

TECHNICAL NOTE

COMBINED SENSITIVITIES OF T2K-II AND NO ν A EXPERIMENTS TO CP-VIOLATION IN NEUTRINO SECTOR

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0.1 PHYSICS MOTIVATION

In order to explain the solar neutrino anomaly [1] and atmospheric neutrino anomaly [2], neutrino oscillation phenomenon, in which one type of neutrino can change into another, has been proposed. In 1957, B. Pontecovo [3] firstly suggested a neutrino-antineutrino transition to explain these anomalies. The neutrino flavor oscillation was introduced later in 1962 by Z. Maki, M. Nakagawa and S. Sakata [4]. The neutrino oscillation phenomenon was observed by Super-Kamiokande experiment [21], SNO experiment [22] and later conclusively confirmed by number of neutrino experiments with different detection techniques at different energy range and different baselines. Discovery of neutrino oscillation which indicates that neutrinos have mass and mix among states, matter a lot since this is the only experimental evidence for the incompleteness of the Standard Model of fundamental particles.

Except for some anomalies, the up-to-date (anti-)neutrino data from various experiments can be well described by a 3×3 unitary mixing matrix, so-called PMNS matrix. This unitary matrix, as discussed more details in the next section, are parameterized by three mixing angles (θ_{12} , θ_{13} and θ_{23}) and one single Dirac phase δ_{CP} ¹ which represents CP violation in the lepton sector. The three mixing angles are determined to be non-zero [23] and this allows neutrino experiments to make measurement on the CP violation in the lepton sector, which is one of the most central objective in the present and at near future of neutrino physics. Besides these four parameters, the oscillation probabilities depend on the mass-squared differences among the mass eigenstate, neutrino energy and the distance neutrino travels. At the current landscape of neutrino oscillation physics, two scales of mass-squared differences are determined. However their mass ordering is still unknown and also one of the most important question need to be addressed in the future.

T2K and NOvA are two among the world leading neutrino experiments in searching the CP violation in the lepton sector. The combined sensitivity of these two experiments was performed and shows that this sensitivity can be up to 2σ or higher if the true value δ_{CP} is about $-\pi/2$ [11]. We are revising this analysis with three main updates including (i) possible T2K run extension up to 2026, so-called T2K-II (ii) improvement in selection performance and systematic uncertainties in both experiment and (iii) the ultimate precision on mixing angle θ_{13} can be achieved by the reactor measurements. The first twos are crucial since the measurement is dominated by the statistical errors. The thirds one is needed to break down the $\delta_{CP} - \theta_{13}$ degeneracy with accelerator-based long baseline experiment. These combined critical factors enhances capability to search CP violation to unprecedented level of sensitivity.

The paper is organised as follows, the PMNS formalism of neutrino oscillation is in-

¹If neutrino is Majorana particle, there are two phases added into the PMNS matrix. However the oscillation amplitudes are not sensitive to these two phases.

43 produced with an intense focus on how CP violation can be measured. T2K(-II) and NOvA
 44 experiments are overviewed and their inputs for this analysis are presented in section 0.3.
 45 The outcomes of combined sensitivity between the T2K-II and NOvA experiment with con-
 46 straint from reactor are presented in section 0.4.

47 0.2 NEUTRINO OSCILLATION FRAMEWORK

48 In three-flavor neutrino oscillation framework, the flavor definitive eigenstates are related to
 49 the mass definitive eigenstates by a 3×3 unitary PMNS matrix, shown in Eq. 1,

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{\text{PMNS}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}. \quad (1)$$

50 The unitarity of PMNS matrix $UU^\dagger = I$ yields nine independent parameters. If the PMNS
 51 matrix were real, it could be described by three rotation angles θ_{12} , θ_{13} and θ_{23} via orthogonal
 52 rotation matrix R

$$R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (2)$$

53 where $s_{ij} = \sin \theta_{ij}$ and $c_{ij} = \cos \theta_{ij}$. Since PMNS matrix is unitary and not real, it must
 54 contain six more additional degrees of freedom in term of complex phase $e^{i\delta}$. Five among
 55 these six phases can be absorbed into the definition of the particles and leaves only one single
 56 phase δ . This can be seen as follow.

The charged currents for leptonic weak interaction

$$-i \frac{g_W}{\sqrt{2}} (\bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau) \gamma^\mu \frac{1}{2} (1 - \gamma^5) \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

57 The four-vector currents are unchanged by transformation

$$l_\alpha \rightarrow l_\alpha e^{i\theta_\alpha}, \quad \nu_k \rightarrow \nu_k e^{i\theta_k} \quad \text{and} \quad U_{\alpha k} \rightarrow U_{\alpha k} e^{i(\theta_\alpha - \theta_k)} \quad (3)$$

where l_α is the charged lepton of the type $\alpha = e, \mu, \tau$. Since the phases are arbitrary, all other
 phases can be defined in term of θ_e :

$$\theta_k = \theta_e + \theta'_k$$

The transformation (3) therefore becomes

$$l_\alpha \rightarrow l_\alpha e^{i(\theta_e + \theta'_\alpha)}, \quad \nu_k \rightarrow \nu_k e^{i(\theta_e + \theta'_k)} \quad \text{and} \quad U_{\alpha k} \rightarrow U_{\alpha k} e^{i(\theta'_\alpha - \theta'_k)}$$

For electron $\theta_e = \theta_e + \theta'_e \Rightarrow \theta'_e = 0$. It can be seen now that only five phases are independent and can be absorbed into the particle definitions. The PMNS matrix thus can be parameterized by three mixing angles $(\theta_{12}, \theta_{13}, \theta_{23})$ and a single Dirac phase δ_{CP} , expressed in Eq. 4.

$$\begin{aligned}
U_{PMNS} &= \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \\
&= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \quad (4)
\end{aligned}$$

where $s_{ij} = \sin \theta_{ij}$, $c_{ij} = \cos \theta_{ij}$ and δ_{CP} Dirac phase represents the CP violation in lepton sector². As mentioned before, CP is violated if $U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*$ contains an imaginary component. Therefore, δ is also called the CP-violating phase δ_{CP} .

The oscillation probability from muon neutrino to electron neutrino is

$$\begin{aligned}
P(\nu_\mu \rightarrow \nu_e) &= |\langle \nu_e | \Psi(\vec{x}, t) \rangle|^2 = c_e c_e^* \\
&= |U_{\mu 1}^* U_{e1} e^{-i\phi_1} + U_{\mu 2}^* U_{e2} e^{-i\phi_2} + U_{\mu 3}^* U_{e3} e^{-i\phi_3}|^2 \quad (5)
\end{aligned}$$

In compact form

$$P(\nu_\alpha \rightarrow \nu_\beta) = \left| \sum_i U_{\alpha i}^* U_{\beta i} e^{-i\phi_i} \right|^2 \quad (6)$$

If $\phi_1 = \phi_2 = \phi_3 (\approx \frac{m^2}{2E})$, from unitary condition we have $P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta}$. This means that the oscillations occur if the neutrinos have mass and the masses are not the same.

Using the identity properties of complex number:

$$|z_1 + z_2 + z_3|^2 = |z_1|^2 + |z_2|^2 + |z_3|^2 + 2\text{Re}[z_1 z_2^* + z_1 z_3^* + z_2 z_3^*] \quad (7)$$

Then equation (5) becomes

$$\begin{aligned}
P(\nu_\mu \rightarrow \nu_e) &= |U_{\mu 1}^* U_{e1} e^{-i\phi_1} + U_{\mu 2}^* U_{e2} e^{-i\phi_2} + U_{\mu 3}^* U_{e3} e^{-i\phi_3}|^2 \\
&= |U_{\mu 1}^* U_{e1}|^2 + |U_{\mu 2}^* U_{e2}|^2 + |U_{\mu 3}^* U_{e3}|^2 \\
&\quad + 2\text{Re}[U_{\mu 1}^* U_{e1} U_{\mu 2}^* U_{e2} e^{i(\phi_2 - \phi_1)}] \\
&\quad + 2\text{Re}[U_{\mu 1}^* U_{e1} U_{\mu 3}^* U_{e3} e^{i(\phi_3 - \phi_1)}] \\
&\quad + 2\text{Re}[U_{\mu 2}^* U_{e2} U_{\mu 3}^* U_{e3} e^{i(\phi_3 - \phi_2)}] \quad (8)
\end{aligned}$$

²If neutrino is Majorana particle, the mixing matrix includes two additional phases which do not appear in the expression of oscillation probabilities.

71 In compact form

$$P(\nu_\alpha \rightarrow \nu_\beta) = \sum_i |U_{\alpha i}^* U_{\beta i}|^2 + 2 \sum_{j>i} \text{Re}[U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* e^{i(\phi_j - \phi_i)}] \quad (9)$$

72 From the unitary condition we derive

$$\begin{aligned} & |U_{\mu 1}^* U_{e 1} + U_{\mu 2}^* U_{e 2} + U_{\mu 3}^* U_{e 3}|^2 = 0 \\ \Rightarrow & |U_{\mu 1}^* U_{e 1}|^2 + |U_{\mu 2}^* U_{e 2}|^2 + |U_{\mu 3}^* U_{e 3}|^2 \\ & + 2\text{Re}[U_{\mu 1}^* U_{e 1} U_{\mu 2} U_{e 2}^* + U_{\mu 1}^* U_{e 1} U_{\mu 3} U_{e 3}^* + U_{\mu 2}^* U_{e 2} U_{\mu 3} U_{e 3}^*] \\ = & 0 \end{aligned} \quad (10)$$

73 In compact form

$$\sum_i |U_{\alpha i}^* U_{\beta i}|^2 + 2 \sum_{j>i} \text{Re}[U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*] = \delta_{\alpha\beta} \quad (11)$$

74 It is followed from (8) and (10):

$$\begin{aligned} P(\nu_\mu \rightarrow \nu_e) = & 2\text{Re}\left[U_{\mu 1}^* U_{e 1} U_{\mu 2} U_{e 2}^* \left(e^{i(\phi_2 - \phi_1)} - 1\right)\right] \\ & + 2\text{Re}\left[U_{\mu 1}^* U_{e 1} U_{\mu 3} U_{e 3}^* \left(e^{i(\phi_3 - \phi_1)} - 1\right)\right] \\ & + 2\text{Re}\left[U_{\mu 2}^* U_{e 2} U_{\mu 3} U_{e 3}^* \left(e^{i(\phi_3 - \phi_2)} - 1\right)\right] \end{aligned} \quad (12)$$

75 In compact form

$$P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} + 2 \sum_{j>i} \text{Re}[U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* (e^{i(\phi_j - \phi_i)} - 1)] \quad (13)$$

76 We have

$$\begin{aligned} & \text{Re}[U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* (e^{i(\phi_j - \phi_i)} - 1)] \\ = & \text{Re}[U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* (\cos(\phi_j - \phi_i) - 1 + i \sin(\phi_j - \phi_i))] \\ = & \text{Re}\left\{ (\text{Re}[U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*] + i \text{Im}[U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*]) (-2 \sin^2(\frac{\phi_j - \phi_i}{2}) + i \sin(\phi_j - \phi_i)) \right\} \\ = & -2\text{Re}[U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*] \sin^2(\frac{\phi_j - \phi_i}{2}) - \text{Im}[U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*] \sin(\phi_j - \phi_i) \end{aligned} \quad (14)$$

77 From (14), we can write the oscillation pobability in a normal form

$$\begin{aligned} P(\nu_\mu \rightarrow \nu_e) = & \\ - & 4\text{Re}\left[U_{\mu 1}^* U_{e 1} U_{\mu 2} U_{e 2}^*\right] \sin^2(\frac{\phi_2 - \phi_1}{2}) - 2\text{Im}\left[U_{\mu 1}^* U_{e 1} U_{\mu 2} U_{e 2}^*\right] \sin(\phi_2 - \phi_1) \\ - & 4\text{Re}\left[U_{\mu 1}^* U_{e 1} U_{\mu 3} U_{e 3}^*\right] \sin^2(\frac{\phi_3 - \phi_1}{2}) - 2\text{Im}\left[U_{\mu 1}^* U_{e 1} U_{\mu 3} U_{e 3}^*\right] \sin(\phi_3 - \phi_1) \\ - & 4\text{Re}\left[U_{\mu 2}^* U_{e 2} U_{\mu 3} U_{e 3}^*\right] \sin^2(\frac{\phi_3 - \phi_2}{2}) - 2\text{Im}\left[U_{\mu 2}^* U_{e 2} U_{\mu 3} U_{e 3}^*\right] \sin(\phi_3 - \phi_2) \end{aligned} \quad (15)$$

78

In compact form

$$\begin{aligned}
P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} & - 4 \sum_{j>i} \text{Re} \left[U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \right] \sin^2 \left(\frac{\phi_j - \phi_i}{2} \right) \\
& - 2 \sum_{j>i} \text{Im} \left[U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \right] \sin(\phi_j - \phi_i)
\end{aligned} \quad (16)$$

If the neutrino interacts at a time T at a distance L along its direction of flight, the difference in phase of the three mass eigenstates are written as

$$\phi_j - \phi_i = p_j \cdot x_j - p_i \cdot x_i = (E_j - E_i)T - (p_j - p_i)L$$

79

With assuming that $p_j = p_i = p$ for neutrinos of the same source, then

$$\begin{aligned}
\phi_j - \phi_i &= (E_j - E_i)T \approx \left[p_j \left(1 + \frac{m_j^2}{2p_j^2} \right) - p_i \left(1 + \frac{m_i^2}{2p_i^2} \right) \right] T \\
&= \frac{m_j^2 - m_i^2}{2p} T = \frac{\Delta m_{ji}^2 L}{2E}
\end{aligned} \quad (17)$$

80

In the above calculation, we used the approximation $T \approx L$ and $p \approx E$ for $\nu_\nu \approx c$ and

81

$$m_\nu \ll E_\nu$$

82

We finally get the most common form of the oscillation probability:

$$\begin{aligned}
P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} & - 4 \sum_{j>i} \text{Re} \left[U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \right] \sin^2 \left(\frac{\Delta m_{ji}^2}{4E} L \right) \\
& - 2 \sum_{j>i} \text{Im} \left[U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \right] \sin \left(\frac{\Delta m_{ji}^2}{2E} L \right)
\end{aligned} \quad (18)$$

The probability for a α -flavour neutrino with energy E to change to β -flavour after traveling a distance of L can be calculated as follows

$$P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{i>j} \Re \left[U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \right] \sin^2 \left(\frac{\Delta m_{ij}^2}{4E} L \right) + 2 \sum_{i>j} \Im \left[U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \right] \sin \left(\frac{\Delta m_{ij}^2}{2E} L \right), \quad (19)$$

83

where $\Delta m_{ij}^2 = m_i^2 - m_j^2$. For antineutrinos, the oscillation probabilities can be obtained by

84

replace the mixing matrix elements with their complex conjugate.

85

Equation (19) is completely the same as eq. (18).

86

For antineutrinos, we just take the complex conjugate of the product matrix and get

$$\begin{aligned}
P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) = \delta_{\alpha\beta} & - 4 \sum_{j>i} \text{Re} \left[U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \right] \sin^2 \left(\frac{\Delta m_{ji}^2}{4E} L \right) \\
& + 2 \sum_{j>i} \text{Im} \left[U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \right] \sin \left(\frac{\Delta m_{ji}^2}{2E} L \right)
\end{aligned} \quad (20)$$

87 The probabilities (18) and (20) are called *transition probabilities*, and the *survival*
 88 *probability* for a flavor is

$$P(\nu_\alpha \rightarrow \nu_\alpha) = P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\alpha) = 1 - 4 \sum_{j>i} |U_{\alpha i}|^2 |U_{\alpha j}|^2 \sin^2\left(\frac{\Delta m_{ji}^2}{4E} L\right) \quad (21)$$

89 By using the natural unit conversion for $1eV^{-1}$ of length $= 1.97 \times 10^{-7}m$, we can prac-
 90 tically express the phase in (18), (20) and (21) as

$$\begin{aligned} \frac{\Delta m_{ji}^2 [eV^2] L [eV]}{4E [eV]} &= \frac{\Delta m_{ji}^2 [eV^2] L [m]}{4 \times 1.97 \times 10^{-7} E [eV]} \\ &= 1.269 \frac{\Delta m_{ji}^2 [eV^2] L [m]}{E [MeV]} = 1.269 \frac{\Delta m_{ji}^2 [eV^2] L [km]}{E [GeV]} \end{aligned} \quad (22)$$

91 From (18) and (20), the difference between the neutrino and antineutrino oscillation
 92 probability indicates CP violation in neutrino sector

$$\begin{aligned} \mathcal{A}_{CP} &= P(\nu_\alpha \rightarrow \nu_\beta) - P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) \\ &= 4 \sum_{j>i} \text{Im} \left[U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \right] \sin\left(\frac{\Delta m_{ij}^2}{2E} L\right) \end{aligned} \quad (23)$$

93 If CP is violated, $U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*$ has to contain an imaginary component.
 94 For $\alpha = \mu$ and $\beta = e$, then

$$\begin{aligned} \mathcal{A}_{CP} &= P(\nu_\mu \rightarrow \nu_e) - P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) \\ &= 4 \sum_{j>i} \text{Im} \left[U_{\mu i}^* U_{ei} U_{\mu j} U_{ej}^* \right] \sin\left(\frac{\Delta m_{ij}^2}{2E} L\right) \end{aligned} \quad (24)$$

$$\begin{aligned} &= 4 \text{Im} \left[U_{\mu 1}^* U_{e1} U_{\mu 2} U_{e2}^* \right] \sin\left(\frac{\Delta m_{12}^2}{2E} L\right) \\ &+ 4 \text{Im} \left[U_{\mu 1}^* U_{e1} U_{\mu 3} U_{e3}^* \right] \sin\left(\frac{\Delta m_{13}^2}{2E} L\right) \\ &+ 4 \text{Im} \left[U_{\mu 2}^* U_{e2} U_{\mu 3} U_{e3}^* \right] \sin\left(\frac{\Delta m_{23}^2}{2E} L\right) \end{aligned} \quad (25)$$

95 From the unitary condition we have

$$U_{\mu 1} U_{e1}^* + U_{\mu 2} U_{e2}^* + U_{\mu 3} U_{e3}^* = 0 \quad (26)$$

96 Multiply two sides of the equation (26) with $U_{\mu 1}^* U_{e1}$ and $U_{\mu 2}^* U_{e2}$ respectively and then

97 add them up, we have

$$\begin{aligned}
& U_{\mu 1}^* U_{e 1} U_{\mu 1} U_{e 1}^* + U_{\mu 1}^* U_{e 1} U_{\mu 2} U_{e 2}^* + U_{\mu 1}^* U_{e 1} U_{\mu 3} U_{e 3}^* \\
& + U_{\mu 2}^* U_{e 2} U_{\mu 1} U_{e 1}^* + U_{\mu 2}^* U_{e 2} U_{\mu 2} U_{e 2}^* + U_{\mu 2}^* U_{e 2} U_{\mu 3} U_{e 3}^* = 0 \\
\Leftrightarrow & 0 = |U_{\mu 1}|^2 |U_{e 1}|^2 + |U_{\mu 2}|^2 |U_{e 2}|^2 \\
& + \operatorname{Re}[U_{\mu 1}^* U_{e 1} U_{\mu 2} U_{e 2}^*] + \operatorname{Re}[U_{\mu 2}^* U_{e 2} U_{\mu 1} U_{e 1}^*] + \operatorname{Re}[U_{\mu 1}^* U_{e 1} U_{\mu 3} U_{e 3}^*] + \operatorname{Re}[U_{\mu 2}^* U_{e 2} U_{\mu 3} U_{e 3}^*] \\
& + i \left\{ \operatorname{Im}[U_{\mu 1}^* U_{e 1} U_{\mu 2} U_{e 2}^*] + \operatorname{Im}[U_{\mu 2}^* U_{e 2} U_{\mu 1} U_{e 1}^*] + \operatorname{Im}[U_{\mu 1}^* U_{e 1} U_{\mu 3} U_{e 3}^*] + \operatorname{Im}[U_{\mu 2}^* U_{e 2} U_{\mu 3} U_{e 3}^*] \right\} \\
\Rightarrow & \operatorname{Im}[U_{\mu 1}^* U_{e 1} U_{\mu 2} U_{e 2}^*] + \operatorname{Im}[U_{\mu 2}^* U_{e 2} U_{\mu 1} U_{e 1}^*] + \operatorname{Im}[U_{\mu 1}^* U_{e 1} U_{\mu 3} U_{e 3}^*] + \operatorname{Im}[U_{\mu 2}^* U_{e 2} U_{\mu 3} U_{e 3}^*] = 0
\end{aligned} \tag{27}$$

Note that

$$[U_{\mu 1}^* U_{e 1} U_{\mu 2} U_{e 2}^*]^* = U_{\mu 2}^* U_{e 2} U_{\mu 1} U_{e 1}^* \Rightarrow \operatorname{Im}[U_{\mu 1}^* U_{e 1} U_{\mu 2} U_{e 2}^*] = -\operatorname{Im}[U_{\mu 2}^* U_{e 2} U_{\mu 1} U_{e 1}^*]$$

98 Therefore, from (27) we get

$$\operatorname{Im}[U_{\mu 1}^* U_{e 1} U_{\mu 3} U_{e 3}^*] = -\operatorname{Im}[U_{\mu 2}^* U_{e 2} U_{\mu 3} U_{e 3}^*] \tag{28}$$

99 Multiply two sides of the equation (26) with $U_{\mu 1}^* U_{e 1}$ and $U_{\mu 3}^* U_{e 3}$ respectively and then
100 add them up, we have

$$\begin{aligned}
& U_{\mu 1}^* U_{e 1} U_{\mu 1} U_{e 1}^* + U_{\mu 1}^* U_{e 1} U_{\mu 2} U_{e 2}^* + U_{\mu 1}^* U_{e 1} U_{\mu 3} U_{e 3}^* \\
& + U_{\mu 3}^* U_{e 3} U_{\mu 1} U_{e 1}^* + U_{\mu 3}^* U_{e 3} U_{\mu 2} U_{e 2}^* + U_{\mu 3}^* U_{e 3} U_{\mu 3} U_{e 3}^* = 0 \\
\Leftrightarrow & 0 = |U_{\mu 1}|^2 |U_{e 1}|^2 + |U_{\mu 3}|^2 |U_{e 3}|^2 \\
& + \operatorname{Re}[U_{\mu 1}^* U_{e 1} U_{\mu 3} U_{e 3}^*] + \operatorname{Re}[U_{\mu 3}^* U_{e 3} U_{\mu 1} U_{e 1}^*] + \operatorname{Re}[U_{\mu 1}^* U_{e 1} U_{\mu 2} U_{e 2}^*] + \operatorname{Re}[U_{\mu 3}^* U_{e 3} U_{\mu 2} U_{e 2}^*] \\
& + i \left\{ \operatorname{Im}[U_{\mu 1}^* U_{e 1} U_{\mu 3} U_{e 3}^*] + \operatorname{Im}[U_{\mu 3}^* U_{e 3} U_{\mu 1} U_{e 1}^*] + \operatorname{Im}[U_{\mu 1}^* U_{e 1} U_{\mu 2} U_{e 2}^*] + \operatorname{Im}[U_{\mu 3}^* U_{e 3} U_{\mu 2} U_{e 2}^*] \right\} \\
\Rightarrow & \operatorname{Im}[U_{\mu 1}^* U_{e 1} U_{\mu 3} U_{e 3}^*] + \operatorname{Im}[U_{\mu 3}^* U_{e 3} U_{\mu 1} U_{e 1}^*] + \operatorname{Im}[U_{\mu 1}^* U_{e 1} U_{\mu 2} U_{e 2}^*] + \operatorname{Im}[U_{\mu 3}^* U_{e 3} U_{\mu 2} U_{e 2}^*] = 0
\end{aligned} \tag{29}$$

Note that

$$[U_{\mu 1}^* U_{e 1} U_{\mu 3} U_{e 3}^*]^* = U_{\mu 3}^* U_{e 3} U_{\mu 1} U_{e 1}^* \Rightarrow \operatorname{Im}[U_{\mu 1}^* U_{e 1} U_{\mu 3} U_{e 3}^*] = -\operatorname{Im}[U_{\mu 3}^* U_{e 3} U_{\mu 1} U_{e 1}^*]$$

and

$$[U_{\mu 3}^* U_{e 3} U_{\mu 2} U_{e 2}^*]^* = U_{\mu 2}^* U_{e 2} U_{\mu 3} U_{e 3}^* \Rightarrow \operatorname{Im}[U_{\mu 3}^* U_{e 3} U_{\mu 2} U_{e 2}^*] = -\operatorname{Im}[U_{\mu 2}^* U_{e 2} U_{\mu 3} U_{e 3}^*]$$

101 Therefore, from (29) we get

$$\operatorname{Im}[U_{\mu 1}^* U_{e 1} U_{\mu 2} U_{e 2}^*] = \operatorname{Im}[U_{\mu 2}^* U_{e 2} U_{\mu 3} U_{e 3}^*] \tag{30}$$

By using (28) and (30), we can rewrite (29) as

$$\begin{aligned}\mathcal{A}_{\text{CP}} &= P(\nu_\mu \rightarrow \nu_e) - P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) \\ &= 4\text{Im} \left[U_{\mu 1}^* U_{e 1} U_{\mu 3} U_{e 3}^* \right] (\sin \Delta_{13} - \sin \Delta_{12} - \sin \Delta_{23})\end{aligned}\quad (31)$$

Where $\Delta_{13} = \frac{\Delta m_{13}^2}{2E}L$, $\Delta_{12} = \frac{\Delta m_{12}^2}{2E}L$ and $\Delta_{23} = \frac{\Delta m_{23}^2}{2E}L = \Delta_{13} - \Delta_{12}$

By a simple trigonometry calculation, we have

$$\begin{aligned}\sin \Delta_{13} - \sin \Delta_{12} - \sin(\Delta_{13} - \Delta_{12}) &= -4 \sin \frac{\Delta_{12}}{2} \sin \frac{\Delta_{13}}{2} \sin \frac{\Delta_{23}}{2} \\ &= 4 \sin \frac{\Delta_{21}}{2} \sin \frac{\Delta_{31}}{2} \sin \frac{\Delta_{32}}{2}\end{aligned}$$

Then we can rewrite (31) as

$$\begin{aligned}\mathcal{A}_{\text{CP}} &= P(\nu_\mu \rightarrow \nu_e) - P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) \\ &= 16\text{Im} \left[U_{\mu 1}^* U_{e 1} U_{\mu 3} U_{e 3}^* \right] \left(\sin \frac{\Delta_{21}}{2} \sin \frac{\Delta_{31}}{2} \sin \frac{\Delta_{32}}{2} \right) \\ &= 16\text{Im} \left[U_{\mu 1}^* U_{e 1} U_{\mu 3} U_{e 3}^* \right] \sin \left(\frac{\Delta m_{21}^2 L}{4E} \right) \sin \left(\frac{\Delta m_{31}^2 L}{4E} \right) \sin \left(\frac{\Delta m_{32}^2 L}{4E} \right)\end{aligned}\quad (32)$$

The general form in term of oscillation parameters can be obtained as shown in Eq. 33.

The coefficients in the denominator should be 4 instead of 2?

$$\begin{aligned}\mathcal{A}_{\text{CP}} &= P(\nu_\alpha \rightarrow \nu_\beta) - P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) = 4 \sum_{j>i} \Im \left[U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \right] \sin \left(\frac{\Delta m_{ji}^2}{2E} L \right), \\ &= \pm 2\delta_{\alpha\beta} \cos \theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \sin \delta_{CP} \sin \frac{\Delta m_{21}^2 L}{2E} \sin \frac{\Delta m_{32}^2 L}{2E} \sin \frac{\Delta m_{13}^2 L}{2E}\end{aligned}\quad (33)$$

in which $\{\alpha, \beta\} = \{e, \mu, \tau\}$; $\{i, j\} = \{1, 2, 3\}$, $j > i$ and $\Delta m_{ji}^2 = m_j^2 - m_i^2$; the positive (negative) sign is applied based on (anti-) cyclic permutation of ordered flavor (e, μ, τ). Apparently CP violation can be measured via the neutrino oscillation phenomenon if only three mixing angles are non-zero. The up-to-date neutrino data shows that Nature supports this scenario and it opens the door to search CP violation in the lepton sector with neutrino oscillation measurements. This CP violation source might be a promising explanation for the matter asymmetry in the Universe.

In practical, CP violation can be measured by comparing the rate of electron neutrinos appearance from muon neutrinos, $P(\nu_\mu \rightarrow \nu_e)$, with its of electron antineutrinos appearance from muon anti-neutrinos, $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ in accelerator-based experiments or comparing the first with electron antineutrino disappearance in the reactor-based experiments³

³Accelerator-based measurements leads to an intrinsic $\delta_{CP} - \theta_{13}$ degeneracy while reactor-based measurement can precisely measure θ_{13} . Their combined information thus can provide constraint on δ_{CP} .

117 * **Evolution of neutrino flavors in matter**

118 The relation between mass eigenstates and flavor eigenstates

$$|\nu_\alpha\rangle = \sum_k U_{\alpha k}^* |\nu_k\rangle$$

119 The total Hamiltonian in matter is

$$H = H_0 + H_1$$

120 Where

$$\begin{aligned} H_0 |\nu_k\rangle &= E_k |\nu_k\rangle; \quad \text{with} \quad E_k = \sqrt{\vec{p}_k^2 + m_k^2} \approx p_k + \frac{m_k^2}{2p_k} \\ H_1 |\nu_\alpha\rangle &= V_\alpha |\nu_\alpha\rangle \end{aligned}$$

121 The Schrodinger equation for neutrino in matter is

$$\begin{aligned} i \frac{d}{dt} |\nu_\alpha(t)\rangle &= H |\nu_\alpha(t)\rangle \\ &= (H_0 + H_1) |\nu_\alpha(t)\rangle \\ &= (E_k + V_\alpha) |\nu_\alpha(t)\rangle \\ &= \left[\left(p_k + \frac{m_k^2}{2p_k} \right) + V_\alpha \right] |\nu_\alpha(t)\rangle \end{aligned}$$

122 For $v \approx c$ (means $t \approx x$) we have $p_k \approx E$. We can see that $E + V_{NC}$ is the same for all neutrinos.

123 They generate a phase common to all flavors and will cancel out in transition. Hence we can

124 ignore them here for simplicity. So we rewrite the above equation as

$$i \frac{d}{dt} |\nu_\alpha(t)\rangle = \left(\frac{m_k^2}{2E} + V_{CC} \delta_{\alpha e} \right) |\nu_\alpha(t)\rangle$$

125 Or in explicit form

$$i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \left[\frac{1}{2E} U \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} U^\dagger + \begin{pmatrix} V_{CC} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right] \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \quad (34)$$

126 Where U is an unitary matrix.

127 * **Complete oscillation probability in matter**

Since $v \approx c$ so $x \approx ct = t$ for $c = 1$. We can rewrite the Schrodinger equation in matter as

$$i \frac{d\nu}{dx} = H\nu$$

128 Where

$$\begin{aligned} H &= H_0 + H_1 \\ &= \frac{1}{2E} U \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \Delta m_{31}^2 \end{pmatrix} U^\dagger + \frac{1}{2E} \left[U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & 0 \end{pmatrix} U^\dagger + \begin{pmatrix} a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right] \end{aligned}$$

129 and $a = 2EV_{CC} = 2\sqrt{2}G_F E N_e$.

130 Since Δm_{21}^2 and $a \ll \Delta m_{31}^2$, we can treat H_1 as a perturbation.

131 The Schrodinger equation has a solution of Dyson series form

$$\nu(x) = S(x)\nu(0) \quad (35)$$

With

$$S(x) \equiv T e^{\int_0^x H(s) ds}$$

132 T is the symbol of time ordering. The oscillation probability at distance L then can be

133 calculate through $S(x)$

$$P(\nu_\alpha \rightarrow \nu_\beta) = |S_{\beta\alpha}(L)|^2 \quad (36)$$

We can calculate the perturbation to the first order in a and Δm_{21}^2 . We have

$$S_0(x) = e^{-iH_0 x}$$

and

$$S_1(x) = e^{-iH_0 x} (-i) \int_0^x ds H_1(s) = e^{-iH_0 x} (-i) \int_0^x ds e^{iH_0 s} H_1 e^{-iH_0 s}$$

134 We now calculate $S_0(x)$ and $S_1(x)$ as the following

$$\begin{aligned} (S_0(x))_{\beta\alpha} &= \left[U e^{-i\frac{x}{2E} \text{diag}(0,0,\Delta m_{31}^2)} U^\dagger \right]_{\beta\alpha} \\ &= \sum_{i,j} \left[U_{\beta i} \left(e^{-i\frac{x}{2E} \text{diag}(0,0,\Delta m_{31}^2)} \right)_{ij} U_{\alpha j}^* \right] \end{aligned}$$

Note that

$$e^{-i\frac{x}{2E} \text{diag}(0,0,\Delta m_{31}^2)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\frac{\Delta m_{31}^2 x}{2E}} \end{pmatrix}$$

135 Hence

$$\begin{aligned} (S_0(x))_{\beta\alpha} &= U_{\beta 1} U_{\alpha 1}^* \cdot 1 + U_{\beta 1} U_{\alpha 2}^* \cdot 0 + U_{\beta 1} U_{\alpha 3}^* \cdot 0 \\ &\quad + U_{\beta 2} U_{\alpha 1}^* \cdot 0 + U_{\beta 2} U_{\alpha 2}^* \cdot 1 + U_{\beta 2} U_{\alpha 3}^* \cdot 0 \\ &\quad + U_{\beta 3} U_{\alpha 1}^* \cdot 0 + U_{\beta 3} U_{\alpha 2}^* \cdot 0 + U_{\beta 3} U_{\alpha 3}^* \cdot e^{-i\frac{\Delta m_{31}^2 x}{2E}} \\ &= U_{\beta 1} U_{\alpha 1}^* + U_{\beta 2} U_{\alpha 2}^* + U_{\beta 3} U_{\alpha 3}^* \cdot e^{-i\frac{\Delta m_{31}^2 x}{2E}} \end{aligned}$$

136 By using $U_{\beta 1}U_{\alpha 1}^* + U_{\beta 2}U_{\alpha 2}^* + U_{\beta 3}U_{\alpha 3}^* = \delta_{\alpha\beta} \Rightarrow$

$$(S_0(x))_{\beta\alpha} = \delta_{\alpha\beta} + U_{\beta 3}U_{\alpha 3}^* \left(e^{-i\frac{\Delta m_{31}^2 x}{2E}} - 1 \right) \quad (37)$$

137 Similarity for calculating $(S_1(x))_{\beta\alpha}$. We have

$$\begin{aligned} (S_1(x))_{\beta\alpha} &= \left(e^{-iH_0 x} (-i) \int_0^x ds e^{iH_0 s} H_1 e^{-iH_0 s} \right)_{\beta\alpha} \\ &= -i \int_0^x ds \left(e^{-iH_0(x-s)} H_1 e^{-iH_0 s} \right)_{\beta\alpha} \\ &= -i \int_0^x ds \left(U e^{-i\frac{x-s}{2E} \text{diag}(0,0,\Delta m_{31}^2)} U^\dagger H_1 U e^{-i\frac{s}{2E} \text{diag}(0,0,\Delta m_{31}^2)} U^\dagger \right)_{\beta\alpha} \\ &= -i \int_0^x ds \sum_{i,j',i',j} \left[U_{\beta i} \left(e^{-i\frac{x-s}{2E} \text{diag}(0,0,\Delta m_{31}^2)} \right)_{ij'} U_{\gamma j'}^* (H_1)_{\gamma\sigma} U_{\sigma i'} \left(e^{-i\frac{s}{2E} \text{diag}(0,0,\Delta m_{31}^2)} \right)_{i'j} U_{\beta j}^* \right] \end{aligned}$$

138 Since $\left(e^{-i\frac{x-s}{2E} \text{diag}(0,0,\Delta m_{31}^2)} \right)_{ij'} = 0$ for $j' \neq i$

139 and $\left(e^{-i\frac{s}{2E} \text{diag}(0,0,\Delta m_{31}^2)} \right)_{i'j} = 0$ for $i' \neq j \Rightarrow$

$$\begin{aligned} (S_1(x))_{\beta\alpha} &= -i \int_0^x ds \sum_{i,j} \left[U_{\beta i} \left(e^{-i\frac{x-s}{2E} \text{diag}(0,0,\Delta m_{31}^2)} \right)_{ii} U_{\gamma i}^* (H_1)_{\gamma\sigma} U_{\sigma j} \left(e^{-i\frac{s}{2E} \text{diag}(0,0,\Delta m_{31}^2)} \right)_{jj} U_{\beta j}^* \right] \\ &= -i \sum_{i,j} U_{\beta i} U_{\gamma i}^* (H_1)_{\gamma\sigma} U_{\sigma j} U_{\beta j}^* \int_0^x ds \left(e^{-i\frac{\Delta m_{31}^2}{2E} [(x-s)\delta_{i3} + s\delta_{j3}]} \right) \end{aligned} \quad (38)$$

Let

$$X_{ij} = U_{\beta i} U_{\gamma i}^* (H_1)_{\gamma\sigma} U_{\sigma j} U_{\beta j}^*$$

and

$$Y_{ij} = \int_0^x ds \left(e^{-i\frac{\Delta m_{31}^2}{2E} [(x-s)\delta_{i3} + s\delta_{j3}]} \right) = \int_0^x ds \left(e^{-i\frac{\Delta m_{31}^2 x}{2E} \delta_{i3}} e^{-i\frac{\Delta m_{31}^2 s}{2E} (\delta_{j3} - \delta_{i3})} \right)$$

140 We first calculate the X_{ij} term

We see that

$$U_{\gamma i}^* (H_1)_{\gamma\sigma} U_{\sigma j} = \frac{1}{2E} [U_{\gamma i}^* (V_{12})_{\gamma\sigma} U_{\sigma j} + U_{\gamma i}^* (V_a)_{\gamma\sigma} U_{\sigma j}]$$

Since $(V_{12})_{22} = \Delta m_{21}^2$ and $(V_{12})_{\gamma\sigma} = 0$ for $\gamma \neq 2$ or $\sigma \neq 2$ then

$$\frac{1}{2E} U_{\gamma i}^* (V_{12})_{\gamma\sigma} U_{\sigma j} = \frac{\Delta m_{21}^2}{2E} \delta_{2i} \delta_{2j}$$

Since $(V_a)_{11} = a$ and $(V_a)_{\gamma\sigma} = 0$ for $\gamma \neq 1$ or $\sigma \neq 1$ then

$$\frac{1}{2E} U_{\gamma i}^* (V_a)_{\gamma\sigma} U_{\sigma j} = \frac{a}{2E} U_{1i}^* U_{1j}$$

Therefore

$$U_{\gamma i}^*(H_1)_{\gamma\sigma}U_{\sigma j} = \frac{\Delta m_{21}^2}{2E}\delta_{2i}\delta_{2j} + \frac{a}{2E}U_{1i}^*U_{1j}$$

141 and

$$\begin{aligned} X_{ij} &= U_{\beta i}U_{\gamma i}^*(H_1)_{\gamma\sigma}U_{\sigma j}U_{\beta j}^* \\ &= U_{\beta i}\left[\frac{\Delta m_{21}^2}{2E}\delta_{2i}\delta_{2j} + \frac{a}{2E}U_{1i}^*U_{1j}\right]U_{\alpha j}^* \\ &= \frac{\Delta m_{21}^2}{2E}U_{\beta i}U_{\alpha j}^*\delta_{2i}\delta_{2j} + \frac{a}{2E}U_{\beta i}U_{1i}^*U_{1j}U_{\alpha j}^* \end{aligned} \quad (39)$$

142 and the Y_{ij} integral is

$$\begin{aligned} Y_{11} &= Y_{12} = Y_{21} = Y_{22} = x \\ Y_{13} &= Y_{23} = Y_{31} = Y_{32} = \left(-i\frac{\Delta m_{31}^2}{2E}\right)^{-1} \left(e^{-i\frac{\Delta m_{31}^2}{2E}x} - 1\right) \\ Y_{33} &= xe^{-i\frac{\Delta m_{31}^2}{2E}x} \end{aligned}$$

143 In general

$$\begin{aligned} Y_{ij} &= (1 - \delta_{i3})(1 - \delta_{j3})x + \delta_{i3}\delta_{j3}xe^{-i\frac{\Delta m_{31}^2}{2E}x} \\ &\quad + [(1 - \delta_{i3})\delta_{j3} + \delta_{i3}(1 - \delta_{j3})] \left(-i\frac{\Delta m_{31}^2}{2E}\right)^{-1} \left(e^{-i\frac{\Delta m_{31}^2}{2E}x} - 1\right) \end{aligned} \quad (40)$$

144 Insert (40) and (39) into (37) we get

$$\begin{aligned} (S_1(x))_{\beta\alpha} &= -i\frac{ax}{2E}e^{-i\frac{\Delta m_{31}^2}{2E}x}U_{\beta 3}U_{\alpha 3}^*|U_{13}|^2 \\ &\quad -ix\left[\frac{\Delta m_{21}^2}{2E}U_{\beta 2}U_{\alpha 2}^* + \frac{a}{2E}(U_{\beta 1}U_{11}^*U_{11}U_{\alpha 1}^* + U_{\beta 1}U_{11}^*U_{12}U_{\alpha 2}^* \right. \\ &\quad \left. + U_{\beta 2}U_{12}^*U_{11}U_{\alpha 1}^* + U_{\beta 2}U_{12}^*U_{12}U_{\alpha 2}^*)\right] \\ &\quad -i\left(-i\frac{\Delta m_{31}^2}{2E}\right)^{-1}\left(e^{-i\frac{\Delta m_{31}^2}{2E}x} - 1\right)\frac{a}{2E}(U_{\beta 1}U_{11}^*U_{13}U_{\alpha 3}^* + U_{\beta 2}U_{12}^*U_{13}U_{\alpha 3}^* \\ &\quad + U_{\beta 3}U_{13}^*U_{11}U_{\alpha 1}^* + U_{\beta 3}U_{13}^*U_{12}U_{\alpha 2}^*) \end{aligned}$$

145 Note that $\sum_{k=1}^2 U_{\alpha k}^*U_{1k} = \delta_{\alpha 1} - U_{\alpha 3}^*U_{13}$. We now can calculate the factors that are relevant
146 to matrix elements as the following

$$\begin{aligned} &U_{\beta 1}U_{11}^*U_{11}U_{\alpha 1}^* + U_{\beta 1}U_{11}^*U_{12}U_{\alpha 2}^* + U_{\beta 2}U_{12}^*U_{11}U_{\alpha 1}^* + U_{\beta 2}U_{12}^*U_{12}U_{\alpha 2}^* \\ &= (U_{\beta 1}U_{11}^* + U_{\beta 2}U_{12}^*)(U_{11}U_{\alpha 1}^* + U_{12}U_{\alpha 2}^*) \\ &= (\delta_{\beta 1} - U_{\beta 3}U_{13}^*)(\delta_{\alpha 1} - U_{13}U_{\alpha 3}^*) \\ &= \delta_{\alpha 1}\delta_{\beta 1} - \delta_{\alpha 1}U_{\beta 3}U_{13}^* - \delta_{\beta 1}U_{13}U_{\alpha 3}^* + U_{\beta 3}U_{\alpha 3}^*|U_{13}|^2 \\ &= \delta_{\alpha 1}\delta_{\beta 1} + U_{\beta 3}U_{\alpha 3}^*(|U_{13}|^2 - \delta_{\alpha 1} - \delta_{\beta 1}) \end{aligned}$$

$$\begin{aligned}
& U_{\beta 1} U_{11}^* U_{13} U_{\alpha 3}^* + U_{\beta 2} U_{12}^* U_{13} U_{\alpha 3}^* + U_{\beta 3} U_{13}^* U_{11} U_{\alpha 1}^* + U_{\beta 3} U_{13}^* U_{12} U_{\alpha 2}^* \\
&= U_{13} U_{\alpha 3}^* (\delta_{\beta 1} - U_{\beta 3} U_{13}^*) + U_{\beta 3} U_{13}^* (\delta_{\alpha 1} - U_{13} U_{\alpha 3}^*) \\
&= \delta_{\alpha 1} U_{\beta 3} U_{13}^* + \delta_{\beta 1} U_{13} U_{\alpha 3}^* - 2 U_{\beta 3} U_{\alpha 3}^* |U_{13}|^2 \\
&= U_{\beta 3} U_{\alpha 3}^* (\delta_{\alpha 1} + \delta_{\beta 1} - 2|U_{13}|^2)
\end{aligned}$$

148 Therefore

$$\begin{aligned}
(S_1(x))_{\beta\alpha} &= -i \frac{ax}{2E} e^{-i \frac{\Delta m_{31}^2 x}{2E}} U_{\beta 3} U_{\alpha 3}^* |U_{13}|^2 \\
&\quad - i \frac{x}{2E} [\Delta m_{21}^2 U_{\beta 2} U_{\alpha 2}^* + a(\delta_{\alpha 1} \delta_{\beta 1} + U_{\beta 3} U_{\alpha 3}^* (|U_{13}|^2 - \delta_{\alpha 1} - \delta_{\beta 1}))] \\
&\quad - \frac{a}{\Delta m_{31}^2} \left(e^{-i \frac{\Delta m_{31}^2 x}{2E}} - 1 \right) (2|U_{13}|^2 - \delta_{\alpha 1} - \delta_{\beta 1}) U_{\beta 3} U_{\alpha 3}^* \quad (41)
\end{aligned}$$

149 From (41) and (37) and note that $\sin X/2 = \frac{e^{iX/2} - e^{-iX/2}}{2i}$ with $X = \frac{\Delta m_{31}^2}{2E}$ we get

$$\begin{aligned}
(S(x))_{\beta\alpha} &= (S_0(x))_{\beta\alpha} + (S_1(x))_{\beta\alpha} \\
&= \delta_{\alpha\beta} + U_{\beta 3} U_{\alpha 3}^* \left(e^{-i \frac{\Delta m_{31}^2 x}{2E}} - 1 \right) - \frac{a}{\Delta m_{31}^2} \left(e^{-i \frac{\Delta m_{31}^2 x}{2E}} - 1 \right) (2|U_{13}|^2 - \delta_{\alpha 1} - \delta_{\beta 1}) U_{\beta 3} U_{\alpha 3}^* \\
&\quad - i \frac{ax}{2E} e^{-i \frac{\Delta m_{31}^2 x}{2E}} U_{\beta 3} U_{\alpha 3}^* |U_{13}|^2 + \left(i \frac{ax}{2E} U_{\beta 3} U_{\alpha 3}^* |U_{13}|^2 - i \frac{ax}{2E} U_{\beta 3} U_{\alpha 3}^* |U_{13}|^2 \right) \\
&\quad - i \frac{\Delta m_{31}^2 x}{2E} \left[\frac{\Delta m_{21}^2}{\Delta m_{31}^2} U_{\beta 2} U_{\alpha 2}^* + \frac{a}{\Delta m_{31}^2} (\delta_{\alpha 1} \delta_{\beta 1} + U_{\beta 3} U_{\alpha 3}^* (|U_{13}|^2 - \delta_{\alpha 1} - \delta_{\beta 1})) \right]
\end{aligned}$$

150 By rearranging the common terms the above equation becomes

$$\begin{aligned}
(S(x))_{\beta\alpha} &= \delta_{\alpha\beta} - i2e^{-i\Delta_{31}} \sin \Delta_{31} U_{\beta 3} U_{\alpha 3}^* \left[(1 - C) - \frac{iax}{2E} |U_{13}|^2 \right] \\
&\quad - i2\Delta_{31} \left[\varepsilon U_{\beta 2} U_{\alpha 2}^* + \frac{a}{\Delta m_{31}^2} \delta_{\alpha 1} \delta_{\beta 1} + C U_{\beta 3} U_{\alpha 3}^* \right] \\
&= \delta_{\alpha\beta} + A + B \quad (42)
\end{aligned}$$

151 Where $\Delta_{31} = \frac{\Delta m_{31}^2 x}{4E}$; $\varepsilon = \frac{\Delta m_{21}^2}{\Delta m_{31}^2}$ and $C = \frac{a}{\Delta m_{31}^2} (2|U_{13}|^2 - \delta_{\alpha 1} - \delta_{\beta 1})$

152 The oscillation probability now can be calculated

$$\begin{aligned}
P(\nu_\alpha \rightarrow \nu_\beta) &= |(S(x))_{\beta\alpha}|^2 \\
&= \delta_{\alpha\beta} (1 + A + A^* + B + B^*) + AA^* + BB^* + A^*B + AB^* \quad (43)
\end{aligned}$$

- The term which is relevant to $\delta_{\alpha\beta}$

$$\begin{aligned}
& \delta_{\alpha\beta}(1 + A + A^* + B + B^*) \\
= & \delta_{\alpha\beta} \left\{ 1 - i2e^{-i\Delta_{31}} \sin \Delta_{31} U_{\beta 3} U_{\alpha 3}^* \left[(1 - C) - \frac{iax}{2E} |U_{13}|^2 \right] \right. \\
& + i2e^{i\Delta_{31}} \sin \Delta_{31} U_{\beta 3}^* U_{\alpha 3} \left[(1 - C) + \frac{iax}{2E} |U_{13}|^2 \right] \\
& - i2\Delta_{31} \left[\varepsilon U_{\beta 2} U_{\alpha 2}^* + \frac{a}{\Delta m_{31}^2} \delta_{\alpha 1} \delta_{\beta 1} + C U_{\beta 3} U_{\alpha 3}^* \right] \\
& \left. + i2\Delta_{31} \left[\varepsilon U_{\beta 2}^* U_{\alpha 2} + \frac{a}{\Delta m_{31}^2} \delta_{\alpha 1} \delta_{\beta 1} + C U_{\beta 3}^* U_{\alpha 3} \right] \right\} \\
= & \delta_{\alpha\beta} \left[1 - 4(1 - C) |U_{\alpha 3}|^2 \sin^2 \Delta_{31} - \frac{ax}{E} |U_{\alpha 3}|^2 |U_{13}|^2 \sin 2\Delta_{31} \right] \\
= & \delta_{\alpha\beta} \left[1 - 4 |U_{\alpha 3}|^2 \sin^2 \Delta_{31} \left(1 - \frac{2a}{\Delta m_{31}^2} (|U_{13}|^2 - \delta_{\alpha 1}) \right) - \frac{ax}{E} |U_{\alpha 3}|^2 |U_{13}|^2 \sin 2\Delta_{31} \right]
\end{aligned}$$

- In order to calculate the term which is irrelevant to $\delta_{\alpha\beta}$, we first calculate its com-

$$\begin{aligned}
AA^* &= 4 \sin^2 \Delta_{31} |U_{\beta 3}|^2 |U_{\alpha 3}|^2 \left[(1 - 2C + C^2) + \left(\frac{ax}{2E} \right)^2 |U_{13}|^4 \right] \\
BB^* &= 4(\Delta_{31})^2 \left[\varepsilon^2 |U_{\beta 2}|^2 |U_{\alpha 2}|^2 + \varepsilon \frac{2a}{\Delta m_{31}^2} |U_{13}|^2 \delta_{\alpha 1} \delta_{\beta 1} \right. \\
& \quad + 2\varepsilon C \operatorname{Re}(U_{\beta 3}^* U_{\alpha 3} U_{\beta 2} U_{\alpha 2}^*) + C^2 |U_{\beta 3}|^2 |U_{\alpha 3}|^2 \\
& \quad \left. + \frac{2aC}{\Delta m_{31}^2} |U_{13}|^2 \delta_{\alpha 1} \delta_{\beta 1} + \left(\frac{a}{\Delta m_{31}^2} \right)^2 \right] \\
A^*B + AB^* &= 2\operatorname{Re}(AB^*) \\
&= 4\varepsilon(1 - C)\Delta_{31} \sin 2\Delta_{31} \operatorname{Re}(U_{\beta 3}^* U_{\alpha 3} U_{\beta 2} U_{\alpha 2}^*) - 8\varepsilon(1 - C)\Delta_{31} \sin^2 \Delta_{31} \operatorname{Im}(U_{\beta 3}^* U_{\alpha 3} U_{\beta 2} U_{\alpha 2}^*) \\
& \quad - 8\varepsilon \left(\frac{ax}{2E} \right) \Delta_{31} \sin^2 \Delta_{31} |U_{13}|^2 \operatorname{Re}(U_{\beta 3}^* U_{\alpha 3} U_{\beta 2} U_{\alpha 2}^*) \\
& \quad - 4\varepsilon \left(\frac{ax}{2E} \right) \Delta_{31} \sin^2 \Delta_{31} |U_{13}|^2 \operatorname{Im}(U_{\beta 3}^* U_{\alpha 3} U_{\beta 2} U_{\alpha 2}^*) \\
& \quad + 4(1 - C) \frac{a}{\Delta m_{31}^2} \Delta_{31} \sin 2\Delta_{31} |U_{13}|^2 \delta_{\alpha 1} \delta_{\beta 1} - 8 \frac{a^2 x}{2E \Delta m_{31}^2} \Delta_{31} \sin^2 \Delta_{31} |U_{13}|^2 \delta_{\alpha 1} \delta_{\beta 1} \\
& \quad + 4(1 - C) C \Delta_{31} \sin 2\Delta_{31} |U_{\beta 3}|^2 |U_{\alpha 3}|^2 - 8 \left(\frac{axC}{2E} \right) \Delta_{31} \sin^2 \Delta_{31} |U_{13}|^2 |U_{\beta 3}|^2 |U_{\alpha 3}|^2
\end{aligned}$$

Since $C \propto a$, $\varepsilon \Delta_{31} = \Delta_{21} = \frac{\Delta m_{21}^2}{4E}$ and we have made the approximations:

$$\frac{ax}{2E} \ll 1; \quad \frac{\Delta m_{21}^2}{2E} \ll 1$$

156 we can neglect all the terms that contain $a^2, C^2, aC, \varepsilon a, \varepsilon C, \varepsilon C \Delta_{31}, \varepsilon a \Delta_{31}$ and leave

$$\begin{aligned}
AA^* &= 4 \sin^2 \Delta_{31} |U_{\beta 3}|^2 |U_{\alpha 3}|^2 \left[1 - 2 \frac{a}{\Delta m_{31}^2} (2|U_{13}|^2 - \delta_{\alpha 1} - \delta_{\beta 1}) \right] \\
BB^* &= 4 \Delta_{21}^2 |U_{\beta 2}|^2 |U_{\alpha 2}|^2 \\
A^* B + AB^* &= 2 \operatorname{Re}(AB^*) \\
&= 4 \Delta_{21} \sin 2 \Delta_{31} \operatorname{Re}(U_{\beta 3}^* U_{\alpha 3} U_{\beta 2} U_{\alpha 2}^*) - 8 \Delta_{21} \sin^2 \Delta_{31} \operatorname{Im}(U_{\beta 3}^* U_{\alpha 3} U_{\beta 2} U_{\alpha 2}^*) \\
&\quad + 4 \frac{ax}{4E} \sin 2 \Delta_{31} |U_{13}|^2 \delta_{\alpha 1} \delta_{\beta 1} + 4 \frac{ax}{4E} \sin 2 \Delta_{31} |U_{\beta 3}|^2 |U_{\alpha 3}|^2 (2|U_{13}|^2 - \delta_{\alpha 1} - \delta_{\beta 1})
\end{aligned}$$

157 The general form of oscillation probability is therefore

$$\begin{aligned}
P(\nu_\alpha \rightarrow \nu_\beta) &= \delta_{\alpha\beta} (1 + A + A^* + B + B^*) + AA^* + BB^* + A^* B + AB^* \\
&= \delta_{\alpha\beta} \left[1 - 4|U_{\alpha 3}|^2 \sin^2 \Delta_{31} \left(1 - \frac{2a}{\Delta m_{31}^2} (|U_{13}|^2 - \delta_{\alpha 1}) \right) - \frac{ax}{E} |U_{\alpha 3}|^2 |U_{13}|^2 \sin 2 \Delta_{31} \right] \\
&\quad + 4 \sin^2 \Delta_{31} |U_{\beta 3}|^2 |U_{\alpha 3}|^2 \left[1 - 2 \frac{a}{\Delta m_{31}^2} (2|U_{13}|^2 - \delta_{\alpha 1} - \delta_{\beta 1}) \right] \\
&\quad - 8 \Delta_{21} \sin^2 \Delta_{31} \operatorname{Im}(U_{\beta 3}^* U_{\alpha 3} U_{\beta 2} U_{\alpha 2}^*) \\
&\quad + 4 \sin 2 \Delta_{31} \left[\Delta_{21} \operatorname{Re}(U_{\beta 3}^* U_{\alpha 3} U_{\beta 2} U_{\alpha 2}^*) \right. \\
&\quad \left. + \frac{ax}{4E} (|U_{13}|^2 \delta_{\alpha 1} \delta_{\beta 1} + |U_{\beta 3}|^2 |U_{\alpha 3}|^2 (2|U_{13}|^2 - \delta_{\alpha 1} - \delta_{\beta 1})) \right] \\
&\quad + 4 \Delta_{21}^2 |U_{\beta 2}|^2 |U_{\alpha 2}|^2
\end{aligned} \tag{44}$$

158 * **Survival probability $P(\nu_\mu \rightarrow \nu_\mu)$ in matter**

159 For $\alpha = \beta = \mu$ we have

$$\begin{aligned}
P(\nu_\mu \rightarrow \nu_\mu) &= 1 + 4 \sin^2 \Delta_{31} |U_{\mu 3}|^2 \left[(|U_{\mu 3}|^2 - 1) - \frac{2a}{\Delta m_{31}^2} |U_{e 3}|^2 (2|U_{\mu 3}|^2 - 1) \right] \\
&\quad + 4 \Delta_{31} \sin 2 \Delta_{31} |U_{\mu 3}|^2 \left[\frac{\Delta m_{21}^2}{\Delta m_{31}^2} |U_{\mu 2}|^2 + \frac{a}{\Delta m_{31}^2} |U_{e 3}|^2 (2|U_{\mu 3}|^2 - 1) \right] \\
&\quad + 4 \Delta_{21}^2 |U_{\mu 2}|^4
\end{aligned} \tag{45}$$

From the PMNS matrix we see that

$$U_{e2} = s_{12} c_{13}; \quad U_{e3} = s_{13} e^{-i\delta}$$

and

$$U_{\mu 2} = c_{12} c_{23} - s_{12} s_{13} s_{23} e^{i\delta}; \quad U_{\mu 3} = s_{23} c_{13}$$

160 Therefore

$$\begin{aligned}
P(\nu_\mu \rightarrow \nu_\mu) = & 1 + 4s_{23}^2 c_{13}^2 (s_{23}^2 c_{13}^2 - 1) \sin^2 \Delta_{31} \\
& + 4s_{23}^2 c_{13}^2 s_{13}^2 (2s_{23}^2 c_{13}^2 - 1) \frac{2a}{\Delta m_{31}^2} \sin^2 \Delta_{31} \\
& + 4s_{23}^2 c_{13}^2 s_{13}^2 (2s_{23}^2 c_{13}^2 - 1) \frac{a}{\Delta m_{31}^2} \Delta_{31} \sin 2\Delta_{31} \\
& + 4s_{23}^2 c_{13}^2 (c_{12}^2 c_{23}^2 + s_{12}^2 s_{13}^2 s_{23}^2 - 2s_{12}s_{13}s_{23}c_{12}c_{23} \cos \delta) \Delta_{21} \sin 2\Delta_{31} \\
& + 4(c_{12}^2 c_{23}^2 + s_{12}^2 s_{13}^2 s_{23}^2 - 2s_{12}s_{13}s_{23}c_{12}c_{23} \cos \delta)^2 \Delta_{21}^2 \quad (46)
\end{aligned}$$

161 As you can see in the equation (46), the second term dominates. The third and the forth
162 are related to matter effect.

163 * **Transition probability $P(\nu_\mu \rightarrow \nu_e)$ in matter**

164 For $\alpha = \mu$ and $\beta = e$ we have

$$\begin{aligned}
P(\nu_\mu \rightarrow \nu_e) = & 4 \sin^2 \Delta_{31} |U_{e3}|^2 |U_{\mu 3}|^2 \\
& - 8 \sin^2 \Delta_{31} |U_{e3}|^2 |U_{\mu 3}|^2 \frac{a}{\Delta m_{31}^2} (2|U_{e3}|^2 - 1) \\
& + 4 \sin 2\Delta_{31} \frac{ax}{4E} |U_{e3}|^2 |U_{\mu 3}|^2 (2|U_{e3}|^2 - 1) \\
& - 8 \Delta_{21} \sin^2 \Delta_{31} \text{Im}(U_{e3}^* U_{\mu 3} U_{e2} U_{\mu 2}^*) \\
& + 4 \Delta_{21} \sin 2\Delta_{31} \text{Re}(U_{e3}^* U_{\mu 3} U_{e2} U_{\mu 2}^*) \\
& + 4 \Delta_{21}^2 |U_{e2}|^2 |U_{\mu 2}|^2 \quad (47)
\end{aligned}$$

From the PMNS matrix we see that

$$U_{e2} = s_{12}c_{13}; \quad U_{e3} = s_{13}e^{-i\delta}$$

and

$$U_{\mu 2} = c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta}; \quad U_{\mu 3} = s_{23}c_{13}$$

165 Therefore

$$\begin{aligned}
P(\nu_\mu \rightarrow \nu_e) = & 4s_{13}^2 s_{23}^2 c_{13}^2 \sin^2 \Delta_{31} \\
& - 8s_{13}^2 s_{23}^2 c_{13}^2 \frac{a}{\Delta m_{31}^2} (2s_{13}^2 - 1) \sin^2 \Delta_{31} \\
& + 4s_{13}^2 s_{23}^2 c_{13}^2 \frac{ax}{4E} (2s_{13}^2 - 1) \sin 2\Delta_{31} \\
& - 8s_{12}s_{13}s_{23}c_{12}c_{13}^2 c_{23} \sin \delta \Delta_{21} \sin^2 \Delta_{31} \\
& + 4s_{12}s_{13}s_{23}c_{13}^2 (c_{12}c_{23} \cos \delta - s_{12}s_{13}s_{23}) \Delta_{21} \sin 2\Delta_{31} \\
& + 4s_{12}^2 c_{13}^2 (c_{12}^2 c_{23}^2 + s_{12}^2 s_{13}^2 s_{23}^2 - 2s_{12}s_{13}s_{23}c_{12}c_{23} \cos \delta) \Delta_{21}^2 \quad (48)
\end{aligned}$$

For $\frac{\Delta m_{21}^2}{4E} \ll 1$ and $\Delta m_{31}^2 \approx \Delta m_{32}^2$, you can make a replacement with: $\Delta_{21} = \sin \Delta_{21}$; $\cos \Delta_{31} = \cos \Delta_{32}$; $\sin \Delta_{31} = \sin \Delta_{32}$ and the probability of $\nu_\mu \rightarrow \nu_e$ oscillation can be written as follows

$$\begin{aligned}
P(\nu_\mu \rightarrow \nu_e) \approx & 4s_{13}^2 s_{23}^2 c_{13}^2 \sin^2 \Delta_{31} \\
& - 8s_{13}^2 s_{23}^2 c_{13}^2 \frac{a}{\Delta m_{31}^2} (2s_{13}^2 - 1) \sin^2 \Delta_{31} \\
& + 8s_{13}^2 s_{23}^2 c_{13}^2 \frac{aL}{4E} (2s_{13}^2 - 1) \sin \Delta_{31} \cos \Delta_{32} \\
& - 8s_{12}s_{13}s_{23}c_{12}c_{13}^2 c_{23} \sin \delta_{CP} \sin \Delta_{21} \sin \Delta_{31} \sin \Delta_{32} \\
& + 8s_{12}s_{13}s_{23}c_{13}^2 (c_{12}c_{23} \cos \delta_{CP} - s_{12}s_{13}s_{23}) \sin \Delta_{21} \sin \Delta_{31} \cos \Delta_{32} \\
& + 4s_{12}^2 c_{13}^2 (c_{12}^2 c_{23}^2 + s_{12}^2 s_{13}^2 s_{23}^2 - 2s_{12}s_{13}s_{23}c_{12}c_{23} \cos \delta_{CP}) \sin^2 \Delta_{21},
\end{aligned} \tag{49}$$

where $\Delta_{ji} = \frac{\Delta m_{ji}^2}{4E} L$, and $a = 2\sqrt{2}G_F n_e E = 7.56 \times 10^{-5} [eV^2] (\frac{\rho}{g/cm^3}) (\frac{E}{GeV})$, n_e is the electron density of the matter and ρ is the density of the Earth. The appearances of a in the equation (50) is due to the matter effect which is rooted from the fact that electron neutrino when passing through ordinary matter will interact weakly with electrons. For anti-neutrino counterpart, $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ can be obtained from Eq. 49 by replacing $\delta \rightarrow -\delta$ and $a \rightarrow -a$. In Eq. (49), the first term dominates with current long-baseline neutrino experiments and about 0.043 at the maximum of $\sin^2 \Delta_{31}$. The matter effect, represented by a constant, involves to the second and third terms. While the term proportional to $\sin \delta_{CP}$ is called *CP-violating* since their contribution for total probability are opposite for neutrino and antineutrino, the fifth, which contains $\cos \delta_{CP}$, is called *CP-conserving term* since their contributions are the same for neutrino and antineutrino. The last one depends on Δ_{21}^2 and can be ignored in the case of long baseline experiments. At present landscape of neutrino oscillations, this channels is the only hope to provide information about δ_{CP} . However challenges for this channel measurement are the smallness of oscillation amplitude and its degeneracy with other oscillation parameters. Along with the appearance channels, the accelerator-based long-baseline neutrino experiments typically can measure precisely the probability of $\nu_\mu \rightarrow \nu_\mu$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu$, which, to the first order approximation of matter effect, can be expressed as:

$$\begin{aligned}
P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu) \approx & 1 + 4s_{23}^2 c_{13}^2 (s_{23}^2 c_{13}^2 - 1) \sin^2 \Delta_{31} \\
& \pm 4s_{23}^2 c_{13}^2 s_{13}^2 (2s_{23}^2 c_{13}^2 - 1) \frac{2a}{\Delta m_{31}^2} \sin^2 \Delta_{31} \\
& \pm 4s_{23}^2 c_{13}^2 s_{13}^2 (2s_{23}^2 c_{13}^2 - 1) \frac{a}{\Delta m_{31}^2} \Delta_{31} \sin 2\Delta_{31} \\
& + 4s_{23}^2 c_{13}^2 (c_{12}^2 c_{23}^2 + s_{12}^2 s_{13}^2 s_{23}^2 - 2s_{12}s_{13}s_{23}c_{12}c_{23} \cos \delta) \Delta_{21} \sin 2\Delta_{31},
\end{aligned} \tag{50}$$

where positive (negative) signs are taken for neutrino (antineutrino) oscillations respectively. Due to relative smallness of θ_{13} the first term is dominated in the accelerator-based long-baseline neutrino experiment and measurement with this channel is essentially sensitive to

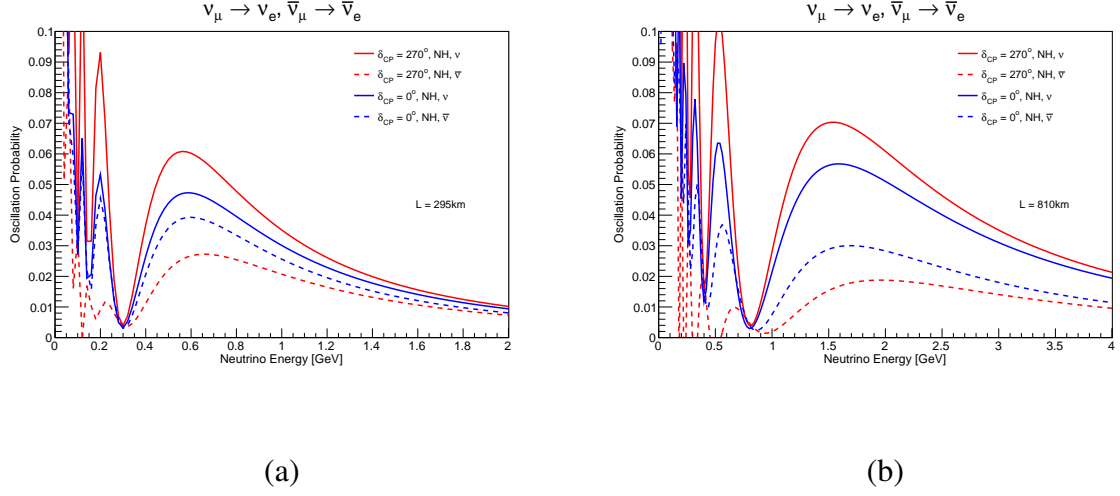


Figure 1: Transition probabilities $P(\nu_\mu \rightarrow \nu_e)$ and $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ as a function of neutrino energy for T2K baseline $L = 295$ km (a) and NOvA baseline $L = 810$ km (b).

189 mixing angle θ_{23} and Δm_{31}^2 . In practice, neutrino oscillation analyses take advance of com-
 190 bining both appearance channel and disappearance channel in order to provide the most
 191 precise measurements of oscillation parameters and explore CP violation from constraints
 192 on δ_{CP} . Fig. 1 (a) and Fig. 1 (b) show the oscillation probabilities of $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ as a function
 193 of neutrino energy at different true value of δ_{CP} for T2K baseline $L = 295$ km (with peak of
 194 neutrino flux at 0.6 GeV) and NOvA baseline $L = 810$ km (with peak of neutrino flux at 2
 195 GeV), respectively. These two leading accelerator-based long-baseline neutrino experiments
 196 will be discussed in detail in Section 0.3. In the figures, the difference between solid and
 197 dashed blue lines indicates the matter effect, and the difference between solid and dashed
 198 red lines shows the combined effect of both matter and CP-violation. In the case of T2K
 199 experiment, the matter effect is much smaller than the CP-violation effect. For NOvA, due
 200 to its longer baseline the matter effect is larger. The plots are made with assumed values of
 201 oscillation parameters as listed Table 1:

Table 1: Input values of oscillation parameters, taken from [18].

	$\sin^2 2\theta_{12}$	$\sin^2 2\theta_{13}$	$\sin^2 \theta_{23}$	Δm_{21}^2	Δm_{32}^2
Value	0.8704	0.085	0.5	$7.6 \times 10^{-5} eV^2/c^4$	$2.5 \times 10^{-3} eV^2/c^4$

0.3 T2K(-II) AND NOvA EXPERIMENTS

At present, T2K and NOvA are two leading accelerator-based long baseline neutrino exper-
 iment in the world. We briefly describe these two experiments and inputs we use to study

their combined sensitivity on CP violation search.

T2K (Tokai-to-Kamioka) [10] is an accelerator-based long-baseline neutrino oscillation experiment placed in Japan with three main complexes: (i) the J-PARC accelerator, (ii) the near detector suite placed 280 m from the neutrino production target, and (iii) the far detector, Super-Kamiokande, situated 295 km away from target. The J-PARC, one of the most intense proton beam in the world, is used to produce a nearly pure $\bar{\nu}_\mu$ source. The near detector suite is designed to characterize the unoscillated neutrino beam while the far detector is used to observe the oscillation patterns. The primary goal of T2K is to observe oscillation from muon neutrinos to electron neutrinos, which has been achieved in 2013. With relatively large value of mixing angle θ_{13} , the physics potential of T2K is revisited and CP violation search is placed as the central target. For the latest results [25], based on a total exposure of 2.23×10^{21} POT, consisting of 1.47×10^{21} POT in ν -mode and 0.76×10^{21} POT in $\bar{\nu}$ -mode, T2K firstly reports that CP conserving value (0 and π) of δ_{CP} is out of the 2σ C.L. range of the measurement for both normal and inverted mass hierarchies. By the year 2021, with a fully approved exposure of 7.8×10^{21} POT collected, T2K will have sensitivity to the CP-violating phase δ_{CP} at 90% C.L. or higher over a significant range [11]. To intensively explore CP violation in the lepton sector, T2K-II, extension of T2K operation up to 2026, is proposed to collect 20×10^{21} POT [12]. This amount of data in combination with expected improvement in the neutrino beamline and neutrino oscillation analysis allows T2K to have 3σ or higher significant sensitivity to CP violation. Also the oscillation parameters θ_{23} and Δm_{31}^2 can be measured at the unprecedented level.

Rescaling T2K flux

T2K flux in GLoBES which is corresponding to 2.0 degree off-axis is out of date. The new flux (50MeV wide bins) corresponding to actual 2.5 degree off-axis angle provided by Dr. Cao Son (KEK) is, however, different in format with default in GLoBES (100MeV wide bins). We therefore have to rebin it in order to be consistent with GLoBES. The rebin flux is in 100MeV wide bins up to 10GeV. All fluxes are normalized to 1×10^{21} protons delivered to the T2K production target. The code used to rebin flux from root file and then store in txt file is provided in [rebinflux.cxx](#).

In this study, we try to follow as closely as possible the paper. The event rates reconstructed from GLoBES for the case true $\delta_{CP} = -\pi/2$ are shown in the Table 2 and Table 3. The channels $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ for ν -mode, $\nu_\mu \rightarrow \nu_e$ for $\bar{\nu}$ -mode in Table 2, and Beam CC $\bar{\nu}_\mu$ for ν -mode, Beam CC ν_μ for $\bar{\nu}$ -mode in Table 3 are zero because they have not been included in GLoBES yet.

Table 2: The ν_e and $\bar{\nu}_e$ event rates of T2K-II with 20×10^{21} POT with $\delta_{CP} = -\pi/2$.

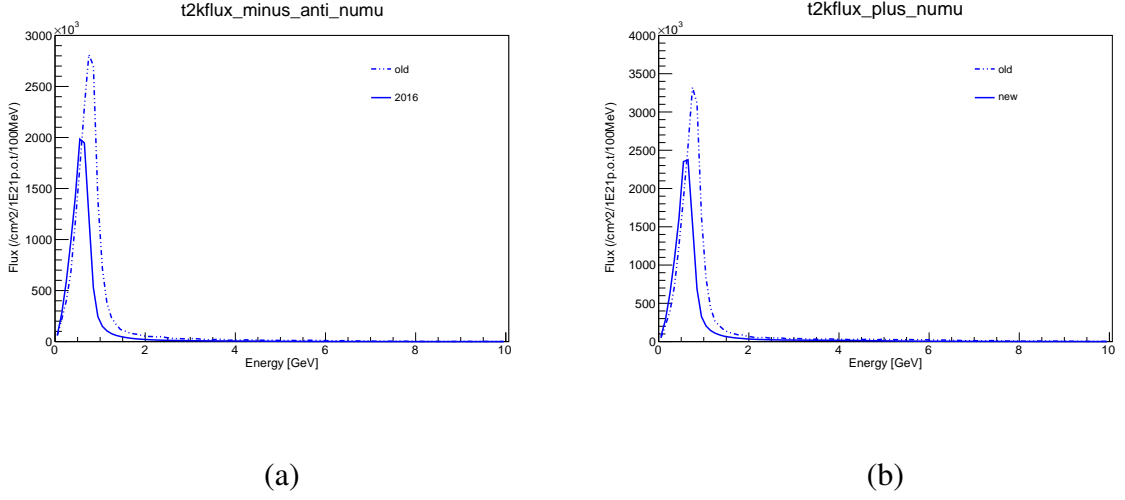


Figure 2: Rebin flux compared with default in GLoBES for muon anti-neutrino beam (a) and muon neutrino beam (b).

	Total	Signal $\nu_\mu \rightarrow \nu_e$	Signal $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$	Beam CC $\nu_e + \bar{\nu}_e$	Beam CC $\nu_\mu + \bar{\nu}_\mu$	NC
ν -mode ν_e sample	556.0	448.6	0.0	73.3	1.8	32.3
$\bar{\nu}$ -mode $\bar{\nu}_e$ sample	96.0	0.0	52.3	29.2	0.4	14.1

Table 3: The ν_μ and $\bar{\nu}_\mu$ event rates of T2K-II with $20 \times 10^{21} (\nu\text{-mode}:\bar{\nu}\text{-mode} = 1:1)$ POT with $\delta_{CP} = -\pi/2$.

	Total	Beam CC ν_μ	Beam CC $\bar{\nu}_\mu$	NC
ν -mode ν_μ sample	2568.0	2393.0	0.0	175.0
$\bar{\nu}$ -mode $\bar{\nu}_\mu$ sample	774.1	0.0	707.9	66.2

In addition, systematic uncertainties of T2K-II is anticipated to go down to 4% compared with 5.5% – 6.8% of current level. This can be achieved by reducing errors from neutrino flux, neutrino interaction models and detector model uncertainties.

NOvA (NuMI Off-axis ν_e Appearance) experiment [13] is an accelerator-based long-baseline neutrino experiment placed in United State. NOvA uses the intense and nearly pure

$\bar{\nu}_\mu$ beam from NuMI (Neutrino at Main Injector), Fermilab and study oscillations with two functionally identical detectors: near detector (0.3 kton) situated underground at Fermilab, Illinois and far detector (14 kton) installed on the surface in Ash River, Minnesota, 810 km from the neutrino production target. The detectors are optimized for observing ν_e signal, and placed at an angular offset of 14 mrad in order to achieve a narrow neutrino spectrum with peak at 2 GeV and suppress the neutral current π^0 background. With 810 km baseline, the matter effect can change $\nu_\mu \rightarrow \nu_e$ appearance rate up to $\pm 30\%$. In 2017, with an equivalent exposure of 6.05×10^{20} POT, 33 ν_e candidate was observed, clearly excess from 8.2 ± 0.8 background expected from MC. One of the most significant improvement in NOvA oscillation analysis is adopting convolutional visual network [26] for the event-by-event classification. The gain from this new approach for event selection is equivalent to 30% effectively statistic increase.

We also update the event classification in GLOBES for NOvA from its latest announcement [14]. With an operation up to the year 2024, NOvA is expected to accumulate a total exposure of 53×10^{20} POT. We define the event rates for full operation of NOvA as shown in the Table 6. Here we omitted the ν_τ CC and cosmic events channels since they have not been incorporated in GLOBES yet. The systematic uncertainty of NOvA is kept to be 5% in GLOBES as it is provided.

NOvA event rates and efficiency for 8.85×10^{20} POT, NH, $\delta_{CP} = -\pi/2$.

- ν_e mode: efficiency = 0.62
- $\bar{\nu}_e$ mode: efficiency = 0.67
- ν_μ mode: efficiency = 0.68
- $\bar{\nu}_\mu$ mode: efficiency = 0.62

Table 4: The ν_e and $\bar{\nu}_e$ event rates of NOvA with 8.85×10^{21} POT with $\delta_{CP} = -\pi/2$.

	Total	Signal	ν_μ beam CC	ν_μ beam NC	ν_e beam
ν -mode ν_e sample		48.0	1.1	6.6	7.1
$\bar{\nu}$ -mode $\bar{\nu}_e$ sample		6.5	0.4	2.5	2.7

Table 5: The ν_μ and $\bar{\nu}_\mu$ event rates of NOvA with 8.85×10^{21} POT with $\delta_{CP} = -\pi/2$.

	Signal	ν_μ beam NC
ν -mode ν_μ sample	129.0	3.46
$\bar{\nu}$ -mode $\bar{\nu}_\mu$ sample	39.0	1.2

277 Table 6: The ν_e and $\bar{\nu}_e$ event rates of NOvA with 53×10^{21} POT with $\delta_{CP} = -\pi/2$.

	Total	Signal	ν_μ beam CC	ν_μ beam NC	ν_e beam
ν -mode ν_e sample	376.8	288.0	6.6	39.6	42.6
$\bar{\nu}$ -mode $\bar{\nu}_e$ sample	72.4	38.8	2.4	15.0	16.2

279 Table 7: The ν_μ and $\bar{\nu}_\mu$ event rates of NOvA with 53×10^{21} POT with $\delta_{CP} = -\pi/2$.

	Signal	ν_μ beam NC
ν -mode ν_μ sample	774.0	20.8
$\bar{\nu}$ -mode $\bar{\nu}_\mu$ sample	234.0	7.2

281 0.4 SENSITIVITY TO CP-VIOLATION

282 In this paper, we use GLoBES software package [15] to study the physics potential of the
283 T2K-II with updated constraint from reactor [16] and NOvA event classification [14].

284 0.4.1 Constraint on θ_{13} from reactor

285 As mentioned before, determination of θ_{13} plays an important role in measuring δ_{CP} . Current
286 precision on $\sin^2 2\theta_{13}$ is 6% [17] with best fit $\sin^2 2\theta_{13} = 0.085$ [18]. Daya Bay reactor
287 experiment has recently approved that it can achieve 3% precision on $\sin^2 2\theta_{13}$ by the year
288 2020 [16]. We examine here the two above possible cases.

289 In order to get plots as shown in Fig. 3, we used **Reactor2.glb**. Different constraints on
290 θ_{13} can be obtained by changing value at **@time**.

- 291 • @time = 60 years corresponds to 6% precision on $\sin^2 2\theta_{13}$.
- 292 • @time = 300 years corresponds to 3% precision on $\sin^2 2\theta_{13}$.

293 To make file, use **theta13_proj_reactor.c**:

- 294 • cd to current directory.
- 295 • run file by typing: **./theta13_proj_reactor.c**

296 You can change output filename in **theta13_proj_reactor.c** at

```
297 /* Output file */
298 char MYFILE[]="theta13_proj_reactor.dat";
```

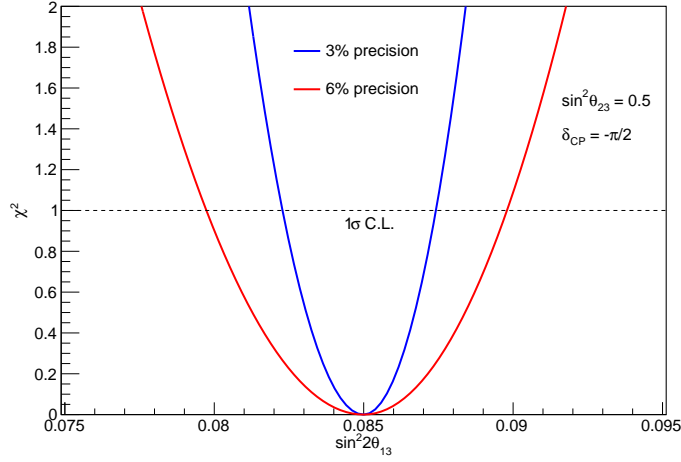



Figure 3: Precision of θ_{13} from reactor. The green line corresponds to 6% precision and the red line corresponds to 3% precision of the current best fit value of $\sin^2 2\theta_{13}$.

299 To make plot, use *plot_theta13_proj_reactor.C*
 300 To make graph containing two plots, first run *theta13_proj_reactor.c* with *@time = 60*
 301 and output file *theta13_proj_reactor_current.dat* for 6% precision. Then run *theta13_proj_reactor.c*
 302 with *@time = 300* and output file *theta13_proj_reactor.dat* for 3% precision.
 303 The code to get simultaneously two plots in the same graph is provided in
 304 *plot_theta13_proj_reactor.C*

305 0.4.2 The analysis method and results

306 The analysis method is followed as in the paper [11]. The physics outcomes are based
 307 on the signal efficiency, background and systematic errors established for T2K-II [12] and
 308 NOvA [14] for both ν -mode and $\bar{\nu}$ -mode with normal mass hierarchy and $\delta_{CP} = -\pi/2$. The
 309 GLoBES package is used to combine the two experiments with constraining from reactor.
 310 At first, the minimizing $\Delta\chi^2$ to exclude $\delta_{CP} = 0$ and $\delta_{CP} = \pm 180^\circ$ are obtained individually.
 311 Their minimum values are then plotted as a function of true δ_{CP} in the meaning to exclude
 312 $\sin \delta_{CP} = 0$. All the plots are made for the case $\sin^2 \theta_{23} = 0.5$ as close as the latest T2K
 313 result. The detail procedure is as follow:

- 314 • The GLoBES files are provided to define the experiments: *T2K.glb*, *NOvA.glb* and
 315 *Reactor2.glb* are corresponding to T2K-II, NOvA and Reactor experiments, respectively.
- 316 • The execute file in GLoBES is *deltacp_proj_wreactor.c*. In this file, you can initial-
 317 ize individual experiment or combined two or three experiments at section

```
318 /* Initialize experiment */
319 glbInitExperiment("T2K.glb",&glb_experiment_list[0],&glb_num_of_exps);
```

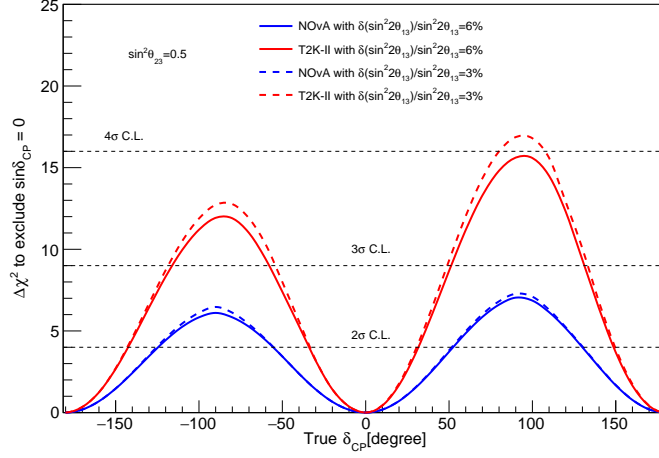


Figure 4: Sensitivity to CP-violation as a function of true δ_{CP} for T2K-II and NOvA (solid red and blue lines, respectively), for T2K-II and NOvA with ultimate constraint on θ_{13} (dashed red and blue lines, respectively).

- In this analysis, we run GLoBES with 10^o-step of true δ_{CP} from -180^o to 180^o .
- The result is exported in file

deltacp_proj_wreactor.dat

after running the

deltacp_proj_wreactor.c

The three-column output format is: first column corresponds to δ_{CP} , the second corresponds to $\Delta\chi^2$ values for two-parameter correlation (minimizer over θ_{13} only), and the last one corresponds to $\Delta\chi^2$ values for all-parameter correlation (minimize over all but δ_{CP}). To make plot, we choose the minimum value of the third column which is corresponding to each true δ_{CP} value:

δ_{CP}	$\Delta\chi_1^2$	$\Delta\chi_2^2$
-180	35.2988	26.0074
0	33.1935	26.4975
180	35.2988	26.0074

Three cases are demonstrated as: (1) Effect of constraint on θ_{13} , (2) Effect of combined T2K-II and NOvA, and (3) Effect of reducing systematic uncertainties.

Fig. 4. shows the CP sensitivity ($\Delta\chi^2$ for resolving $\sin\delta_{CP} \neq 0$) plotted as a function of true δ_{CP} for T2K-II and NOvA (solid red and blue lines, respectively), for T2K-II and NOvA with ultimate constraint on θ_{13} (dashed red and blue lines, respectively). The study

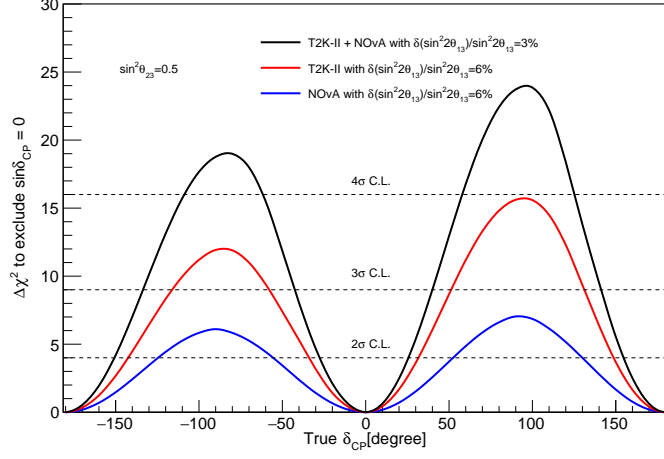


Figure 5: Sensitivity to CP-violation as a function of true δ_{CP} for NOvA (solid green lines), T2K-II (solid red line) and T2K-II + NOvA with ultimate constraint on θ_{13} (dashed blue line).

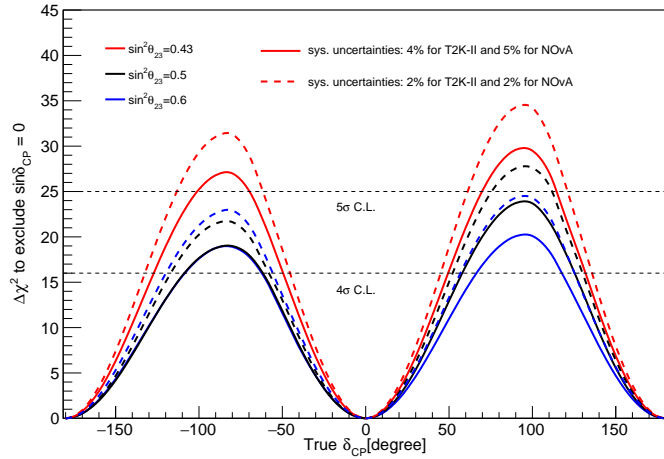


Figure 6: Sensitivity to CP-violation as a function of true δ_{CP} for T2K-II + NOvA with expected sys. uncertainties (solid red line), and 2% sys. uncertainties (dashed blue line).

339 shows that by updating constraint on θ_{13} from reactor by the year 2020, the sensitivity will
340 increase by 22% - 28% for NOvA and by 29% – 37% for T2K-II.

341 By combining with NOvA, the sensitivity can be achieved slightly near 5σ C.L. It will
342 increase by 43% - 46% compared to T2K alone and by 213% – 224% compared to NOvA
343 alone as showed in Fig. 5.

344 The Fig. 6 presents the significance of improving systematic uncertainty. By assuming
345 that the systematic uncertainties of both T2K-II and NOvA are simultaneously equal to 2%,
346 we pointed out that the sensibility to CP-violation will be enlarged by 16% – 18%.

347 **0.5 CONCLUSIONS**

348 In this paper, we have studied the sensitivity to CP-violation by combining the T2K-II and
349 NOvA experiments with constraint from reactor. With this combination, the sensitivity will
350 significantly increase to exceed 4σ C.L. The study shows the importance of constraint on
351 θ_{13} and improvements in statistics and systematic uncertainties of each experiment indepen-
352 dently as well as cooperation between the experiments.

353 In the near future, GLoBES will be improved by including new channels as mentioned
354 in sections IV.2 and IV.3, and updating the latest flux as well as cross section of the two
355 experiments. These may helps the prospective results to be closer with realistic experiments
356 and can achieve higher sensitivity.

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