Robust Median Reversion Strategy

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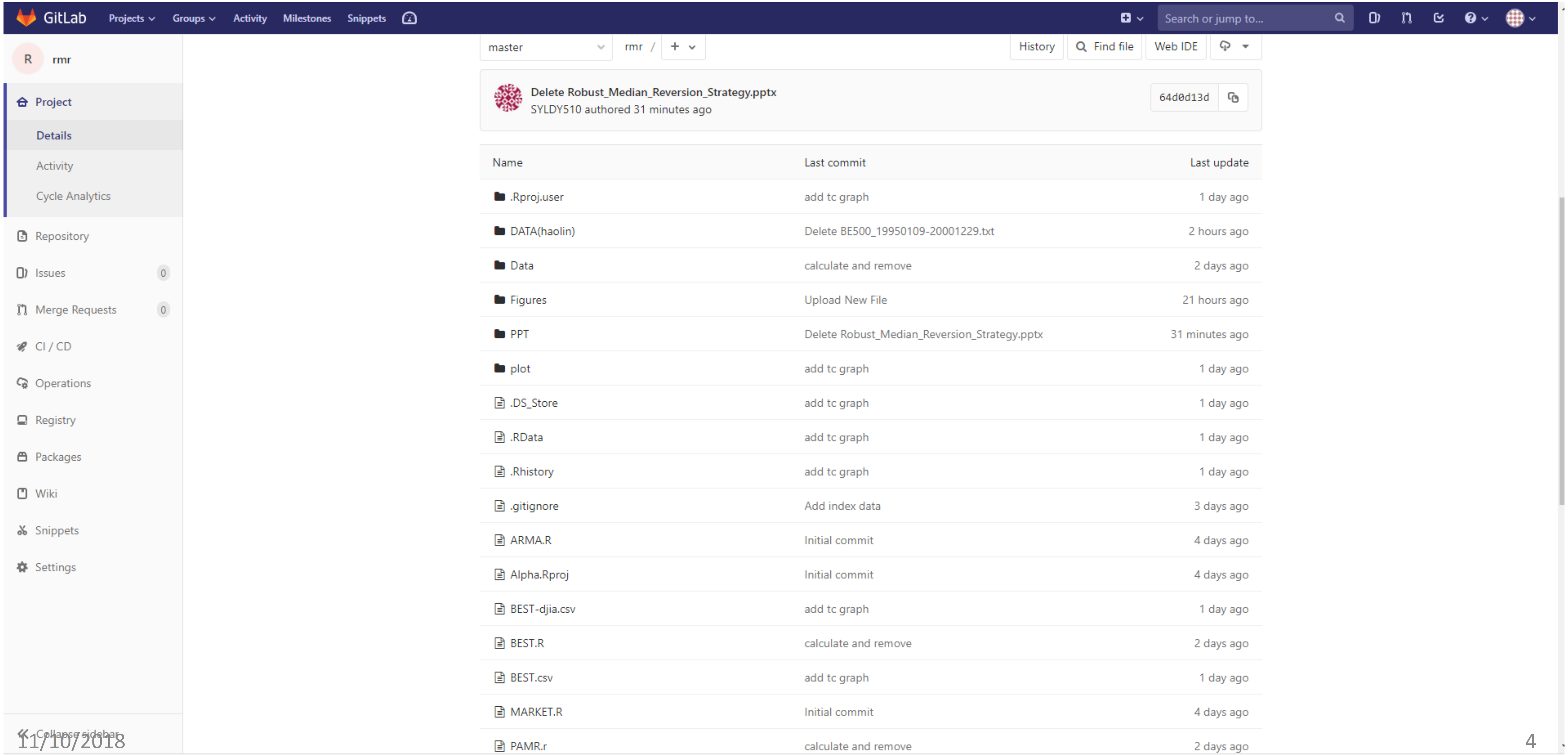
Content

- Portfolio Selection
- Median Reversion
- Robust Median Reversion (‘RMR’)
- RMR Algorithm
- Four other models
- Experiment-1
- Experiment-2
- Conclusion

Challenges

Our solution



Portfolio Selection

- Efficiently allocate wealth among a set of assets so as to achieve certain financial objectives in the long run.
- A financial market with d assets for n trading periods to be invested.
  - Close price :On the t_{th} period, the asset prices are represented by a close price.
  - Relative price : The change ratio of asset prices.
  - Portfolio strategy: self-financed and no margin/short is allowed, allocate the capital among the d assets.
  - Portfolio cumulative wealth : the wealth increases multiplicatively by the product of relative price and portfolio strategy.

Portfolio Selection

- Close price vector :
  - On the t_{th} period, the close price vector $P_t \in \mathrm{R}_{+}^{d}$, and each element p_t^i represents the close price of asset i.
- Relative price vector :
  - $\mathrm{x}_t = (x_t^1, \dots, x_t^d) \in R_{+}^{d}$ where x_t^i expresses the ratio of close price to last close price of asset j at the t_{th} period, i.e., $x_t^i = p_t^i / p_{t-1}^i$.
- Portfolio Vector :
  - $b_t = (b_t^1, \dots, b_t^d)$ where b_t^i represents the proportion of wealth invested in the j_{th} asset.
- Portfolio cumulative wealth :
  - Our initial wealth is $\boldsymbol{S_0}$, after n trading period, the portfolio cumulative wealth $\boldsymbol{S_n(b_1^n, x_1^n) = S_0 \prod_t^T (b_t^T x_t)}$.

Mean Reversion

- Estimate the next price relative by a single value prediction based on mean reversion or moving average reversion.
- From the price perspective , they implicitly assume that next price revert to last price.
  - $\tilde{x}_{t+1} = \frac{1}{x_t} \Rightarrow \frac{\tilde{P}_{t+1}}{P_t} = \frac{P_{t-1}}{P_t} \Rightarrow \tilde{P}_{t+1} = P_{t-1}$

- Weakness:
  - Single-period mean reversion assumption is not always satisfied in the real world.
  - Data contain a lot of noise and outliers and thus substantially influences the effectiveness of the algorithm and even the final cumulative wealth.

Robust Mean Reversion ('RMR')

- 1^{st} Optimization Problem
  - Considering noise and outliers in the real market data, price distribution has a long tail.
  - Price vector and relative price vector
    - The next price by Robust L_1-Median Estimator at the end of t^{th} period is $\tilde{P}_{t+1} = L_1 med_{t+1}(\omega) = \mu$, where ω is the window size, μ denotes the L_1-Median Estimator optimal value of Optimization problems below:
      - $\tilde{x}_{t+1}(\omega) = \frac{L_1\, med_{t+1}(\omega)}{P_t} = \frac{\mu}{P_t}$
      - $\mu = argmin_{\mu} \sum_{i=0}^{\mathrm{k}-1} \|P_{t-i} - \mu\|$, where $\| \,.\, \|$denotes the Euclidean norm.

Robust Mean Reversion ('RMR')

- 2^{nd} Optimization Problem
  - Portfolio Vector
    - $b_{t+1} = \ argmin_{b \in \Delta d} \frac{1}{2} \|b - b_t\|^2 \ s.t. \ b^T \tilde{x}_{t+1} \geq \varepsilon.$
  - The above formulation attempts to find an optimal portfolio by minimizing the deviation from last portfolio b_t under the condition of $b^T \tilde{x}_{t+1} \geq \varepsilon$.

RMR Algorithm

1. Optimization problem 1 $\mu = argmin_{\mu} \sum_{i=0}^{k-1} \|P_{t-i} - \mu\|$, where $\| . \|$denotes the Euclidean norm.

Proposition 1. *The solution of L_1-median-MAADM optimization problem 1 is calculated through iteration, and the iteration process is described as:*

$$\boldsymbol{\mu} \to T(\boldsymbol{\mu}) = \left(1 - \frac{\eta(\boldsymbol{\mu})}{\gamma(\boldsymbol{\mu})}\right)^{+} \tilde{T}(\boldsymbol{\mu}) + \min\left(1, \frac{\eta(\boldsymbol{\mu})}{\gamma(\boldsymbol{\mu})}\right)\boldsymbol{\mu},$$

where

$$\eta(\boldsymbol{\mu}) = \begin{cases} 1 & if\ \boldsymbol{\mu} = \mathbf{p}_{t-i}, \quad i = 0, \ldots, k-1 \\ 0 & otherwise \end{cases},$$

$$\gamma(\boldsymbol{\mu}) = \|\tilde{R}(\boldsymbol{\mu})\|, \quad \tilde{R}(\boldsymbol{\mu}) = \sum_{\mathbf{p}_{t-i} \neq \boldsymbol{\mu}} \frac{\mathbf{p}_{t-i} - \boldsymbol{\mu}}{\|\mathbf{p}_{t-i} - \boldsymbol{\mu}\|},$$

$$\tilde{T}(\boldsymbol{\mu}) = \left\{\sum_{\mathbf{p}_{t-i} \neq \boldsymbol{\mu}} \frac{1}{\|\mathbf{p}_{t-i} - \boldsymbol{\mu}\|}\right\}^{-1} \sum_{\mathbf{p}_{t-i} \neq \boldsymbol{\mu}} \frac{\mathbf{p}_{t-i}}{\|\mathbf{p}_{t-i} - \boldsymbol{\mu}\|}.$$

Input: data $\mathbf{p}_t, \mathbf{p}_{t-1}, \ldots, \mathbf{p}_{t-w+1}$; iteration maximum m; toleration level τ
Output: estimated $\hat{\mathbf{x}}_{t+1}$
Procedure:
Initialize $\boldsymbol{\mu}_1 = median(\mathbf{p}_t, \mathbf{p}_{t-1}, \ldots, \mathbf{p}_{t-w+1})$.
for $i = 2$ **to** m **do**
  $\boldsymbol{\mu}_i = T(\boldsymbol{\mu}_{i-1})$
  if $\|\boldsymbol{\mu}_{i-1} - \boldsymbol{\mu}_i\|_1 \leq \tau \|\boldsymbol{\mu}_i\|_1$ **then**
    break
  end if
end for
$\hat{\mathbf{p}}_{t+1} = \boldsymbol{\mu}_i$
$\hat{\mathbf{x}}_{t+1} = \hat{\mathbf{p}}_{t+1} / \mathbf{p}_t$

RMR Algorithm

Alpha

2. Optimization problem 2 $b_{t+1} = argmin_{b\in\Delta d}\frac{1}{2}\|b-b_t\|^2\ s.t.\ b^T\tilde{x}_{t+1}\geq\varepsilon.$

If $\hat{\mathbf{x}}_{t+1}^T\mathbf{b}_t-\varepsilon\geq 0$, then $\mathbf{b}=\mathbf{b}_t$. If $\hat{\mathbf{x}}_{t+1}^T\mathbf{b}_t-\varepsilon<0$, then

$$L(\mathbf{b},\alpha,\lambda)=\frac{1}{2}\|\mathbf{b}-\mathbf{b}_t\|^2+\alpha(\hat{\mathbf{x}}_{t+1}^T\mathbf{b}_t-\varepsilon)+\lambda(\mathbf{b}^T\mathbf{1}-1) \quad (9)$$

Solution:

$$\mathbf{b}_{t+1}=\mathbf{b}_t-\alpha_{t+1}(\hat{\mathbf{x}}_{t+1}-\overline{x}_{t+1}\cdot\mathbf{1}),$$

where $\overline{x}_{t+1}=\frac{1}{d}(\mathbf{1}\cdot\hat{\mathbf{x}}_{t+1})$ *denotes the average predicted price relative and* α_{t+1} *is the Lagrangian multiplier calculated as,*

$$\alpha_{t+1}=\min\left\{0,\frac{\hat{\mathbf{x}}_{t+1}^T\mathbf{b}_t-\epsilon}{\|\hat{\mathbf{x}}_{t+1}-\overline{x}_{t+1}\cdot\mathbf{1}\|^2}\right\}.$$

1: **Input:** reversion threshold $\epsilon>1$; predicted the next price relative vector $\hat{\mathbf{x}}_{t+1}$; current portfolio $\mathbf{b}_t$;
2: **Output:** next portfolio $\mathbf{b}_{t+1}$
3: **Procedure:**
4: Calculate the following variable:

$$\alpha_{t+1}=\min\left\{0,\frac{\hat{\mathbf{x}}_{t+1}^T\mathbf{b}_t-\epsilon}{\|\hat{\mathbf{x}}_{t+1}-\overline{x}_{t+1}\cdot\mathbf{1}\|^2}\right\}$$

5: Update the portfolio:

$$\mathbf{b}_{t+1}=\mathbf{b}_t-\alpha_{t+1}(\hat{\mathbf{x}}_{t+1}-\overline{x}_{t+1}\cdot\mathbf{1})$$

6: Normalize $\mathbf{b}_{t+1}$: $\mathbf{b}_{t+1}=argmin_{\mathbf{b}\in\Delta_d}\|\mathbf{b}-\mathbf{b}_{t+1}\|^2$

RMR Algorithm

3. Summary

Input: reversion threshold $\epsilon > 1$; iteration maximum m; window size $w \geq 2$; toleration level τ; market sequence $\mathbf{x}_1^n$

Output: S_n: Cumulative wealth after nth periods

Procedure:

Initialization: $b_1 = \frac{1}{d}\mathbf{1}, S_0 = 1, \mathbf{p}_0 = \mathbf{1}$

for $t = 1, 2, \ldots, n$ **do**

  Receive stock price: $\mathbf{x}_t$

  Update cumulative return: $S_t = S_{t-1} \times (\mathbf{b}_t \cdot \mathbf{x}_t)$

  Predict the next price relative vector:

$$\hat{\mathbf{x}}_{t+1} = \begin{cases} \frac{L_1 medianMAADM_{t+1}(w)}{\mathbf{p}_t} & \text{MR} \\ \frac{L_1 medianHLFBM_{t+1}(w)}{\mathbf{p}_t} & \text{MR-Variant} \end{cases}$$

  Update the portfolio:

$$\mathbf{b}_{t+1} = \mathrm{R}MR(\epsilon, \hat{\mathbf{x}}_{t+1}, \mathbf{b}_t)$$

end for

Four other models

- Best-stock ('BEST')
  - Buys the best stock over the period.
  - Obviously, a hindsight strategy.
- Passive aggressive mean reversion ('PAMR')
  - Estimate the next price relative as the inverse of last price relative
  - Adopt the single-period mean reversion assumption, not satisfied with reality
  - Cannot exempt from the influence of noise and outliers

Four other models

- Online Moving Average Reversion (‘OLMAR’)
  - Predicts the next price relative using moving averages and explores the multi-period mean reversion
  - Cannot exempt from the influence of noise and outliers
- Market
  - Buys assets according to a pre-defined weight and holds until the end.
  - In the experiment, we used equal weight.

Assumptions

- 1) Transaction cost:
  - We assume no transaction cost or taxes in this PS model;
- 2) Market liquidity:
  - We assume that one can buy and sell required quantities at last closing price of any given trading period;
- 3) Impact cost:
  - We assume that market behavior is not affected by a PS strategy.

Experiments

- Parameter Settings for RMR
  - two key parameters
    - ω represents the length of the window, ϵ is about sensitivity parameter.
  - Roughly speaking, the best values for these parameters are often dataset dependent.
  - In the experiments, we simply set these parameters empirically without tuning for each dataset separately.
  - Specifically, for all datasets and experiments, we set ω to 5 and ϵ to 5.

Experiments-1

- Datasets

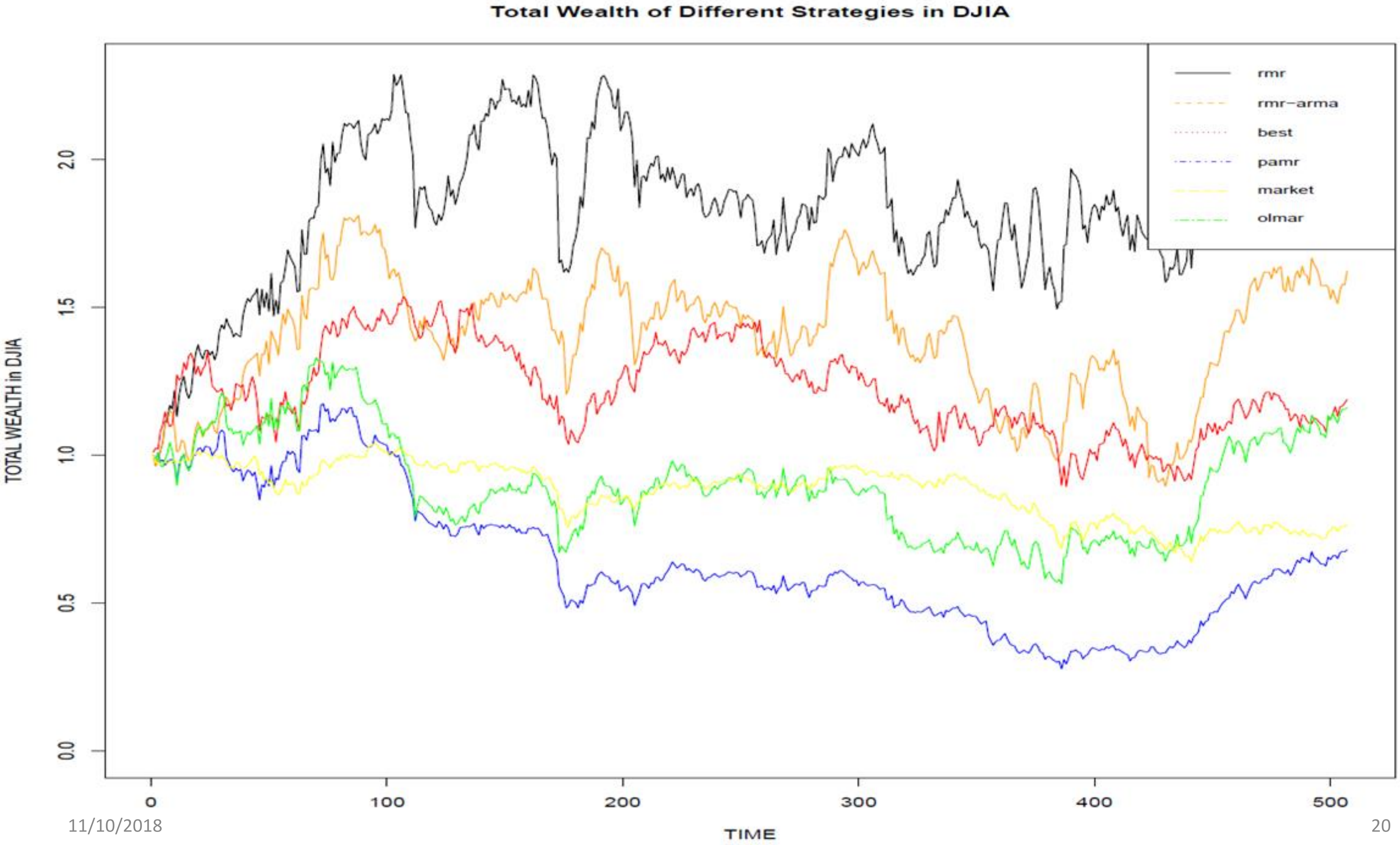
Data set	Region	Time Frame	#days	#assets
DJIA	US	14/01/2001 - 14/01-2003	507	30
SP500	US	02/01/1998 - 31/01/2003	1276	25
TSE	CA	04/01/1994 - 31/12/1998	1259	88
MSCI	Global	01/04/2006 – 31/03/2010	1043	24
NYSE(O)	US	03/04-1962 – 31/12/1984	5651	36
NYSE(N)	US	01/01/1985 – 30/06/2010	6431	23

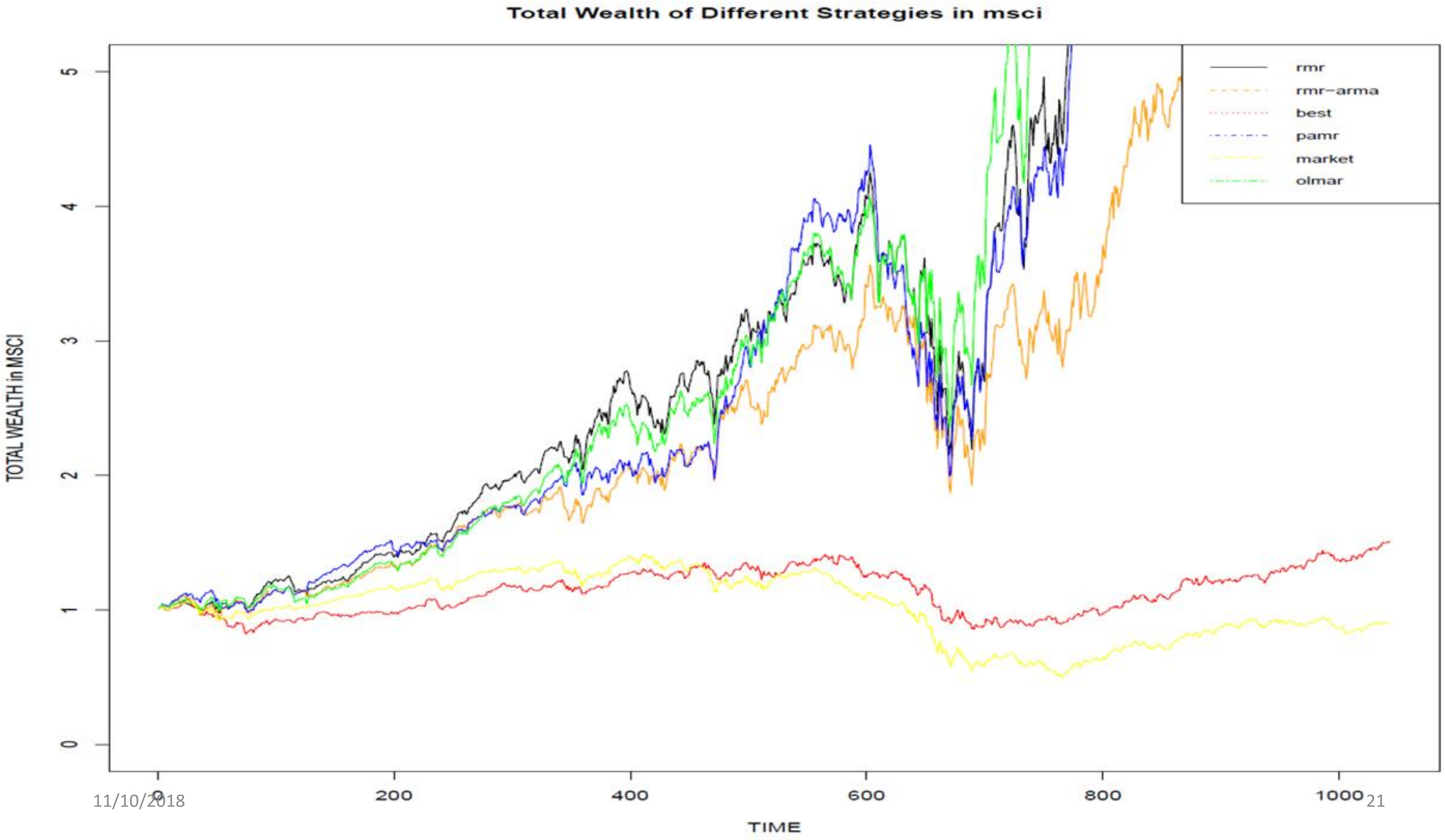
Experiments

- Performance Measure
  - Portfolio Cumulative Wealth
    - The initial wealth $S_0 = 1$
    - S_n denotes portfolio cumulative wealth at the end of the n_{th} trading day
  - Annualized Percentage Yield (APY)
    - Takes the compounding effect into account
    - $APY = \sqrt[y]{S_n}$ where y is the number of years corresponding to n trading days
  - Winning Ratio (WT)
    - Percentage of cases when the proposed strategy beats the BAH strategy

Experiments

- Performance Measure
  - Sharpe Ratio
    - Evaluates the risk-adjusted return
    - SR = (APY-R_f)/σ_p
  - Maximum Drawdown (MDD)
    - Measures the down side risk of different strategies.
  - Calmar Ratios(CR)
    - Indicates performance of a trading strategy concerning the drawdown risk
    - CR = APY/MDD


Total Wealth of Different Strategies in DJIA
TOTAL WEALTH in DJIA
TIME
0.0
0.5
1.0
1.5
2.0
0
100
200
300
400
500
rmr
rmr−arma
best
pamr
market
olmar


Total Wealth of Different Strategies in msci
TOTAL WEALTH in MSCI
TIME
rmr
rmr−arma
best
pamr
market
olmar
0
1
2
3
4
5
0
200
400
600
800
1000

Experiments-1

- The Comparison of APY, WT, Sharpe Ratio, MDD and CR among BEST, OLMAR, PAMR, and RMR strategy.

Experiment 1					
Dataset	Criteria	BEST	OLMAR	PAMR	RMR
DJIA	APY	0.0901	0.0776	-0.1753	0.6334
	WT	0.5049	0.5207	0.5187	0.5503
	SR	0.1174	0.0728	-0.4637	1.1597
	MDD	0.4187	0.5762	0.7646	0.3469
	CR	0.2152	0.1346	-0.2293	1.8258
MSCI	APY	0.1074	1.1783	0.9756	1.0234
	WT	0.5197	0.6002	0.5916	0.5916
	SR	0.3391	2.8418	2.5647	2.5128
	MDD	0.3935	0.4203	0.5528	0.4933
	CR	0.2730	2.8035	1.7648	2.0747

Experiments-1

- Conclusion
  - RMR strategy is promising and reliable PS technique to achieve high return
    1. RMR strategy works very well on almost all the datasets.
    2. Compared with the existing mean reversion strategies (PAMR and OLMAR), RMR strategies obtained higher cumulative wealth on the datasets NYSE(O), NYSE(N) and DJA.
    3. RMR in combination with ARMA did good job in the original.

Experiments-2

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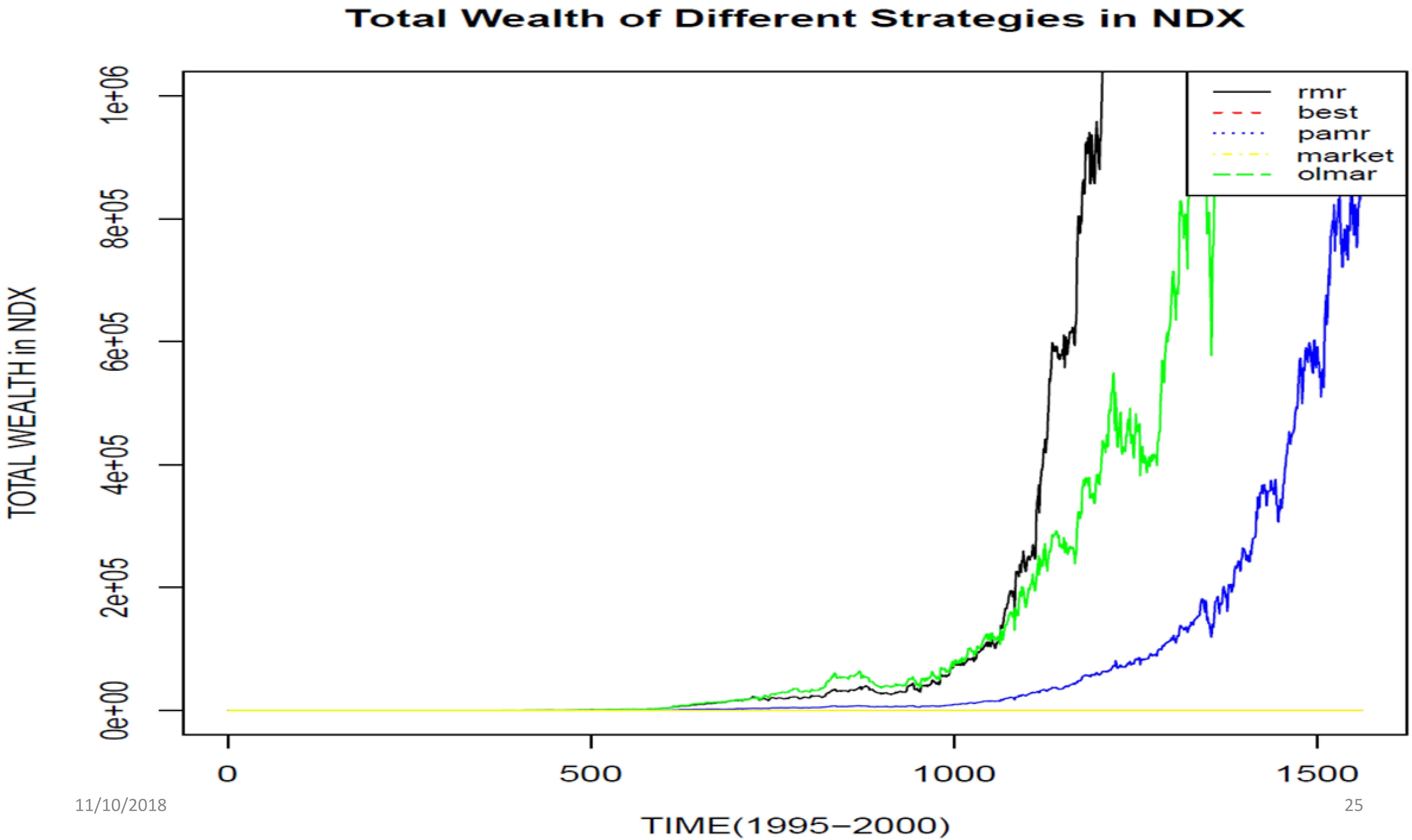
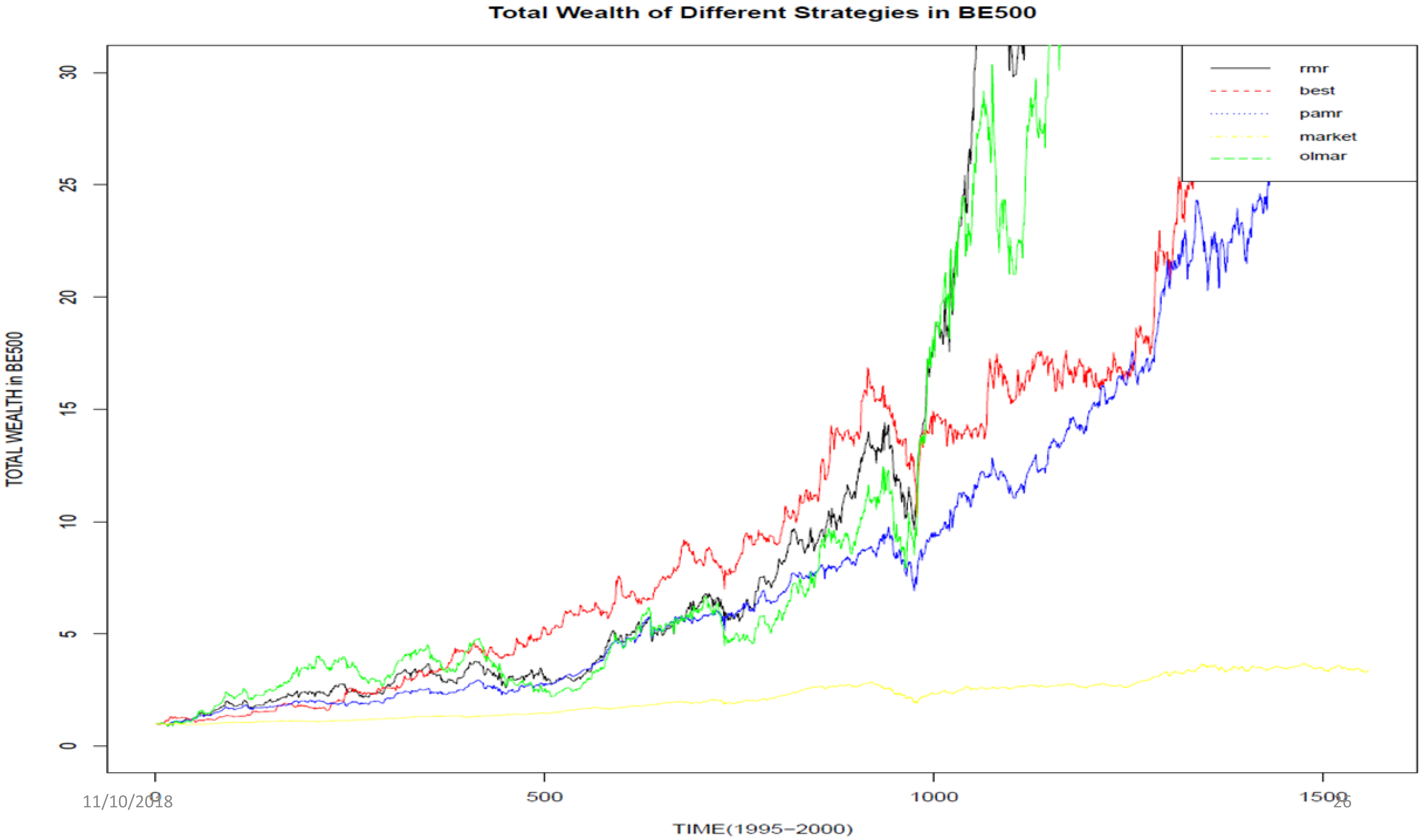
- Datasets

Data set	Region	Time Frame	#days	#assets
BE500	UK	09/01/1995 - 01/01-2018	5993	435
IBOV	BR	04/01/1995 - 28/12/2017	5988	56
NDX	US	04/01/1995 – 29/12/2017	5995	85
NKY	JP	04/01/1995 – 29/12/2017	5990	213
TWSE	TW	06/01/1995 – 29/12/2017	5992	733

- We divided the time series into four parts deliberately.
- 1995-2000/2001-2006/2007-2009/2010-2017


Total Wealth of Different Strategies in NDX
TOTAL WEALTH in NDX
0e+00
2e+05
4e+05
6e+05
8e+05
1e+06
0
500
1000
1500
TIME(1995−2000)
rmr
best
pamr
market
olmar


Total Wealth of Different Strategies in BE500
TOTAL WEALTH in BE500
TIME(1995−2000)
rmr
best
pamr
market
olmar
0
5
10
15
20
25
30
0
500
1000
1500

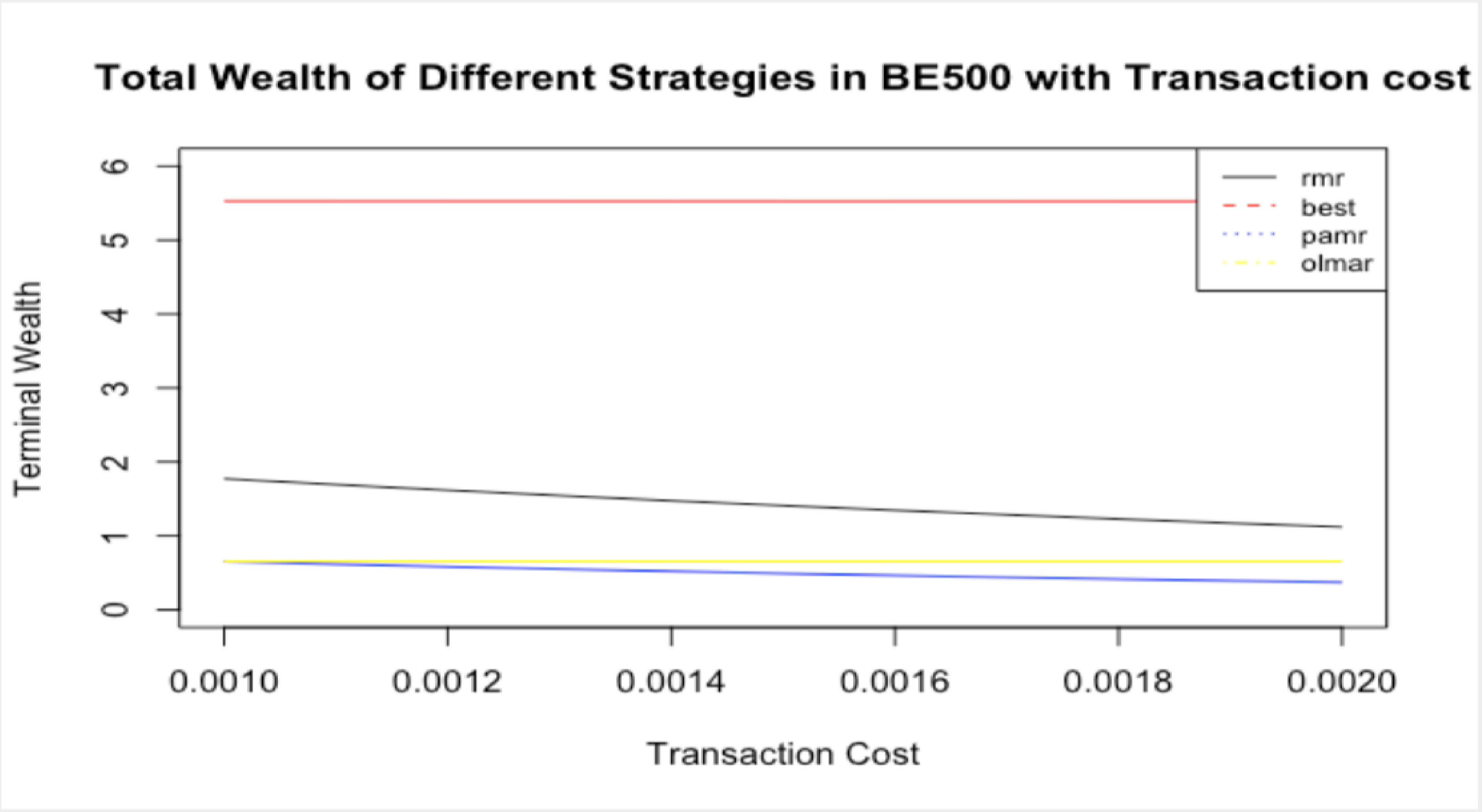
Experiments

- The Comparison of APY, WT, Sharpe Ratio, MDD and CR among BEST, OLMAR, PAMR, and RMR strategy.

Experiment 2					
Dataset	Criteria	BEST	OLMAR	PAMR	RMR
NDX (1995-2000)	APY	1.3091	12.0080	8.7290	13.9750
	WT	0.5013	0.5730	0.5973	0.5775
	Sharpe Ratio	1.6342	12.7110	12.1172	15.3026
	MDD	0.6651	0.4944	0.3706	0.4133
	CR	1.9682	24.2886	23.5520	33.8171
BE500 (1995-2000)	APY	1.1019	2.6569	1.6890	2.0003
	WT	0.5677	0.5469	0.6094	0.5729
	Sharpe Ratio	2.3713	3.9008	5.2323	3.5157
	MDD	0.1994	0.3302	0.1033	0.2134
	CR	5.5265	8.0456	16.3451	9.3727

Experiments-2 (add tc)

- Improvement:
  - Add transaction cost to observe the performance of RMR.

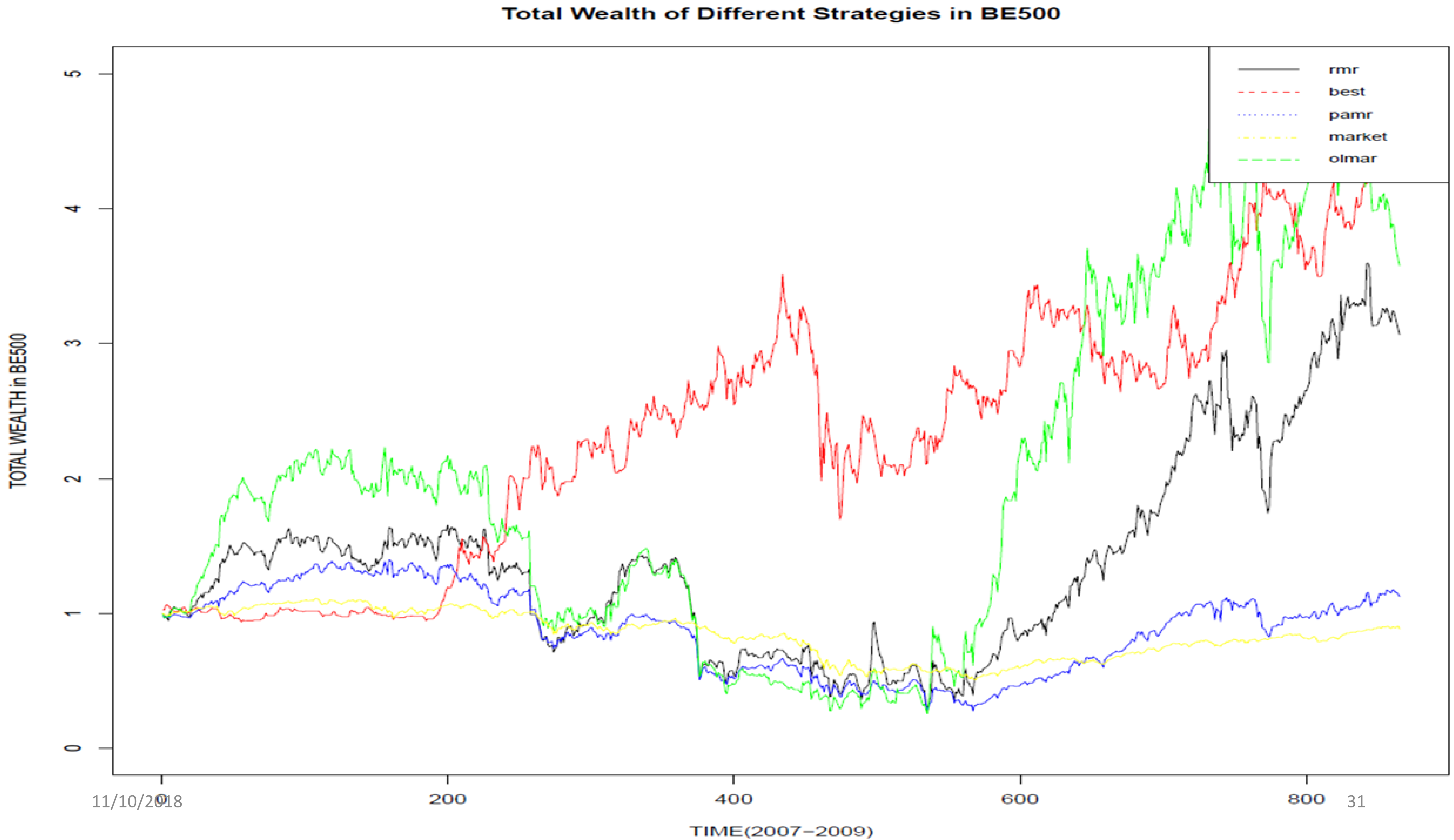


Experiments-2 (add tc)

- Result Analysis:
  - 1. Because BEST strategy has the lowest turn-over rate, which means the lowest transaction cost, BEST has very good performance.
  - 2. RMR's performance is largely affected by transaction cost and it's suitable for market with low tc, like US.

Experiments-2

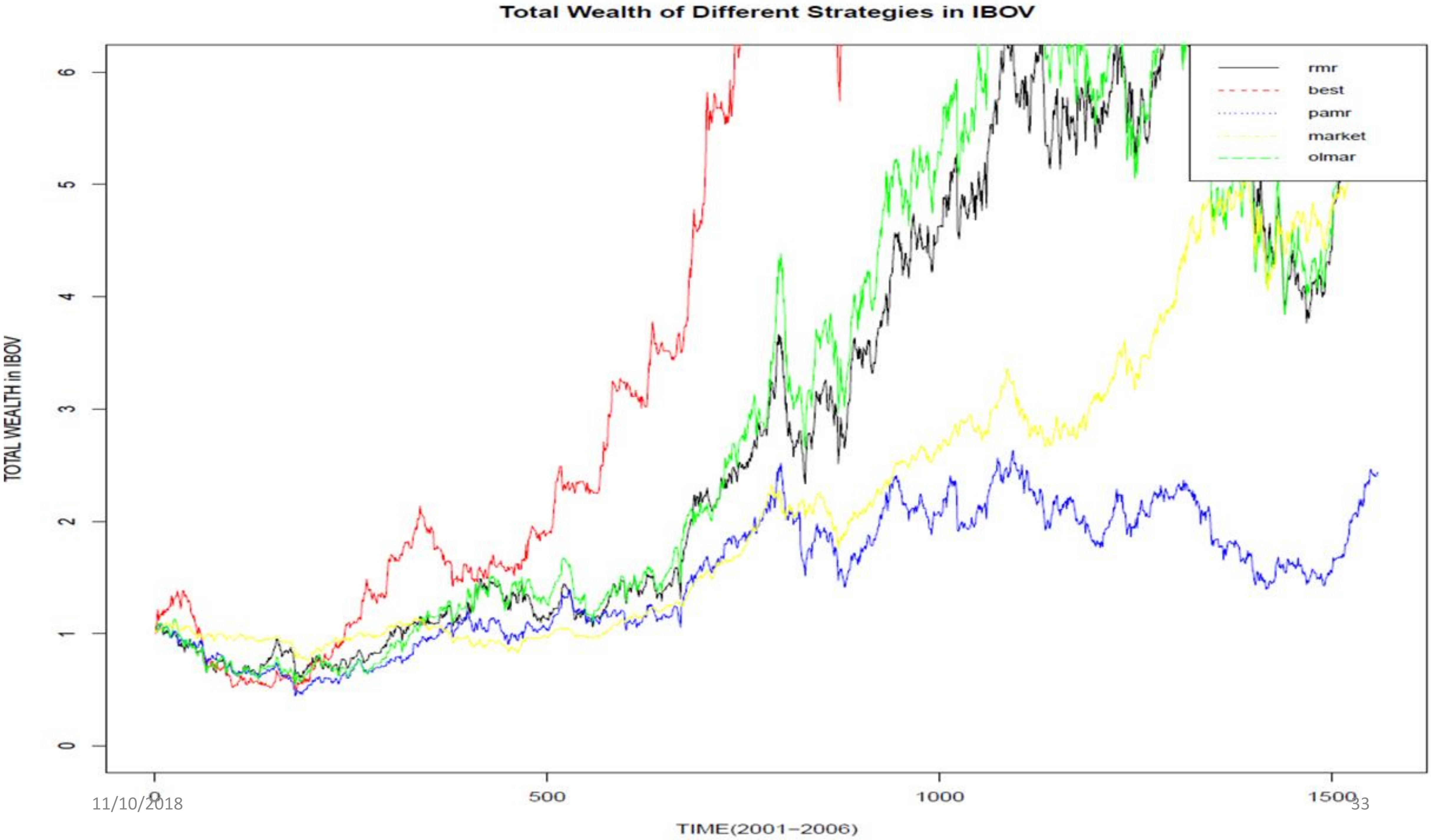
- Exception – BE500 P3
  - Performances during 2017-2018 financial crisis



Total Wealth of Different Strategies in BE500
TOTAL WEALTH in BE500
TIME(2007−2009)
rmr
best
pamr
market
olmar
0
1
2
3
4
5
0
200
400
600
800

Experiments-2

- Exception – IBOV P2
  - Performances in emerging market


Total Wealth of Different Strategies in IBOV
TOTAL WEALTH in IBOV
0
1
2
3
4
5
6
0
500
1000
1500
TIME(2001–2006)
rmr
best
pamr
market
olmar

Experiments-2

- The Comparison of APY, WT, Sharpe Ratio, MDD and CR among BEST, OLMAR, PAMR, and RMR strategy.

Experiment 2					
Dataset	Criteria	BEST	OLMAR	PAMR	RMR
BE500 (2007-2009)	APY	0.7329	0.5293	0.0404	0.4526
	WT	0.5156	0.5075	0.5168	0.5098
	Sharpe Ratio	1.5068	0.4879	0.0007	0.4589
	MDD	0.5174	0.8853	0.8016	0.8322
	CR	1.4167	0.5978	0.0504	0.5439
IBOV (2001-2006)	APY	0.7882	0.4424	0.1764	0.3963
	WT	0.4987	0.5160	0.4859	0.5064
	Sharpe Ratio	1.7534	0.8126	0.3260	0.7485
	MDD	0.6475	0.5152	0.5903	0.4528
	CR	1.2173	0.8587	0.2988	0.8753

Experiments-2

- Conclusion
  - 1. RMR's performance is highly impacted by transaction cost.
  - 2. RMR perform well in most markets but there're some exceptions.
    a) During financial crisis, momentum strategy has better performance.
    b) Mean reversion strategy did not perform best in emerging market, which lacks market efficiency.

Reference

Qlpha

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Q&A