**Abstract**

Our project models two underlying future contract namely, the WTI Light Sweet Crude Oil Futures and Brent Crude Futures. [Formula]. The [Formula] comes from PCA. The additional sigma\_t is calibrated on t=date using WTI European Option to reflect current market volatilities.

At the end, we use F(t, T\_wti), F(t, T\_Brent) as starting point and find the terminal price F(t+1,T\_wti) and F(t+1, T\_brent) according our two forward curve model and use 3 different methods namely, Monte Carlo, Moment Matching and Kirk’s approximation to price a spread option whose value is max(F(t+1,T\_wti) - F(t+1, T\_brent) - K, 0).

**PCA**

Forward contract data mentioned above are used in two separate PCA. Delta t of both PCA is set to be 1 day. Tau of PCA\_WTI is set to be [Formula]. Tau of PCA\_Brent is set to be [Formula]. The Tau of PCA\_WTI is chosen so that F(te,te+tau\_1) can be determined which is later used in calibration.

**Calibration**

[Table]

This entry of WTI European option is used to calibrate the additional sigma(t). As there are no Brent European option who shares the same expiry date with WTI European option, we choose to calibrate again WTI European option and assume the two models share the same additional sigma(t).

However, under such setting, model implied volatility won’t match market volatilities and use similar calculation (total variance divided by tau\_1\_brent) as in calibration, we car derive the model implied volatility for Kirk’s for brent is 0.4965824.

**Comparison**

When comparing Monte Carlo and Moment matching, we can see minor price difference. It should be noted that it is only when there are “enough” samples should Monte Carlo converge to true distribution of our models, therefore there is the first reason of error. The second error may come from the assumption of Moment matching which is we are trying to fit our model with lognormal distribution which may not be a perfect fit.