**FE5222 ADP Project One**

Ho Ngok Chao, Gao Jichen, Cheng Tuoyuan

1. **Introduction**

In this project, we would implement the least square Monte Carlo (LSMC) method for American put options pricing, and employ the binomial Black-Scholes model with Richardson extrapolation (BBSR) as a benchmark. Results from both methods are compared and discussed in various scenarios. The effects of numerical parameters such as the number of simulation paths as well as the number of discrete time steps are further investigated and visualized.

Merits of LSMC: can be extended with factor models, allows for path-dependent & early exercise features, permits parallelization; simple, transparent, flexible

Challenges: simulation & regression are time-consuming; needs to balance between MC simulation counts & time step counts.

Merits of BBSR: BBS is improved from traditional binomial trees by applying BSM for option values at the m-2 step, which were difficult for OTM options to consider their time value. BBSR further involved the Richardson extrapolation technique to cancel out higher-order error terms.

Challenges: disadvantages from binomial trees; not flexible

1. **Materials and Methods**
   1. **LSMC**

Cashflow vector

Exercise value and continuation value

Least square regression

Backward induction and discounting

* 1. **BBSR**
* Binomial tree
* Black-Scholes binomial tree method

BBS method is a modification to the binomial method where the Black-Scholes formula replaces the usual “continuation value” at the time step just before option maturity. At time t\_(N-1), the continuation value is equivalent to the price of a European put option, replace it with BS formula for put option, that is:

* Richardson extrapolation
  1. **Comparison and visualization**

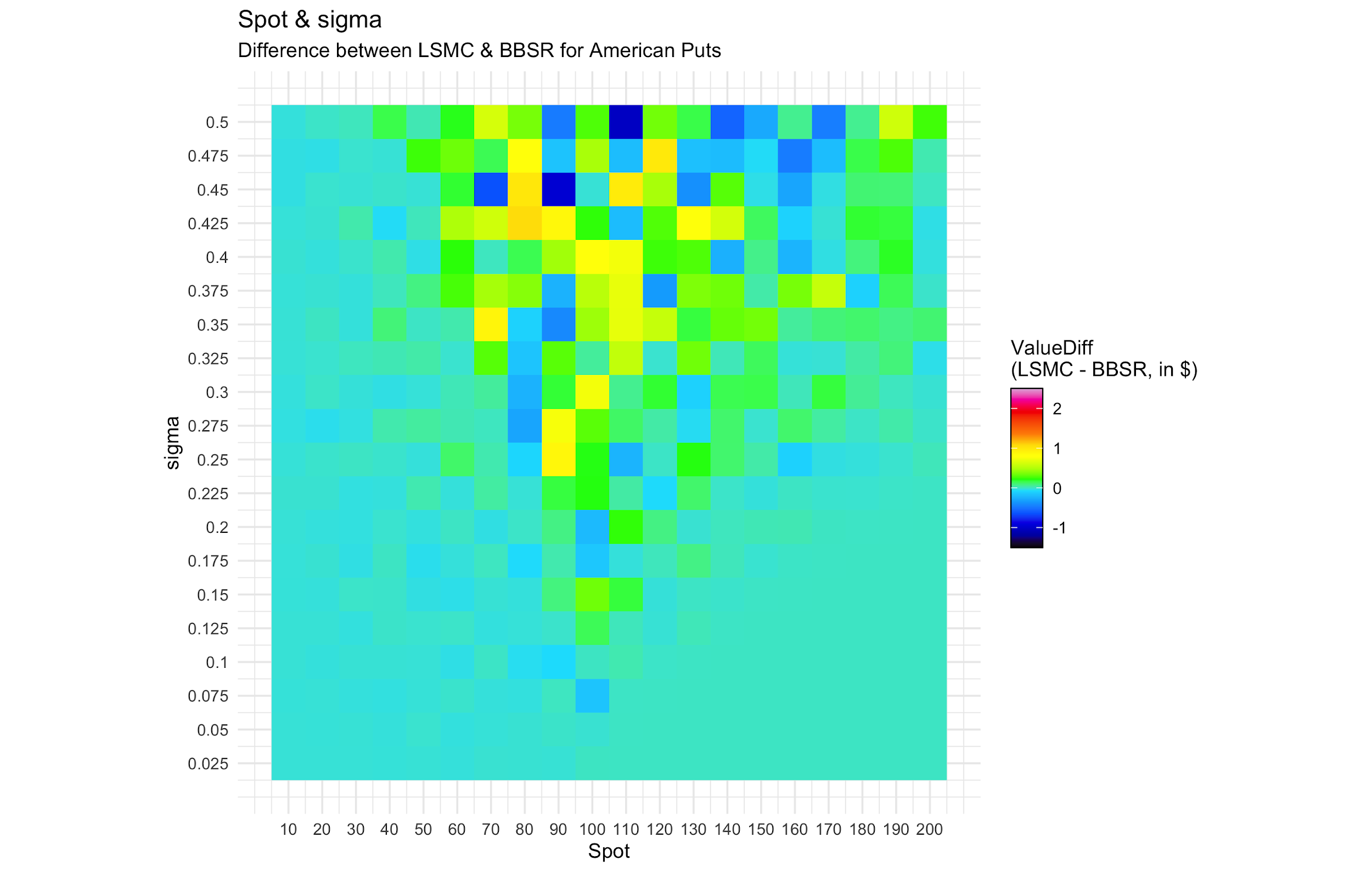
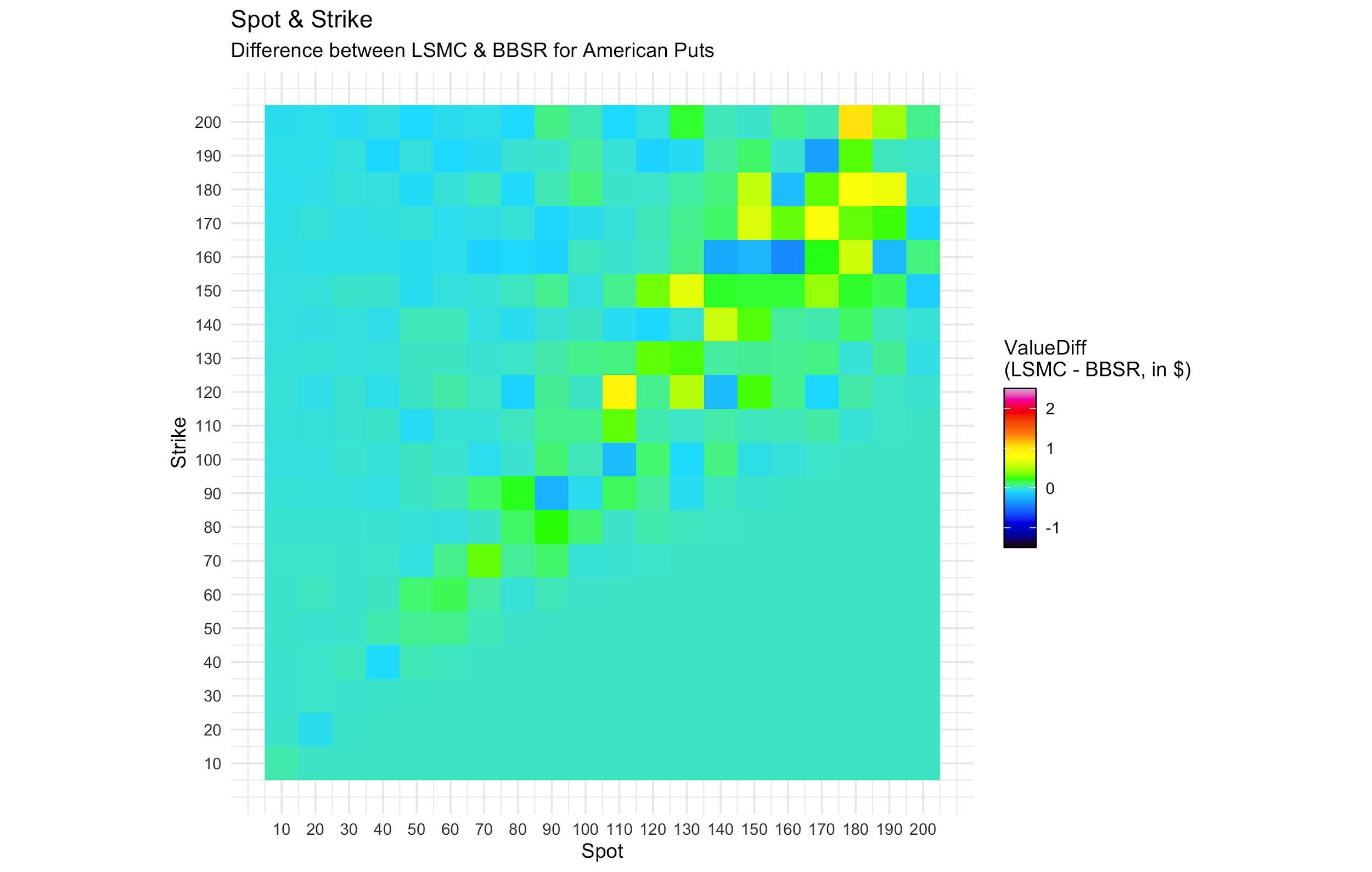
Investigated pricing parameters: Spot, Strike, volatility (sigma), interest rate (r), time to maturity (mT)

Pairwise grid comparison between LSMC and BBSR.

Investigated numerical parameters: number of Monte Carlo paths (n), number of time steps (m)

LSMC performances at different n & m; BBSR performances at different m.

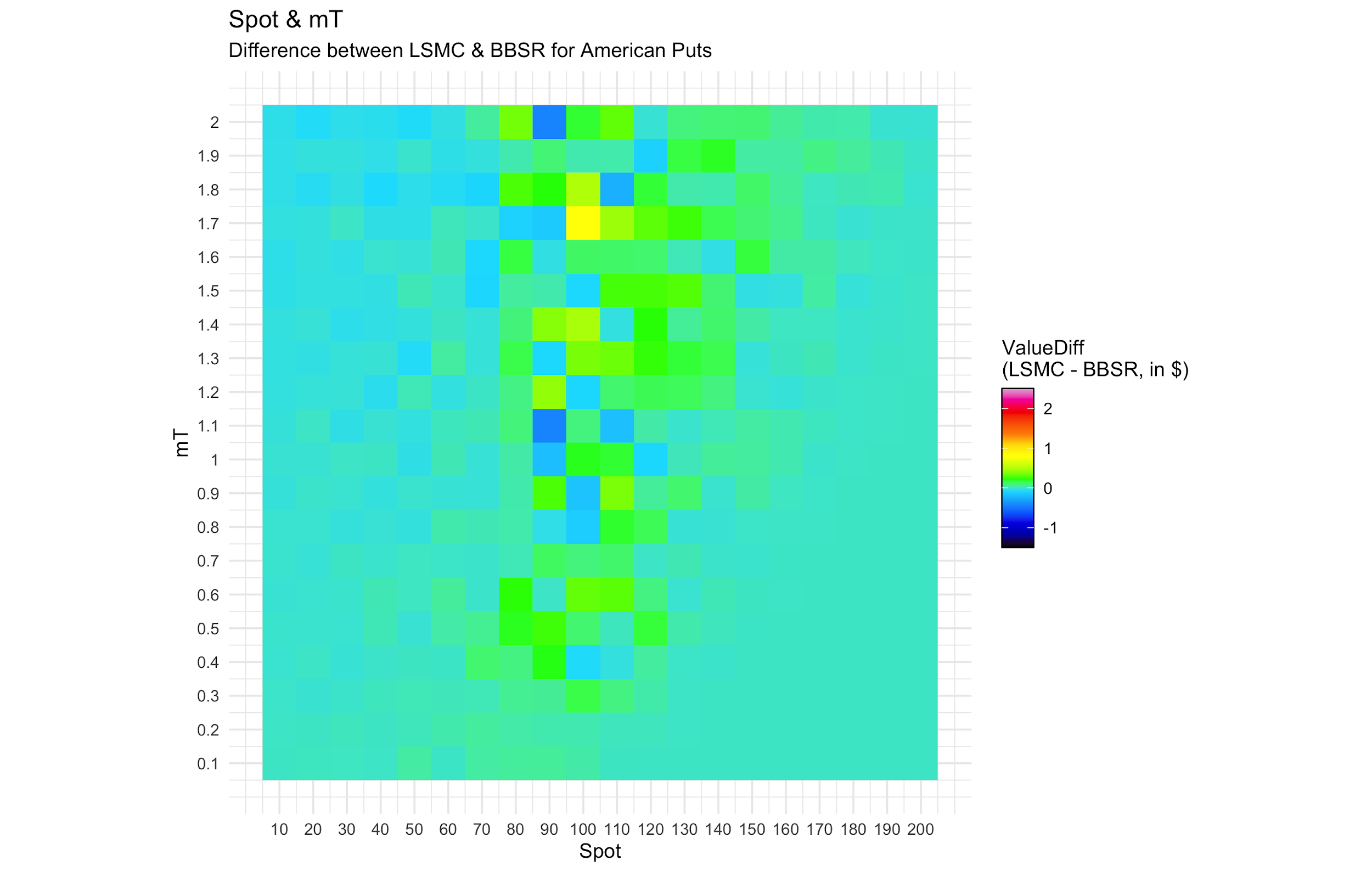
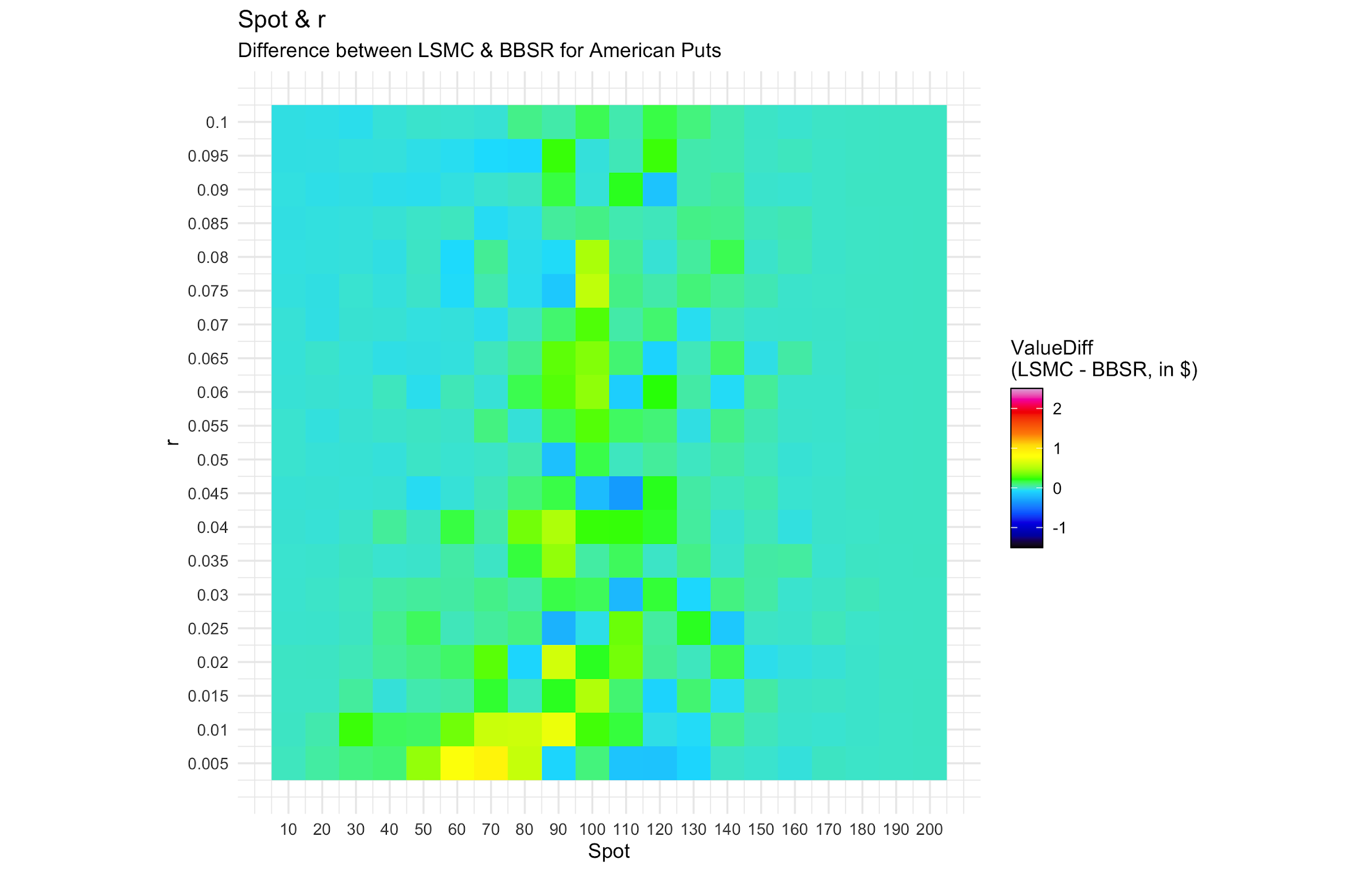
1. **Results and Discussion**
   1. **Grid comparison for pricing parameters**

****

(a) (b)

Figure 1. The differences between LSMC and BBSR among various

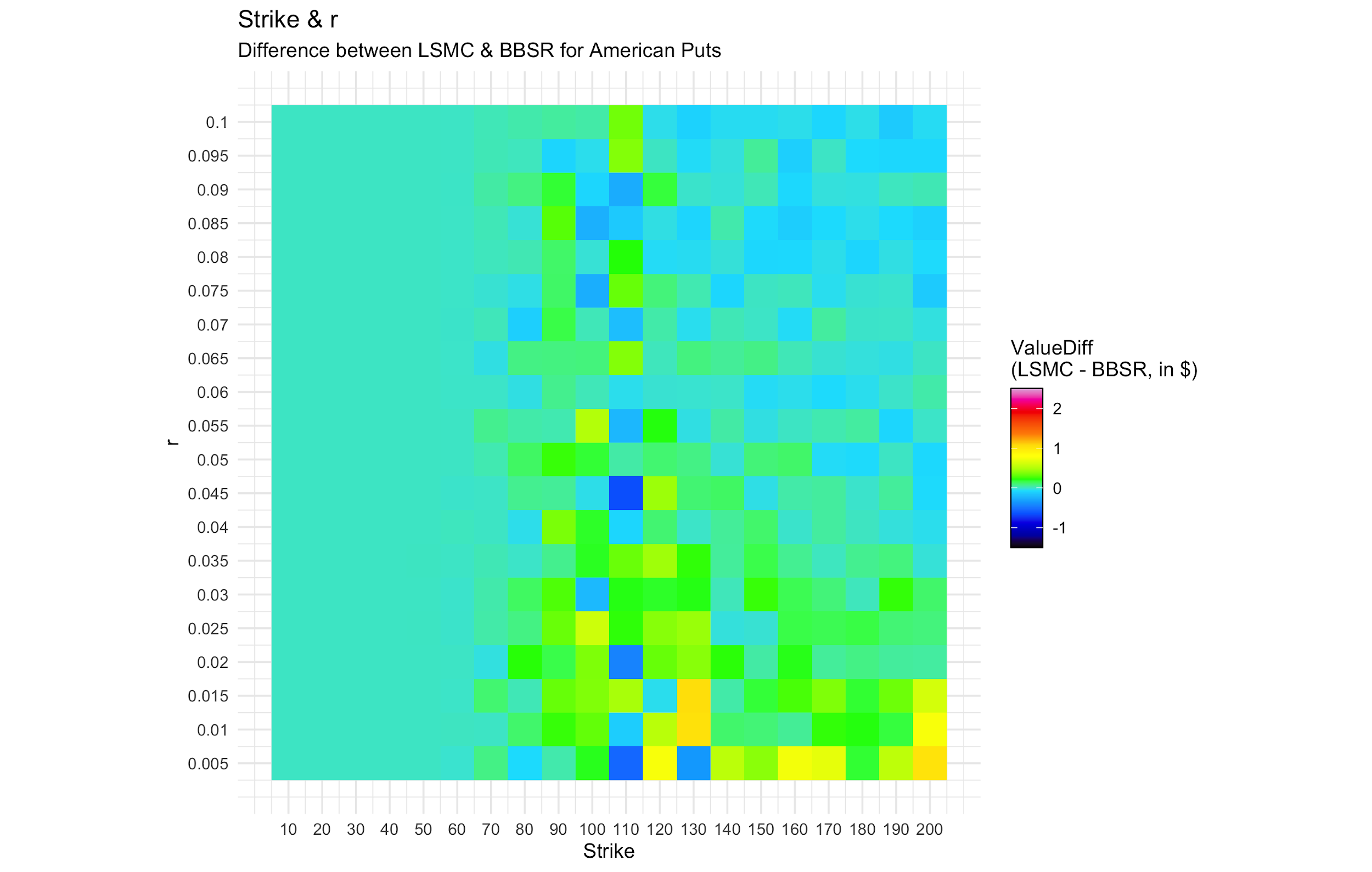
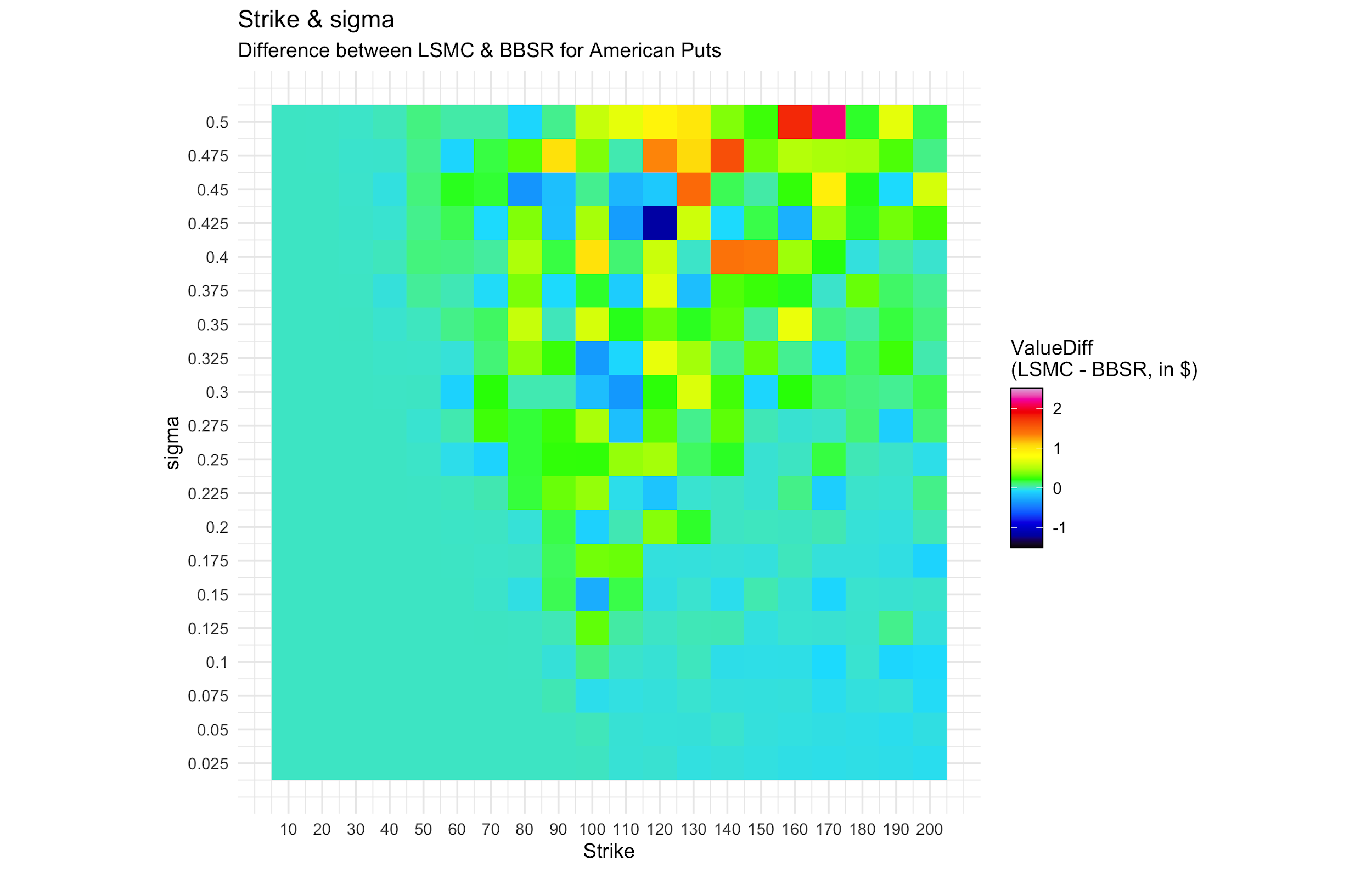
(a) spot and strike, as well as (b) spot and volatility (sigma).

****

(a) (b)

Figure 2. The differences between LSMC and BBSR among various

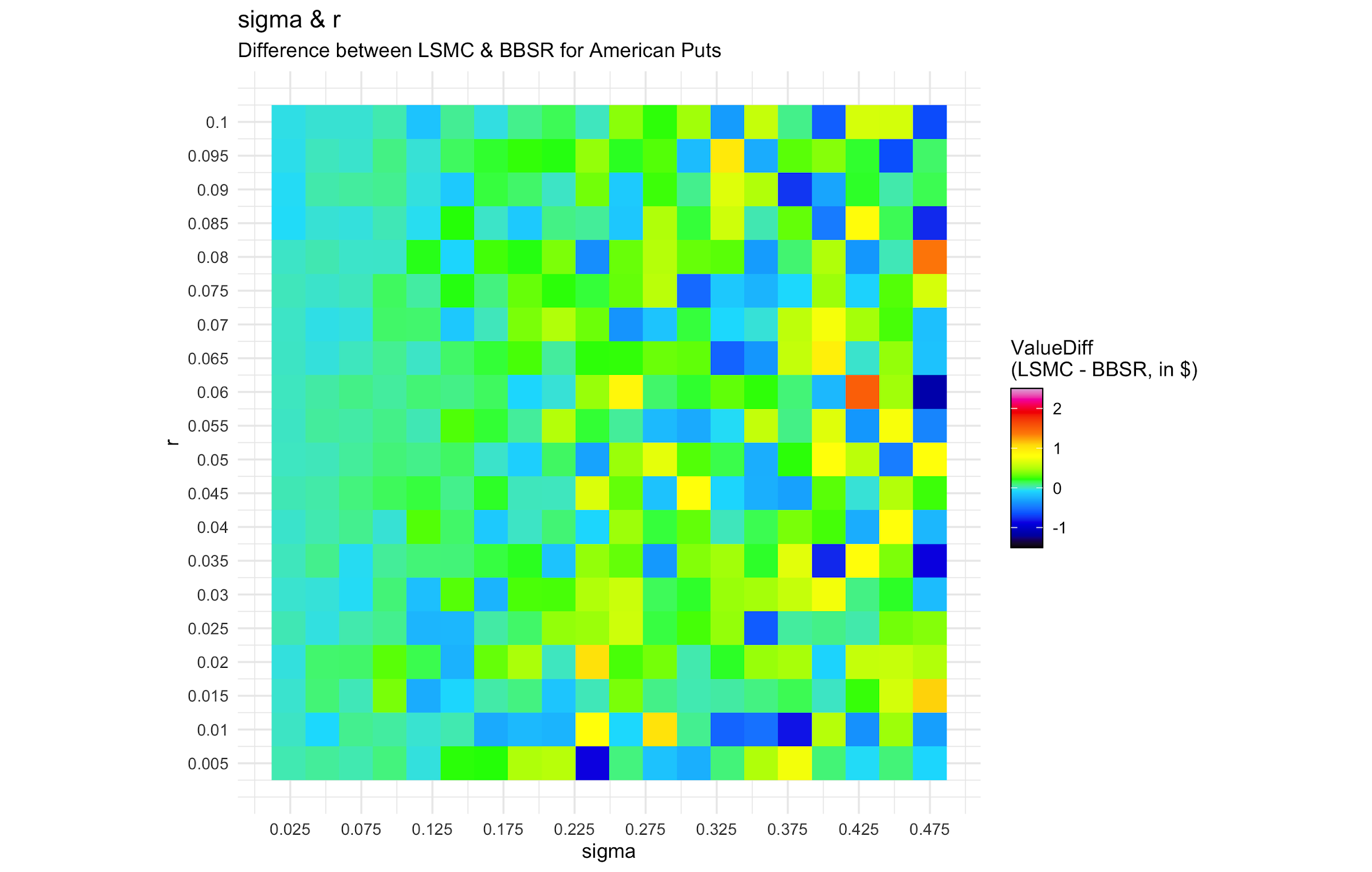
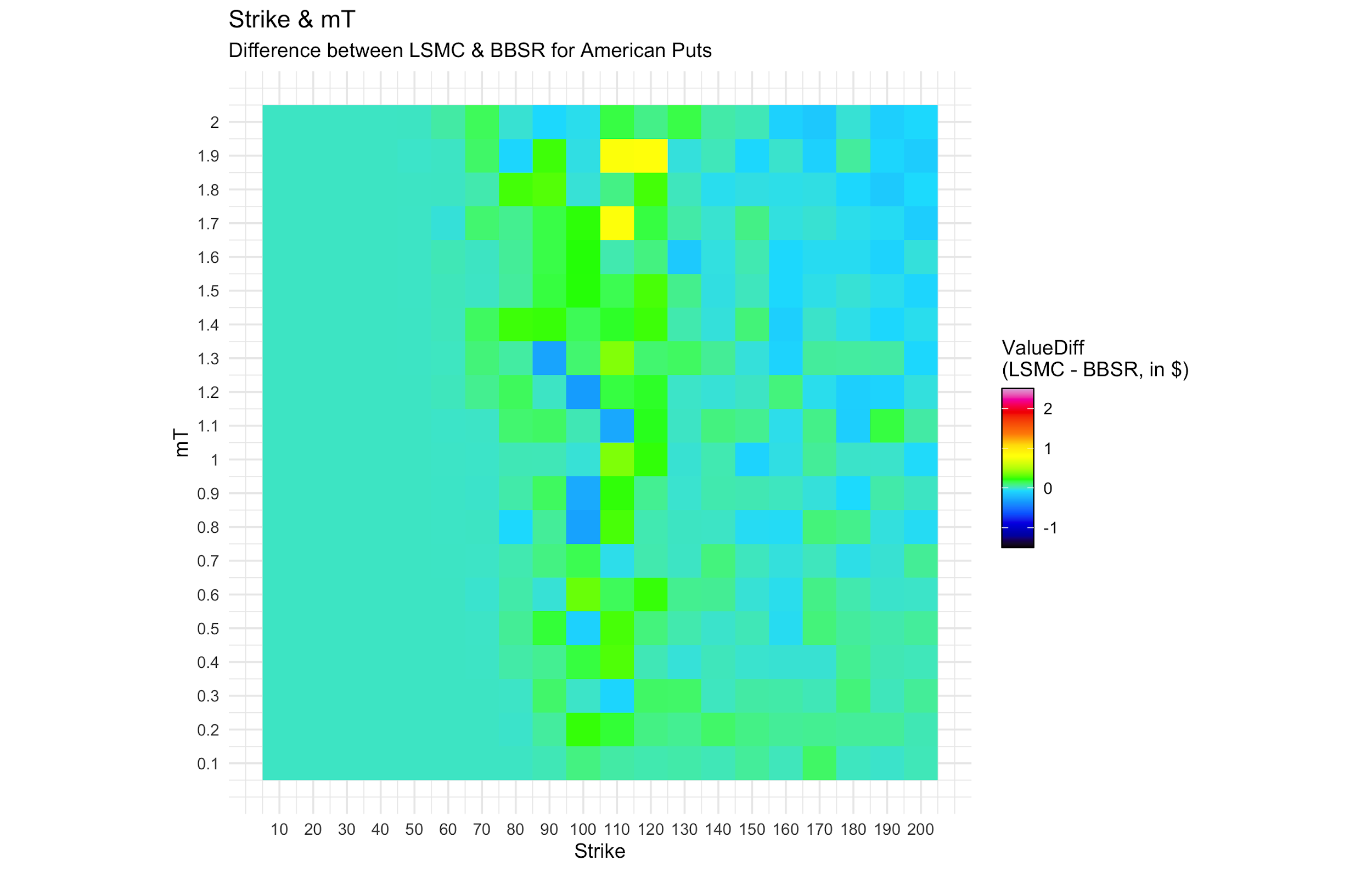
(a) spot and interest rate (r), as well as (b) spot and time to expiry (mT).

****

(a) (b)

Figure 3. The differences between LSMC and BBSR among various

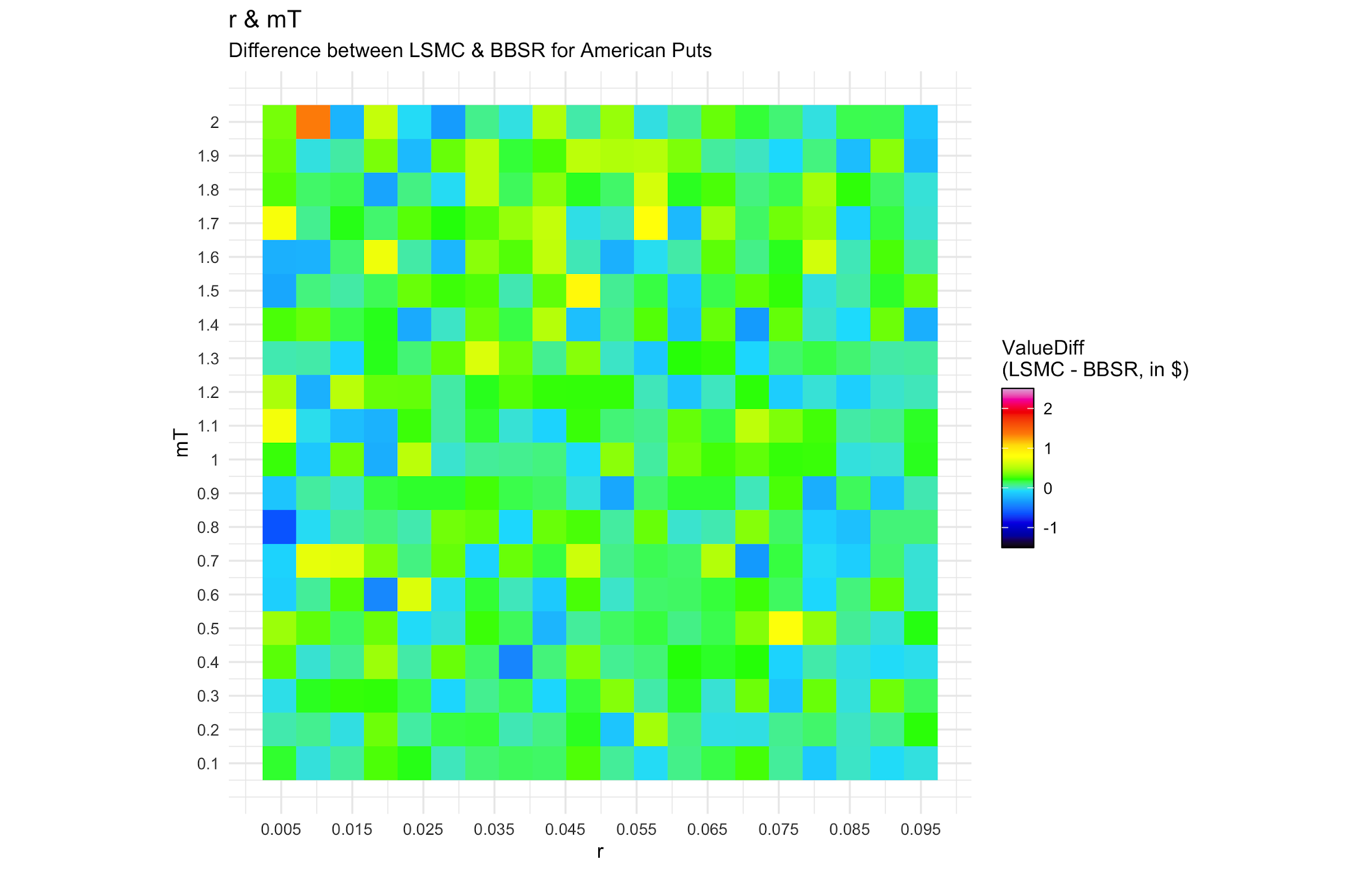
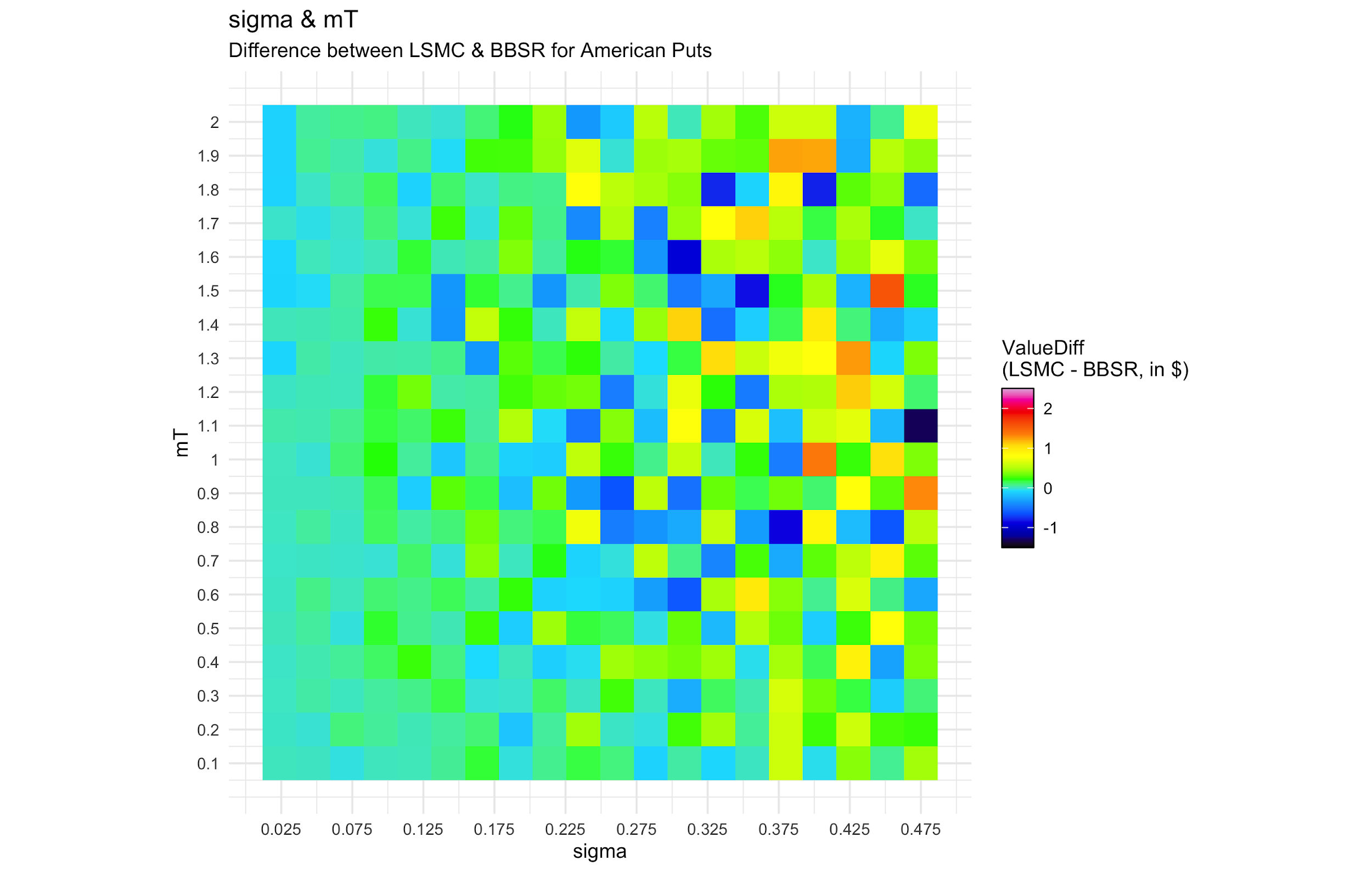
(a) strike and sigma, as well as (b) strike and interest rate.

****

(a) (b)

Figure 4. The differences between LSMC and BBSR among various

(a) strike and time to expiry, as well as (b) volatility and interest rate.

****

(a) (b)

Figure 5. The differences between LSMC and BBSR among various

(a) volatility and time to expiry, as well as (b) interest rate and time to expiry.

* 1. **Grid comparison for numerical parameters**

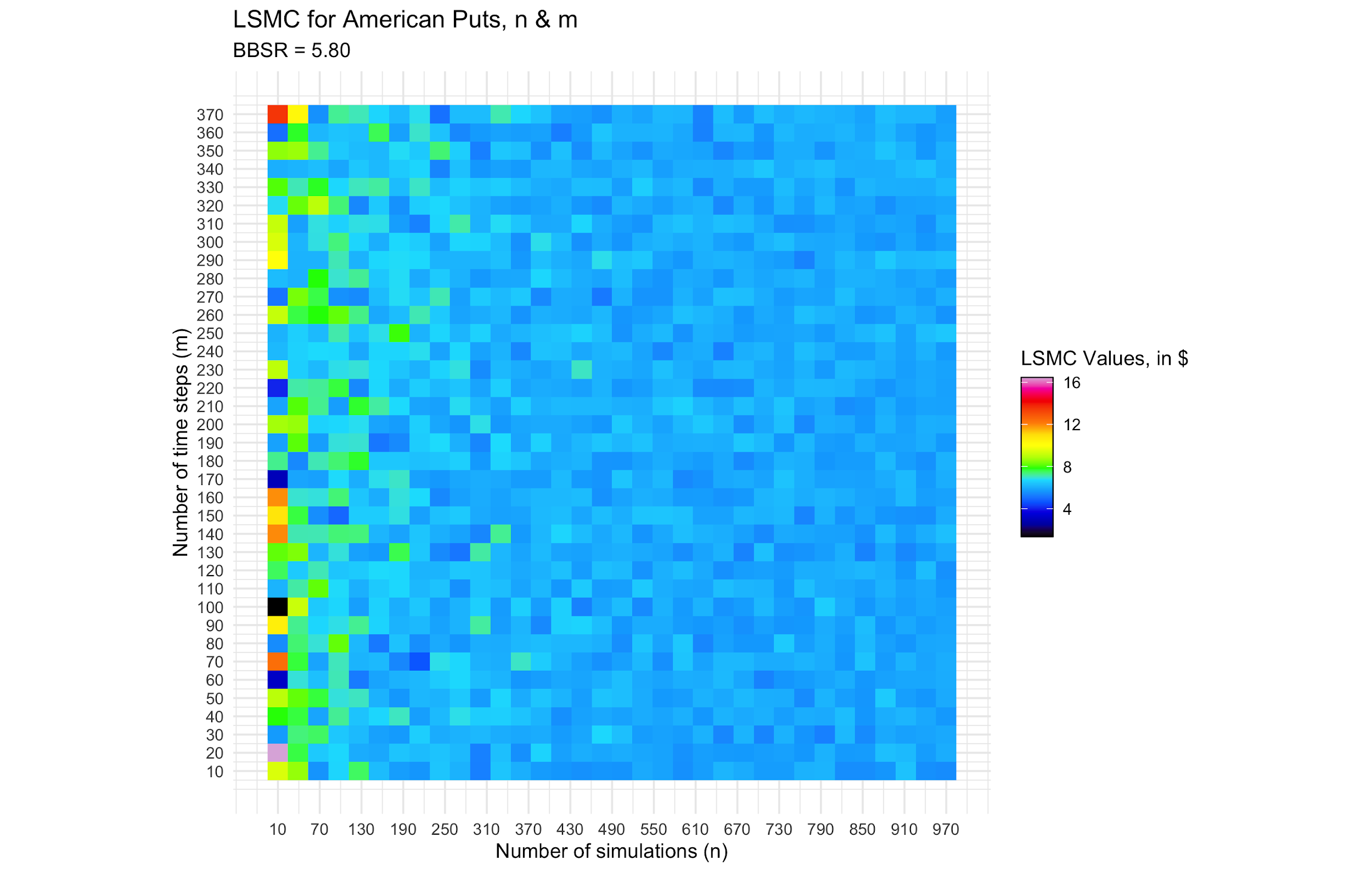


Figure 6. The LSMC results among various numbers of Monte Carlo simulations (n) and numbers of time steps (m). We are assuming Spot = 100, Strike = 100, sigma = 0.2, r = 0.06, mT=1, And the BBSR benchmark value is $ 5.80.

Needs a trade-off between time steps and simulations to price with satisfactory accuracy and acceptable computational complexity.

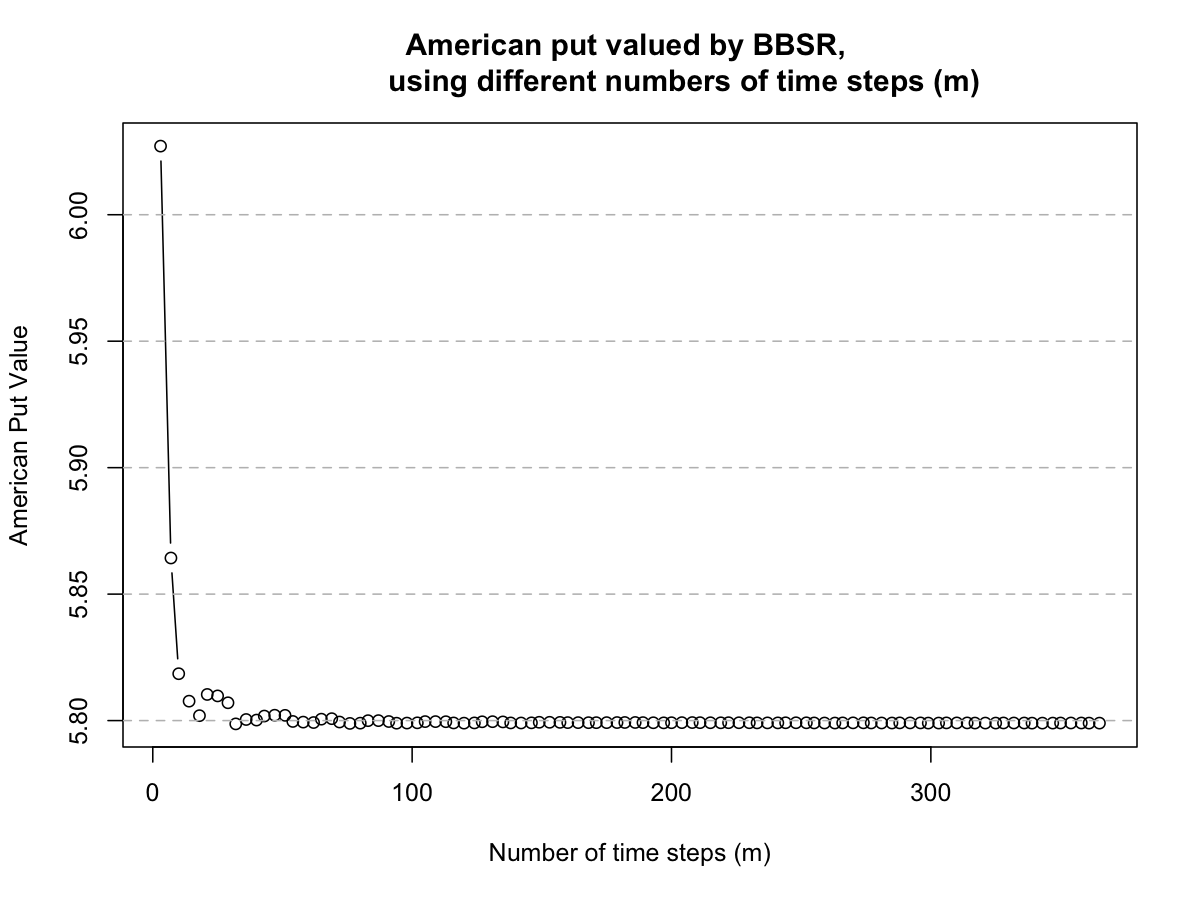


Figure 7. The BBSR results among various numbers of time steps (m). We are assuming Spot = 100, Strike = 100, sigma = 0.2, r = 0.06, mT=1, And the BBSR benchmark value is $ 5.80.

BBSR converges efficiently within a limited number of time steps, and remains stable afterward. It is serving as a good benchmark for vanilla American option pricing.

1. **Conclusion and Future Research**

Different basis functions

LSMC application on more path-dependent options

Comparison among trees

1. **References**

Longstaff, F.A., and Schwartz, E.S., 2001. Valuing American options by simulation: a simple least-squares approach. *The review of financial studies*, *14*(1), pp.113-147.

Haug, E.G., 2007. *The complete guide to option pricing formulas*. McGraw-Hill Companies.

Wilmott, P., 2007. *Paul Wilmott introduces quantitative finance*. John Wiley & Sons.

1. **Appendix:** 
   1. **LSMC in Python Jupyter notebook, by Ho Ngok Chao**
   2. **BBSR in Python Codes, by Gao Jichen**
   3. **Comparison & Visualization in R Codes, by Cheng Tuoyuan**