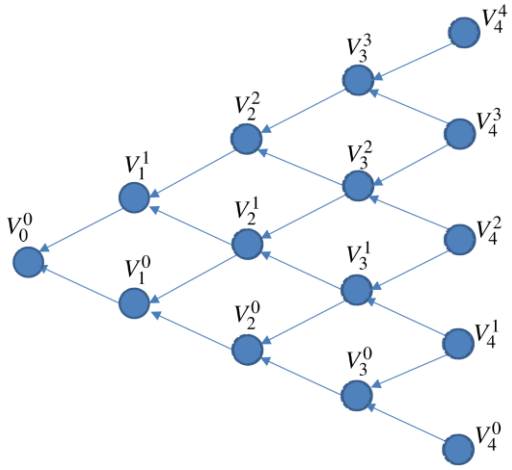
Binomial Tree

Binomial tree method is frequently used in the calculation of option prices. The time between intervals is Δt, time to final maturity is t_N. A multi-period binomial can be shown as:



For European options, the option price at node (i,j) is calculated by $V_{i,j} = e^{-r\Delta t}(pV_{i+1,j+1} + (1-p)V_{i+1,j})$, where $i < N$. For the American put option that we need to price in the project, its option price is calculated by $V_{i,j} = \max\{e^{-r\Delta t}\left(pV_{i+1,j+1} + (1-p)V_{i+1,j}\right),\ K - S_{i,j}\}$.

Binomial Black–Scholes Method

BBS method is a modification to the binomial method where the Black-Scholes formula replaces the usual "continuation value" at the time step just before option maturity.

To compute the price of an American put option, we start with the leaf nodes $i = N$. These nodes correspond to $t_i = T$ and the value of option is $V_{N,j} = (K - S_{N-1,j})^+$. At time t_{N-1}, the continuation value is equivalent to the price of a European put option, replace it with BS formula for put option, that is:

$$V_{N-1,j} = \max\{p\left(\Delta t, S_{N-1,j}\right), K - S_{N-1,j}\}$$

Notice that $p\left(\Delta t, S_{N-1,j}\right)$ is the price of a Europe put option with time to maturity Δt and spot $S_{N-1,j}$.

Binomial Black–Scholes Method with Richardson Extrapolation

In numerical analysis, Richardson extrapolation is a sequence acceleration method, used to improve the rate of convergence of a sequence of estimates of some value A^* The formula is:

$$A^* = A(h) + a_0 h^{k_0} + O(h^{k_1})$$

Using the step sizes h and $\frac{h}{t}$ for some constant t, the two formulas for A^* are:

$$A^* = A(h) + a_0 h^{k_0} + O(h^{k_1})$$

$$A^* = A\left(\frac{h}{t}\right) + a_0(\frac{h}{t})^{k_0} + O(h^{k_1})$$

Multiplying the second equation and subtracting the first equation gives:

$$(t^{k_0} - 1)A^* = t^{k_0} A\left(\frac{h}{t}\right) - A(h) + (t^{k_0} - 1)O(h^{k_1})$$

Then we can solve the equation to get A^*. Combined BBS model and Richardson Extrapolation,

the option price we want is A^* in the above formula. That is BBSR model.