Problem Statement 1: [100 marks]

A company manufactures LED bulbs with a faulty rate of 30%. If I randomly select 6 chosen LEDs, what is the probability of having 2 faulty LEDs in my sample? Calculate the average value of this process. Also evaluate the standard deviation associated with it.

Solution 1: Let X be the random variable representing faulty bulbs in a sample

a) $X \sim Bin(6, 0.3)$, therefore the probability is given by

$$P(X = 2) = \binom{n}{x} * p^x * (1-p)^{n-x}$$

$$= \binom{6}{2} * 0.3^2 * (1-0.3)^{6-2}$$

$$= 15 * 0.09 * 0.2401 = \mathbf{0.324135}$$

b) The average value is given by $E(X) = n\,p = 6 * 0.3 = \mathbf{1.8}$

c) The standard deviation associated with it is given by $s = \sqrt{Var(X)} = n\,p(1-p)$

$$= \sqrt{6 * 0.3 * (1-0.3)}$$

$$= \sqrt{1.8 * 0.7} = \mathbf{1.122}\ (to\ 3dp)$$

Problem Statement 2: [100 marks]

Gaurav and Barakha are both preparing for entrance exams. Gaurav attempts to solve 8 questions per day with a correction rate of 75%, while Barakha averages around 12 questions per day with a correction rate of 45%. What is the probability that each of them will solve 5 questions correctly? What happens in cases of 4 and 6 correct solutions? What do you infer from it? What are the two main governing factors affecting their ability to solve questions correctly? Give a pictorial representation of the same to validate your answer.

Problem Statement 3: [100 marks]

Customers arrive at a rate of 72 per hour to my shop. What is the probability of k customers arriving in 4 minutes? a) 5 customers, b) not more than 3 customers, c) more than 3 customers.

Give a pictorial representation of the same to validate your answer.

Solution 3: Let X be the random variable that represents the number of customers who arrive at a shop. $\mu = \left(\frac{72}{60}\right) * 4 = 4.8$, thus $X \sim Po(4.8)$

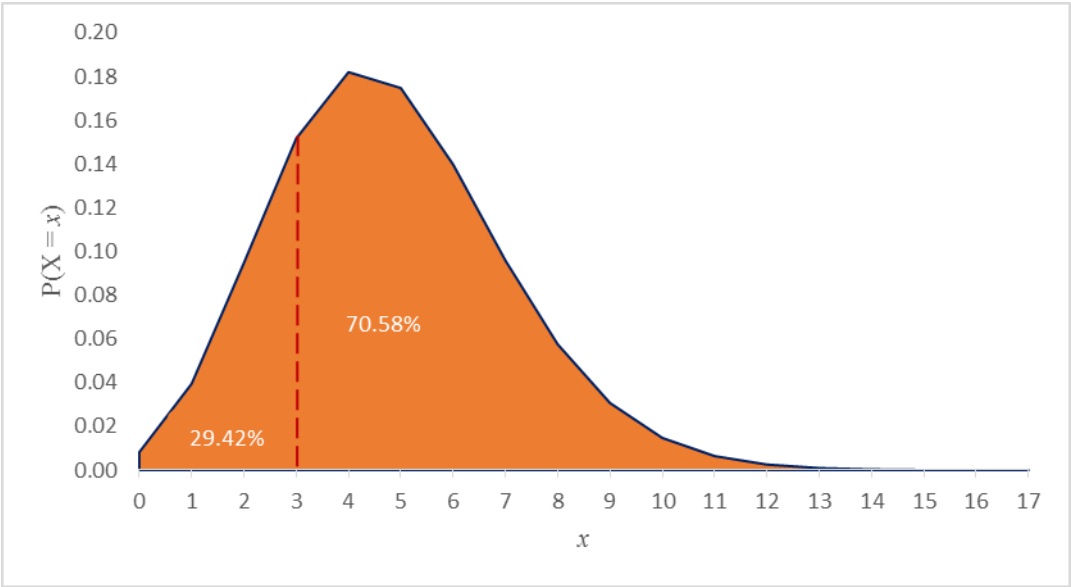
a) $P(X = 5) = \frac{e^{-\mu} * \mu^x}{x!} = \frac{e^{-4.8} * 4.8^5}{5!} = \mathbf{0.1747}$

b) $P(X \leq 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$

$$= \frac{e^{-4.8} * 4.8^0}{0!} + \frac{e^{-4.8} * 4.8^1}{1!} + \frac{e^{-4.8} * 4.8^2}{2!} + \frac{e^{-4.8} * 4.8^3}{3!} = \mathbf{0.2942}$$

c) $P(X > 3) = 1 - P(X \leq 3) = 1 - 0.2942 = \mathbf{0.7058}$

d) The pictorial view of this is given by



Problem Statement 4: [100 marks]

I work as a data analyst in Aeon Learning Pvt. Ltd. After analysing data, I make reports, where I have the efficiency of entering 77 words per minute with 6 errors per hour. What is the probability that I will commit 2 errors in a 455-word financial report? What happens when the no. of words increases (in case of 1000 words) or decreases (255 words)? How is the λ affected? How does it influence the PMF? Give a pictorial representation of the same to validate your answer.

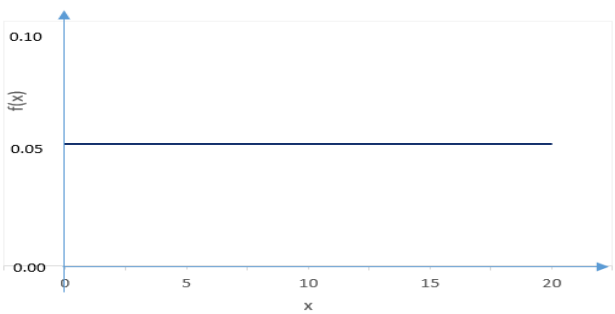
Problem Statement 5: [100 marks]

The current measured in a copper wire is modelled by a continuous random variable (is in mA.) Assume that the range of X is [0, 20mA]. The probability density function is given by $f(x) = 0.05$ *for* $0 \leq x \leq 20$. *What is the probability that a current measurement is less than 10 milliamperes? Draw the PDF and the CDF diagrams as well.*

a)

$$P(X < 10) = \int_0^{10} 0.05\,dx$$
$$= 0.05x\Big|_0^{10}$$
$$= 0.05(10) - 0.05(0)$$
$$= 0.5$$

b) The PDF diagram is as follows



c) The CDF diagram is given below