Solving a 1D Second-Order ODE
Due: Wednesday, November 1.

A one-dimensional, damped, driven, harmonic oscillator of mass m is subject to a *restoring force* proportional to the displacement of the mass from its equilibrium position, a frictional *damping force* proportional to the speed of the mass, and a time-dependent, harmonic *driving force*:

$$\sum F_x = -kx - b\dot{x} + F_\circ \cos(2\pi f t) = m\ddot{x}$$

where k is the linear restoring force constant, b is the frictional force constant, $F_\circ$ is the constant amplitude of the driving force, f is the driving force frequency, $x(t)$ is the displacement from equilibrium, $\dot{x} = dx/dt = v_x(t)$ is the x-component of the velocity, and $\ddot{x} = d^2x/dt^2 = a_x(t)$ is the x-component of the acceleration. We will study how this system evolves subsequent to the initial conditions of $x(0) = 12.4$ cm and $v_x(0) = 0$.

- Write a `Matlab` function that can be used with `rk2.m` to calculate both $x(t)$ and $v_x(t)$ for the following values

  $$k = 1.00 \text{ N/m} \qquad b = 0.800 \text{ kg/s} \qquad F_\circ = 0.800 \text{ N} \qquad f = 1.50 \text{ Hz} \qquad m = 1.20 \text{ kg}$$

- Calculate both $x(t)$ and $v_x(t)$ over a time interval for which a *steady state* behavior is reached, *i.e.*, when the oscillations stop varying appreciably. Choose your time step to fully represent all the time-dependent behavior of the system.

- Change the frequency of the driving force f so that *resonance* is achieved, *i.e.*, when your calculations of both $x(t)$ and $v_x(t)$ have their maximum steady-state amplitudes. You may need a different time step and/or time interval than for the off-resonant case. Your value for the resonant frequency need only be correct to within 50%.

- For comparison, obtain the analytic solution for both $x(t)$ and $v_x(t)$ for the *simple harmonic oscillator* ($b = F_\circ = 0$) for our initial conditions.

- Construct three plots — $x(t)$, $v_x(t)$, and the phase portrait $v_x(x)$ — with three, overlaid curves each — the off-resonance case at 1.50-Hz, the on-resonance case, and the analytic solution for the simple harmonic oscillator.

To submit HW09 to D2L for grading:

1. Deposit a copy of your function(s) and the three plots you generated (in JPEG format, with axes labeled) in your HW09 Assignments Submission Folder. There is no need to submit `rk2.m`.

2. Complete the HW09 Survey.

This homework is worth 25 points. The analytic solution to the damped, driven harmonic oscillator is presented in most intermediate-level mechanics texts.