Solving a 2D Second-Order ODE
Due: Wednesday, November 8.

Many scientific spacecraft bound for the distant reaches of our solar system make use of a *gravitational assist* to gain speed. For example, the Cassini spacecraft passed within 1200 km of the surface of the Earth in 2000 and gained 5.5 km/s in speed on its way to Saturn. Consider the gravitational force on an object of mass m at some location $\vec{r}$ with respect to the Earth:

$$m\frac{d^2\vec{r}}{dt^2} = -G\frac{mM_E}{r^3}\vec{r}$$

where G is the Newtonian gravitational constant and M_E is the mass of the Earth. The initial conditions for this exercise are

$$\vec{r}(t=0) = (x_\circ, y_\circ) = (-100.0R_E, 1.500R_E) \qquad \vec{v}(t=0) = (v_{x\circ}, v_{y\circ}) = (25.00 \text{ km/s}, 0)$$

where R_E is the radius of the Earth.

- Write a `Matlab` function for use with `rk2.m` to calculate $x(t)$, $y(t)$, $v_x(t)$, and $v_y(t)$.
- Write a `Matlab` driver program that determines the *distance of closest approach* of the spacecraft to the Earth.
- Make a plot of y versus x, spanning from the initial position to a final position an approximately equal distance from Earth. Indicate on your plot the point of closest approach.
- Write a `Matlab` driver program that determines the speed of the spacecraft after it has passed Earth and is again at a distance approximately equal to the initial distance. Because of conservation of energy, you should find that the two speeds are equal. What?!
- The gravity assist doesn't come from the Earth alone — it comes from the Earth's motion around the Sun. The velocity of the spacecraft with respect to the Sun: $\vec{v}' = \vec{v} + \vec{v}_E$, where $\vec{v}_E$ is the velocity of the Earth in its orbit around the Sun, which we'll take to be exactly in the $-\hat{y}$ direction. *Assume that the Earth's velocity is constant during the time interval under consideration.* Make an overlaid plot of the speeds v and v' versus t, spanning from the initial distance to an approximately equal final distance.

To submit HW10 to D2L for grading:

1. Deposit a copy of your functions and the two plots you generated (in JPEG format, with axes labeled) in your HW10 Assignments Submission Folder. There is no need to submit `rk2.m`.
2. Complete the HW10 Survey.

This homework is worth 25 points.