Distributed Algorithms
6.046J, Spring, 2015
Part 2

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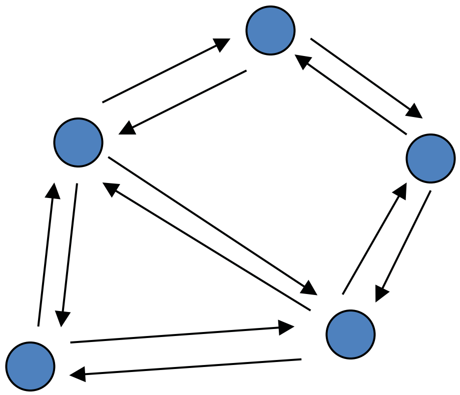
This Week

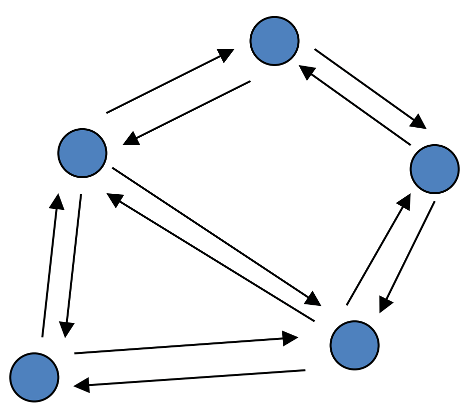
- Synchronous distributed algorithms:
  - Leader Election
  - Maximal Independent Set
  - Breadth-First Spanning Trees
  - Shortest Paths Trees   (started)

  - Shortest Paths Trees (finish)
- Asynchronous distributed algorithms:
  - Breadth-First Spanning Trees
  - Shortest Paths Trees

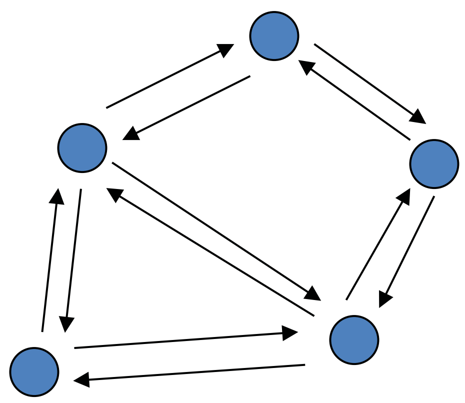


Distributed Networks

- Based on undirected graph $G = (V, E)$.
  - $n = |V|$
  - $\Gamma(u)$, set of neighbors of vertex u.
  - $\deg(u) = |\Gamma(u)|$, number of neighbors of vertex u.
- Associate a process with each graph vertex.
- Associate two directed communication channels with each edge.



Synchronous Distributed Algorithms

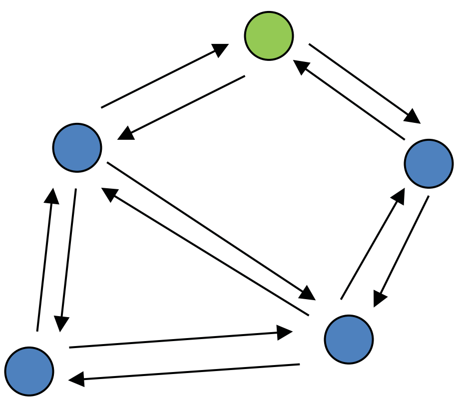


Synchronous Network Model

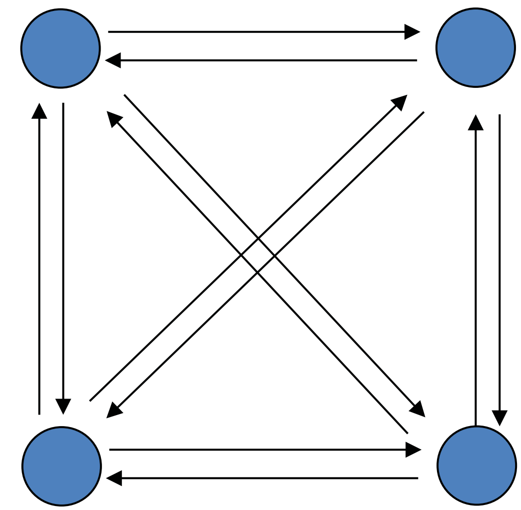
- Processes at graph vertices, communicate using messages.
- Each process has output ports, input ports that connect to communication channels.

- Algorithm executes in synchronous rounds.
- In each round:
  - Each process sends messages on its ports.
  - Each message gets put into the channel, delivered to the process at the other end.
  - Each process computes a new state based on the arriving messages.

Leader Election



n-vertex Clique

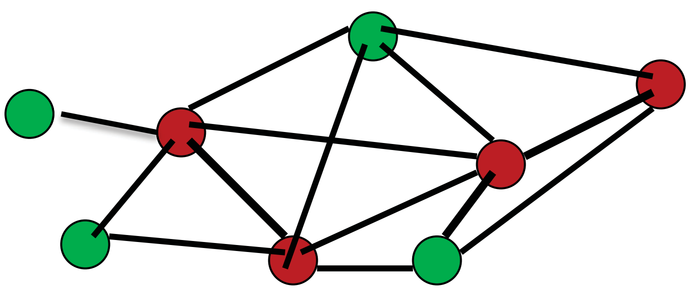


- Theorem: There is no algorithm consisting of deterministic, indistinguishable processes that is guaranteed to elect a leader in G.

- Theorem: There is an algorithm consisting of deterministic processes with UIDs that is guaranteed to elect a leader.
  - 1 round, n^2 messages.

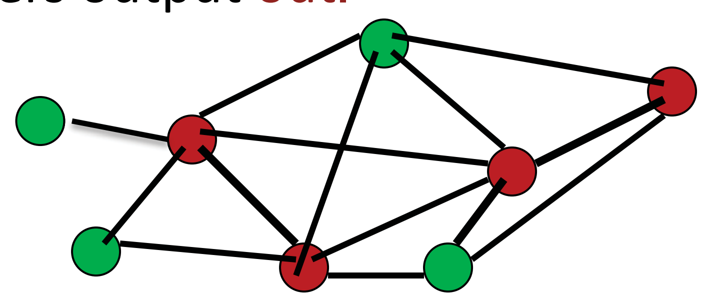
- Theorem: There is an algorithm consisting of randomized, indistinguishable processes that eventually elects a leader, with probability 1.
  - Expected time $\leq \frac{1}{1-\epsilon}$.
  - With probability $\geq 1 - \epsilon$, finishes in one round.

Maximal Independent Set (MIS)



MIS

- Independent: No two neighbors are both in the set.
- Maximal: We can't add any more nodes without violating independence.
- Every node is either in S or has a neighbor in S.
- Assume:
  - No UIDs
  - Processes know a good upper bound on n.
- Require:
  - Compute an MIS S of the network graph.
  - Each process in S should output in, others output out.



Luby's Algorithm

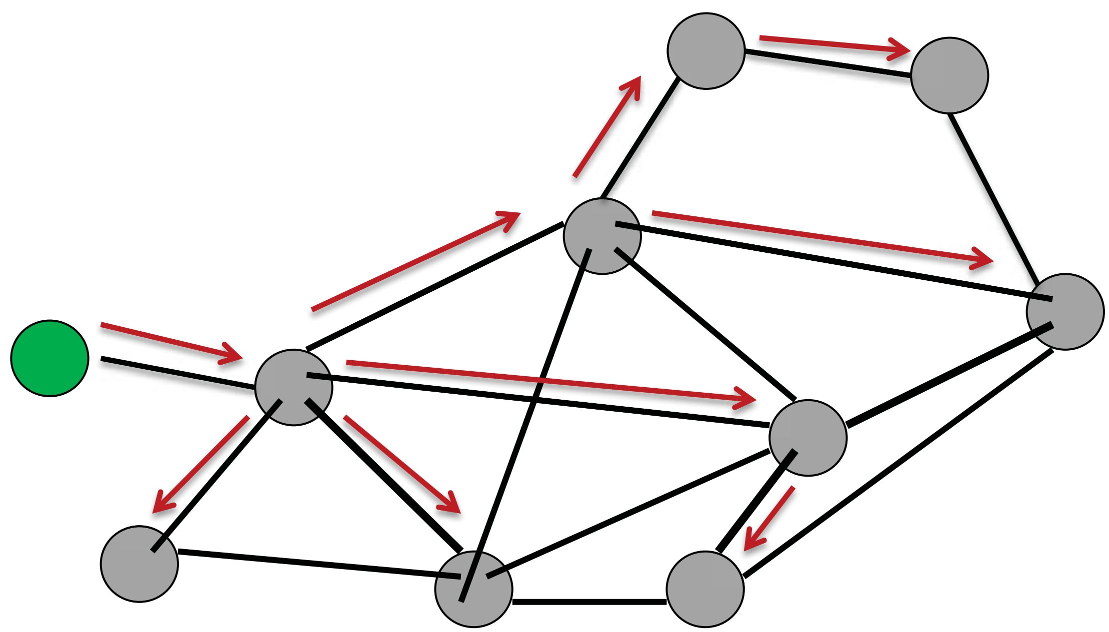
- Initially all nodes are active.
- At each phase, some active nodes decide to be in, others decide to be out, the rest continue to the next phase.

- Behavior of active node at a phase:
- Round 1:
  - Choose a random value r in $\{1,2,\ldots,n^5\}$, send it to all neighbors.
  - Receive values from all active neighbors.
  - If r is strictly greater than all received values, then join the MIS, output in.
- Round 2:
  - If you joined the MIS, announce it in messages to all (active) neighbors.
  - If you receive such an announcement, decide not to join the MIS, output out.
  - If you decided one way or the other at this phase, become inactive.

Luby's Algorithm

- Theorem: If Luby's algorithm ever terminates, then the final set S is an MIS.
- Theorem: With probability at least $1 - \frac{1}{n}$,all nodes decide within $4 \log n$ phases.

Breadth-First Spanning Trees



Breadth-First Spanning Trees

- Distinguished vertex v_0.
- Processes must produce a Breadth-First Spanning Tree rooted at vertex v_0.
- Assume:
  - UIDs.
  - Processes have no knowledge about the graph.
- Output: Each process $i \neq i_0$ should output $parent(j)$.

Simple BFS Algorithm

- Processes mark themselves as they get incorporated into the tree.
- Initially, only i_0 is marked.
- Algorithm for process i:
  - Round 1:
    - If $i = i_0$ then process i sends a $search$ message to its neighbors.
    - If process i receives a message, then it:
      - Marks itself.
      - Selects i_0 as its parent, outputs $parent(i_0)$.
      - Plans to send at the next round.
  - Round $r > 1$:
    - If process i planned to send, then it sends a $search$ message to its neighbors.
    - If process i is not marked and receives a message, then it:
      - Marks itself.
      - Selects one sending neighbor, j, as its parent, outputs $parent(j)$.
      - Plans to send at the next round.

Correctness

- State variables, per process:
  - $marked$, a Boolean, initially true for i_0, false for others
  - $parent$, a UID or undefined
  - $send$, a Boolean, initially true for i_0, false for others
  - uid
- Invariants:
  - At the end of r rounds, exactly the processes at distance $\leq r$ from v_0 are marked.
  - A process $\neq i_0$ has its $parent$ defined iff it is marked.
  - For any process at distance d from v_0, if its $parent$ is defined, then it is the UID of a process at distance $d-1$ from v_0.

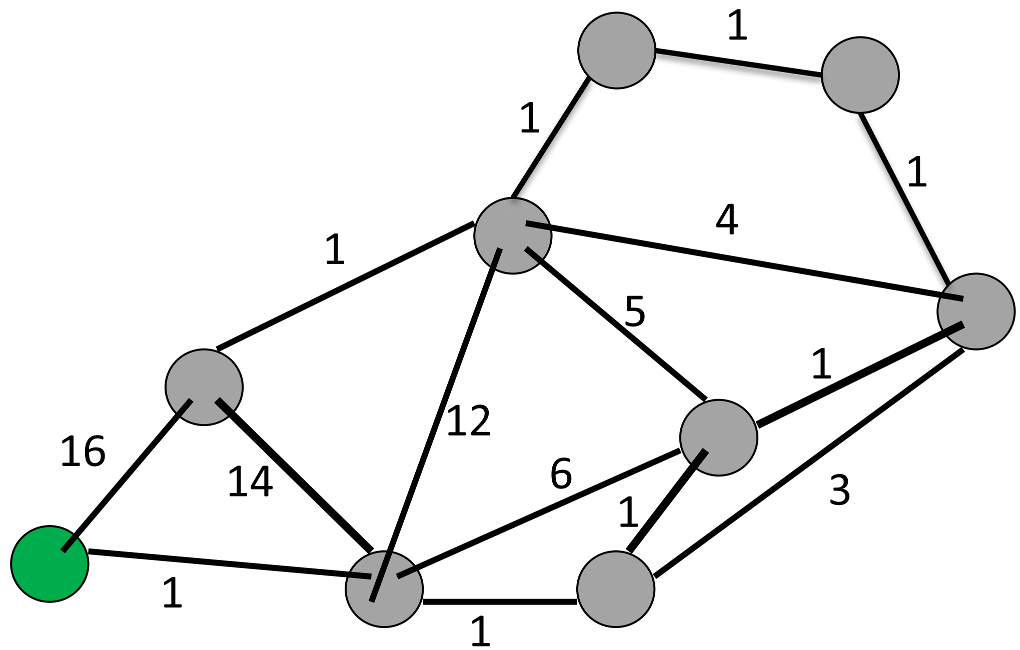
Complexity

- Time complexity:
  - Number of rounds until all nodes outputs their parent information.
  - Maximum distance of any node from v_0, which is $\leq diam$
- Message complexity:
  - Number of messages sent by all processes during the entire execution.
  - $O(|E|)$

Bells and Whistles

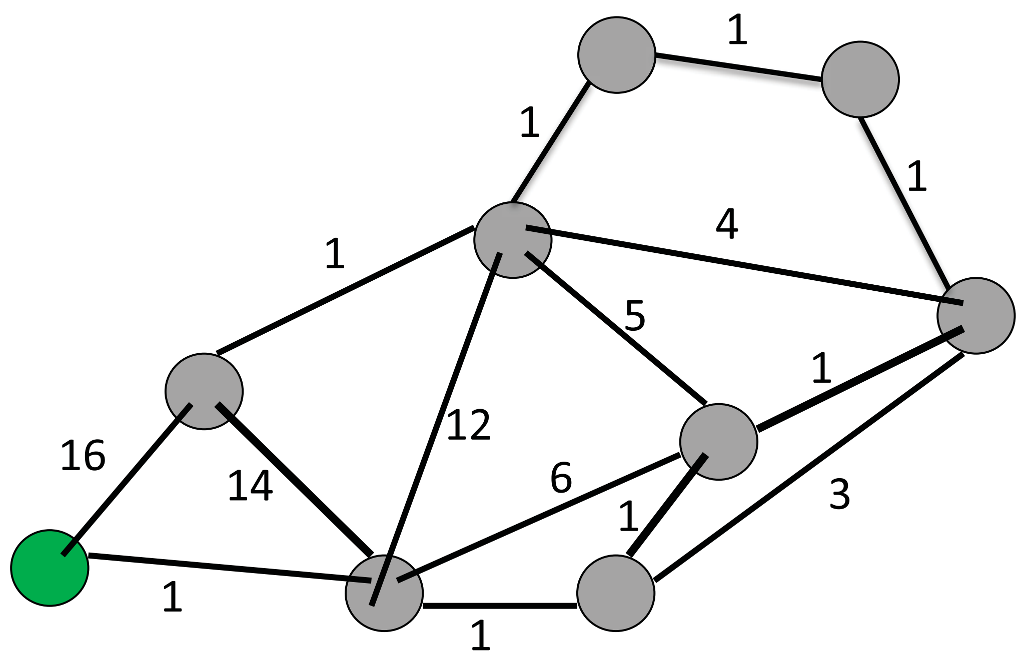
- Child pointers:
  – Send $parent/nonparent$ responses to search messages.
- Distances:
  – Piggyback distances on $search$ messages.
- Termination:
  – Convergecast starting from the leaves.
- Applications:
  – Message broadcast from the root
  – Global computation

Shortest Paths Trees



Shortest Paths

- Generalize the BFS problem to allow weights on the graph edges, $weight_{\{u,v\}}$ for edge $\{u, v\}$
- Connected graph $G = (V, E)$, root vertex v_0, process i_0.
- Processes have UIDs.
- Processes know their neighbors and the weights of their incident edges, but otherwise have no knowledge about the graph.



Shortest Paths

- Processes must produce a Shortest-Paths Spanning Tree rooted at vertex v_0.
- Branches are directed paths from v_0.
  - Spanning: Branches reach all vertices.
  - Shortest paths: The total weight of the tree branch to each node is the minimum total weight for any path from v_0 in G.
- Output: Each process $i \neq i_0$ should output $parent(j), distance(d),$ meaning that:
  - j's vertex is the parent of i's vertex on a shortest path from v_0,
  - d is the total weight of a shortest path from v_0 to j.

Bellman-Ford Shortest Paths Algorithm

- State variables:
  - $dist$, a nonnegative real or ∞, representing the shortest known distance from v_0. Initially 0 for process i_0, ∞ for the others.
  - $parent$, a UID or undefined, initially undefined.
  - uid
- Algorithm for process i:
  - At each round:
    - Send a $distance(dist)$ message to all neighbors.
    - Receive messages from neighbors; let d_j be the distance received from neighbor j.
    - Perform a relaxation step:
      $dist := \min(dist, \min_j(d_j + weight_{\{i,j\}}))$.
    - If $dist$ decreases then set $parent := j$, where j is any neighbor that produced the new $dist$.

Correctness

- Claim: Eventually, every process i has:
  - $dist$ = minimum weight of a path from i_0 to i, and
  - if $i \neq i_0$, $parent$ $=$ the previous node on some shortest path from i_0 to i.

- Key invariant:
  - For every r, at the end of r rounds, every process $i \neq i_0$ has its $dist$ and $parent$ corresponding to a shortest path from i_0 to i among those paths that consist of at most r edges; if there is no such path, then $dist = \infty$ and $parent$ is undefined.

Complexity

- Time complexity:
  - Number of rounds until all the variables stabilize to their final values.
  - $n - 1$ rounds
- Message complexity:
  - Number of messages sent by all processes during the entire execution.
  - $O(n \cdot |E|)$
- More expensive than BFS:
  - $diam$ rounds,
  - $O(|E|)$ messages

- Q: Does the time bound really depend on n?

Child Pointers

- Ignore repeated messages.
- When process i receives a message that it does not use to improve $dist$, it responds with a $nonparent$ message.
- When process i receives a message that it uses to improve $dist$, it responds with a $parent$ message, and also responds to any previous parent with a $nonparent$ message.
- Process i records nodes from which it receives $parent$ messages in a set $children.$
- When process i receives a $nonparent$ message from a current child, it removes the sender from its $children.$
- When process i improves $dist,$ it empties $children.$

Termination

- Q: How can the processes learn when the shortest-paths tree is completed?
- Q: How can a process even know when it can output its own $parent$ and $distance$?

- If processes knew an upper bound on n, then they could simply wait until that number of rounds have passed.
- But what if they don't know anything about the graph?

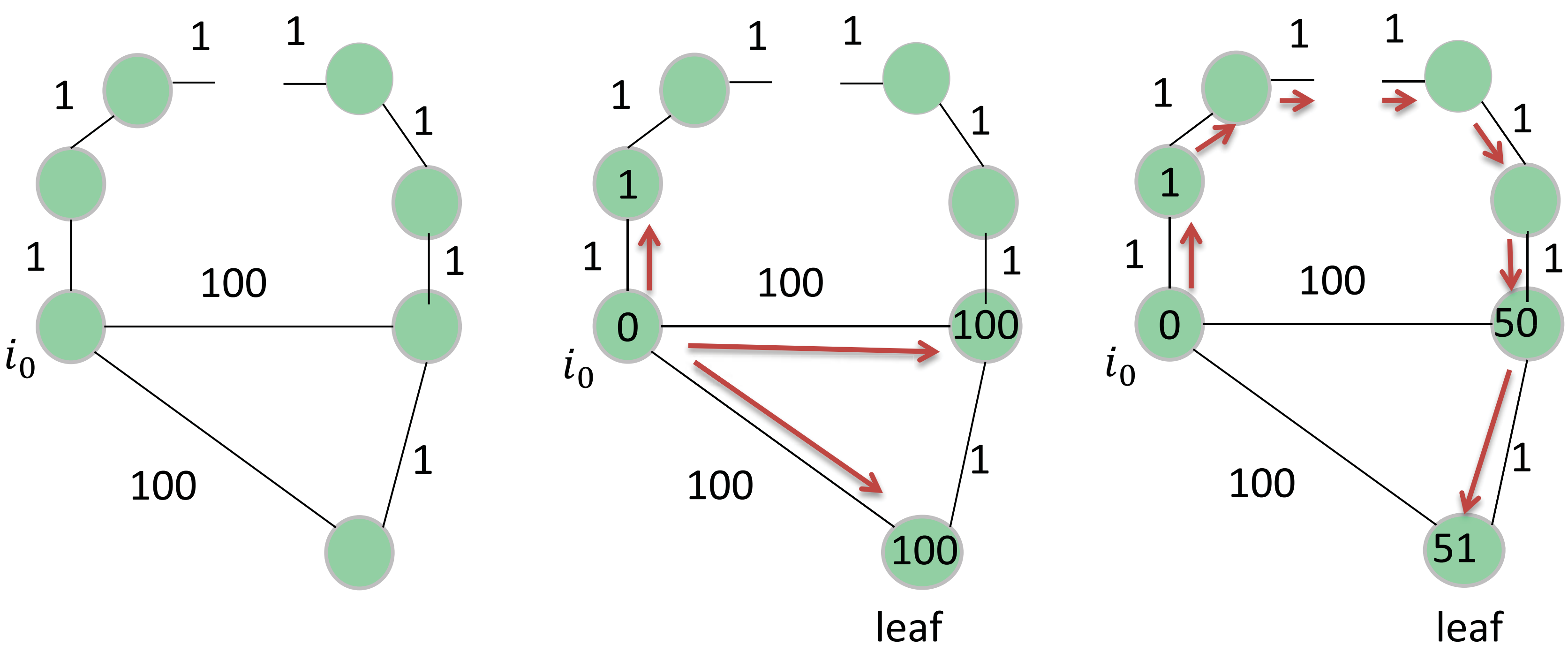
- Recall termination for BFS: Used convergecast.
- Q: Does that work here?

Termination

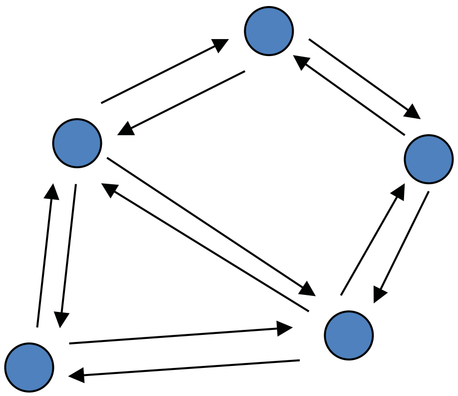
- Q: How can the processes learn when the shortest-paths tree is completed?
- Q: Does convergecast work here?
- Yes, but it's trickier, since the tree structure changes.

- Key ideas:
  - A process $\neq i_0$ can send a $done$ message to its current parent after:
    - It has received responses to all its $distance$ messages, so it believes it knows who its children are, and
    - It has received $done$ messages from all of those children.
  - The same process may be involved several times in the convergecast, based on improved estimates.

Termination

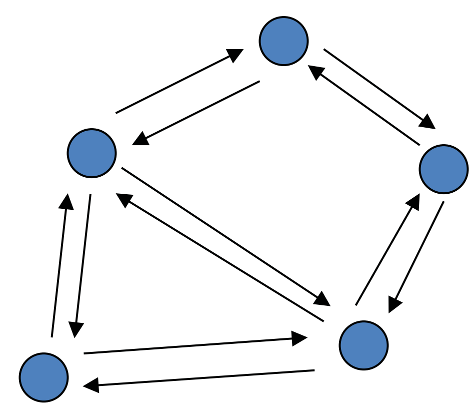


Asynchronous Distributed Algorithms



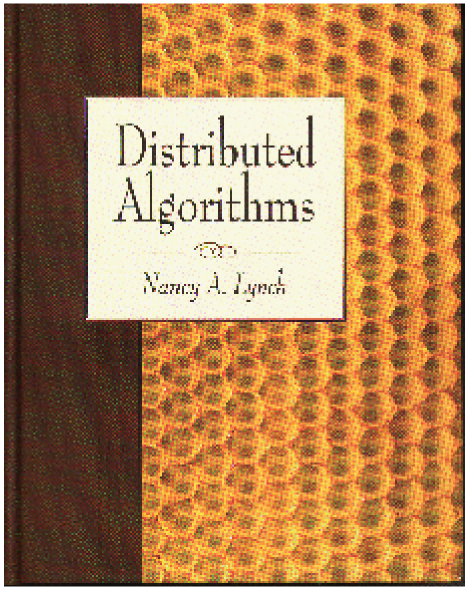
Asynchronous Network Model

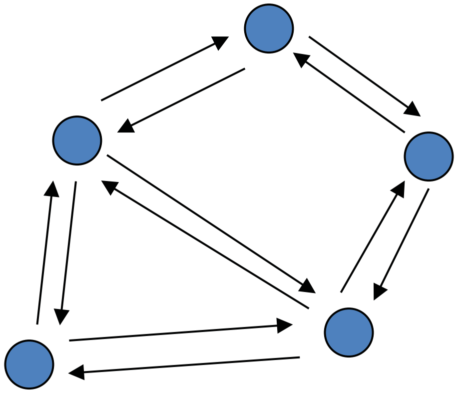
- Complications so far:
  - Processes act concurrently.
  - A little nondeterminism.
- Now things get much worse:
  - No rounds---process steps and message deliveries happen at arbitrary times, in arbitrary orders.
  - Processes get out of synch.
  - Much more nondeterminism.
- Understanding asynchronous distributed algorithms is hard because we can't understand exactly how they execute.
- Instead, we must understand abstract properties of executions.



Aynchronous Network Model

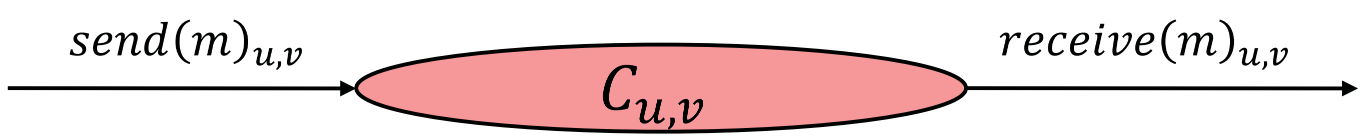
- Lynch, Distributed Algorithms, Chapter 8.
- Processes at nodes of an undirected graph $G = (V, E)$, communicate using messages.
- Communication channels associated with edges (one in each direction on each edge).
  - $C_{u,v}$, channel from vertex u to vertex v.
- Each process has output ports and input ports that connect it to its communication channels.
- Processes need not be distinguishable.





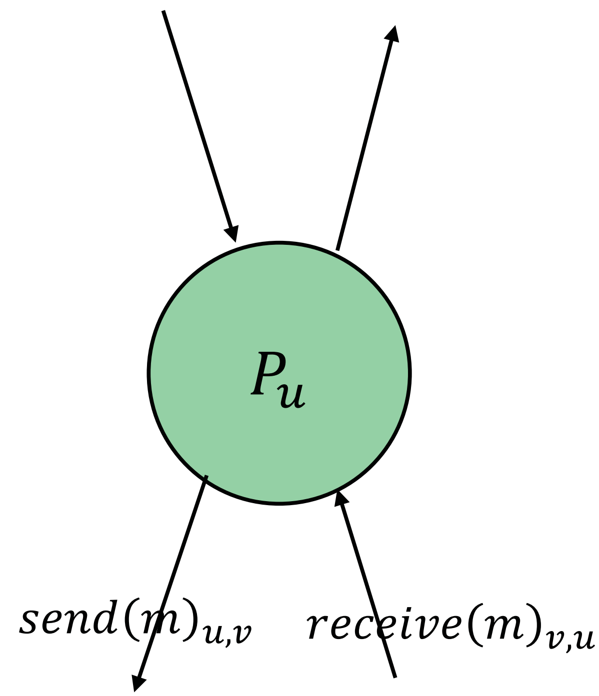
Channel Automaton $C_{u,v}$

- Formally, an input/output automaton.
- Input actions: $send(m)_{u,v}$
- Output actions: $receive(m)_{u,v}$
- State variable:
  - $mqueue$, a FIFO queue, initially empty.
- Transitions:
  - $send(m)_{u,v}$
    - Effect: add m to $mqueue$.
  - $receive(m)_{u,v}$
    - Precondition: m = head($mqueue$)
    - Effect: remove head of $mqueue$



Process Automaton P_u

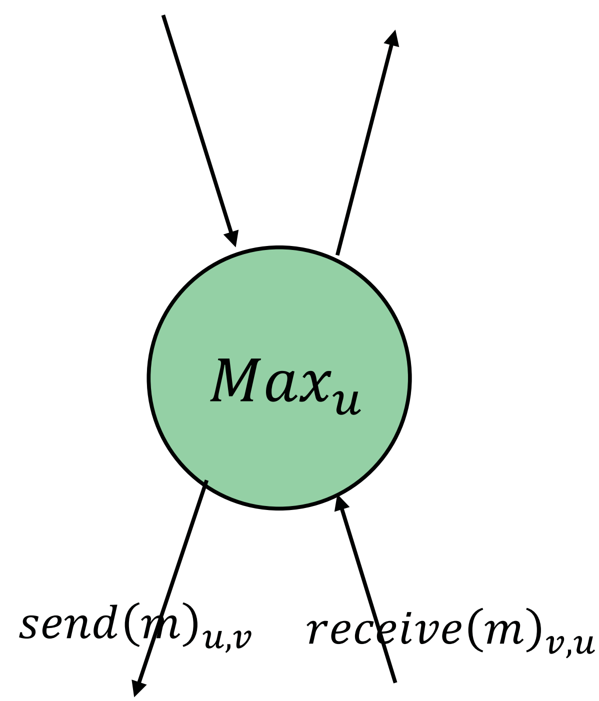
- Associate a process automaton with each vertex of G.
- To simplify notation, let P_u denote the process automaton at vertex u.
  - But the process does not "know" u.



- P_u has $send(m)_{u,v}$ outputs and $receive(m)_{v,u}$ inputs.
- May also have external inputs and outputs.
- Has state variables.
- Keeps taking steps (eventually).

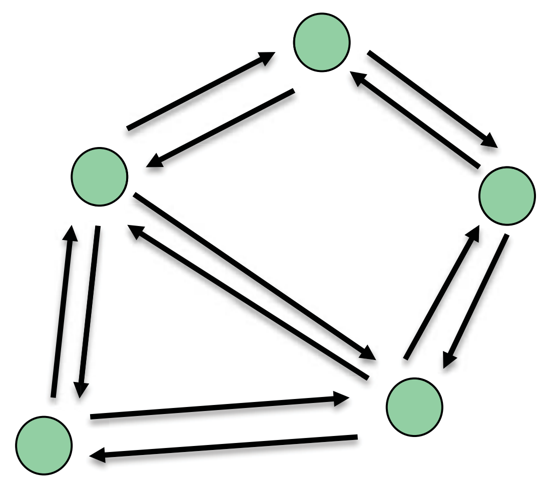
Example: Max_u Process Automaton

- Input actions: $receive(m)_{v,u}$
- Output actions: $send(m)_{u,v}$
- State variables:
  - max, a natural number, initially x_u
  - For each neighbor v:
    - $send(v)$, a Boolean, initially $true$
- Transitions:
  - $receive(m)_{v,u}$
    - Effect: if $m > max$ then
      - $max := m$
      - for every w, $send(w) := true$
  - $send(m)_{u,v}$
    - Precondition: $send(v) = true$ and $m = max$
    - Effect: $send(v) := false$



Combining Processes and Channels

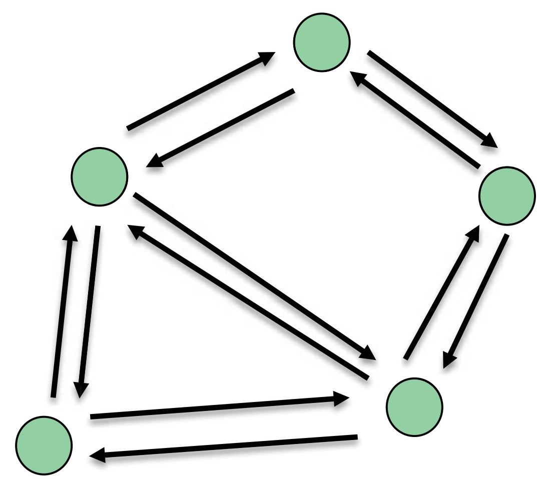
- Undirected graph $G = (V, E)$.
- Process P_u at each vertex u.
- Channels $C_{u,v}$ and $C_{v,u}$, associated with each edge $\{u, v\}$.
- $send(m)_{u,v}$ output of process P_u gets identified with $send(m)_{u,v}$ input of channel $C_{u,v}$.
- $receive(m)_{v,u}$ output of channel $C_{v,u}$ gets identified with $receive(m)_{v,u}$ input of process P_u.
- Steps involving such a shared action involve simultaneous state transitions for a process and a channel.



Execution

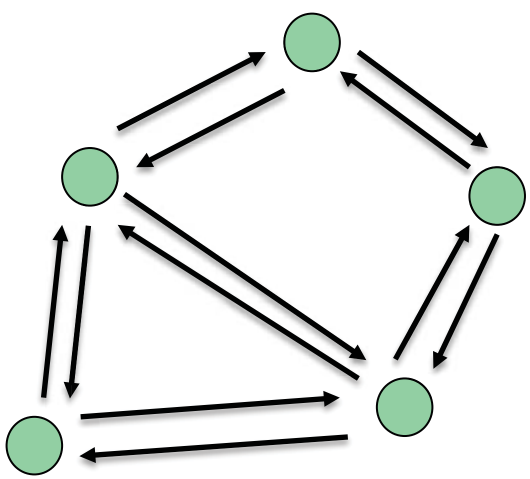
- No synchronous rounds anymore.
- The system executes by performing enabled steps, one at a time, in any order.
- Formally, an execution is modeled as a sequence of individual steps.
- Different from the synchronous model, in which all processes take steps concurrently at each round.

- Assume enabled steps eventually occur:
  - Each channel always eventually delivers the first message in its queue.
  - Each process always eventually performs some enabled step.

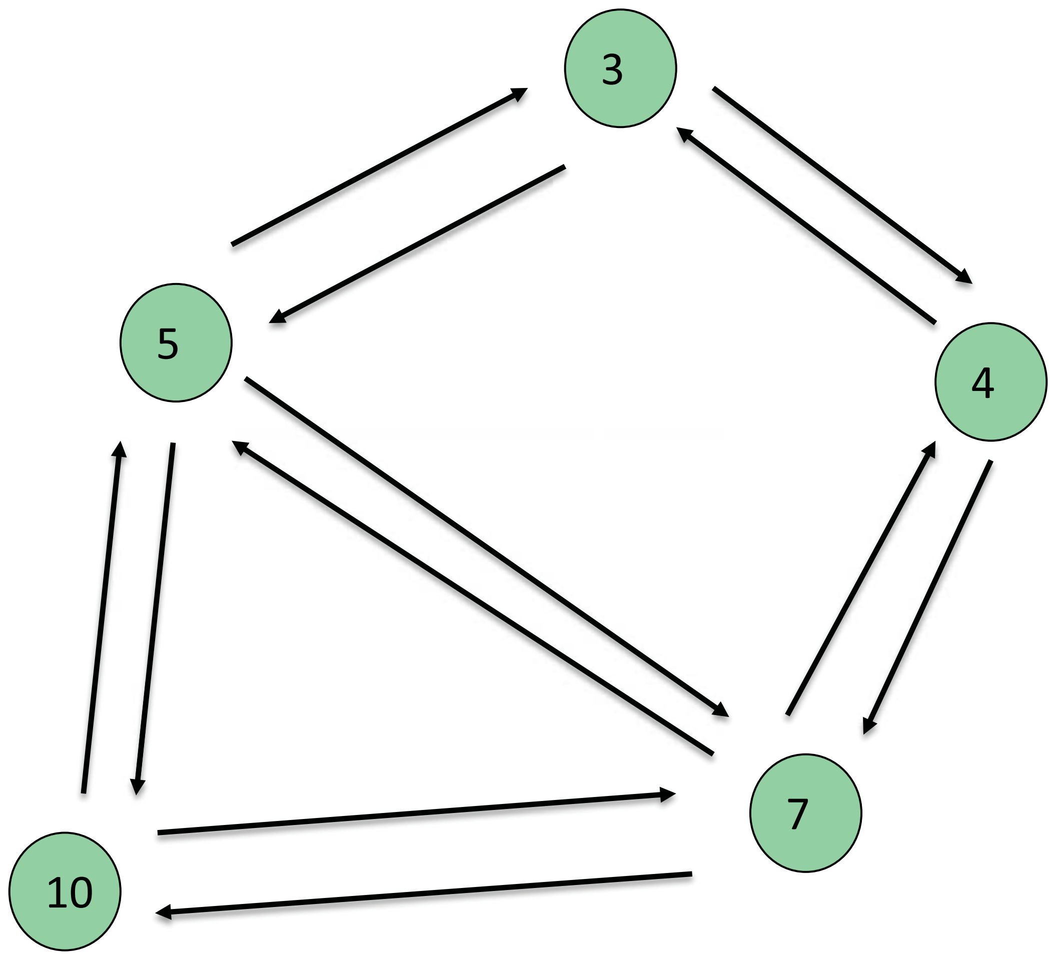


Combining Max Processes and Channels

- Each process Max_u starts with an initial value x_u.
- They all send out their initial values, and propagate their max values, until everyone has the globally-maximum value.
- Sending and receiving steps can happen in many different orders, but in all cases the global max will eventually arrive everywhere.



Max System



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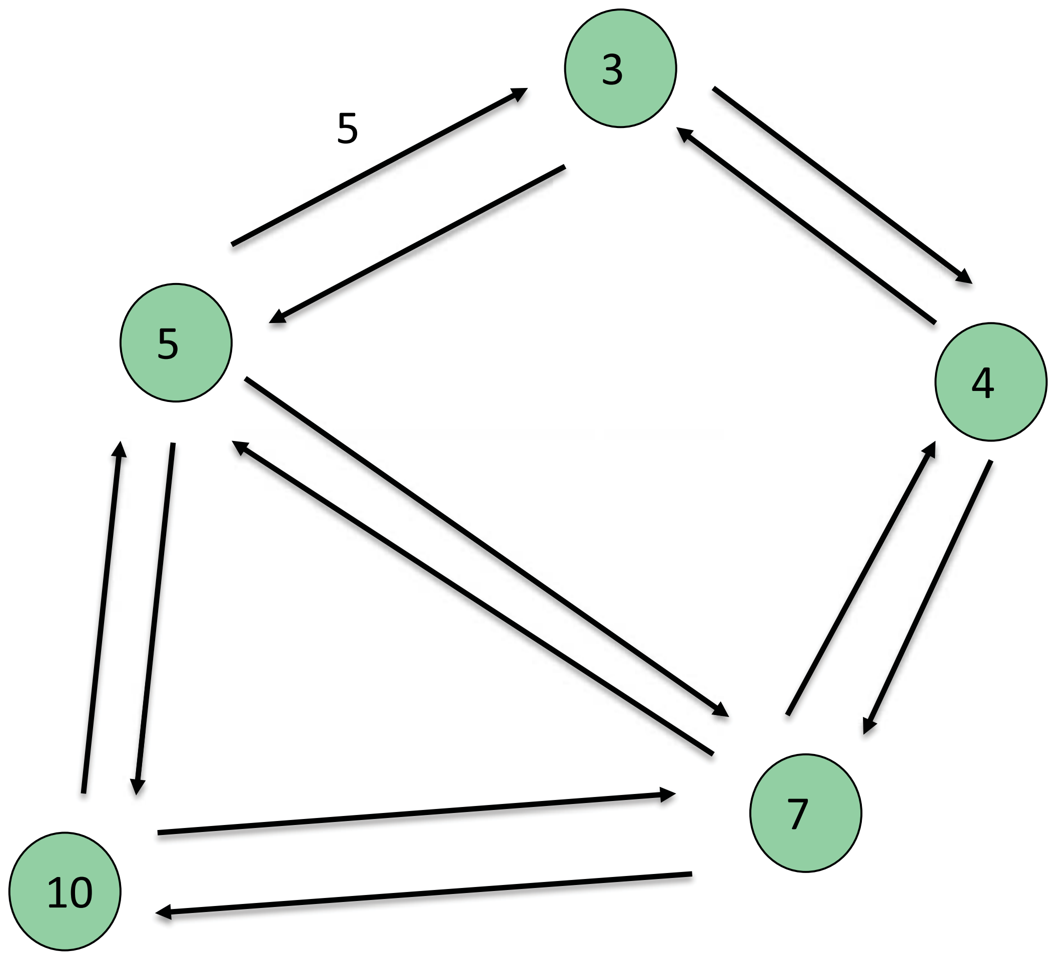

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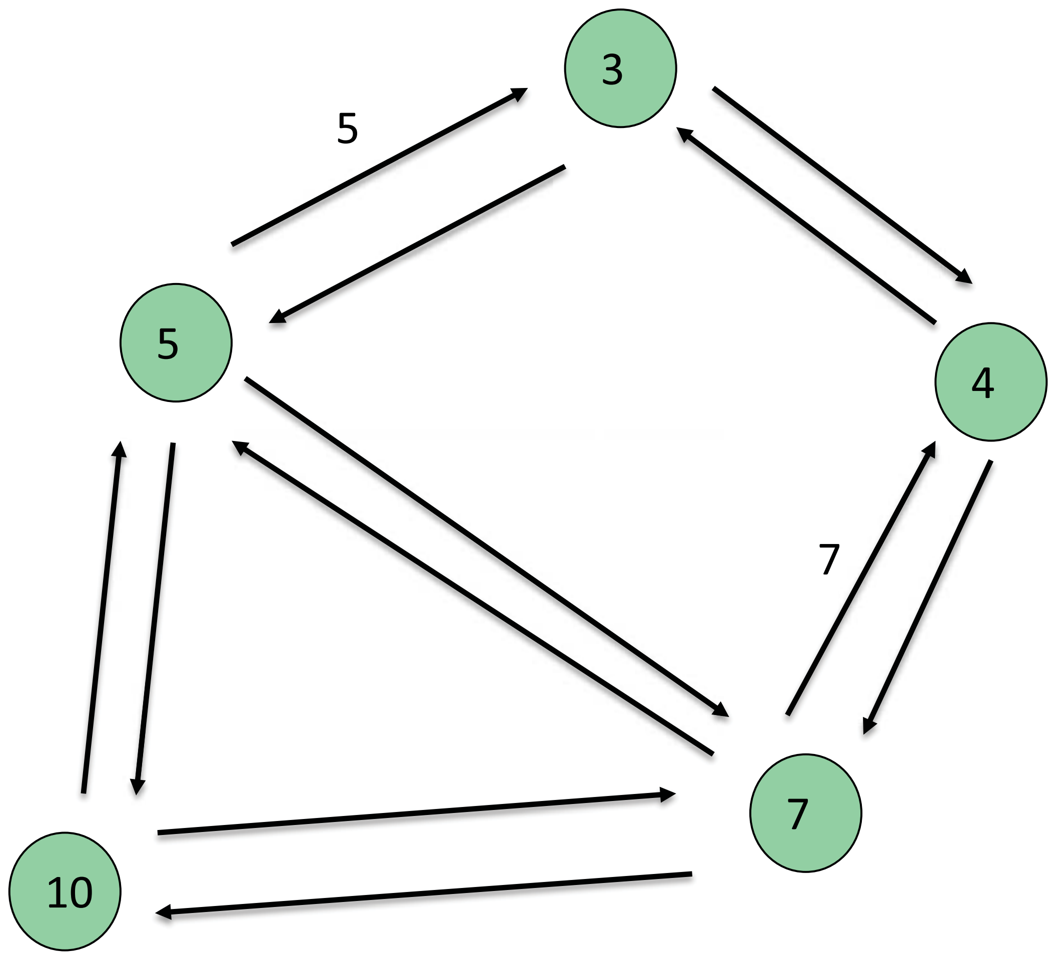

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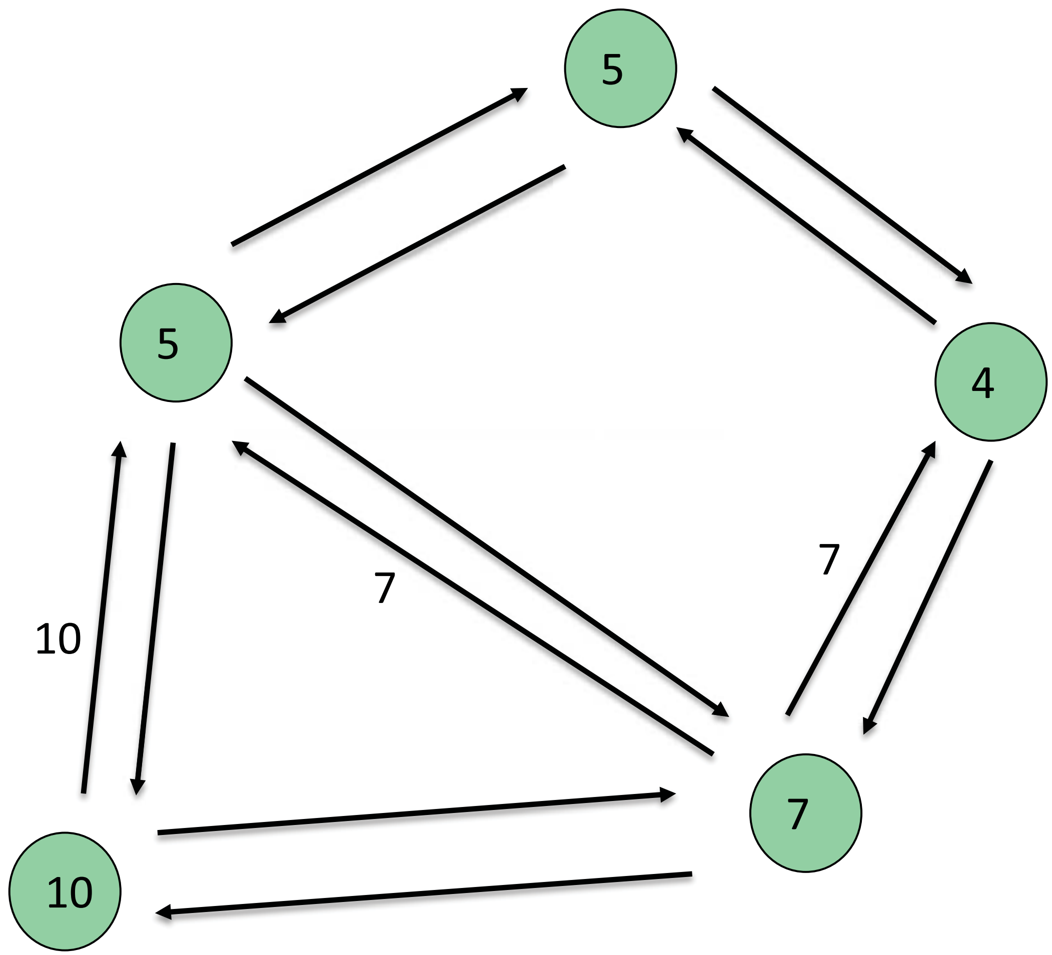

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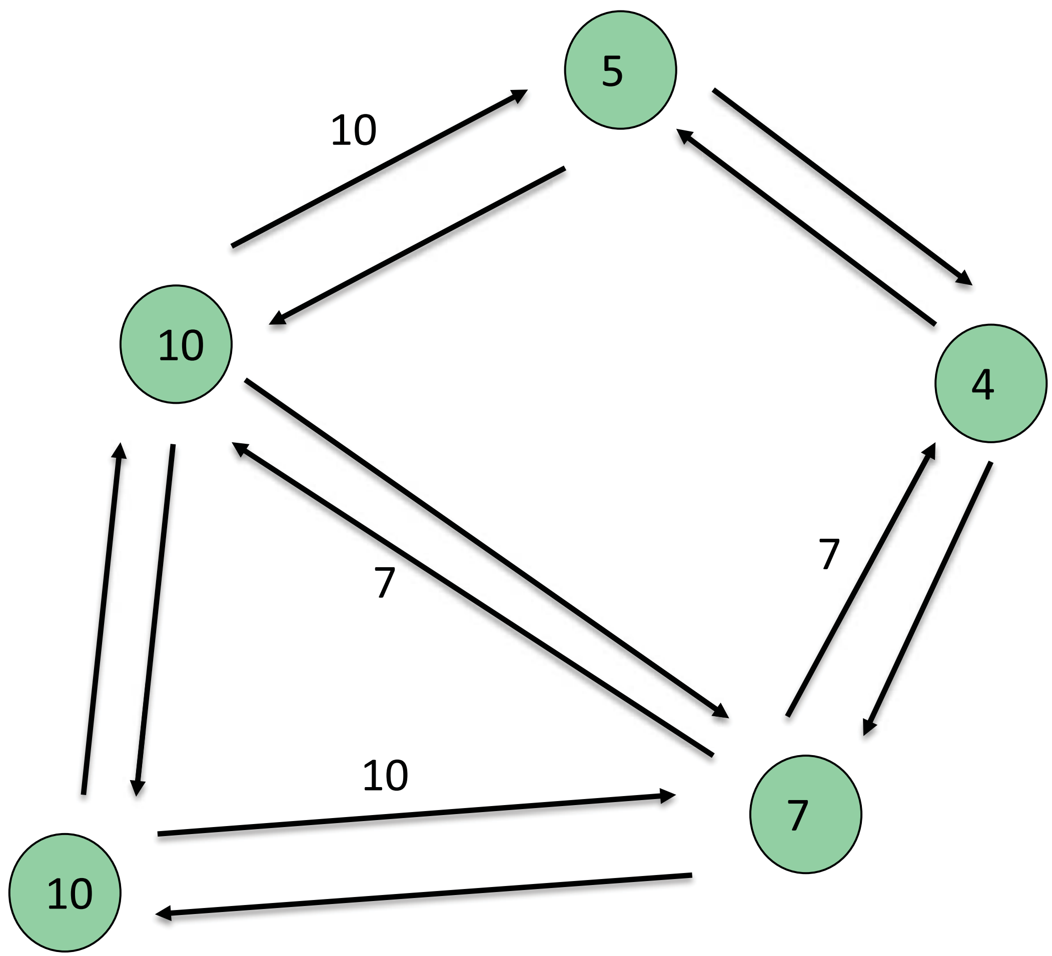

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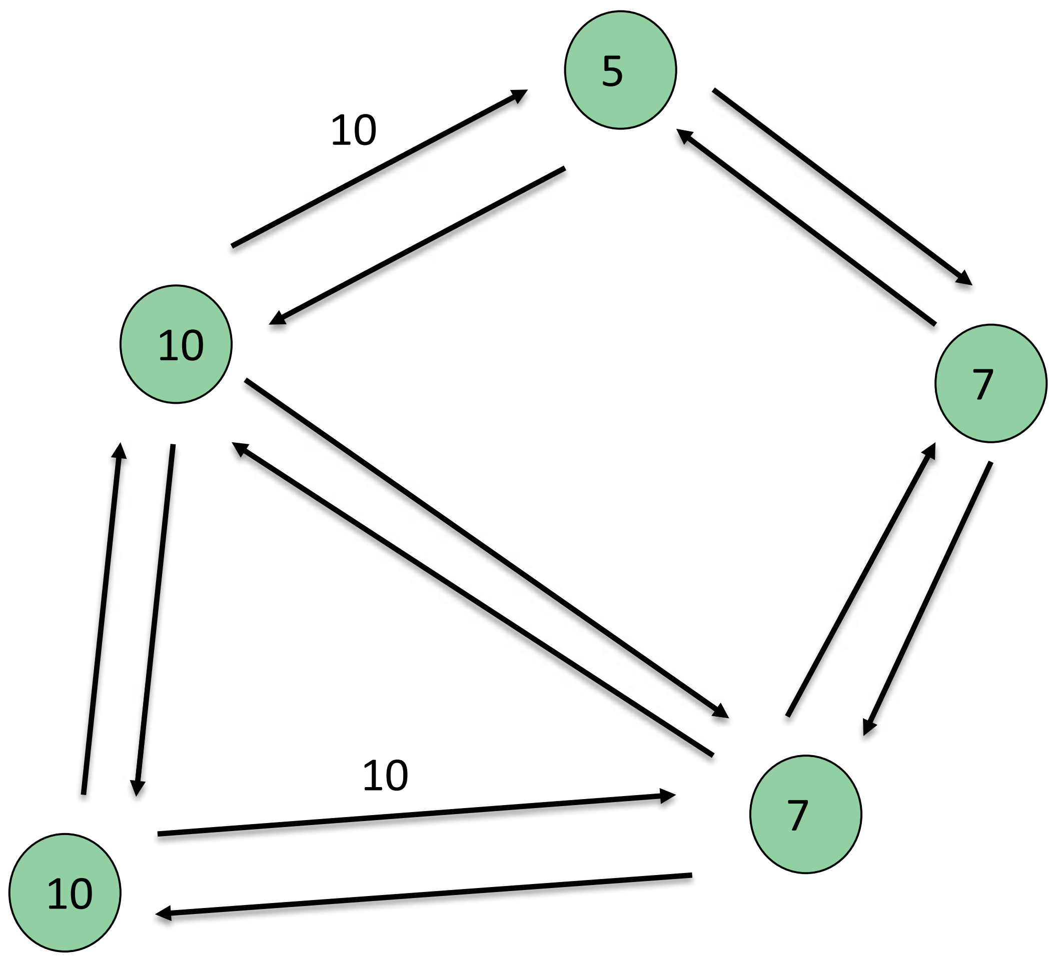

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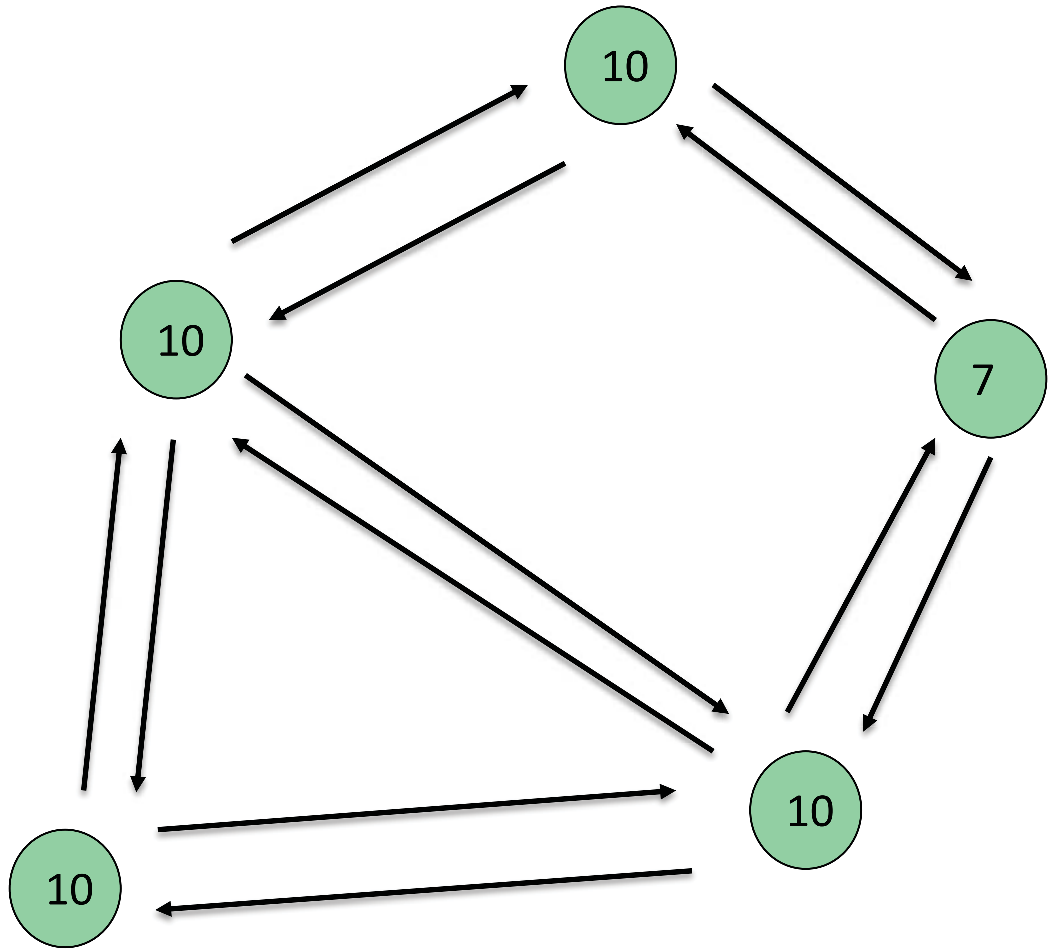

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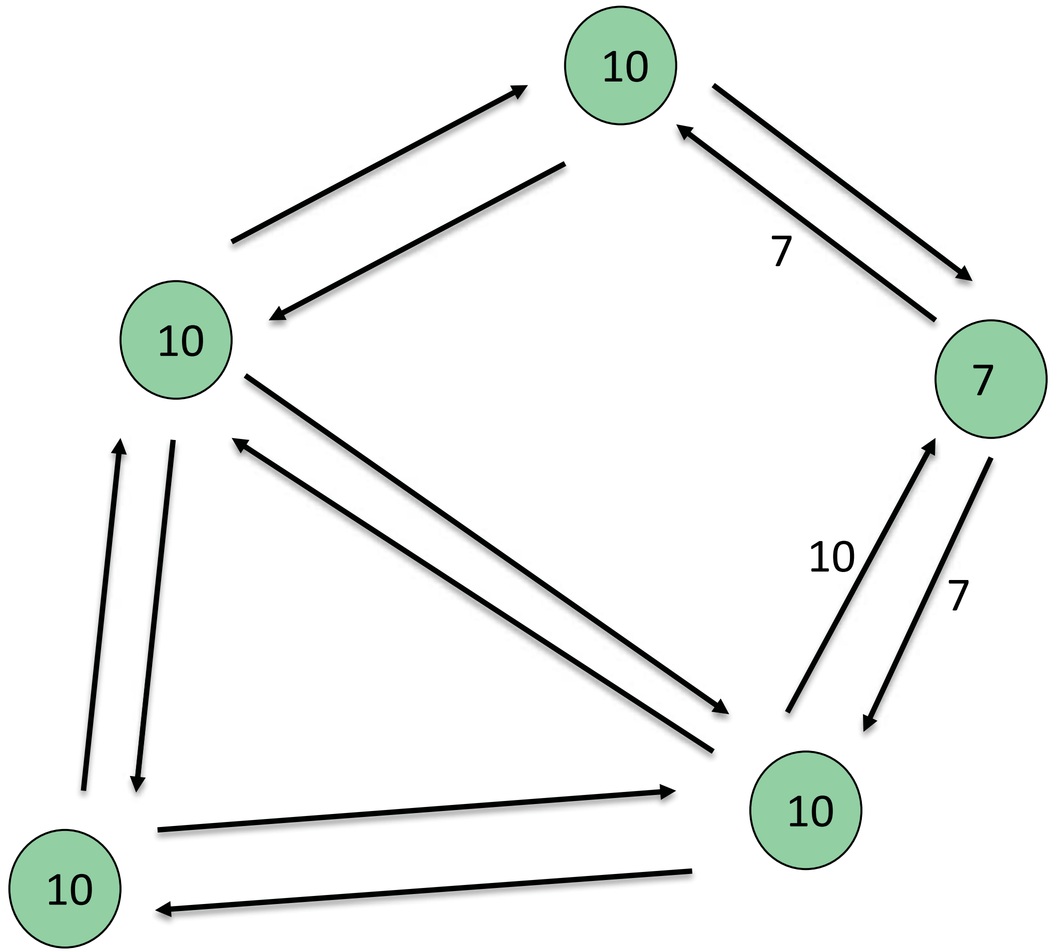

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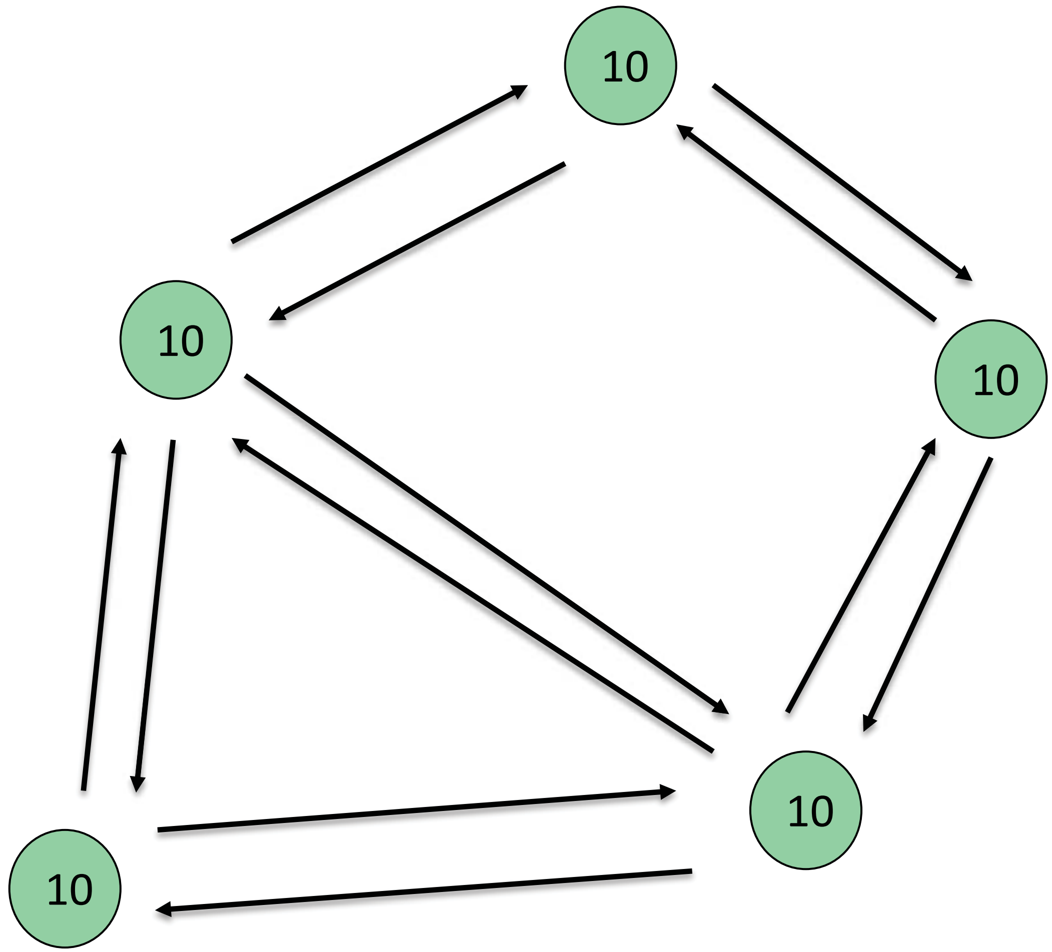


Complexity

- Message complexity:
  - Number of messages sent by all processes during the entire execution.
  - $O(n \cdot |E|)$

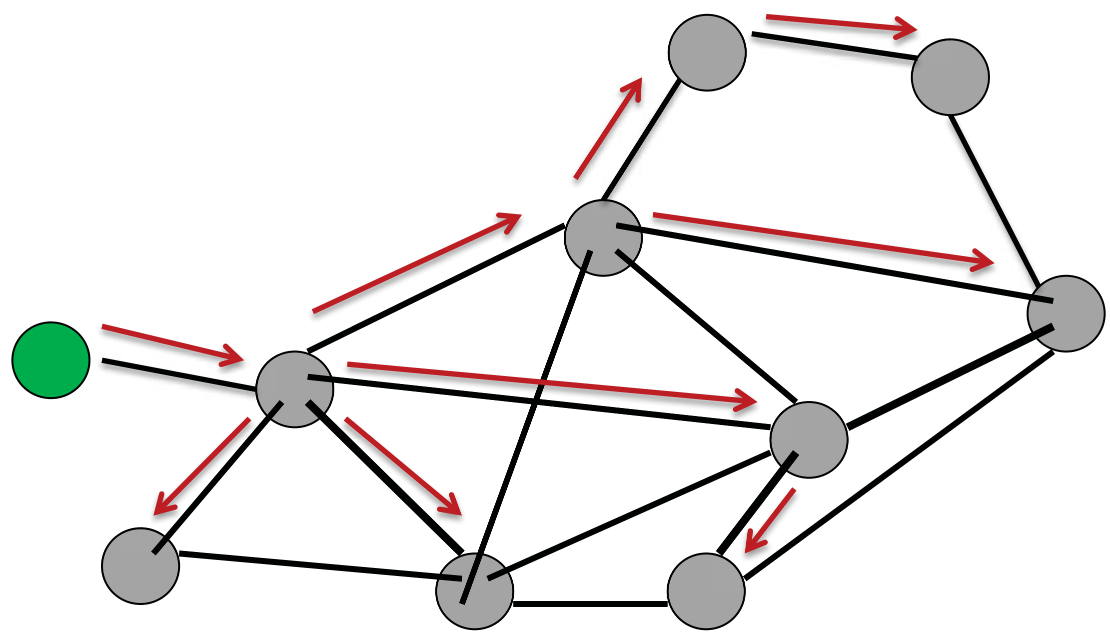
- Time complexity:
  - Q: What should we measure?
  - Not obvious, because the various components are taking steps in arbitrary orders---no “rounds”.
  - A common approach:
    - Assume real-time upper bounds on the time to perform basic steps:
      - d for a channel to deliver the next message, and
      - l for a process to perform its next step.
    - Infer a real-time upper bound for solving the overall problem.

Complexity

- Time complexity:
  - Assume real-time upper bounds on the time to perform basic steps:
    - d for a channel to deliver the next message, and
    - l for a process to perform its next step.
  - Infer a real-time upper bound for solving the problem.

- For the Max system:
  - Ignore local processing time (l = 0), consider only channel sending time.
  - Straightforward upper bound: $O(diam \cdot n \cdot d)$
    - Consider the time for the max to reach any particular vertex u, along a shortest path in the graph.
    - At worst, it waits in each channel on the path for every other value, which is at most time $n \cdot d$ for that channel.

Breadth-First Spanning Trees



Breadth-First Spanning Trees

- Problem: Compute a Breadth-First Spanning Tree in an asynchronous network.
- Connected graph $G = (V, E)$.
- Distinguished root vertex v_0.
- Processes have no knowledge about the graph.
- Processes have UIDs
  - i_0 is the UID of the root v_0.
  - Processes know UIDs of their neighbors, and know which ports are connected to each neighbor.
- Processes must produce a BFS tree rooted at v_0.
- Each process $i \neq i_0$ should output $parent(j)$, meaning that j's vertex is the parent of i's vertex in the BFS tree.

First Attempt

- Just run the simple synchronous BFS algorithm asynchronously.
- Process i_0 sends $search$ messages, which everyone propagates the first time they receive it.
- Everyone picks the first node from which it receives a $search$ message as its parent.

- Nondeterminism:
  - No longer any nondeterminism in process decisions.
  - But plenty of new nondeterminism: orders of message deliveries and process steps.

Process Automaton P_u

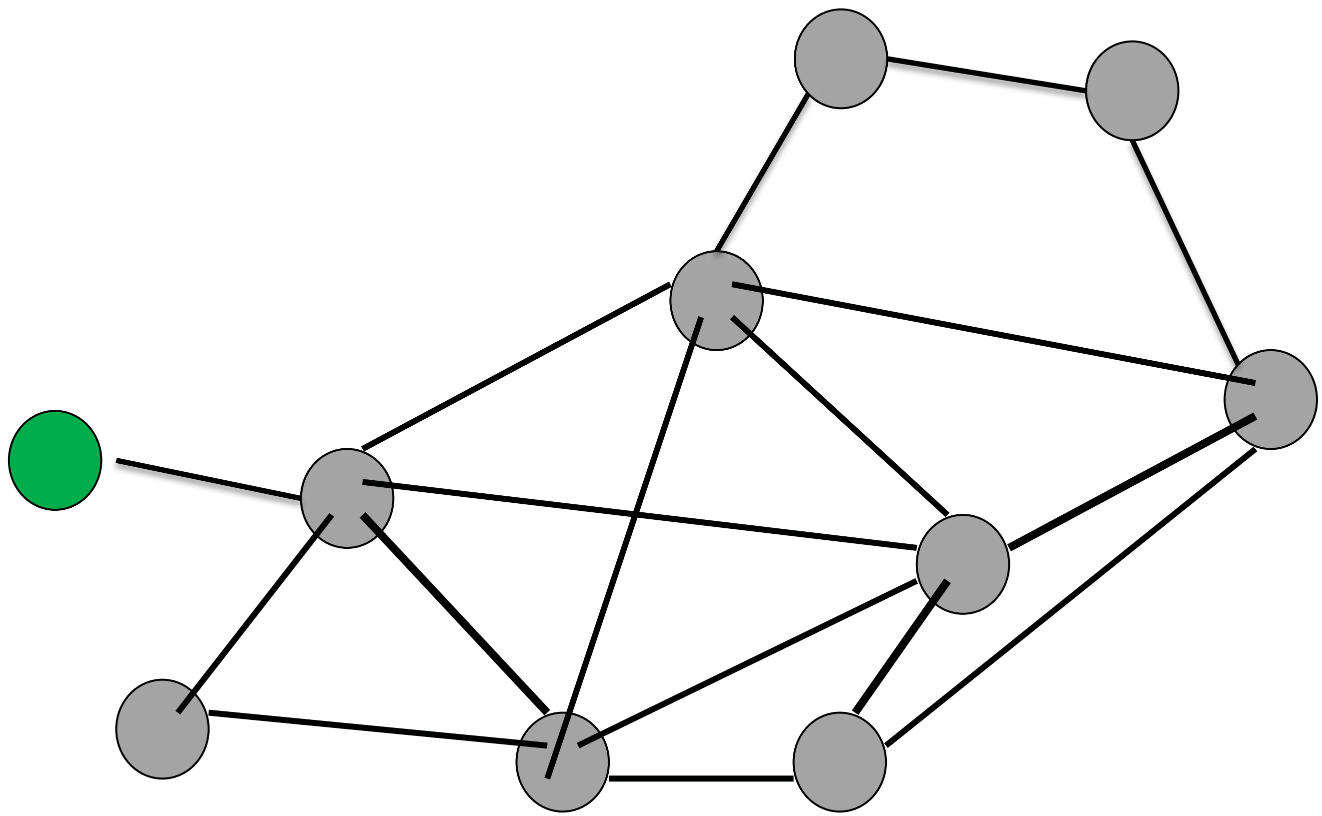
- Input actions: $receive(search)_{v,u}$
- Output actions: $send(search)_{u,v}$; $parent(v)_u$
- State variables:
  - $parent$: $\Gamma(u) \cup \{\perp\}$, initially $\perp$
  - $reported$: Boolean, initially false
  - For every $v \in \Gamma(u)$:
    - $send(v) \in \{search, \perp\}$, initially $search\ if\ u = v_0$, else $\perp$

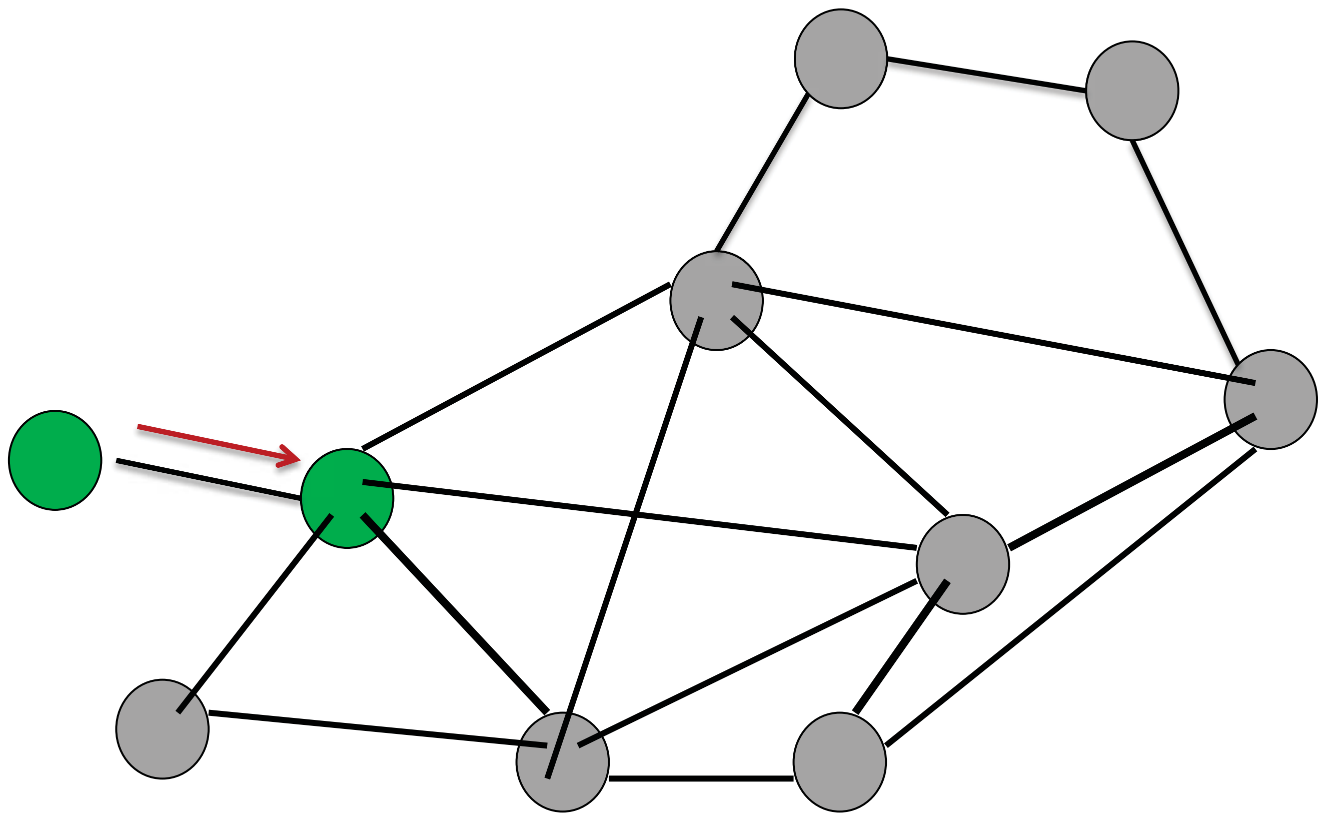
- Transitions:
  - $receive(search)_{v,u}$
    - Effect: if $u \neq v_0$ and $parent = \perp$ then
      - $parent := v$
      - for every w, $send(w) :=\ search$

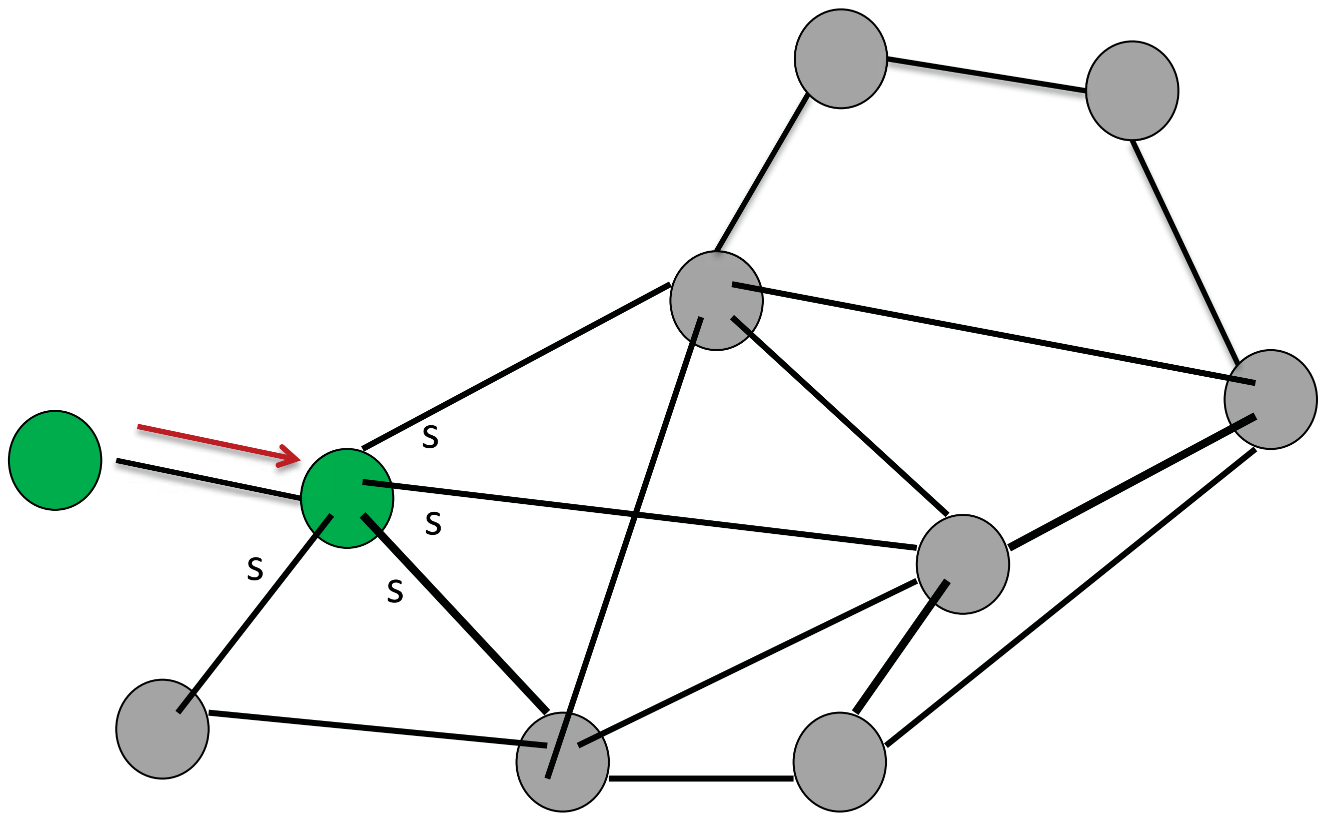
Process Automaton P_u

- Transitions:
  - $receive(search)_{v,u}$
    - Effect: if $u \neq v_0$ and $parent = \perp$ then
      - $parent := v$
      - for every w, $send(w) := search$
  - $send(search)_{u,v}$
    - Precondition: $send(v) = search$
    - Effect: $send(v) := \perp$
  - $parent(v)_u$
    - Precondition: $parent = v$ and $reported = false$
    - Effect: $reported := true$

Running Simple BFS Asynchronously



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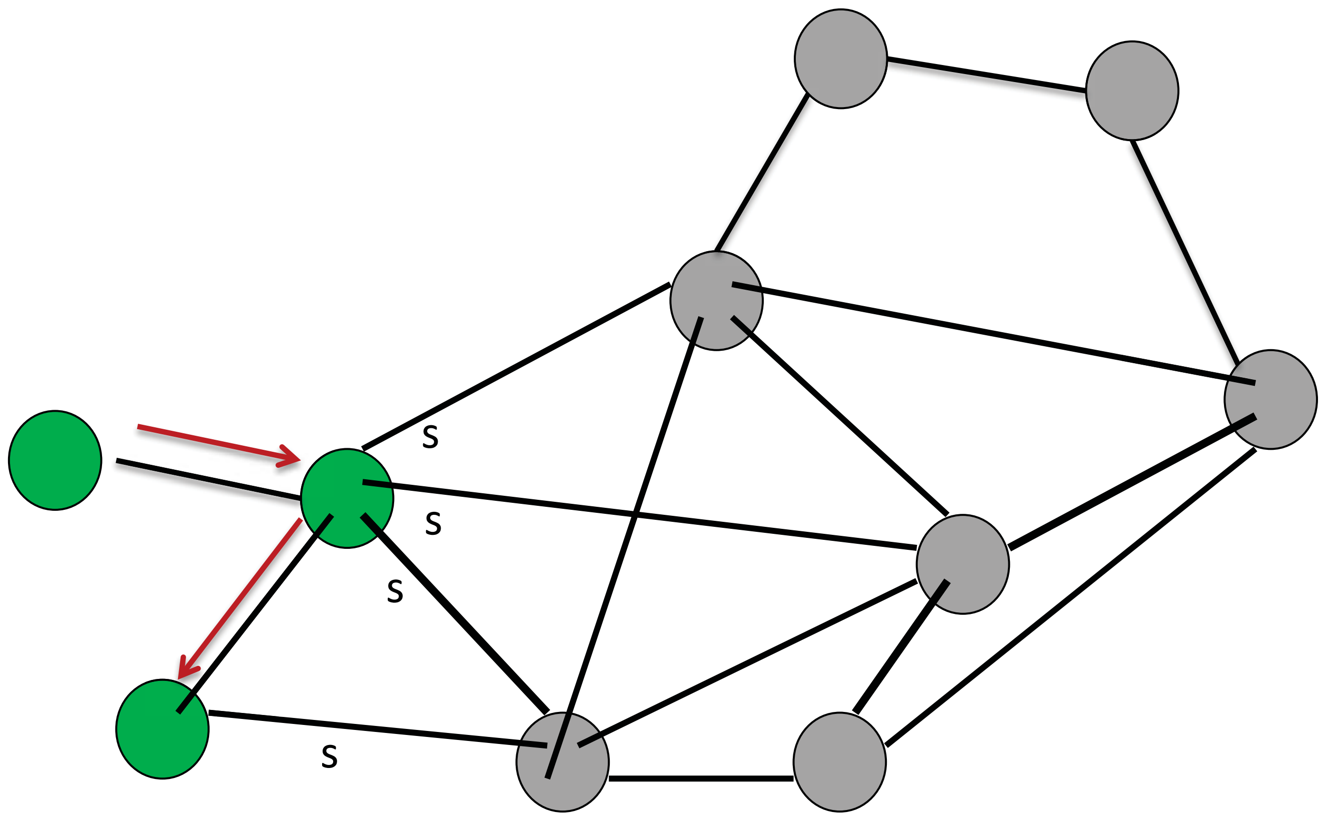
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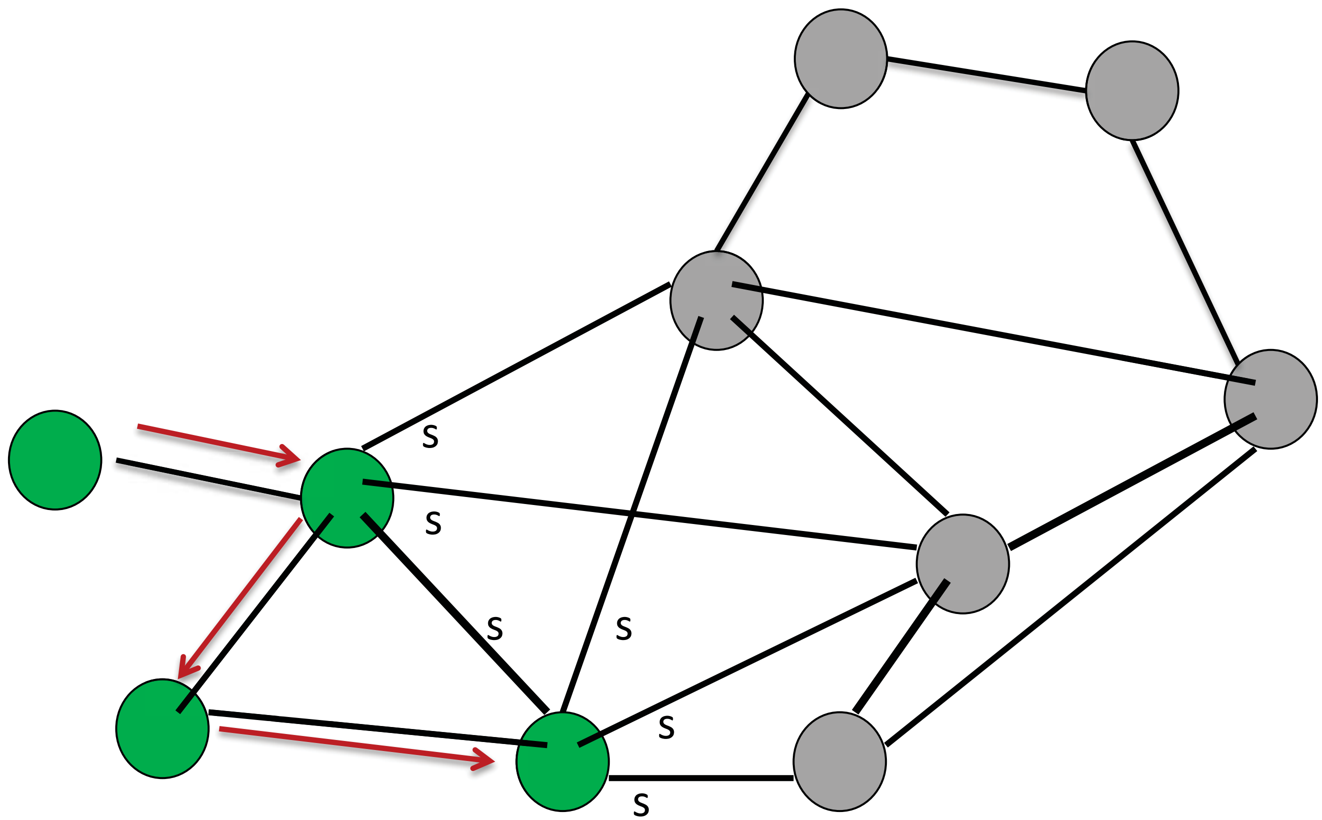
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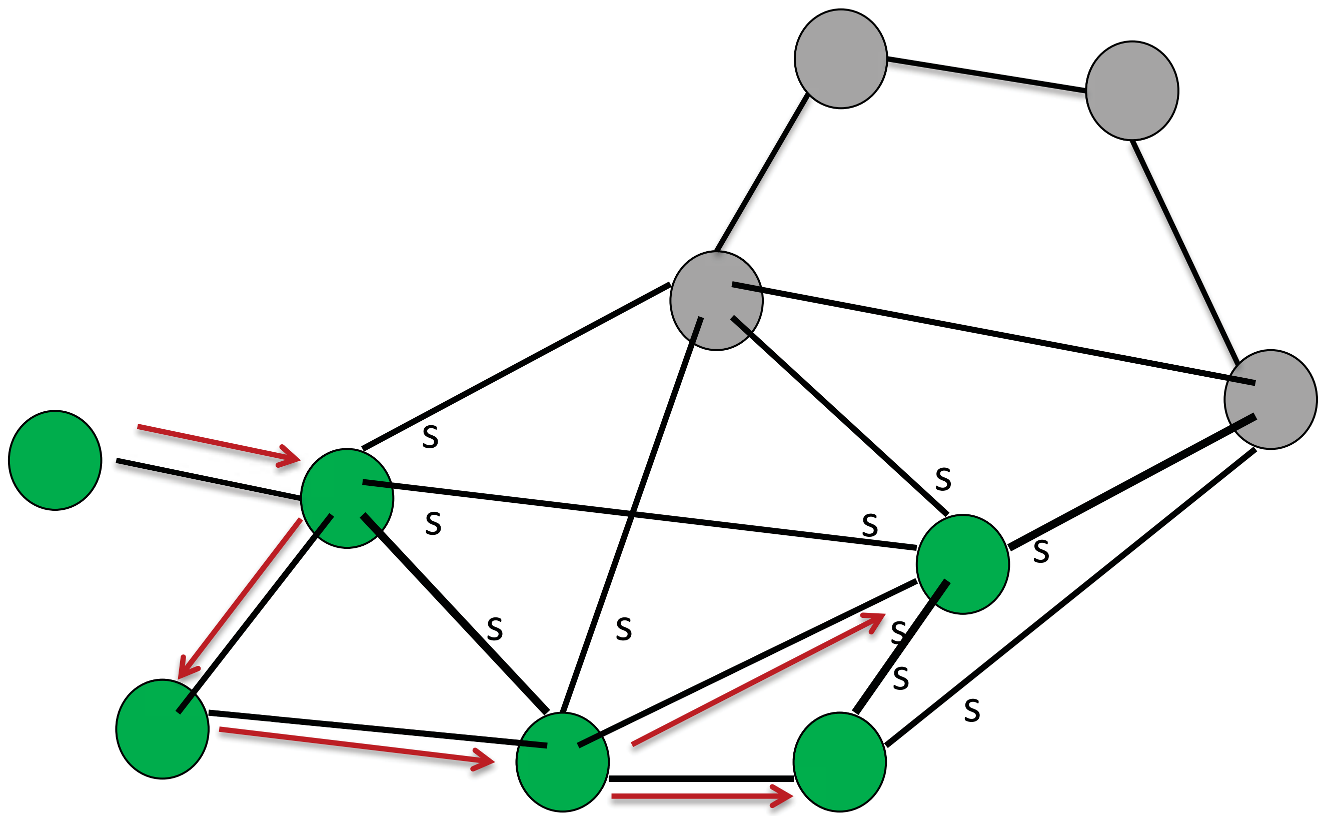
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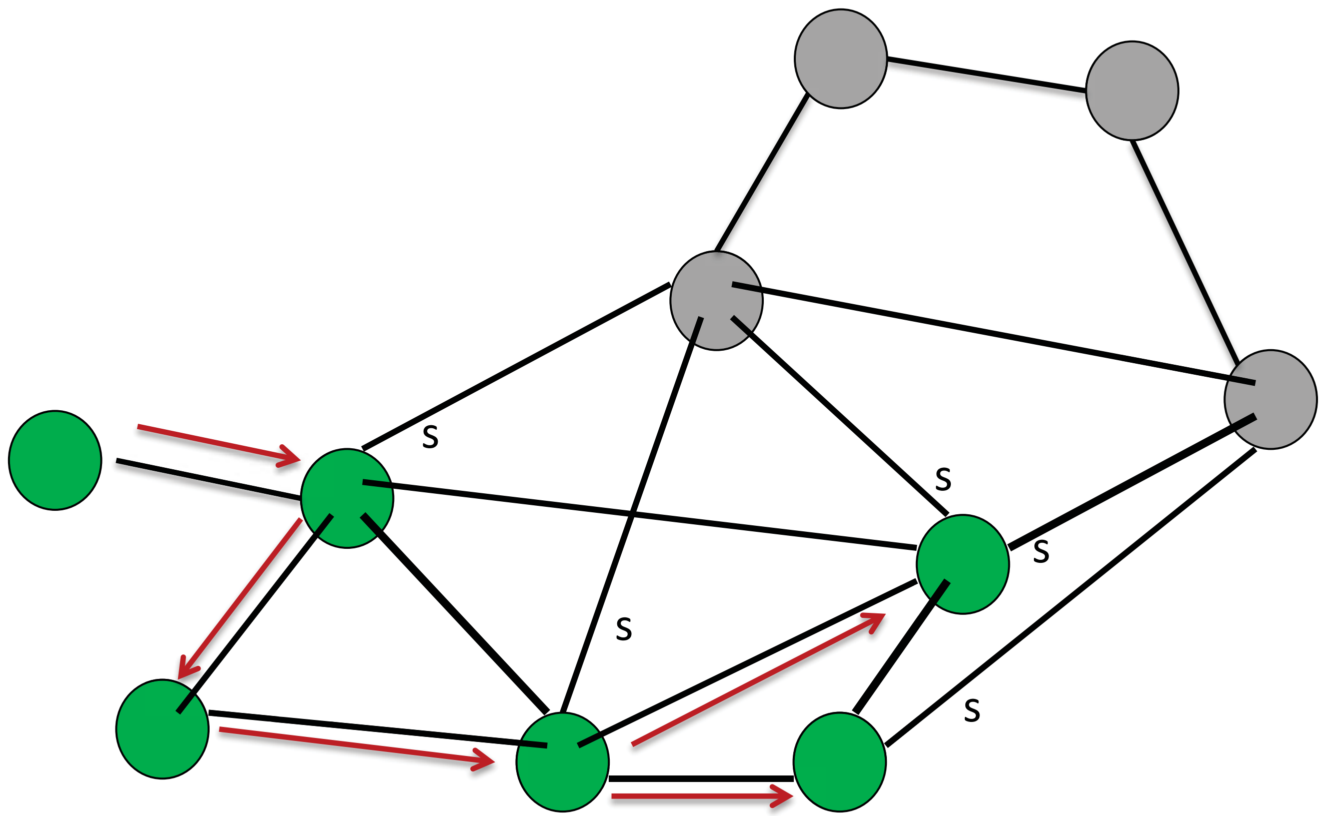

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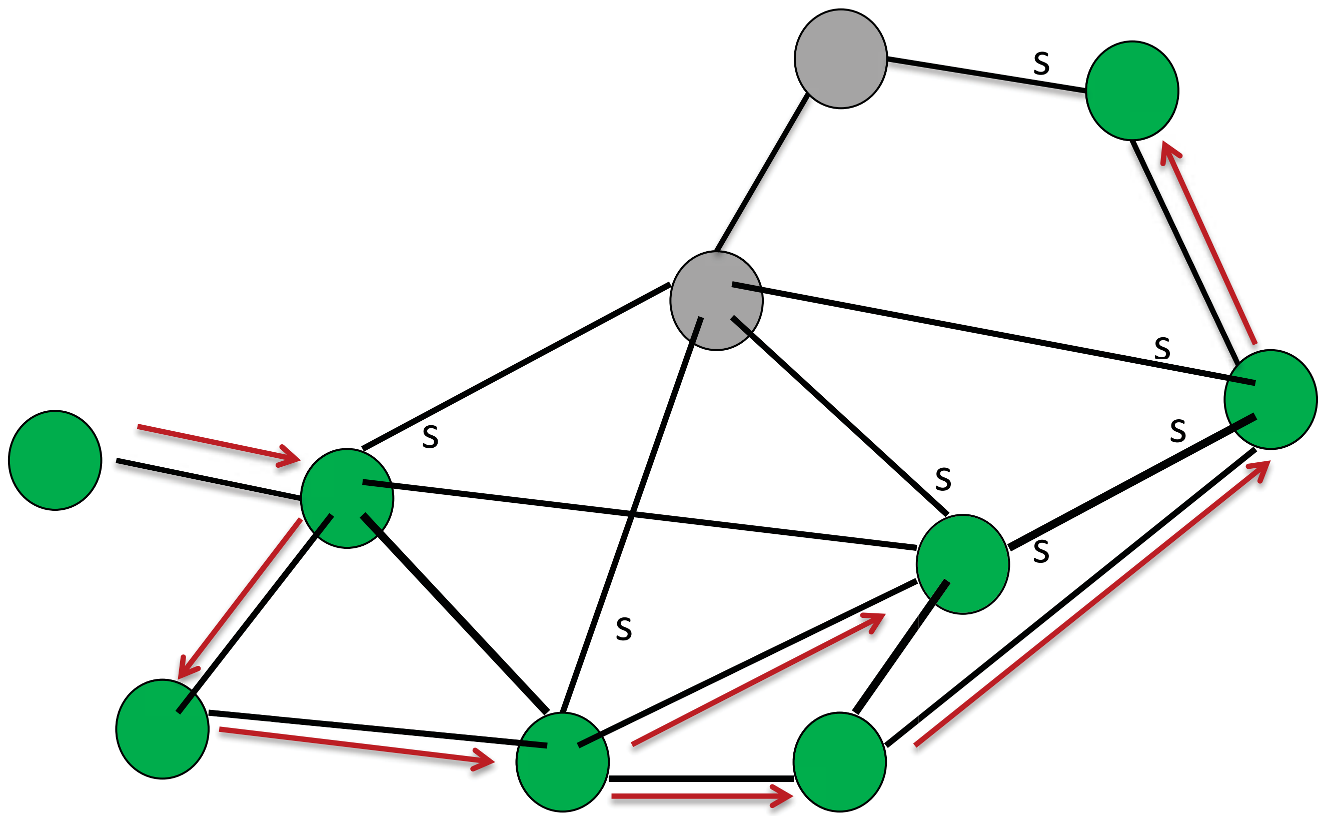
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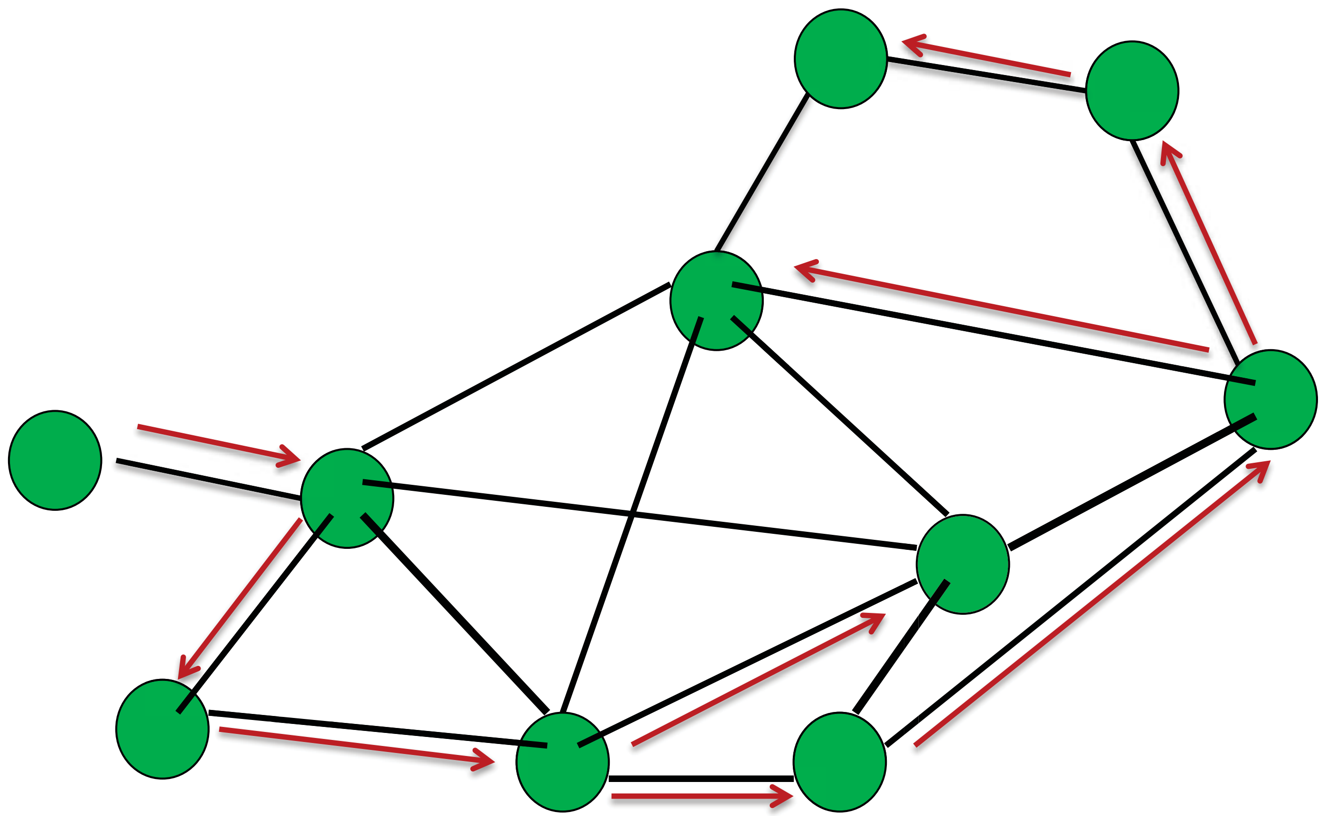
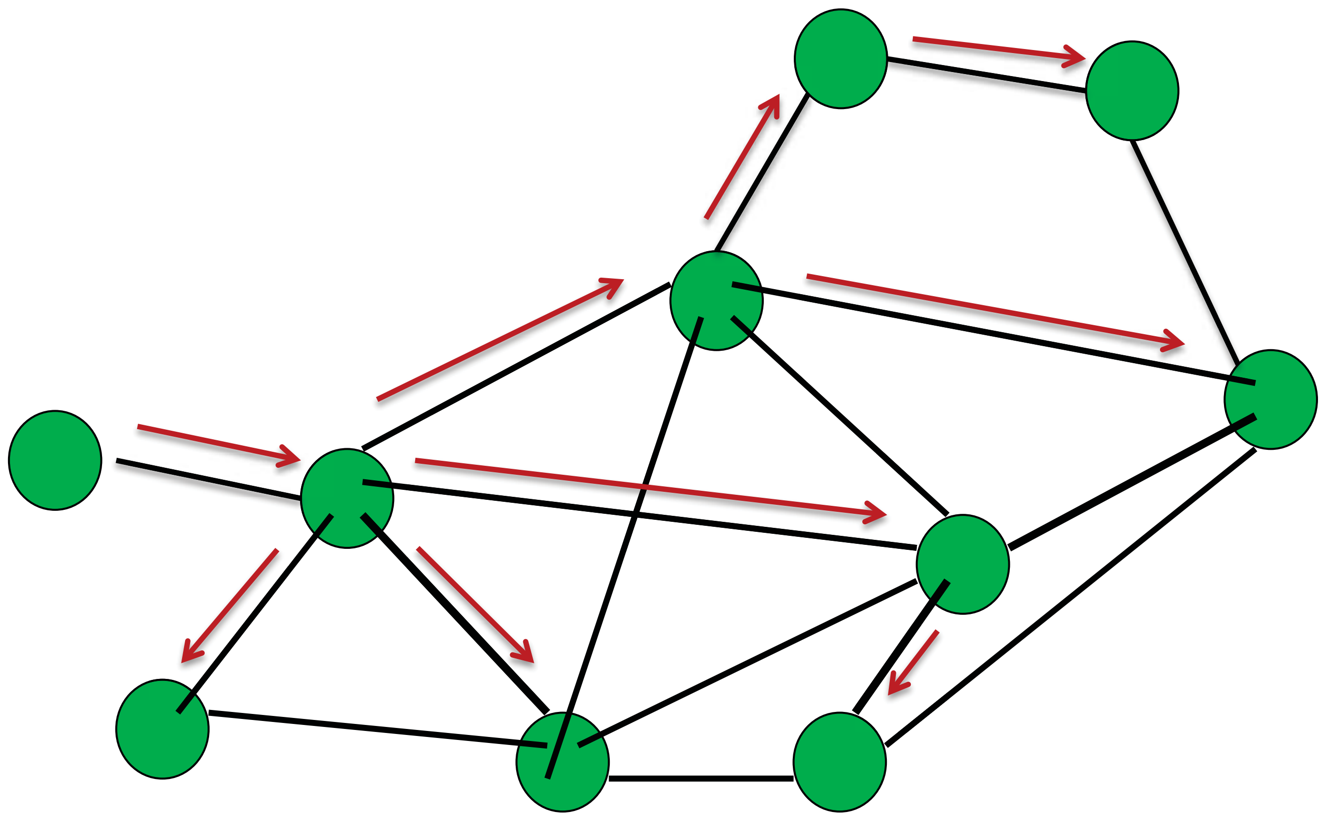
Final Spanning Tree

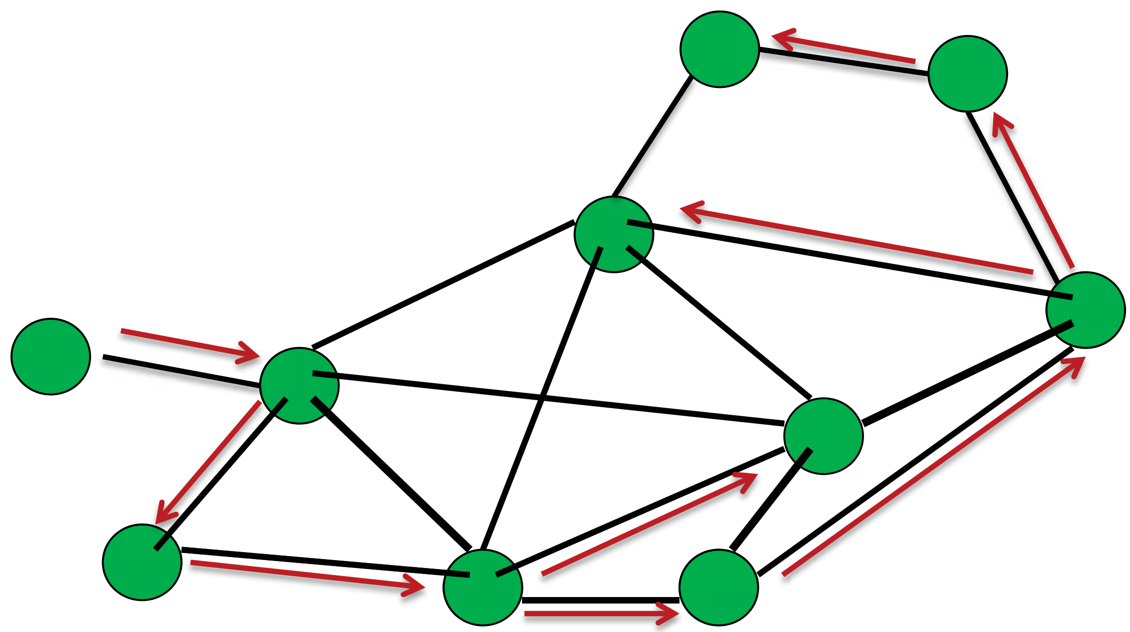


Actual BFS



Anomaly

- Paths produced by the algorithm may be longer than the shortest paths.
- Because in asynchronous networks, messages may propagate faster along longer paths.



Complexity

- Message complexity:
  - Number of messages sent by all processes during the entire execution.
  - $O(|E|)$

- Time complexity:
  - Time until all processes have chosen their parents.
  - Neglect local processing time.
  - $O(\,diam \cdot d)$
  - Q: Why $diam$, when some of the paths are longer?
  - The time until a node receives a $search$ message is at most the time it would take on a shortest path.

Extensions

- Child pointers:
  - As for synchronous BFS.
  - Everyone who receives a $search$ message sends back a $parent$ or $nonparent$ response.
- Termination:
  - After a node has received responses to all its $search$ its messages, it knows who its children are, and knows they are marked.
  - The leaves of the tree learn who they are.
  - Use a convergecast strategy, as before.
  - Time complexity: After the tree is done, it takes time $O(n \cdot d)$ for the $done$ information to reach i_0.
  - Message complexity: $O(n)$

Applications

- Message broadcast:
  - Process i_0 can use the tree (with child pointers) to broadcast a message.
  - Takes $O(n \cdot d)$ time and n messages.
- Global computation:
  - Suppose every process starts with some initial value, and process i_0 should determine the value of some function of the set of all processes' values.
  - Use convergecast on the tree.
  - Takes $O(n \cdot d)$ time and n messages.

Second Attempt

- A relaxation algorithm, like synchronous Bellman-Ford.
- Before, we corrected for paths with many hops but low weights.
- Now, instead, correct for errors caused by asynchrony.
- Strategy:
  - Each process keeps track of the hop distance, changes its parent when it learns of a shorter path, and propagates the improved distances.
  - Eventually stabilizes to a breadth-first spanning tree.

Process Automaton P_u

- Input actions: $receive(m)_{v,u}$, m a nonnegative integer
- Output actions: $send(m)_{u,v}$, m a nonnegative integer

- State variables:
  - $parent$: $\Gamma(u) \cup \{\perp\}$, initially $\perp$
  - $dist \in N \cup \{\infty\}$, initially 0 if $u = v_0$, ∞ otherwise
  - For every $v \in \Gamma(u)$:
    - $send(v)$, a FIFO queue of N, initially (0) if $u = v_0$, else empty

- Transitions:
  - $receive(m)_{v,u}$
    - Effect: if $m + 1 < dist$ then
      - $dist := m + 1$
      - $parent := v$
      - for every w, add $dist$ to $send(w)$

Process Automaton P_u

- Transitions:
  - $receive(m)_{v,u}$
    - Effect: if $m + 1 < dist$ then
      - $dist := m + 1$
      - $parent := v$
      - for every w, add $m + 1$ to $send(w)$
  - $send(m)_{u,v}$
    - Precondition: $m = \text{head}(send(v))$
    - Effect: remove head of $send(v)$

- No terminating actions…

Correctness

- For synchronous BFS, we characterized precisely the situation after r rounds.
- We can't do that now.
- Instead, state abstract properties, e.g., invariants and timing properties, e.g.:
- Invariant: At any point, for any node $u \neq v_0$, if its $dist \neq \infty$, then it is the actual distance on some path from v_0 to u, and its $parent$ is u's predecessor on such a path.
- Timing property: For any node u, and any r, $0 \leq r \leq diam$, if there is an at-most-r-hop path from v_0 to u, then by time $r \cdot n \cdot d$, node u's $dist$ is $\leq r$.

Complexity

- Message complexity:
  - Number of messages sent by all processes during the entire execution.
  - $O(n\ |E|)$

- Time complexity:
  - Time until all processes' $dist$ and $parent$ values have stabilized.
  - Neglect local processing time.
  - $O(diam \cdot n \cdot d)$
    - Time until each node receives a message along a shortest path, counting time $O(n \cdot d)$ to traverse each link.

Termination

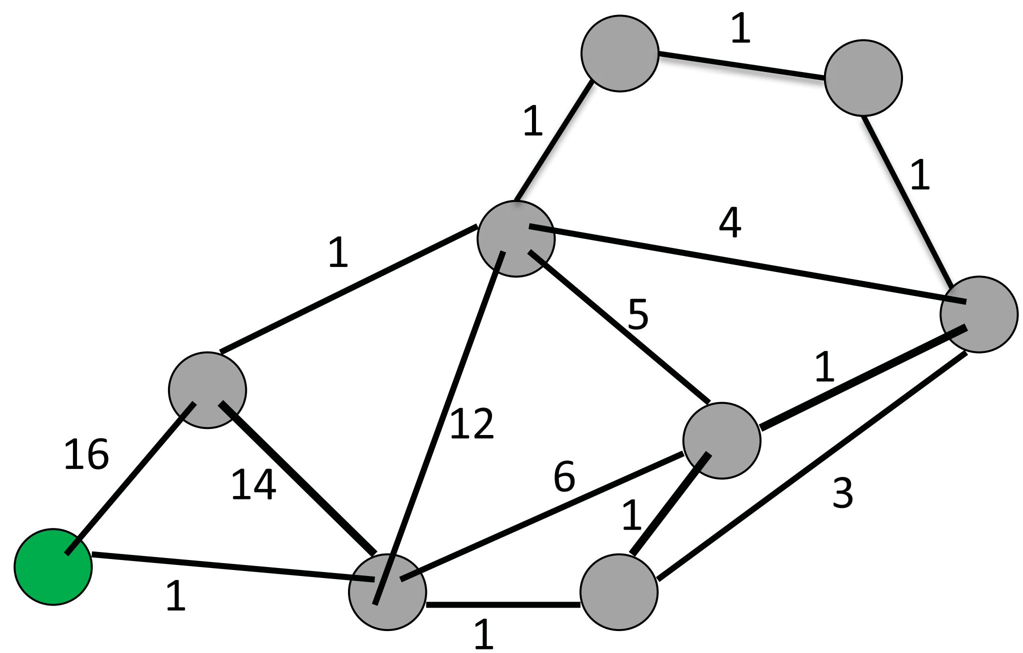
- Q: How can processes learn when the tree is completed?
- Q: How can a process know when it can output its own $dist$ and $parent$?
- Knowing a bound on n doesn't help here: can't use it to count rounds.

- Can use convergecast, as for synchronous Bellman-Ford:
  - Compute and recompute child pointers.
  - Process $\neq v_0$ sends $done$ to its current parent after:
    - It has received responses to all its messages, so it believes it knows all its children, and
    - It has received $done$ messages from all of those children.
  - The same process may be involved several times, based on improved estimates.

Uses of Breadth-First Spanning Trees

- Same as in synchronous networks, e.g.:
  - Broadcast a sequence of messages
  - Global function computation
- Similar costs, but now count time d instead of one round.

Shortest Paths Trees



Shortest Paths

- Problem: Compute a Shortest Paths Spanning Tree in an asynchronous network.
- Connected weighted graph, root vertex v_0.
- $weight_{\{u,v\}}$ for edge $\{u, v\}$.
- Processes have no knowledge about the graph, except for weights of incident edges.
- UIDs

- Processes must produce a Shortest Paths spanning tree rooted at v_0.
- Each process $u \neq v_0$ should output its distance and parent in the tree.

Shortest Paths

- Use a relaxation algorithm, once again.
- Asynchronous Bellman-Ford.

- Now, it handles two kinds of corrections:
  - Because of long, small-weight paths (as in synchronous Bellman-Ford).
  - Because of asynchrony (as in asynchronous Breadth-First search).
- The combination leads to surprisingly high message and time complexity, much worse than either type of correction alone (exponential).

Asynch Bellman-Ford, Process P_u

- Input actions: $receive(m)_{v,u}$, m a nonnegative integer
- Output actions: $send(m)_{u,v}$, m a nonnegative integer

- State variables:
  - $parent$: $\Gamma(u) \cup \{\bot\}$, initially $\bot$
  - $dist \in N \cup \{\infty\}$, initially 0 if $u = v_0$, ∞ otherwise
  - For every $v \in \Gamma(u)$:
    - $send(v)$, a FIFO queue of N, initially (0) if $u = v_0$, else empty

- Transitions:
  - $receive(m)_{v,u}$
    - Effect: if $m + weight_{\{v,u\}} < dist$ then
      - $dist := m + weight_{\{v,u\}}$
      - $parent := v$
      - for every w, add $dist$ to $send(w)$

Asynch Bellman-Ford, Process P_u

- Transitions:
  - $receive(m)_{v,u}$
    - Effect: if $m + weight_{\{v,u\}} < dist$ then
      - $dist := m + weight_{\{v,u\}}$
      - $parent := v$
      - for every w, add $dist$ to $send(w)$
  - $send(m)_{u,v}$
    - Precondition: $m = \text{head}(send(v))$
    - Effect: remove head of $send(v)$

- No terminating actions…

Correctness: Invariants and Timing Properties

- Invariant: At any point, for any node $u \neq v_0$, if its $dist \neq \infty$, then it is the actual distance on some path from v_0 to u, and its $parent$ is u's predecessor on such a path.

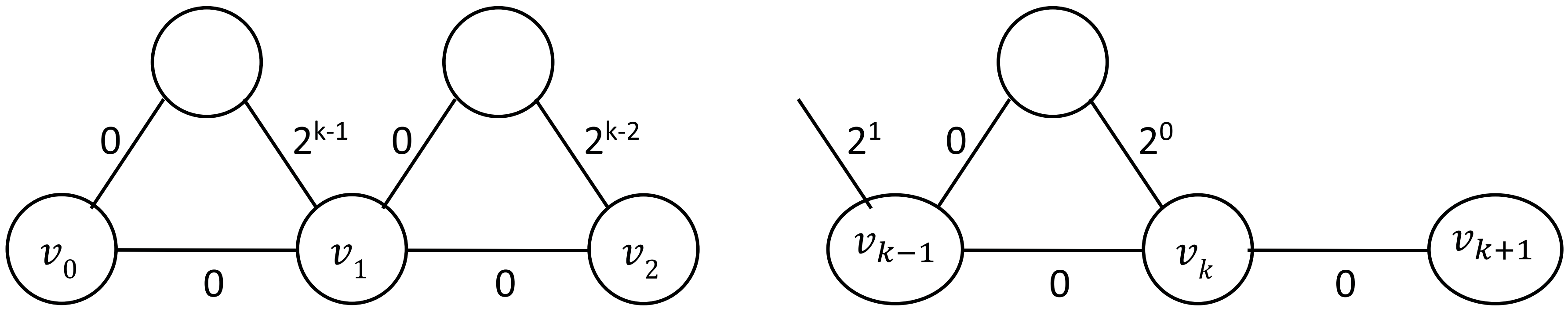
- Timing property: For any node u, and any r, $0 \leq r \leq diam$, if p is any at-most-r-hop path from v_0 to u, then by time ???, node u's $dist$ is $\leq$ total weight of p.

- Q: What is ??? ?
- It depends on how many messages might pile up in a channel.
- This can be a lot!

Complexity

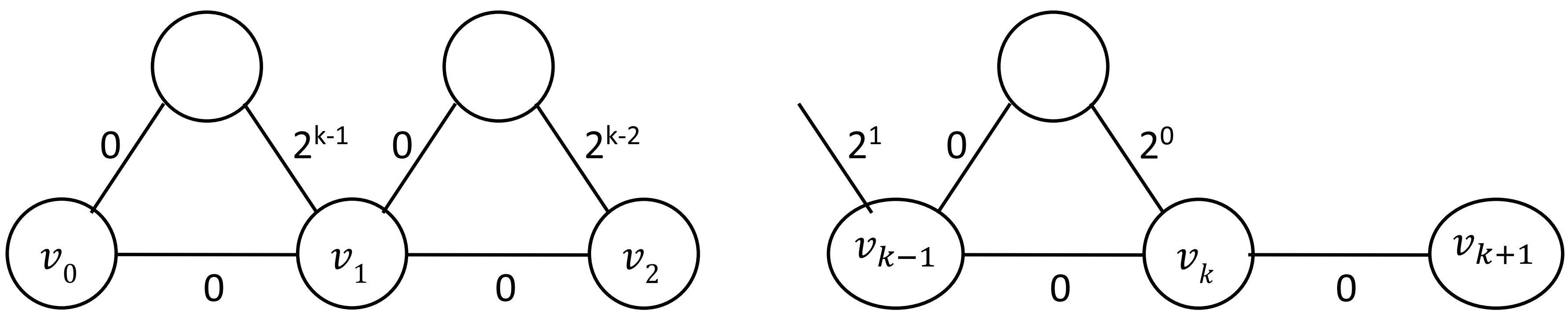
- $O(n!)$ simple paths from v_0 to any other node u, which is $O(n^n)$.
- So the number of messages sent on any channel is $O(n^n)$.
- Message complexity: $O(n^n\,|E|)$.
- Time complexity: $O(n^n \cdot n \cdot d)$.

- Q: Are such exponential bounds really achievable?



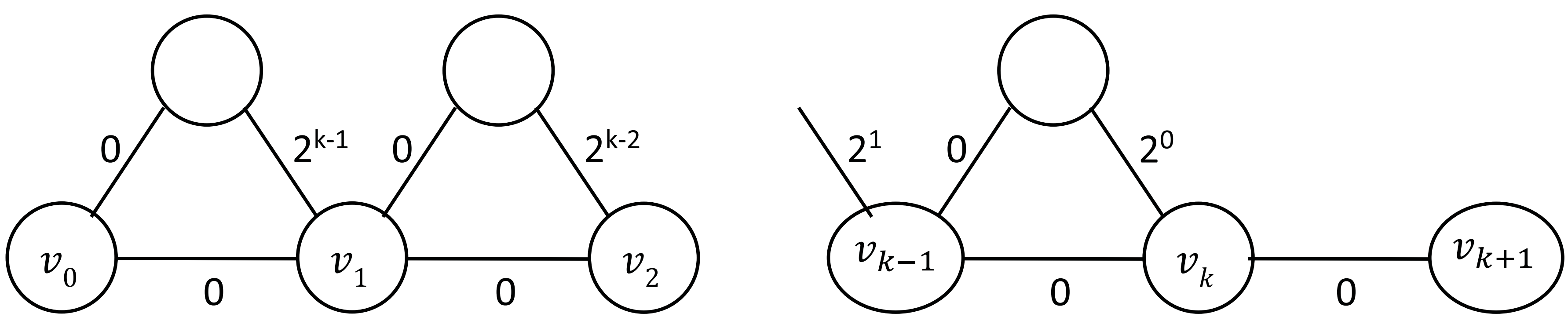
Complexity

- Q: Are such exponential bounds really achievable?
- Example:
  - There is an execution of the network below in which node v_k sends $2^k \approx 2^{n/2}$ messages to node v_{k+1}.
  - Message complexity is $\Omega(2^{n/2})$.
  - Time complexity is $\Omega(2^{n/2}\, d)$.



Complexity

- Execution in which node v_k sends 2^k messages to node v_{k+1}.
- Possible distance estimates for v_k are $2^k - 1, 2^k - 2, \ldots, 0$.
- Moreover, v_k can take on all these estimates in sequence:
  - First, messages traverse upper links, $2^k - 1$.
  - Then last lower message arrives at v_k, $2^k - 2$.
  - Then lower message $v_{k-2} \rightarrow v_{k-1}$ arrives, reduces v_{k-1}'s estimate by 2, message $v_{k-1} \rightarrow v_k$ arrives on upper links, $2^k - 3$.
  - Etc. Count down in binary.
  - If this happens quickly, get pileup of 2^k search messages in $C_{k,k+1}$.



Termination

- Q: How can processes learn when the tree is completed?
- Q: How can a process know when it can output its own $dist$ and $parent$?

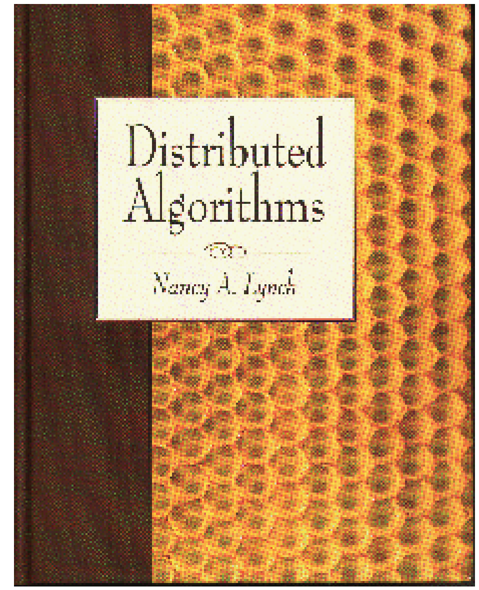
- Convergecast, once again
  - Compute and recompute child pointers.
  - Process $\neq v_0$ sends $done$ to its current parent after:
    - It has received responses to all its messages, so it believes it knows all its children, and
    - It has received $done$ messages from all of those children.
  - The same process may be involved several (many) times, based on improved estimates.

Shortest Paths

- Moral: Unrestrained asynchrony can cause problems.
- What to do?

- Find out in 6.852/18.437, Distributed Algorithms!

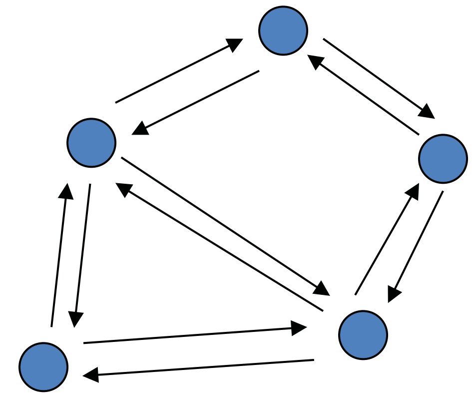
What's Next?



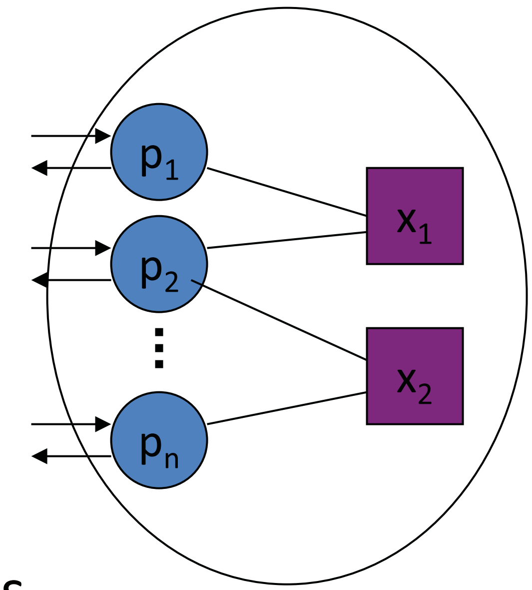
- 6.852/18.437 Distributed Algorithms
- Basic grad course
- Covers synchronous, asynchronous, and timing-based algorithms

- Synchronous algorithms:
  - Leader election
  - Building various kinds of spanning trees
  - Maximal Independent Sets and other network structures
  - Fault tolerance
  - Fault-tolerant consensus, commit, and related problems

Asynchronous Algorithms



- Asynchronous network model
- Leader election, network structures.
- Algorithm design techniques:
  - Synchronizers
  - Logical time
  - Global snapshots, stable property detection.
- Asynchronous shared-memory model
- Mutual exclusion, resource allocation



- Fault tolerance
- Fault-tolerant consensus and related problems
- Atomic data objects, atomic snapshots
- Transformations between models.
- Self-stabilizing algorithms

And More

- Timing-based algorithms
  - Models
  - Revisit some problems
  - New problems, like clock synchronization.
- Newer work (maybe):
  - Dynamic network algorithms
  - Wireless networks
  - Insect colony algorithms and other biological distributed algorithms

6.046J / 18.410J Design and Analysis of Algorithms
Spring 2015