, \end{aligned} \tag{49}$$

where $\Delta_{ji} = \frac{\Delta m^2_{ji}}{4E}L$, and $a = 2\sqrt{2}G_F n_e E = 7.56\times10^{-5}[eV^2](\frac{\rho}{g/cm^3})(\frac{E}{GeV})$, n_e is the electron density of the matter and ρ is the density of the Earth. The appearances of a in the equation (50) is due to the matter effect which is rooted from the fact that electron neutrino when passing through ordinary matter will interact weakly with electrons. For anti-neutrino counterpart,$P(\overline{\nu}_\mu \to \overline{\nu}_e)$ can be obtained from Eq. 49 by replacing $\delta \to -\delta$ and $a \to -a$. In Eq. (49), the first term dominates with current long-baseline neutrino experiments and about 0.043 at the maximum of $\sin^2\Delta_{31}$. The matter effect, represented by a constant, involves to the second and third terms. While the term proportional to $\sin\delta_{CP}$ is called *CP-violating* since their contribution for total probability are opposite for neutrino and antineutrino, the fifth ,which contains $\cos\delta_{CP}$, is called *CP-conserving term* since their contributions are the same for neutrino and antineutrino. The last one depends on Δ^2_{21} and can be ignored in the case of long baseline experiments. At present landscape of neutrino oscillations, this channels is the only hope to provide information about δ_{CP}. However challenges for this channel measurement are the smallness of oscillation amplitude and its degeneracy with other oscillation parameters. Along with the appearance channels, the accelerator-based long-baseline neutrino experiments typically can measure precisely the probability of $\nu_\mu \to \nu_\mu$ and $\overline{\nu}_\mu \to \overline{\nu}_\mu$, which, to the first order approximation of matter effect, can be expressed as:

$$\begin{aligned} P(\bar{\nu}_\mu \to \bar{\nu}_\mu) \approx \quad & 1 + 4s^2_{23}c^2_{13}(s^2_{23}c^2_{13} - 1)\sin^2\Delta_{31} \\ &\pm 4s^2_{23}c^2_{13}s^2_{13}\left(2s^2_{23}c^2_{13} - 1\right)\tfrac{2a}{\Delta m^2_{31}}\sin^2\Delta_{31} \\ &\pm 4s^2_{23}c^2_{13}s^2_{13}\left(2s^2_{23}c^2_{13} - 1\right)\tfrac{a}{\Delta m^2_{31}}\Delta_{31}\sin 2\Delta_{31} \\ &+4s^2_{23}c^2_{13}(c^2_{12}c^2_{23} + s^2_{12}s^2_{13}s^2_{23} - 2s_{12}s_{13}s_{23}c_{12}c_{23}\cos\delta)\Delta_{21}\sin 2\Delta_{31}, \end{aligned} \tag{50}$$

where positive (negative) signs are taken for neutrino (antineutrino) oscillations respectively. Due to relative smallness of θ_{13} the first term is dominated in the accelerator-based long-baseline neutrino experiment and measurement with this channel is essentially sensitive to



(a) (b)

Figure 1: Transition probabilities $P(\nu_\mu \to \nu_e)$ and $P(\bar{\nu}_\mu \to \bar{\nu}_e)$ as a function of neutrino energy for T2K baseline $L = 295$ km (a) and NOνA baseline $L = 810$ km (b).

mixing angle θ_{23} and Δm^2_{31}. In practice, neutrino oscillation analyses take advance of combining both appearance channel and disappearance channel in order to provide the most precise measurements of oscillation parameters and explore CP violation from constraints on δ_{CP}. Fig. 1 (a) and Fig. 1 (b) show the oscillation probabilities of $\bar{\nu}_\mu \to \bar{\nu}_e$ as a function of neutrino energy at different true value of δ_{CP} for T2K baseline $L = 295$ km (with peak of neutrino flux at 0.6 GeV) and NOνA baseline $L = 810$ km (with peak of neutrino flux at 2 GeV), respectively. These two leading accelerator-based long-baseline neutrino experiments will be discussed in detail in Section 0.3. In the figures, the difference between solid and dashed blue lines indicates the matter effect, and the difference between solid and dashed red lines shows the combined effect of both matter and CP-violation. In the case of T2K experiment, the matter effect is much smaller than the CP-violation effect. For NOνA, due to its longer baseline the matter effect is larger. The plots are made with assumed values of oscillation parameters as listed Table 1:

Table 1: Input values of oscillation parameters, taken from [18].

	$\sin^2 2\theta_{12}$	$\sin^2 2\theta_{13}$	$\sin^2 \theta_{23}$	Δm^2_{21}	Δm^2_{32}
Value	0.8704	0.085	0.5	$7.6 \times 10^{-5} eV^2/c^4$	$2.5 \times 10^{-3} eV^2/c^4$

0.3 T2K(-II) AND NOνA EXPERIMENTS

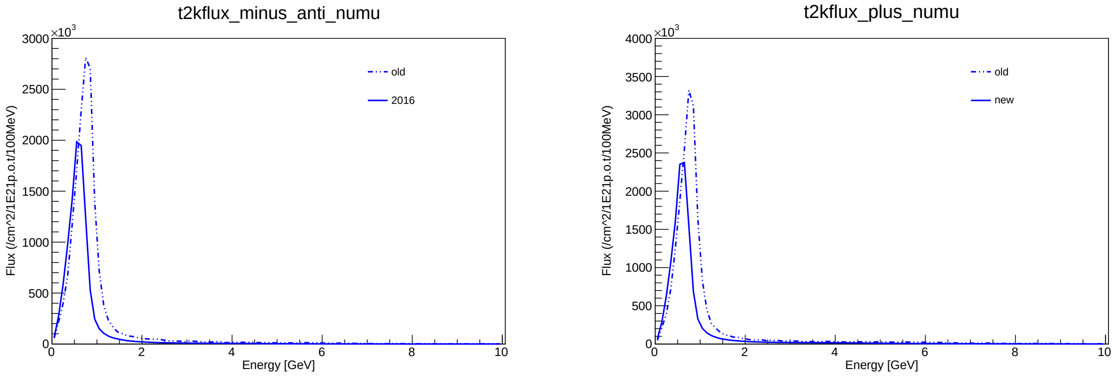
At present, T2K and NOvA are two leading accelerator-based long baseline neutrino experiment in the world. We briefly describe these two experiments and inputs we use to study their combined sensitivity on CP violation search.

T2K (Tokai-to-Kamioka) [10] is an accelerator-based long-baseline neutrino oscillation experiment placed in Japan with three main complexes: (i) the J-PARC accelerator, (ii) the near detector suite placed 280 m from the neutrino production target, and (iii) the far detector, Super-Kamiokande, situated 295 km away from target. The J-PARC, one of the most intense proton beam in the world, is used to produce a nearly pure $\bar{\nu}_\mu$ source. The near detector suite is designed to characterize the unoscillated neutrino beam while the far detector is used to observe the oscillation patterns. The primary goal of T2K is to observe oscillation from muon neutrinos to electron neutrinos, which has been achieved in 2013. With relatively large value of mixing angle θ_{13}, the physics potential of T2K is revisited and CP violation search is placed as the central target. For the latest results [25], based on a total exposure of 2.23×10^{21} POT, consisting of 1.47×10^{21} POT in ν-mode and 0.76×10^{21} POT in $\bar{\nu}$-mode, T2K firstly reports that CP conserving value (0 and π) of δ_{CP} is out of the 2σ C.L. range of the measurement for both normal and inverted mass hierarchies. By the year 2021, with a fully approved exposure of 7.8×10^{21} POT collected, T2K will have sensitivity to the CP-violating phase δ_{CP} at 90% C.L. or higher over a significant range [11]. To intensively explore CP violation in the lepton sector, T2K-II, extension of T2K operation up to 2026, is proposed to collect 20×10^{21} POT [12]. This amount of data in combination with expected improvement in the neutrino beamline and neutrino oscillation analysis allows T2K to have 3σ or higher significant sensitivity to CP violation. Also the oscillation parameters θ_{23} and Δm^2_{31} can be measured at the unprecedented level.

Rescaling T2K flux

T2K flux in GLoBES which is corresponding to 2.0 degree off-axis is out of date. The new flux (50MeV wide bins) corresponding to actual 2.5 degree off-axis angle provided by Dr. Cao Son (KEK) is, however, different in format with default in GLoBES (100MeV wide bins). We therefore have to rebin it in order to be consistant with GLoBES. The rebin flux is in 100MeV wide bins up to 10GeV. All fluxes are normalized to 1×10^{21} protons delivered to the T2K production target. The code used to rebin flux from root file and then store in txt file is provided in rebinflux.cxx.

In this paper, we use GLoBES software package [15] to study the physics potential of long-baseline neutrino experiments. For the inputs of T2K configuration in GloBES, we follow closely the information in the paper [12]. The neutrino fluxes for both neutrino-mode and antineutrino-mode operations are updated with the latest fluxes released by T2K collaboration. The full statistics of T2K-II is equivalent to 30×10^{21} POT, which reflects the T2K-II prospect in improving the analysis with a factor of 50%. The efficiencies for detecting ν_e and $\overline{\nu}_e$ signals are set to be 66.3% and 69.7% respectively while the efficiencies for detecting ν_μ and $\overline{\nu}_\mu$ signals are 72.6% and 80.2%, respectively. The event rates for



(a) (b)

Figure 2: Rebin flux compared with default in GLoBES for muon anti-neutrino beam (a) and muon neutrino beam (b).

T2K far detector reconstructed from GLoBES with our T2K-II setup for the cases of true $\delta_{CP} = 0,\ -\pi/2,\ +\pi/2$ are shown in the Table 2 for $\overset{(-)}{\nu}_e$ appearance samples and Table 3 for $\overset{(-)}{\nu}_\mu$ disappearance samples. The value here is consistent with [12] at acceptable level.

Table 2: The ν_e and $\bar{\nu}_e$ event samples predicted to collect in T2K-II far detector with 30×10^{21} POT, sharing same amount for neutrino-mode and antineutrino-mode operations, at three different values of $\delta_{CP} = -\pi/2,\ 0,\ +\pi/2$. The event rates are consistent with result shown in [12].

	δ_{CP}	Total	Signal $\nu_\mu \to \nu_e$	Signal $\bar{\nu}_\mu \to \bar{\nu}_e$	Beam CC $\nu_e + \bar{\nu}_e$	Beam CC $\nu_\mu + \bar{\nu}_\mu$	NC
ν-mode ν_e sample	$-\pi/2$	558.8	448.6	2.8	73.3	1.8	32.3
	0	466.3	354.9	4.0	73.3	1.8	32.3
	$+\pi/2$	370.9	258.6	4.9	73.3	1.8	32.3
$\bar{\nu}$-mode $\bar{\nu}_e$ sample	$-\pi/2$	115.8	19.8	52.3	29.2	0.4	14.1
	0	134.6	16.2	74.7	29.2	0.4	14.1
	$+\pi/2$	149.3	11.8	93.8	29.2	0.4	14.1

Table 3: The ν_μ and $\bar{\nu}_\mu$ event samples predicted to collect in T2K-II far detector with 30×10^{21} POT, sharing same amount of neutrino-mode and antineutrino-mode operations at three values of $\delta_{CP} = -\pi/2,\ 0,\ +\pi/2$. The event rates are consistent with result shown in [12].

	δ_{CP}	Total	Beam CC ν_μ	Beam CC $\bar{\nu}_\mu$	Beam CC $\nu_e + \bar{\nu}_e$	$\nu_\mu \to \nu_e$ + $\bar{\nu}_\mu \to \bar{\nu}_e$	NC
ν-mode ν_μ sample	$-\pi/2$	2735.3	2393.3	158.2	1.6	7.2	175.0
	0	2737.0	2392.4	157.8	1.6	10.2	175.0
	$+\pi/2$	2740.7	2393.3	158.2	1.6	12.6	175.0
$\bar{\nu}$-mode $\overline{\nu}_\mu$ sample	$-\pi/2$	1283.5	507.8	707.9	0.6	1.0	66.2
	0	1280.5	506.8	706.1	0.6	0.8	66.2
	$+\pi/2$	1283.1	507.8	707.9	0.6	0.6	66.2

In addition, systematic uncertainties of all T2K-II signal samples are anticipated to go down to 4% compared with $5.5\% - 6.8\%$ of current level. This can be achieved by reducing errors from neutrino fluxes, neutrino interaction models and detector model uncertainties.

NOvA (NuMI Off-axis ν_e Appearance) experiment [13] is an accelerator-based long-baseline neutrino experiment placed in United State. NOvA uses the intense and nearly pure $\overset{(-)}{\nu}_\mu$ beam from NuMI (Neutrino at Main Injector), Fermilab and studies oscillations with two functionally identical detectors: near detector (0.3 kton) situated underground at Fermilab, Illinois and far detector (14 kton) installed on the surface in Ash River, Minesota, 810 km away from the neutrino production target. The detectors, are placed at an offset angle of 14 mrad from the average neutrino beam line in order to achieve a narrow neutrino spectrum with peak at 2 GeV and suppress the neutral current π^0 background. This configuration is optimized for observing ν_e signal. With 810 km baseline, the matter effect can change $\nu_\mu \to \nu_e$ appearance rate up to $\pm 30\%$. In 2017, with an equivalent exposure of 6.05×10^{20} POT, 33 ν_e candidate was observed, clearly excess from 8.2 ± 0.8 background expected from MC. One of the major improvement in NOvA oscillation analysis is adopting machine learning algorithm, so-called Convolutional Visual Network [26] for the event-by-event classification of ν_e and $\overline{\nu}_e$. The gain from this new selection is equivalent to 30% effectively statistic increase. For the NOvA inputs, 61.0% and 71.5% signal efficiencies are used for selecting ν_e and $\overline{\nu}_e$ appearance samples, respectively. For the disappearance channels, the signal efficiencies for ν_μ and $\overline{\nu}_\mu$ samples are 32.0% and 38.0%, respectively. These number are based on the papers [**?**] and [**?**]. With an proposed operation up to the year 2024, NOvA is expected to accumulate a total exposure of 72×10^{20} POT including 36×10^{20} in $\nu-$ mode operation and same amount for the $\bar{\nu}-$mode operation. The event rates for complete statistics of NOvA are shown in the Table 4. In the NOvA analysis, the background from cosmic ray in the considering signal samples is significant due to the fact that the far detector is on the Earth surface. Since it is not easy to implement the cosmic ray flux in GloBES, this background is manually added into the ν_μ beam neutral-current (NC) channel. The systematic uncertainty by the end of NOνA operation is assumed to be 5% in GLoBES as it is provided.

Table 4: The ν_e and $\bar{\nu}_e$ event rates of full statistics of NOvA operation up to 2024 with 36×10^{20} in $\nu-$ mode and 36×10^{20} in $\bar{\nu}-$mode. The rates are calculated at at three values of $\delta_{CP} = -\pi/2, 0, +\pi/2$.

	δ_{CP}	Total	Signal	ν_μ beam CC	ν_μ beam NC	ν_e beam
ν-mode ν_e sample	$-\pi/2$	266.1	192.4	5.7	39.5	28.5
	0	239.4	165.7	5.7	39.5	28.5
	$+\pi/2$	195.6	121.9	5.7	39.5	28.5
$\bar{\nu}$-mode $\bar{\nu}_e$ sample	$-\pi/2$	63	33.8	2.1	13	14.1
	0	78.9	49.7	2.1	13	14.1
	$+\pi/2$	87.5	58.3	2.1	13	14.1

Table 5: The ν_μ and $\bar{\nu}_\mu$ event rates of NOvA with 36×10^{20} in $\nu-$ mode and 36×10^{20} in $\bar{\nu}-$mode. The rates are calculated at at three values of $\delta_{CP} = -\pi/2, 0, +\pi/2$.

	δ_{CP}	Total	Signal	ν_μ beam NC
ν-mode ν_μ sample	$-\pi/2$	546.3	512.5	33.8
	0	541.9	508.1	33.8
	$+\pi/2$	546.3	512.5	33.8
$\bar{\nu}$-mode $\bar{\nu}_\mu$ sample	$-\pi/2$	276.9	271.3	5.6
	0	275	269.4	5.6
	$+\pi/2$	276.9	271.3	5.6

0.3.1 Constraint on θ_{13} from reactor

As mentioned before, determination of θ_{13} plays an important role in measuring δ_{CP}. Current precision on $\sin^2 2\theta_{13}$ is 6% [17] with best fit $\sin^2 2\theta_{13} = 0.085$ [18]. Daya Bay reactor experiment has recently approved that it can achieve 3% precision on $\sin^2 2\theta_{13}$ by the year 2020 [16]. We examine here the two above possible cases.

In order to get plots as shown in Fig. 3, we used Reactor2.glb. Different constraints on θ_{13} can be obtained by changing value at @time.

- @time = 60 years coressponds to 6% precision on $\sin^2 2\theta_{13}$.
- @time = 300 years corresponds to 3% precision on $\sin^2 2\theta_{13}$.

To make file, use *theta13_proj_reactor.c*:

- cd to current directory.
- run file by typing: *./theta13_proj_reactor.c*

You can change output filename in *theta13_proj_reactor.c* at

```
/* Output file */
char MYFILE[]="theta13_proj_reactor.dat";
```

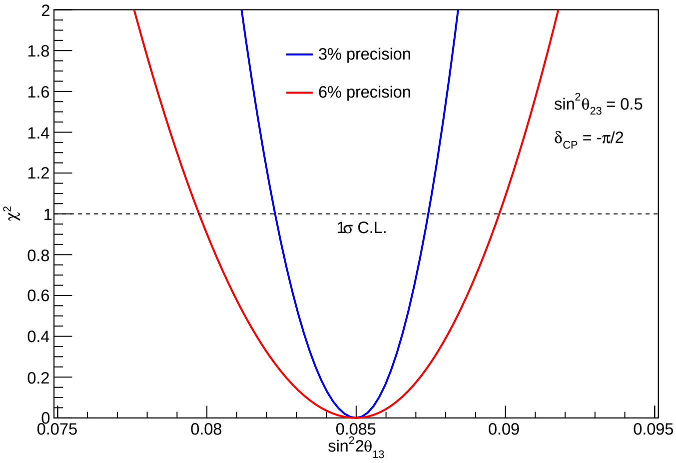


Figure 3: Precision of θ_{13} from reactor. The green line corresponds to 6% precision and the red line corresponds to 3% precision of the current best fit value of $\sin^2 2\theta_{13}$.

To make plot, use *plot_theta13_proj_reactor.C*

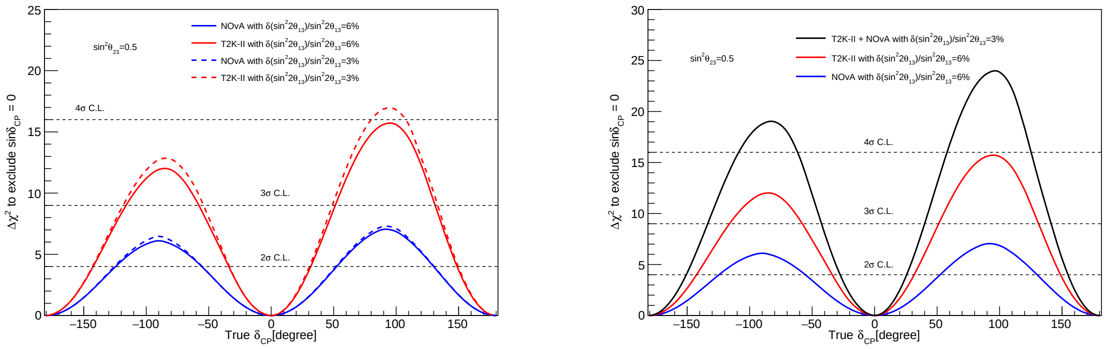
To make graph containing two plots, first run *theta13_proj_reactor.c* with @time = 60 and output file *theta13_proj_reactor_current.dat* for 6% precision. Then run *theta13_proj_reactor.c* with @time = 300 and output file *theta13_proj_reactor.dat* for 3% precision.

The code to get simultanousely two plots in the same graph is provided in *plot_theta13_proj_reactor.C*

0.4 SENSITIVITY TO CP-VIOLATION

At present landscape, the value of δ_{CP} is known with marginal significant. Thus we explore the sensitivity of T2K-II and NOvA experiment on full range of this parameter. At each given value of δ_{CP}, the minimal $\Delta\chi^2$ to exclude $\delta_{CP} = 0$ and $\delta_{CP} = \pm 180°$ are calculated. These values are then plotted as a function of true δ_{CP} in the meaning to exclude $\sin\delta_{CP} = 0$. For all calculations shown below, we assume that the neutrino mass hierarchy would be known by the end of T2K-II and NOvA operations and to be normal. In the future we will explore for the case in which the neutrino mass hierarchy is unknown. Fig. 4(a) shows that T2K-II and NOvA respectively can achieve 3σ C.L. and 2σ C.L. to exclude CP-conserving values with current precision of mixing angle θ_{13} from reactor-based experiments, that agree with their expectations in [12] and [**?**]. Also the figure shows that the sensitivity to CP violation is increased when the ultimate constraint on θ_{13} from reactor-based experiments is taken into account. Particularly, when θ_{13} uncertainty reduces from 6% to 3%, fractional region in which the CP conserving values (0 and $\pm\pi$) can be excluded at 3σ C.L. increases from

39.9% to 42.0% for T2K-II experiment. For NOvA, the 2σ C.L.-excluded factional region increases from 40.8% to 41.4% when the uncertainty of θ_{13} is reduced. When the T2K-II and NOvA signal samples are combined and if the true value of δ_{CP} is close to $-\pi/2$ as indicated by T2K result, the hypothesis of CP conservation in the lepton sector can be excluded at more than 4σ with an fractional δ_{CP} region up to 32.4% as shown in Fig. 4(b).



(a) (b)

Figure 4: (a) Sensitivity to CP-violation as a function of true δ_{CP} for T2K-II and NOvA with 6% precision on $sin^2 2\theta_{13}$ (solid red and blue lines, respectively), for T2K-II and NOvA with 3% precision on $sin^2 2\theta_{13}$ (dashed red and blue lines, respectively).
(b) Sensitivity to CP-violation as a function of true δ_{CP} for NOvA with 6% precision on $sin^2 2\theta_{13}$ (solid blue line), T2K-II with 6% precision on $sin^2 2\theta_{13}$ (solid red line) and T2K-II + NOvA with 3% precision on $sin^2 2\theta_{13}$ (solid black line).

At the present, the sensitivity of CP violation in both T2K and NOvA experiments are limited predominantly by the statistics. However around 2024 to 2026 where we expect that these two experiments collect their full statistics, impact of systematics on the CP violation measurement should be considered seriously. In the above result, 4% and 5% uncertainties are assumed for the signal samples for T2K-II and NOvA respectively. We also check the scenario in which the uncertainties can be reduced to 2% level. The result is shown in Fig. 5. Evidently improving the systematics raises significantly the level of sensitivity to CP violation. We can make discovery of CP violation at 5σ C.L. with fractional regions of 31.2%, 10.4% and 0% for 0.43, 0.5 and 0.6 of mixing angle $\sin^2\theta_{23}$, respectively.

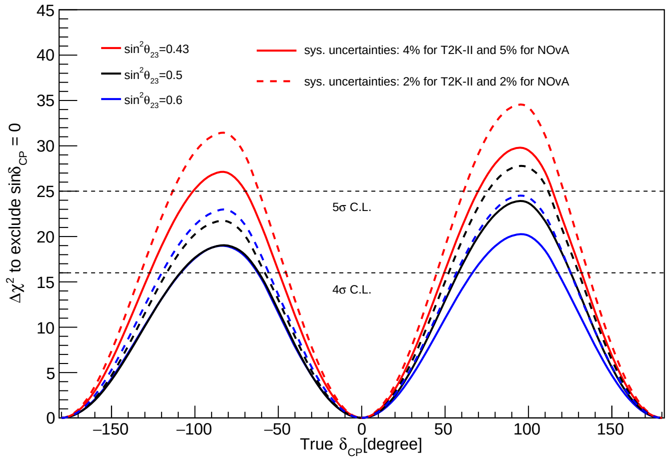


Figure 5: Sensitivity to CP-violation as a function of true δ_{CP} with sys. uncertainties of 4% for T2K-II + 5% for NOvA + ultimate reactor constraint (solid lines), and 2% for T2K-II + 2% for NOvA + ultimate reactor constraint (dashed lines).

0.5 CONCLUSIONS

In this paper, we have studied the sensitivity to CP-violation by combining the experiments T2K-II, which is proposed to run up to 2026, and NOνA, which can run up to 2024, with constraint from reactor-based experiments. With this combination, the CP conservation in lepton sector can be excluded at 4σ C.L. significance if the true values of δ_{CP} is around $-\pi/2$ as indicated by recent T2K measurement. The study also shows that precision measurement of mixing angle θ_{13} from the reactor-based experiments and improvements in the systematic uncertainties of measurement are crucial for the search of CP violation in the lepton sector. We can discovery CP violation at 5σ C.L. significance for a particular range of its value if θ_{13} precision reaches to 3% and the systematic uncertainty reduces to 2%

In the near future, we will consider to improve this study for more realistic description of both T2K-II and NOvA experiments by including efficiency as function of energy, smearing matrixes between true neutrino energy and the reconstructed energy by experiments. Also adding measurements from atmospheric neutrino experiments such as Super-Kamiokande, IceCube, etc... is considered to improve our sensitivity to CP violation measurements.

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