Pseudo-Random Number Generator

Project Report
ELL201

Author
Naman Goel - 2022MT11272
April 27, 2024

Contents

1 Pseudo-Random Number Generators

A **pseudo-random number generator (PRNG)**, also referred to as a **deterministic random bit generator (DRBG)**, is an **algorithm** designed to generate a sequence of numbers that mimic the properties of truly random sequences. Unlike true random sequences, however, the sequence produced by a PRNG is determined by an initial value known as the **seed**. Moreover, PRNG-generated sequences find extensive usage due to their number generation speed and reproducibility.

1.1 Linear Feedback Shift Registers

A **linear-feedback shift register (LFSR)** is a shift register whose input bit is a linear function of its previous state.

The most commonly used linear function of single bits is **exclusive-or (XOR)**. Thus, an LFSR is often a shift register whose input bit is driven by the XOR of some bits of the overall shift register value.

The initial value of the LFSR is called the **seed**. Because the operation of the register is deterministic, the sequence of values produced by the register is completely determined by its current (or previous) state. Likewise, because the register has a finite number of possible states, it must eventually enter a repeating cycle. However, an LFSR with a well-chosen feedback function can produce a sequence of bits that appears random and has a very long cycle.

1.2 Cellular Automata Shift Registers

A **cellular automata shift register (CASR)** is a type of shift register that operates based on principles inspired by cellular automata. It consists of a grid of cells, each representing a storage element (e.g., a bit). The state of each cell, typically binary (0 or 1), is updated based on the states of neighbouring cells and predefined transition rules, similar to the rules governing cellular automata. During a shift operation, the contents of the grid, representing the shift register's data, are shifted, with each cell's state being updated according to the transition rules and neighbouring cell states. The output of the CASR is usually derived from specific cells in the grid, providing a dynamic and adaptable behaviour influenced by the principles of cellular automata.

2 Design

LFSR and CASR may not introduce significant randomness into the sequence of generated numbers individually. Therefore, our approach is to use the bits generated by LFSR and CASR separately, and then XOR these bits together to produce the final random bit of the sequence. All our codes and outputs can be found here.

2.1 Verilog Code

```
1 module pro(clk, reset, number_o);
2 input clk;
3 input reset;
```

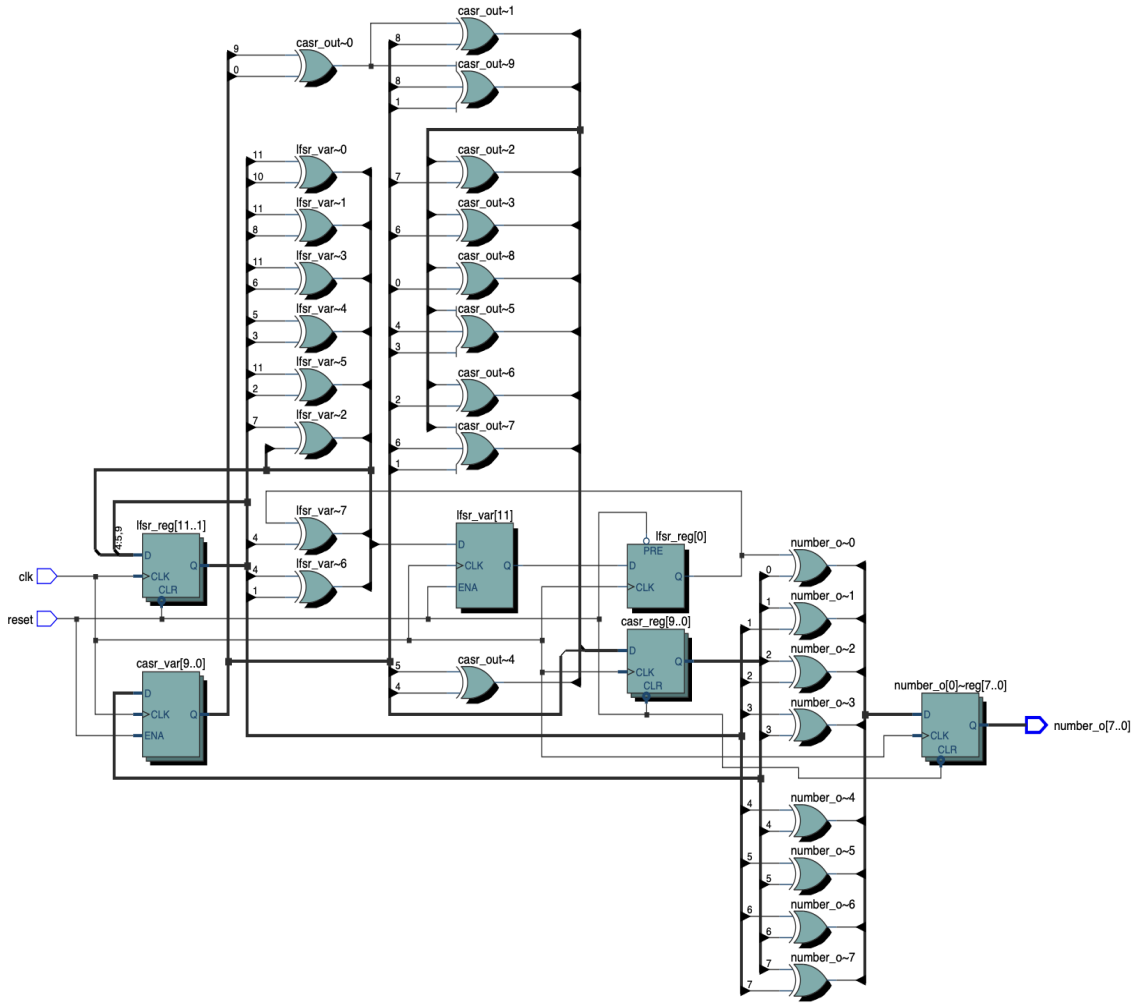
```
output reg[7:0] number_o;

reg[12:0] lfsr_reg;
reg[10:0] casr_reg;
reg[12:0] lfsr_var;
reg outbitlfsr;

always @(posedge clk or negedge reset)
begin
if (!reset)
     begin
        lfsr_reg = 13'b0100100100100;
     end
else
    begin
        lfsr_var = lfsr_reg;
        lfsr_var [12] = lfsr_var[11];
        outbitlfsr = lfsr_var[11];
        lfsr_var[11] = lfsr_var[10]^outbitlfsr;
        lfsr_var[10] = lfsr_var[9];
        lfsr_var[9] = lfsr_var[8]^outbitlfsr;
        lfsr_var[8] = lfsr_var[7]^lfsr_var[11];
        lfsr_var[7] = lfsr_var[6]^outbitlfsr;
        lfsr_var[6] = lfsr_var[5];
        lfsr_var[5] = lfsr_var[4];
        lfsr_var[4] = lfsr_var[3]^lfsr_var[6];
        lfsr_var[3] = lfsr_var[2]^outbitlfsr;
    outbitlfsr = lfsr_var[5];
        lfsr_var[2] = lfsr_var[1]^outbitlfsr;
        lfsr_var[1] = lfsr_var[0]^outbitlfsr;
      lfsr_var[0] = lfsr_var[12];
        lfsr_reg = lfsr_var;
    end
end

reg [10:0] casr_var, casr_out;
always @(posedge clk or negedge reset)
    begin
    if (!reset)
        casr_reg = 11'b00010101010;
    else
        begin
            casr_var = casr_reg;
            casr_out[10] = casr_var[9]^casr_var[0];
                 casr_out[9] = casr_var[8]^casr_out[10];
                 casr_out[8] = casr_var[7]^casr_out[9];
                 casr_out[7] = casr_var[6]^casr_out[8];
                 casr_out[6] = casr_var[5];
                 casr_out[5] = casr_var[4]^casr_out[6];
                 casr_out[4] = casr_var[3]^casr_out[5]^casr_var[4];
                 casr_out[3] = casr_var[2]^casr_out[4];
                 casr_out[2] = casr_var[1]^casr_out[3]^casr_var[6];
                 casr_out[1] = casr_var[0]^casr_out[2];
                 casr_out[0] = casr_var[1]^casr_out[10]^casr_var[8];
            casr_reg = casr_out;
        end
    end

always @(posedge clk or negedge reset)
    begin
         if (!reset)
            begin
                number_o = 8'b0;
            end
         else
            begin
                number_o = (lfsr_reg [7:0] ^ casr_reg [7:0]);
```

```
            end
    end
endmodule
```

2.2 Testbench Code

```
`timescale 1ns / 1ps

module testbench();

  // Inputs
  reg clk;
  reg reset;

  // Outputs
  wire [7:0] number_o;

  pro UUT(
      .clk(clk),
      .reset(reset),
      .number_o(number_o)
  );

  // Clock generation
  always #1 clk = ~clk; // Toggle clock every 1 time unit

  // Initial reset assertion
  initial begin
    clk = 0;
    reset = 0;
    #5 reset = 1;
  end

  integer f;
  // Monitor and write to file
  initial begin
    f = $fopen("out.txt", "w");
    repeat (100000) begin
      @(posedge clk or negedge reset);
      $fwrite(f, "%d\n", number_o);
    end

    $fclose(f);
    $finish;
  end

endmodule
```

2.3 Circuit Diagram



3 Output

In our testbench code, we generated a random sequence consisting of a total of $100k$ numbers and wrote these numbers to a *.txt* file.

3.1 Numbers

Some of the initial numbers of the generated sequence are provided below:

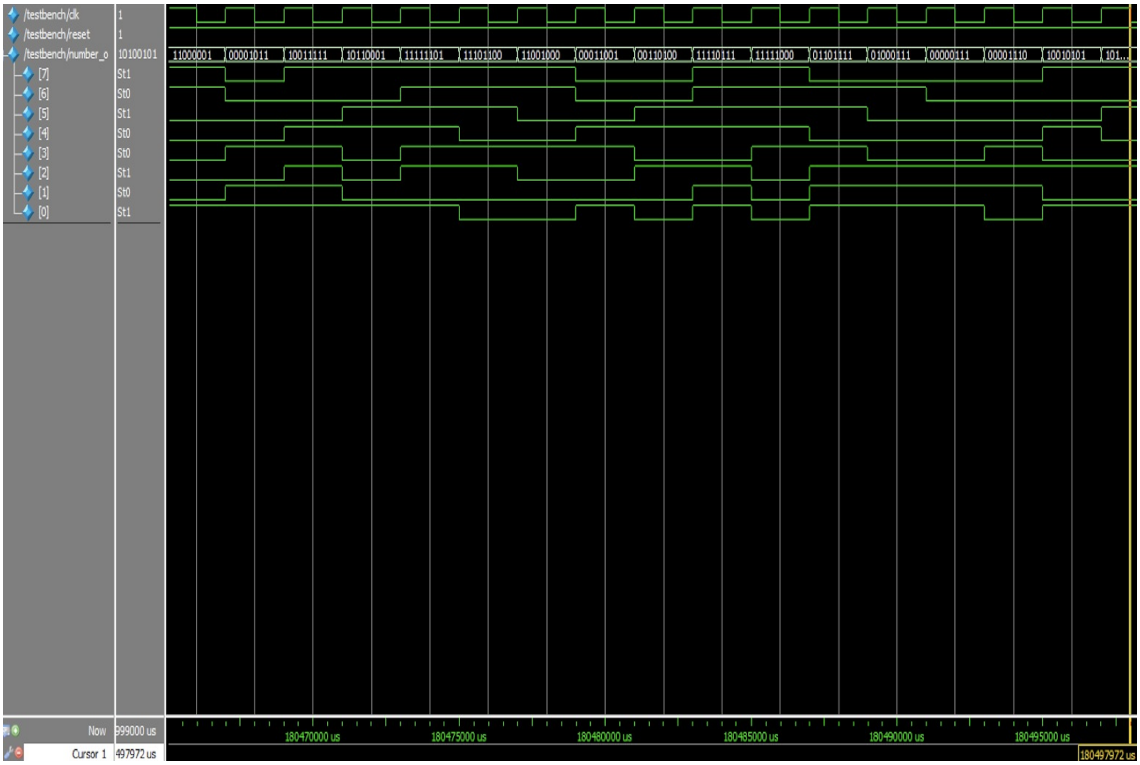
output

```
0, 0, 0, 54, 94, 30, 186, 69, 136, 63, 196, 14, 156, 25, 23, 9, 62, 200, 62, 225, 85, 14, 26, 150, 35,
80, 35, 112, 126, 203, 186, 104, 96, 219, 154, 190, 192, 128, 3, 163, 92, 182, 246, 80, 14, 152, 154,
149, 10, 52, 249, 207, 48, 220, 190, 72, 178, 248, 202, 54, 245, 121, 106, 66, 37, 219, 29, 48, 217, 29,
19, 140, 54, 125, 75, 145, 43, 79, 152, 146, 10, 149, 160, 82, 4, 12, 144, 175, 77, 151, 165, 91, 145,
41, 70, 37, 252, 99, 127, 111, 77, 58, 72, 180, 251, 231, 121, 194, 12, 145, 143, 29, 184, 67, 137, 179,
```

89, 158, 63, 227, 216, 188, 102, 248, 203, 55, 216, 189, 103, 213, 15, 58, 79, 151, 134, 135, 168, 101, 213, 44, 111, 100, 249, 65, 32, 254, 97, 220, 149, 42, 105, 203, 25, 177, 82, 163, 119, 212, 45, 99, 221, 31, 61, 79, 23, 162, 241, 119, 240, 243, 90, 26, 182, 214, 13, 60, 78, 27, 27, 23, 11, 159, 26, 153, 55, 84, 12, 151, 173, 77, 28, 159, 185, 64, 5, 44, 225, 253, 64, 174, 194, 35, 216, 185, 228, 222, 30, 18, 42, 200, 146, 13, 151, 160, 217, 62, 238, 65, 131, 39, 83, 164, 120, 64, 132, 40, 199, 13, 51, 90, 150, 38, 210

3.2 Waveform

The output waveform for the generated random sequence is provided below:



4 Testing

How can we ensure that the generated sequence is truly random? Our sequence of random numbers passed **9 out of 16** standard tests for randomness listed in NIST statistical test suite. The tester can be found here.

4.1 NIST Statistical Test Suite

The **NIST Statistical Test Suite** is a collection of statistical tests developed by the **National Institute of Standards and Technology (NIST)** for evaluating the randomness of random number generators (RNGs) and cryptographic algorithms. These tests are designed to assess the quality and randomness of sequences of binary data produced by RNGs or cryptographic algorithms.

The NIST Statistical Test Suite includes a comprehensive set of tests that evaluate various statistical properties and characteristics of the data, such as:

- **Frequency Test:** Check if the proportion of ones and zeros in the sequence is approximately equal.
- **Frequency Test within a block:** Examine the frequency of predefined patterns or blocks of bits within the sequence.
- **Runs Test:** Assess the presence of runs or sequences of consecutive ones or zeros in the sequence.
- **Test for the Longest Run of Ones in a Block:** Determine the longest consecutive sequence of ones in the sequence.
- **Binary Matrix Rank Test:** Evaluate the rank of matrices derived from the sequence.
- **Discrete Fourier Transform (Spectral) Test:** Analyzes frequency distribution to assess randomness.
- **Non-overlapping Template Matching Test:** Detect the presence of specific patterns or templates within the sequence.
- **Overlapping Template Matching Test:** Similar to non-overlapping template matching, but with overlapping templates.
- **Maurer's "Universal Statistical" Test:** Assess the proportion of ones within certain subsequences of the sequence.
- **Linear Complexity Test:** Measures predictability based on shortest feedback shift register length.
- **Serial Test:** Examine correlations between adjacent bits or sequences of bits within the sequence.
- **Approximate Entropy Test:** Measure the randomness of the sequence by assessing its entropy.
- **Cumulative Sums Test:** Check for deviations from expected cumulative sums of the sequence.
- **Random Excursions Test:** Analyze the number of excursions from a predetermined baseline in the sequence.
- **Random Excursions Variant Test:** Similar to random excursion tests, but with additional criteria.

4.2 Result

The final output displaying the results passed (and failed) along with the respective **p-values** is provided below. It should be noted that the threshold p-value for the "Test for the Longest Run of Ones in a Block" is 0.01, and the value we received is 0.00934. Therefore, this test failed by a narrow margin; otherwise, we would have passed **10 out of 16** standard tests.

```
Type of Test                                        P-Value                Conclusion
01. Frequency (Monobit) Test                        0.9127525207914535     Random
02. Frequency Test within a Block                   1.4024826890989044e-36 Non-Random
03. Runs Test                                       0.8457652178254071     Random
04. Test for the Longest Run of Ones in a Block     0.009338716697701553   Non-Random
05. Binary Matrix Rank Test                         0.0                    Non-Random
06. Discrete Fourier Transform (Spectral) Test      0.0                    Non-Random
07. Non-overlapping Template Matching Test          0.8794434173445763     Random
08. Overlapping Template Matching Test              0.011725941459423392   Random
09. Maurer's "Universal Statistical" Test           0.31817046032235374    Random
10. Linear Complexity Test                          0.0                    Non-Random
11. Serial Test:
                                                                           0.0                    Non-Random
                                                                           0.0                    Non-Random
12. Approximate Entropy Test                        0.0                    Non-Random
13. Cumulative Sums Test (Forward)                  0.9941787759566996     Random
14. Cumulative Sums Test (Backward)                 0.9962003297145519     Random
15. Random Excursions Test:
                              State          Chi Squared            P-Value                Conclusion
                              -4             1.6969935952964255     0.8892782974180536     Random
                              -3             3.1887695817490496     0.6709091906581793     Random
                              -2             4.215180960428109      0.518869403439787      Random
                              -1             12.453738910012675     0.029072604537161813   Random
                              +1             5.916349809885931      0.3144437464444382     Random
                              +2             4.420723215822497      0.4905599452692553     Random
                              +3             7.769149809885934      0.16942554362305548    Random
                              +4             3.652526487432096      0.6004443240451137     Random
16. Random Excursions Variant Test:
                              State          COUNTS                 P-Value                Conclusion
                              -9.0           1569                   0.969005806068061      Random
                              -8.0           1642                   0.7686448409353488     Random
                              -7.0           1664                   0.6711437697170246     Random
                              -6.0           1667                   0.6328872918418516     Random
                              -5.0           1647                   0.6822381849056846     Random
                              -4.0           1632                   0.7163740172053483     Random
                              -3.0           1617                   0.756208208338158      Random
                              -2.0           1630                   0.5930582058703868     Random
                              -1.0           1604                   0.6434989459129046     Random
                              +1.0           1562                   0.7757915939008643     Random
                              +2.0           1505                   0.45311691241123786    Random
                              +3.0           1480                   0.43530844226198684    Random
                              +4.0           1504                   0.618577092058646      Random
                              +5.0           1529                   0.7712493872923101     Random
                              +6.0           1544                   0.8552064425677152     Random
                              +7.0           1527                   0.8012071853421796     Random
                              +8.0           1497                   0.7096836304974316     Random
                              +9.0           1483                   0.6817042766860951     Random
```

5 Applications

PRNGs are versatile tools used in various applications, where randomness and unpredictability are essential for achieving desired outcomes. Some of them are mentioned below:

5.1 Cryptography

PRNGs ensure the creation of cryptographic keys, initialization vectors, and nonces, critical for secure communication. They provide the randomness needed for encryption, decryption, and cryptographic hashing, enhancing the security of data transmission and storage.

5.2 Monte-Carlo Simulations

PRNGs generate random inputs essential for Monte Carlo simulations, enabling the modelling and analysis of complex systems in various fields such as finance, engineering, and physics. They facilitate the estimation of probabilities, solution of optimization problems, and simulation of stochastic processes, contributing to decision-making and problem-solving in diverse domains.

5.3 Testcases Generation

PRNGs are indispensable for generating diverse sets of test cases, ensuring thorough evaluation of software functionality, performance, and robustness. They help simulate real-world scenarios and edge cases, improve test coverage, and identify potential issues or vulnerabilities in software systems.

5.4 Game Simulations

In game development, PRNGs are essential for generating random events, outcomes, and environments, enhancing gameplay dynamics and replayability. They enable the creation of unpredictable gameplay experiences, including random enemy behaviour, procedural level generation, and random item drops, enriching player immersion and engagement.

References

[1] https://en.wikipedia.org/wiki/Pseudorandom_number_generator

[2] https://en.wikipedia.org/wiki/Linear-feedback_shift_register

[3] https://www.yumpu.com/en/document/read/41498041/a-random-number-generator-in-verilog

[4] https://github.com/stevenang/randomness_testsuite