Exploration of an Eigenvalue Problem

Due: Wednesday, November 15.

Consider the quantum-mechanical, energy-eigenstate problem that we studied in class of a particle of mass m confined to a box of length L:

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\varphi(x) = E\varphi(x)$$

where E is the energy eigenvalue. The solutions to this differential equation that satisfy the boundary condition at $x = 0$, namely that $\varphi(0) = 0$, are

$$\varphi(x) = A\sin(kx) \qquad \text{where} \qquad k \equiv \sqrt{\frac{2mE}{\hbar^2}}$$

for *any* value of the constants k and A. In this exercise, we seek to confirm numerically that we can only satisfy the boundary condition at $x = L$, namely that $\varphi(L) = 0$, if k is an integer times π/L regardless of the value of A.

Consider an electron in a box of length 1.00 Å, and take $A = \sqrt{2/L}$ for consistency with our solution in class.

- Write a `Matlab` function that makes a plot of $\varphi(x)$ versus x for $0 < x < L$ for 10 different values of k spanning $0 < k < 2\pi/L$. This should confirm to you that in this range, the only value of k that works is π/L.
- Repeat the above exercise, using a different range of k, to find the next value of k that works.
- Write a `Matlab` function that makes a plot of $\varphi(L)$ as a function of k for $0 < k < 10\pi/L$, treating k as a continuous variable.

To submit HW11 to D2L for grading:

1. Deposit a copy of your functions and the three plots you generated (in JPEG format, with axes labeled) in your HW11 Assignments Submission Folder.
2. Complete the HW11 Survey.

This homework is worth 15 points.