PART 2: LOGIC, RELATIONS AND FUNCTIONS, BOOLEAN ALGEBRA (INDIVIDUAL)

COS10023-Computer and Logic Essentials

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Logic (10 marks)

1. [2 marks] For this question the following statements and symbols should be used:

(a): Andrew plays football.

(d): David plays esports.

(h): Harry plays cricket

** Translate the following into English.

i. H ∧ (d ∨ a)

Harry plays cricket, and David plays esports or Andrew plays football.

ii. d→¬a ∨ h

If David plays esports, then Andew won’t play football or Harry plays cricket.

iii. ¬(h ∨ d)

It is false that Harry plays cricket or David plays esports.

iv. a↔(h ∧ d)

Andrew plays football if and only if Harry plays cricket and David plays esports.

2. [1+2=3 marks] Given the following statements:

A Hawks player is either running at the Glenferrie oval or sitting in the gallery. If he is running, then he is not reading newspaper. The player is reading a newspaper. Therefore, he is sitting in the gallery.

i. [1 mark] * Convert these statements into propositional logic statements

(a): The player is running at the Glenferrie oval

(b): The player is sitting in the gallery

(c): The player is reading newspaper

A Hawks player is either running at the Glenferrie oval or sitting in the gallery.

- ➔ $a \lor b$

If he is running, then he is not reading newspaper.

- ➔ $a \to \neg c$

The player is reading a newspaper. Therefore, he is sitting in the gallery.

- ➔ $c \to b$

ii. [2 marks] ***Without using a truth table, show that this argument is valid using concepts described in the unit. This could be done using contradiction or the laws/axioms of algebra (note the latter is a long process though).

We have: $(a \lor b) \land (c \to b) \land (a \to \neg c)$

I will prove this argument is valid using contradiction.

I will divide this argument into three different parts, and assume I want to prove that $(a \lor b) \land (c \to b) \land (a \to \neg c)$ invalid.

First part: $(a \lor b)$

According to Week 3 content, the only way for this part to be false is if both a and b are false, because the disjunction (or) operator is true as long as at least one of its operands is true. So, we can write:

- ➔ a = false
- ➔ b = false

Second part: $(c \to b)$

According to Week 3 content , b=false so if we want $(c \to b)$ to be false, c have to be true. (1)

- ➔ c = true

Third part: $(a \to \neg c)$

According to Week 3 content, a = false so if we want $(a \to \neg c)$ to be false, $\neg c$ have to be true. (2)

From (1) and (2), we have come to a contradiction:

It is impossible for c and $\neg c$ to be true at the same time. Since our assumption (the argument is invalid) leads to a contradiction, we can understand that the argument is valid

- ➔ $(a \lor b) \land (c \to b) \land (a \to \neg c)$ is valid

The reason why I do not use laws of logic because it is really long.

3. [2 marks] ** Simplify the following statement using the laws and axioms of logic. Clearly state which law or axiom has been used at each step.

We have: $p \land \neg(\neg q \lor p)$

De Morgan's $\equiv p \land \neg\neg q \land \neg p$

Involution $\equiv p \land q \land \neg p \equiv p \land \neg p \land q$ (switch position for better understanding).

Negation $\equiv F \land q$

Domination $\equiv F$

4. [1.5 marks] ** You have a friend who has written the following condition statement in his/her code: (height<=50 or width>10) and (height>50 or width>10) and height<=50. Show using the laws of logic that this condition statement can be simplified to:

width>10 and height<=50

For each step, state which law of logic you have used.

(a): height <=50

(b): width >10

(height<=50 or width>10) and (height>50 or width>10) and height<=50

Which means: $(a \lor b) \land (\neg a \lor b) \land a$

Absorption $\equiv a \land (\neg a \lor b)$

Distributive $\equiv (a \land \neg a) \lor (a \land b)$

Negation $\equiv F \lor (a \land b)$

Identity $\equiv a \land b$ (width>10 and height<=50)

5. [1.5 marks] * Using the truth table find out whether the proposition ((p ∧¬q) V (q -> p)) V q is tautology, contradiction or neither.

p	q	¬q	p ∧¬q	q → p	((p ∧¬q) V (q → p))	((p ∧¬q) V (q → p)) V q
T	T	F	F	T	T	T
T	F	T	T	T	T	T
F	T	F	F	F	F	T
F	F	T	F	T	T	T

The proposition is tautology because is always true, regardless of the variable values.

Functions and Relations (8 marks)

6. [2 marks] Given the following relation S on Z×Z where

Z= {a, b, c, d, e}

S={(a, a),(b,b),(a, b),(b, a),(c, c),(d, d),(e, e),(c, e),(d, e),(e, c),(e, d)}.

Determine whether it is an equivalence relation, by showing whether it satisfies the three criteria needed. State your final answer.

1. **Reflexivity: For every element x in the set Z, (x, x) must be in the relation S**

(a, a) , (b, b) , (c, c) , (d, d) , (e, e) are all in the relation S.

➔ Since (x, x) is in S for every x in Z, S satisfies reflexivity.

2. **Symmetric: If (x, y) is in S, then (y, x) must also be in S.**

(a, b) is in S, but (b, a) is also in S.

(c, e) is in S, but (e, c) is also in S.

(e, d) is in S, but (d, e) is also in S.

➔ Since for every (x, y) in S, we also have (y, x) in S, S satisfies symmetry.

3. Transitivity: if (x, y) and (y, z) are in S, then (x, z) must also be in S.

(c, e) is in S and (e, d) is in relation S, but (c, d) is not in relation S.

➔ S does not satisfy transitivity.

The relation S does not meet all three criteria of reflexivity, symmetry, and transitivity.

In conclusion: The given relation S is not an equivalence relation.

7. [3 marks] **For the domain X={x, y}and co-domain Y={x, y, z}:

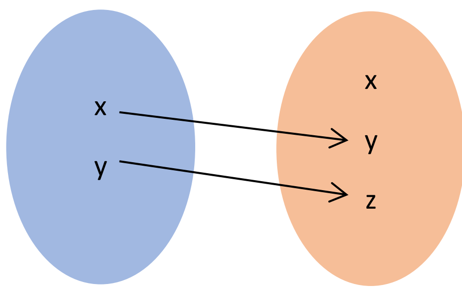
i. How many functions f: X→Y are possible? Provide an example of a function, using formal notation or a diagram.

The first value in the domain X has 3 choices to map in the co-domain Y.

The second value in the domain X has 3 choices to map in the co-domain Y.

The total of functions which are possible will be 3 * 3 = 9 (possible functions).

Example of a function:



Domain Co-domain

ii. How many of the functions in i) are injective? Provide an example that is injective and an example that is not.

Injective functions are where each value in the co-domain is used at most once.

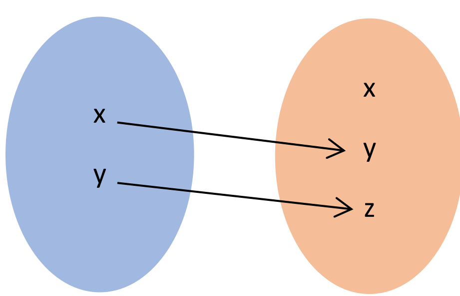
In that case:

The first value “x” in the domain X can have 3 choices from the co-domain.

The second value “y” in the domain X can have 2 choices from the co-domain.

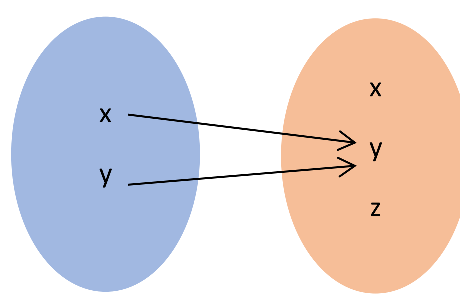
The total number of injective functions will be 3 * 2 = 6 (possible injective functions)

Example of injective function:



Domain Co-domain

Example of not an injective funtion



Domain Co-domain

iii. How many of the functions in i) are bijective? Provide an example if one exists, if not explain why not.

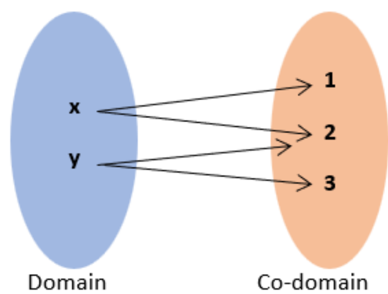
Bijective functions cannot exist because the amount of value in the domain X is lower than the amount of value in the co-domain Y.

As a result, there will always has one value left in the co-domain Y that cannot map to any value in the domain X.

Because bijective functions has to meet the requirements of both injective and surjective, in this case, surjective's requirement cannot be satisfied (each value in the co-domain has to be used at least once).

8. [3 marks] ** Give an example for the following and justify why your example is valid.

i. A function that is surjective but not injective.



This function is surjective but not injective because:

Surjective functions are where each value in the co-domain is used at least once.

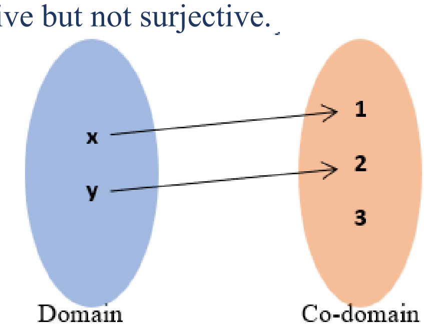
Every value "1", "2", "3", from the co-domain is used at least once.

- ➔ It is surjective.

Injective functions are where each value in the co-domain is used at most once. However in the co-domain, we can see value "2" is used twice.

- ➔ It is not injective.

ii. A function that is injective but not surjective.



This function is injective but not surjective because:

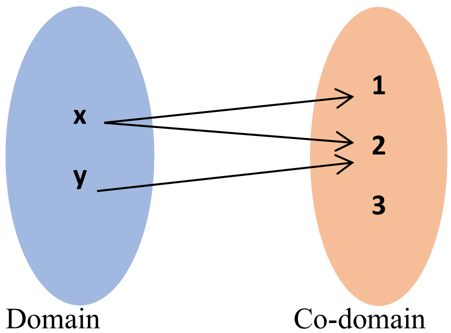
Injective functions are where each value in the co-domain is used at most once. We can see that the maximum number of use of every value is just once.

- ➔ It is injective.

Surjective functions are where each value in the co-domain is used at least once. However, value "3" in the co-domain hasn't been used yet.

- ➔ It is not surjective

iii. A function that is neither injective or surjective.



This function is neither injective or surjective because:

Injective functions are where each value in the co-domain is used at most once.

However, the value "2" is used twice.

- ➔ It is not injective.

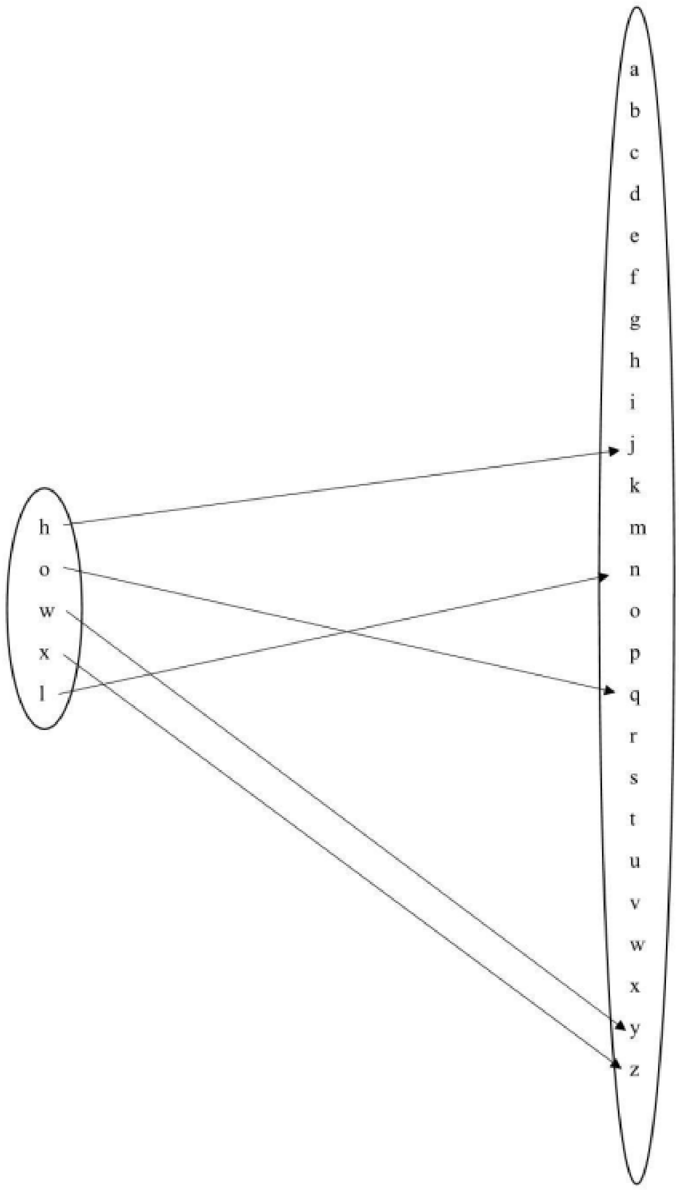
Surjective functions are where each value in the co-domain is used at least once.

However, value "3" in the co-domain hasn't been used yet.

- ➔ It is not surjective

9. [2 marks] *A function could be defined as f(x)={(a,b): b is two characters shifted along from a wrapping at the end}, that is, a Caesar cipher with a right shift of 2. As an example, across the lowercase alphabet, this is enumerated as {(a,c),(b,d)...(y,a),(z,b)}.

i. Given the domain {'h','o','w','x','l'}, draw an arrow diagram to show the ordered pairs produced by f.



Given Domain is {‘h’,’o’,’w’,’x’,’l’}

According to f they are mapped to {j,q,y,z,n}

ii. What is the image for the domain in question i. through f?

Given Domain is {‘h’,’o’,’w’,’x’,’l’}

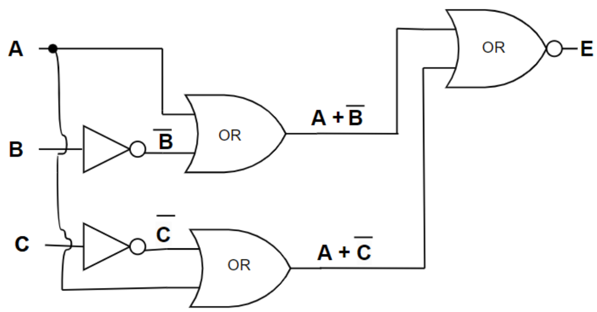
According to f they are mapped to {j,q,y,z,n}, so the image will be {‘j’,’q’,’y’,’z’,’n’}.

Boolean algebra and circuits (8 marks)

10. [1 + 2 + 2 = 5 marks] Given the expression

$$\mathrm{E} = \overline{(A+\bar{B})\ +(A+\bar{C})}$$

i. *draw the circuit that represents this expression as is.



ii. **simplify the expression using Boolean algebra rules. State your steps and the rules used.

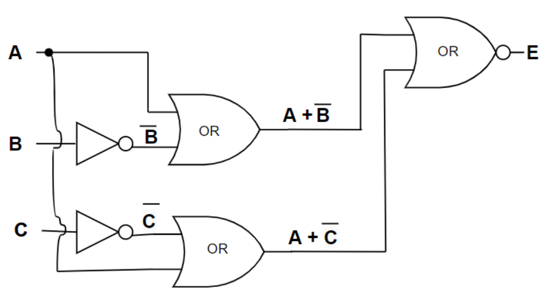
$$\mathrm{E} = \overline{(A+\bar{B})\ +(A+\bar{C})}$$

De Morgan's Law $= (\bar{A}\,.\,\bar{\bar{B}}) + (\bar{A}\,.\,\bar{\bar{C}})$

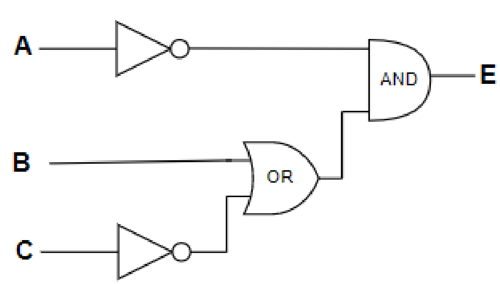
Double Negation Law $= (\bar{A}\,.\,B)\ +\ (\bar{A}\,.\,C)$

Distributive Law $= \bar{A}\,.\,(B\ +\ \bar{C})$

iii. *given the simplified circuit from 10/ ii, state in a sentence how the depth and size of the circuit have changed compared to the original (10/ i). The simplified circuit diagram needs to be included in your submission.



Before simplifying (10/i)



After simplifying (10/ii)

Before(10/i): Size = 5 ; Depth = 3

After(10/ii): Size = 4 ; Depth = 3

11. [1 + 2 = 3 marks] **Given the following circuit:

i. [1 mark] determine the expression for the circuit in Figure 1. Note you do not need to simplify the expression.

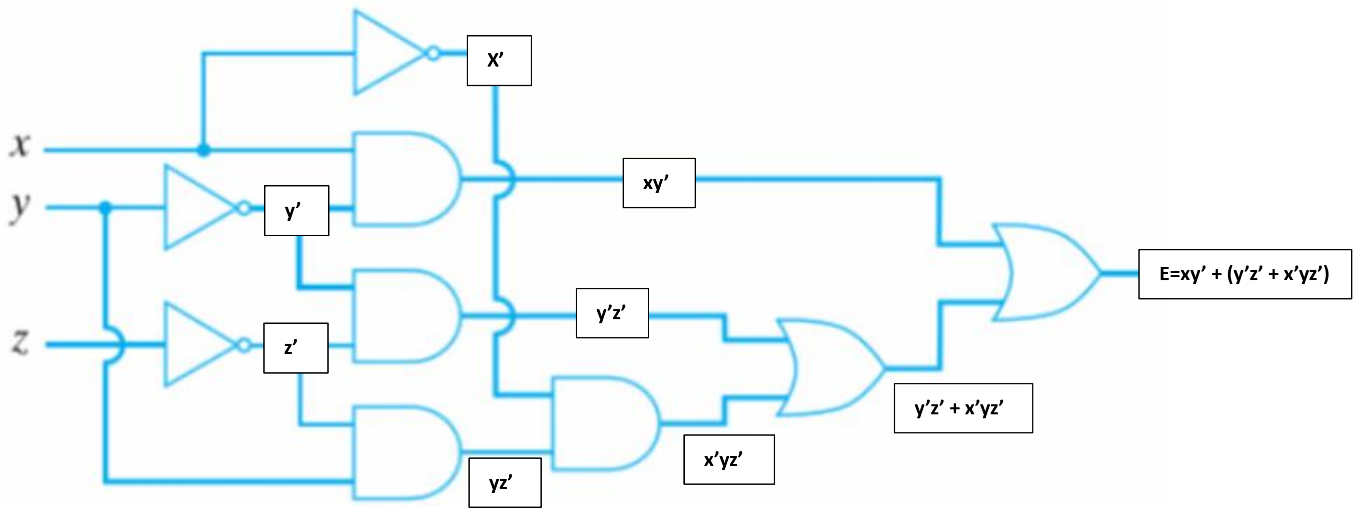


Figure 1 Question 11(i)

As we can see in the diagram, I have written the result for each gate from the beginning to the end.

The answer for this question is : E = xy' + (y'z' + x'yz')

ii. [2 marks] draw a truth table to represent the circuit in Figure 2. Please include intermediate columns to show working; an outcome column alone will incur a deduction.

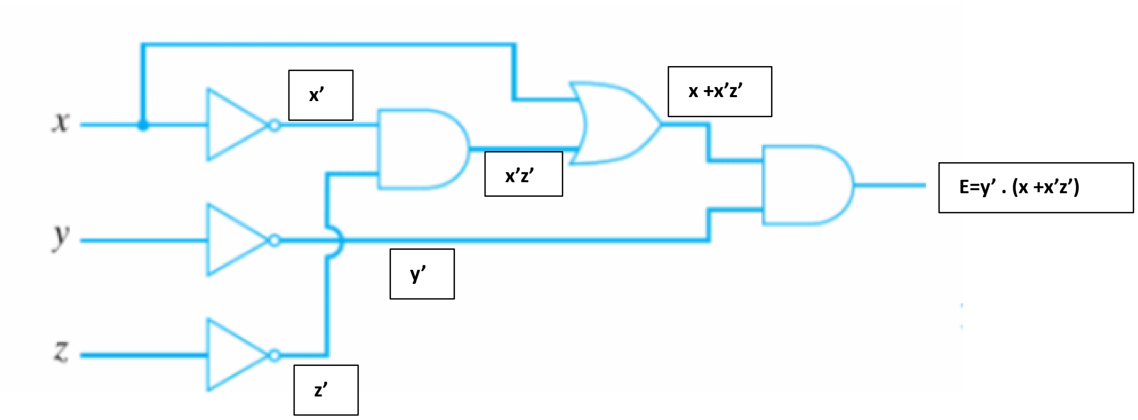


Figure 2 Question 11 (ii)

After doing the same method with 11i., the answer for the expression will be:

E=y’ . (x +x’z’)

The truth table:

x	y	z	x’	y’	z’	x’z’	x +x’z’	y’ . (x +x’z’)
0	0	0	1	1	1	1	1	1
0	0	1	1	1	0	0	0	0
0	1	0	1	0	1	1	1	0
0	1	1	1	0	0	0	0	0
1	0	0	0	1	1	0	1	1
1	0	1	0	1	0	0	1	1
1	1	0	0	0	1	0	1	0
1	1	1	0	0	0	0	1	0