Restructuring Conceptual Intuitions Through Invented Notations: From Path-Making to Map-Making[1]

Jeanne Bamberger

I
INTRODUCTION
Theoretical Background

In this chapter I discuss a single, closely worked out example in which one child (whom I shall call Brad) invents a series of notations, each one a significant transformation of the one before. The transformations occur during one brief session (about 45 minutes) as Brad works with a set of tuned bells to construct, reconstruct, and play the nursery tune, "Hot Cross Buns." Each notation mediates a process of conceptual restructuring for Brad, while at the same time revealing to the observer his changing understanding of the entities and relations of the tune. Except for a few moments of interruption, all of Brad's work was recorded on videotape.

The transformations that occur in Brad's work in this one session are particularly remarkable in that they closely resemble the restructurings that more commonly occur among children working on similar tasks over several months and then only with adult interventions (see Bamberger, 1995). Moreover, the developmental distinctions among children's spontaneously invented notations surprisingly mirror the specific developmental distinctions Piaget finds among children's descriptions of familiar walks through the city. I have called these transformations a process of moving from path-maker to map-maker. In tracing Brad's work in this single session, we follow him through a series of conceptual transformations that encapsulate and compress this process of moving from path-maker toward map-maker. Moreover, I will claim that in following Brad we are able to watch one of those rare instances in which spontaneous and significant learning is actually occurring in real time.

I.A
A Proposal

My proposal for how this process may evolve is borrowed, in part, from Bartlett's seminal book, Remembering:

> An organism which possesses so many avenues of sensory response as man's, and

[1] This chapter originally appeared as "Restructuring Conceptual Intuitions Through Invented Notations: From Path-Making to Map-Making" In E. Teubal,J. Dockrell & L. Tolchinsky (eds) *Notational knowledge:* Developmental and historical perspectives. Rotterdam:Sense Publishers

> which lives in intimate social relationship with numberless other organisms of the same kind, must find some way in which it can break up this chronological order and rove more or less at will in any order over the events which have built up its present momentary 'schemata'. It must find a way of being dominantly determined, not by the immediately preceding reaction, or experience, but by some reaction or experience more remote. (Bartlett, 1932, p. 203)

Following Bartlett, I call these well-trodden, well-practiced action sequences, *felt paths*. Other common examples of felt paths on a small scale might be tying your shoes, starting up your car, or typing your name. If I choose to look selectively at some part of my felt path actions (do I put my shoe lace under or over?), I falter as I "break up this chronological succession." And in doing so, I may also "liberate" from the meld, a new aspect, a new idea.

I focus on Brad's work because it exemplifies the powerful potential for learning in these moments when a practiced, habitual action path is unexpectedly disrupted. As Brad's work will demonstrate, such disruptions may lead to the construction of a new succession of actions and with that to the emergence of new elements and the liberation of new structural relations. Bartlett again:

> There is one way in which an organism could learn how to do this. It may be the only way. At any rate, it is the way that has been discovered and it is continually used. An organism has somehow to acquire the capacity to turn round upon its own 'schemata' and to construct them afresh. This is a crucial step in organic development. (Bartlett, op cit, P. 206)

I argue further that a critical step towards learning to use symbols that populate notation systems is becoming able, at will, to interrupt a unique, contiguous, well-practiced action sequence. This is because all symbolic expressions, those invented by children as well as those associated with a community of professional users, are necessarily partial and they are so in two senses: they are partial in being incomplete, and they are partial in that they favor, or are partial to certain aspects of the phenomena while ignoring others. One way we can come to understand the particular selections implicit in a given notation system and to use the referents appropriately, is by breaking up our own well-practiced contiguous sequences so as to find and focus on the particular selections that constitute a given conventional notation.).

Finally, and perhaps most important, I will argue that the goal of learning is not to overcome behaviors associated with path-making or map-making, but rather to have access to multiple ways of perambulating and seeing the world and thus to be able to choose

depending on when, where, and what for.[2]

II

The Task and The Materials

I begin with a description of the task followed by a brief look at typical musical path-makers in contrast to typical musical map-makers. The task presented to some 50 children between the ages of 8 and 12 and as it was presented to Brad, is as follows:

> "Build Hot Cross Buns with your bells, and then make some instructions so someone else can play the tune on your bells as you have them set up."

In preparation for the task, each child is given a mixed array of seven Montessori bells and a small mallet with which to play them.

Figure 1: Seven bells and a small mallet

The Montessori bells are a rather extraordinary technological invention. Unlike any other musical materials that play different pitches, these bells all look alike. This is in contrast to, for instance, a xylophone bar that is relatively longer and is also lower in pitch or a piano key to the right of another, which is also relatively higher in pitch. Thus, a child working with these bells must find differences in pitch only by listening.

Each individual mushroomed-shaped metal bell is attached to a wooden stem, with bell and stem, in turn, standing on a small wooden base making it easy to move them about. Some stand on brown bases and others on white bases. While this single difference does not influence the pitch properties of the metal bells, themselves, the bells were designed so that a bell with a white base matches one other bell that has a brown base. This feature was useful for certain Montessori projects, but was not told to or used by the participants in this study.

[2] See also, Chapter 1—Revisiting…..

Path-makers

Path-makers build tunes by constructing a cumulating bell-path in which each bell is added in its order of occurrence in the tune. Thus path-makers begin by searching through the mixed array of bells on the table for a bell that they recognize as playing the first event in the tune. They place the found bell in front of the mixed array on the table to begin their cumulating bell path.[3]

Figure 2: The bell for the first event

Going on, they search for and find a bell that sounds the next tune event, placing it (usually) to the right of the previously found bell.

Figure 2a: The first two events

They continue on in this way, adding each found bell to their cumulating bell-path, next-next-next. The result is a bell path where each bell has been added in order of occurrence in the tune (see Figure 3).

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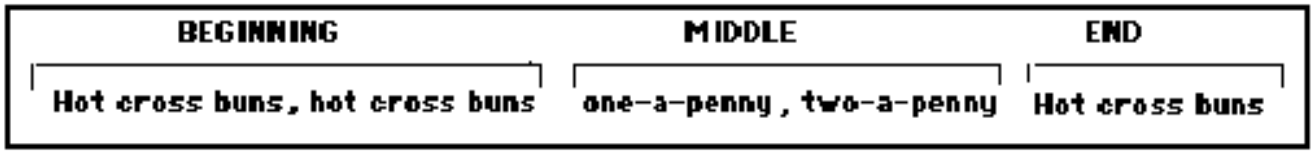
[3] In general after playing through the bells several times, tunebuilders have little trouble finding the bell with which to start the tune. This is another example of how context (here the given pitch collection) can generate structural functions—here, the beginning event..

Figure 3: Hot cross buns--Bell Path

As a consequence of the sequential building procedure, the sequence of bells in a path-maker's bell path makes a unique, "one purpose" instrument--it is made to play just this tune. Further, the spatial structure of the bell-path--three bells, a gap, then two bells-- is a physical embodiment of the motivic grouping or *figural* structure of the tune--a "figure" being the smallest meaningful structural element of a tune (see Figure 4).

Figure 4



The three-part, figural structure of the tune

The first figure, which goes with the words, "Hot cross buns," is embodied and played by the initial group of three bells. The second figure, which goes with the words, "One-a-penny; Two-a-penny," is embodied and played by the following group of two bells.

The whole tune is shown in standard music notation in Figure 5. Notice that the tune includes immediate repetition of the first figure, followed by the second figure which forms the middle of the tune, and ends with the return to the first figure again.

Figure 5

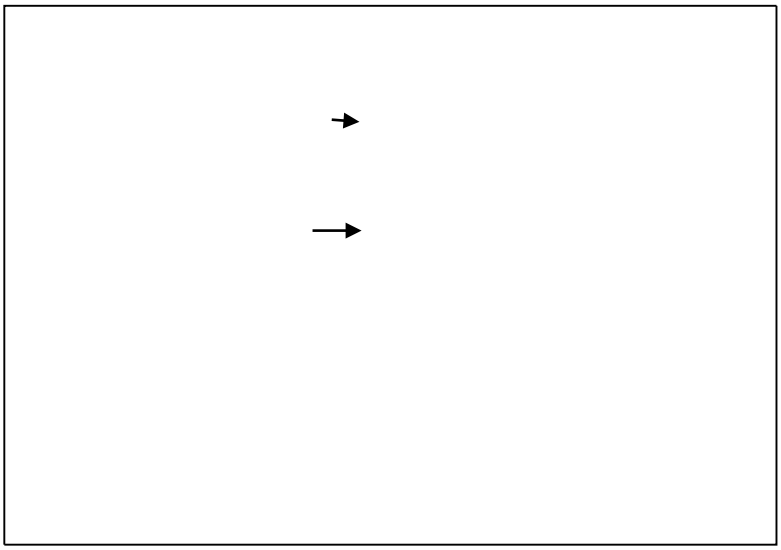
Hot Cross Buns

With regard to classifying pitches, the reader will notice that the second figure includes two of the pitches (C and D) that are already present in the first figure. However, typical path-makers (children and adults) consistently fail to recognize that these bells "sound the same." This is quite understandable, and indeed demonstrates a fundamental characteristic of path-makers: when constructing the tune by adding pitches in order of occurrence and also in playing the tune, the builders are strongly responsive to the context in pitches occur. In this case and most importantly, the respective functions of matching pitches is entirely different within these contexts. For example, the C in the first figure has a longer duration and is last in a descending progression. As a result, this first C-bell functions as an ending, the boundary-marker of the first figure. The C in the second figure has a shorter duration, is repeated, comes after a figural boundary and as a result this C-bell functions as a beginning, a start-up after the "gap" of the previous boundary. It is for all of these musical/functional reasons that path-makers do not recognize the bells as sounding the same pitch. However, when asked specifically to listen for pitch matches, these participants have no trouble hearing that they "sound the same." However, participants are usually surprised to discover that matching bells were actually embedded in the tune. This is strong evidence in support of the view that path-makers do not listen across boundaries of figures and that their focus remains on the function of events within the figures of which they are members. (For more, see Bamberger, 1995).

To play the whole tune, path-makers make an action-path through their bell-path which also has certain special characteristics: As a consequence of building the tune by ordering the bells, left->right, in their order of occurrence, the predominant direction of the action-path is also left->right. However, there are three notable exceptions mediated by the structure of the tune, itself (see Figure 6).

Figure 6[4]

[4] An **x** marks a tap on a bell, while the lines and arrows mark the direction of the path-maker's actions through the bells.



First, the repetition of the beginning figure played again on the first three bells, requires a "turn around" in the action-path, a move "back" or right->left. . A second exception occurs in the middle part of the tune when single bells must be repeated. Thus the player remains in one place even while the tune goes on. And finally, since the tune ends as it began, another turn-around is required to "go back" to play the beginning figure, again.

These moments of exception in the prevailing direction of motion form a sequential series of landmarks which also mark the boundaries of figures. The landmarks thus shape the structure of the action-path that, in turn, coincides with the larger structure of the tune, itself.

Typically, Invented notations for "Hot cross buns" are "copied" on the paper as stick-pictures or sometimes simplified copies of the bells. The notation is basically an "iconic" trail-map, as shown in Figure 7.[5]

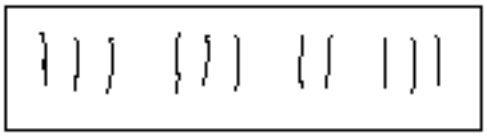


Figure 7: Iconic trail path

However, notice that the five bells on the table have become eleven lines on the paper. This demonstrates what diSessa et al describe as *metarepresentational competence* (diSessa et al, 1991). That is, by learned convention, a temporal/action dimension of a performance--moving on in time by again playing the first figure, is shown in space by drawing the group of three lines again, the second group representing the group of three bells which is to be played again. The three lines are drawn still again at the end to show that the same three bells are to be played once more to complete the tune. So, "coming after" in paper space can substitute

[5] For more on path-makers notations, see Bamberger, 1995)

for “do it again” in action-space. But interestingly, even though the two bells in the middle figure are repeated several times thus going on in time and action, the lines representing the bells are each drawn only once. The difference here may be related to whether the player moves along in space on the bells, as in the first figure, or stays put on one bell and in one place as in the middle figure.[6] If so, the spatial dimension in action (staying put in one place) is reflected by drawing the single bell rather than the actions on it.

Musical and Spatial Path-Makers

As suggested earlier, there are interesting similarities between a musical path-maker's action path through the bells, and a path-maker who is a walker in the city. For instance, Kevin Lynch in his book, "The Image of The City” notices that a path-maker follows:

> "...a sequential series of landmarks, in which one detail calls up anticipation of the next and key details trigger specific moves... In such sequences, there were trigger cues whenever turning decisions must be made and reassuring cues that confirmed the observer in decisions gone by..." (Lynch, 1960: 83).

Thus changes of direction (“turning cues”) help to segment a journey and as such to mark landmarks that form the boundaries of spatial "figures" -- spaces traversed by actions bounded by landmarks. And just as these landmarks are clues for the walker in the city, so changes of direction in actions on the bells form a series of landmarks that mark the path through the tune, "Hot Cross Buns." And like musical path-makers, walkers in the city move ahead within figures but do not construct relations across figures or among landmarks.

The experience of musical path-makers, like path-makers in the real world, is of a journey that is paradoxically always in the immediate present while always going on. And the sense (both as feeling and as meaning) of this passing present is formed by the context of where the path-maker just came from, while the passing present forms the context, in turn, for where he/she is going. Thus, for path-makers, there is no comparing where they are to where they have been because there is no stepping off the continuing path, and no means for comparing events that are distanced from one another in time/space. Christopher Hasty puts the situation this way in relation to musical experience:

> *But because of [an event’s] particularity of unrepeatability, we shall not be able to retrieve such a past event for a postmortem. It would seem, too, that its duration has also vanished. True, the event has not vanished without a trace, but that trace is the*

[6] Children often play the middle part of the tune with only three repetitions on each bell even though the song, properly sung, includes 4 repetitions. This also provides some evidence that the children do not usually know the words for the song—i.e., one-a-pen-ny needing four notes not just three. .

mark the past can make on the present—on a new event or events, each with its own individuality and freedom. [Hasty, 1997: 4]

Map-makers

Musical map-makers differ from path-makers right from the beginning of their work on the task. As if needing to put themselves in order, these players put aside the matching C and D-bells, and arrange the remaining 5 unique bells from lowest to highest proceeding from left (low) to right (high).

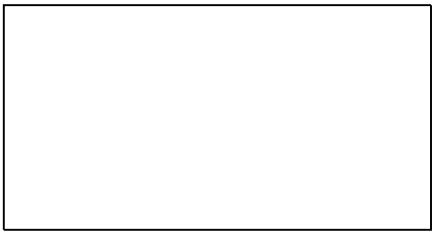


Figure 8: A Fixed Reference Structure

Much like seriating a mixed array of sticks that are graduated in height, each bell added to the right is "higher" than the one to its left, and each bell added to the left is "lower" than the one to its right. Map-makers are "uncomfortable" until they have first built this fixed reference structure--an all-purpose instrument in terms of which they "know where they are" and can plot this and many other tunes. Thus, map-makers initially focus their attention on the de-contextualized, pitch properties of the bells in contrast to the path-maker's focus on situational function within the figures of a particular tune as this function results from their unique order of occurrence.

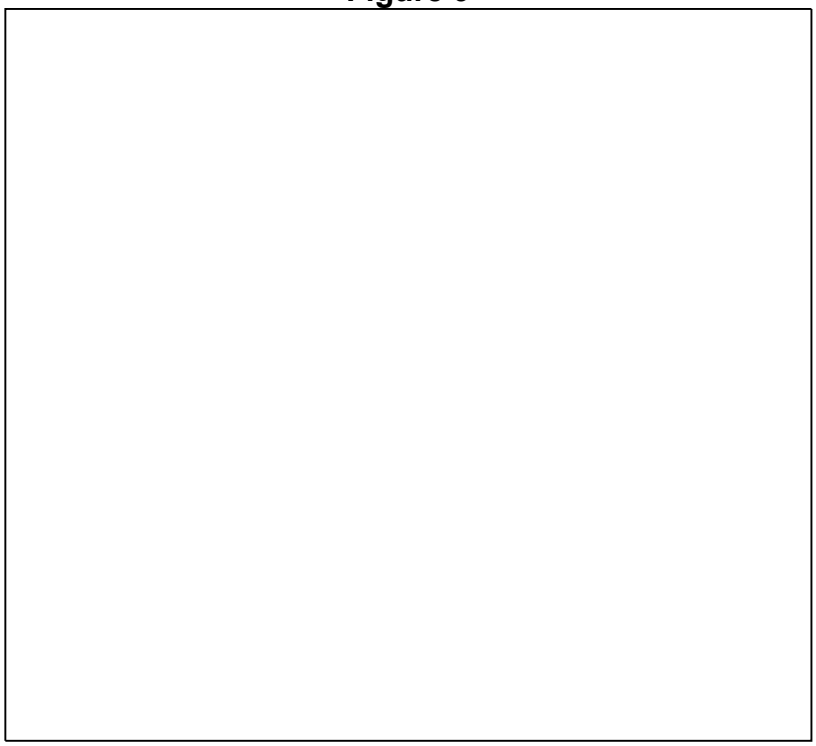
The property-ordered structure is a fixed reference in that it is outside of any one tune and yet, in content, common to many. And because its structure is based on the low-high ordering inherent in pitch properties, themselves, the structure also implies a "unit of pitch distance." This unit can be used to measure, along the reference structure, the distance between any two pitches and through this measure to compare, on at least this dimension, the structures of one tune with another.[7]

Perhaps these tune builders are like travelers who are dependent on their printed map for finding their destination--looking at it instead of the objects and events that, for the path-maker, shape the landmarks, the figures, and the feel of a particular moment along the way. Indeed, compared with a path-maker's bell-path, the pre-ordering of the bells can hardly be called a "path" at all; rather, like a map, it is an ordered terrain on which to trace a particular

[7] Since this collection of pitches includes only a subset of the possible (12) pitch classes used in Western music, strictly speaking, the ordered collection does not provide an invariant unit with which to measure "pitch distance." However, traditionally the “distance” from one pitch in the series to another is measured by counting the inclusive number of steps—e.g., from C to G is an interval of a 5th, while D to F is an interval of a 3rd, etc..

action path (see Figure 9).

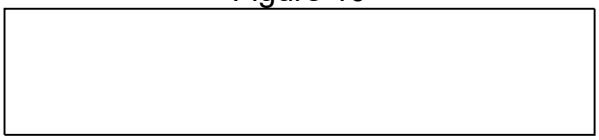
Figure 9



Map-maker’s action-path on the fixed reference terrain

As might be expected, the map maker's notation is no longer iconic but rather symbolic. There are no pictures of bells, only numbers (see Figure 9).

Figure 10



To construct their notation, map-makers number the bells from 1-5 going low to high--assuming the bells organized in the all-purpose fixed reference terrain. Then, playing the tune, they "look up" the number of the serial position in the reference structure for each tune-event in turn. They, so-to-speak, peel off each found number, copying it to the paper. Iterations of this process result in a row of numbers that designate, or point to, a sequence of bell positions and these, in turn, target the new player's moves to the sequence of bells that plays the tune.

Notice that map-makers choose just five bells from the initial seven in the mixed array, the five bells on the table include just one bell of each pitch-type (C D E F G]. Of these five bells they need only three bells (C, D, E) to play the whole tune.

This is in contrast to path-maker's who also have five bells on the table but not at all the same five bells as map-makers. Since path-makers include bells only as they are needed in building up the tune chronologically, they do not include the F and G-bells; they make up the five bells by including doubles--i.e. two C's and two D's. As pointed out earlier, the doubles are necessary for path-makers because each one--each C and each D, has its distinct function within the boundaries of the figure in which it occurs. These differences are critical to the transformations in Brad's work as he moves from path-maker towards map-maker.

The Builders Compared

The differences between musical path-makers and musical map-makers may become more focused by comparing and contrasting them with the differences that Piaget describes between younger and older children's descriptions of journeys through the city. Piaget says of younger children:

> *...each journey shows a particular vantage point and [the children] are unable to bridge the gap between the privileged vantage point of one journey and the next... [E]ach is unique and therefore they cannot coordinate all the features in an area taken as a whole*. (Piaget, 1960: 16).

While with older children:

> ...*each vantage point is no longer unique. The link between any two landmarks can be conceived of as dependent on the system as a whole. [Children] can now relate any one part to all of the remaining parts.* (Piaget, op.cit.: 18)

Coupling Piaget's remarks with those of Lynch (quoted above P. x), we can say that musical path-makers like younger children, construct meanings in relation to the sequence and unique function of contextually situated reference objects or events (landmarks), where the occurrence of each object/event is a necessary condition for triggering the next move. Map-makers depend for meanings on the mental construction of situation-independent reference structures in which objects/events are linked to one another and placed in a single coordinate space, and where distances among them can be measured independently of their occurrence or function in any particular situation or sequence of actions (see also, Bamberger & Schön, 1991).

However, unlike Piaget who associates these differences with age and stage of development, I argue **that experienced musicians make use of both paths and maps and, in fact, move effectively between and within them in order fully to participate (as listeners and performers) in the complexity of a complex piece of music**.

III

The Story of Brad

In his first phase of tune-building, Brad's work clearly resembles that of path-makers--albeit with certain important exceptions. In the later phases of his work we will see transformations that lead him toward becoming a map-maker. While the beginning and ending points of Brad's 45 minute session resemble the differences Piaget finds in children's earlier and later descriptions of their walks, the findings must be differentiated from Piaget's work not only in content, but also in experimental context.

- Time:

  Piaget gives us brief "snapshots" of different children at different times and at different ages and stages of development. The observations and discussion of Brad's work involves this single child and the conceptual changes that take place over a single time period (about 45 minutes).

- Setting:

  The setting is not a neutral one. Brad's work is carried out in the context of The Laboratory for Making Things. His notational inventions are influenced by the work of the five other eight and nine-year-old children who were also working in the Lab on the same task.

- The Lab Culture

  Brad is also influenced by the characteristics of a culture that has developed in the Lab over the seven years of its existence.

As an integral part of this culture, children were accustomed to informal conversations in which they explained to one another or to an adult how they were making sense of their work and their building materials--blocks, foam core, drums, legos, bells. They were also used to inventing some kind of graphic instructions/notations that could help someone else build what they had built. As in Brad's work, this collaborative reflection led to learning from one another--rethinking understandings and descriptions, subsequently even influencing work on later projects that involved quite different materials.

As another part of this culture, children moved freely between building working structures with hands-on materials, and building working structures (graphic designs and also melodies) using the computer as a medium. As a result of this movement back and forth, certain kinds of ideas became part of the culture, influencing and illuminating the children's understanding across all the media—for instance, The sense that it is useful and interesting to look for "patterns," and closely related, the idea of "chunking" or grouping which initially grew out of the children's need to segment the continuous flow of a melody so as to tell one another what they heard as structural elements. Issues around "chunking" became most concrete in the

children's frequently heard, but rather surprising question: As they examined one another's constructions, we would hear them ask the builder, "So what is a THING, here?"

I have grouped Brad's work into six phases. Each phase marks a stage in the transformation of Brad's tune-building strategies and his notation, and these, in turn, are evidence for changes in his way of understanding the tune, itself--its constituents and their relationships to one another.

I describe Brad's work in detail because, at this level of detail, the story provides an unusual glimpse into creativity as learning in real-time generating fundamental cognitive transformation through the interaction of action, symbol, and meaning.

IIIA
PHASE I:
Brad as Path-maker

Labeling the bells

Brad begins the task of building "Hot Cross Buns," by giving himself another task—namely, labeling the bells. (see Figure 9). Cutting out five paper squares, Brad writes a number from 1-5 on each one. Then, without playing them, he puts his 5 bells out on the table in a row just as they come to hand, and puts a numbered square, 1-5, in front of each bell, right-to-left. Brad has invented a novel strategy for naming the bells.

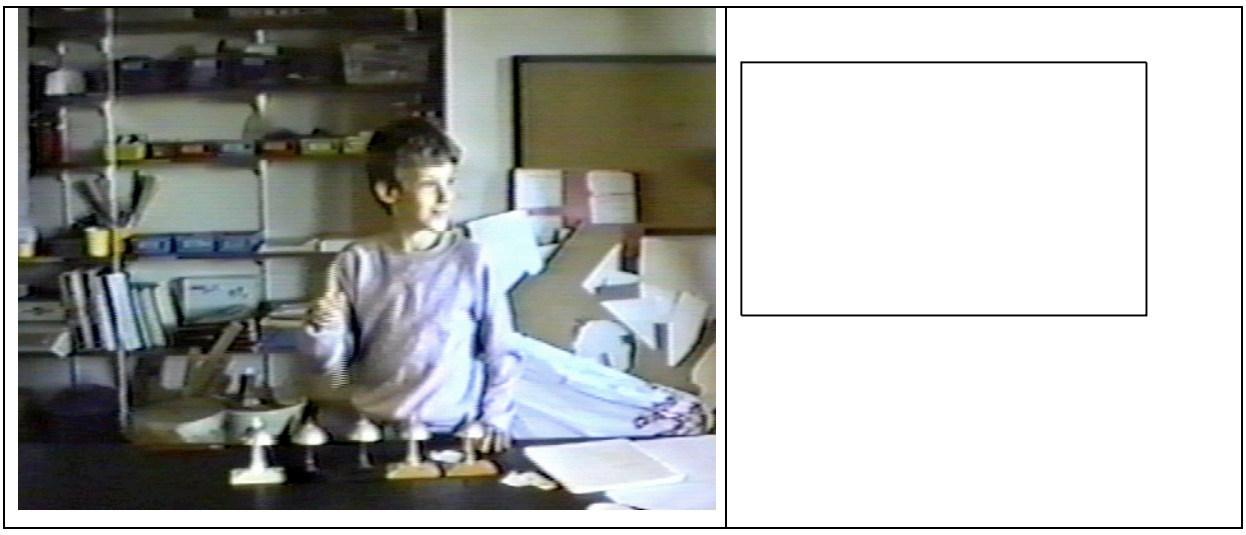


Figure 9: Brad lines up and labels the bells without playing them

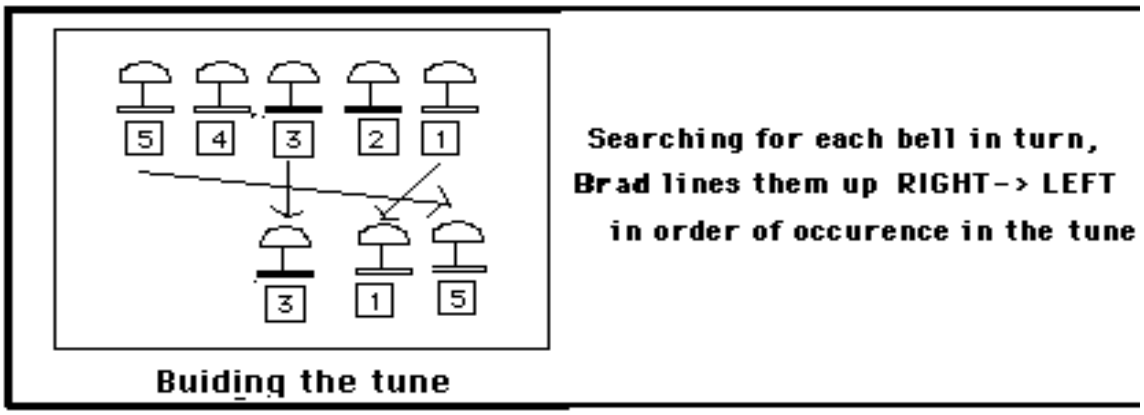
Brad's numbers, 1-5 may look similar to the sequence of map-makers' numbers, but the

meaning of the numbers is entirely different. Brad has not yet played the bells, so at this point the numbers refer simply to the position in the row that the bells happen to have fallen into. Map-makers' ordering and the numbers assigned to the bells refer, of course, specifically to the relations among their perceived but hidden pitch properties: 1 2 3 4 5, refer to the low (1)-high (5) ordering of pitches. While Brad's numbers have no relation to the pitch properties of the bells, as long as a number remains attached to its bell, the number serves as a label with which to identify and refer to it as an individual involved in the tune.

Building the tune.

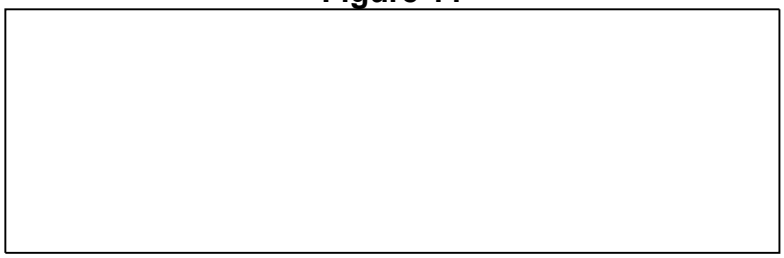
To build the tune, Brad must obviously play and listen closely to the bells. Listening and searching in his line-up, he finds the bell with which to begin the tune—it happens to be the last bell at the end of his arbitrary line-up, the bell labeled 5. Brad places the found bell in front of the others, carrying the label (5) along with it, as the beginning of the path that will play the whole tune. Playing his first bell (5) again, as if starting from the beginning of the tune, Brad searches through the remaining bells to find the bell for the next event. Placing it to the left of the first bell, he again is careful to move its arbitrary label (1) along with it (see Figure 10).

Figure 10



Continuing on in this way, Brad transforms his initially silent line-up into a typical bell-path where each bell is added in its chronological order in the tune. The result is a row of bells accompanied by what could appear to be a meaningless list of numbers, 2 4 3 1 5, going from right-to-left (Figure 11). But the labels will serve Brad's purpose well in making his "instructions."

Figure 11



While Brad's sequence of bells "holds" the sequence of tune events in the order in which they appear in the tune, his resulting bell-path differs from that of the typical path-maker in two ways: he consistently works right-->left while most [ath-makers work left-->right, and his bell-path has a corresponding "number-path" using the labels he initially attached to the bells.

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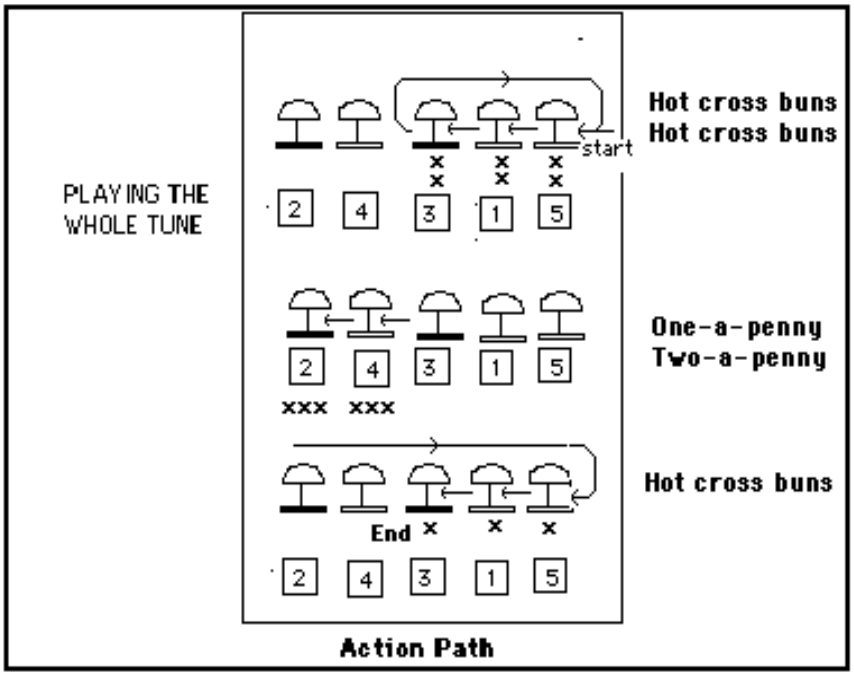
Making an action path

Moving along on his bell-path, playing each bell in turn, Brad makes an action path through his uniquely built instrument to play the whole tune. Brad's action path is exactly the same as the typical path-maker's except for its prevailing right-->left direction (Figure 12).

Figure 12:



Brad's action path

As with other path-makers, Brad's action-path includes three notable exceptions to the prevailing direction (here, right-to-left), each of them mediated by the structure of the tune, itself: first, the repetition of the initial figure requires a move "back" or left -->right; second, the

middle figure requires the repeat of tune events on single bells, and third, another move left-->right to play the first figure again (Figure 12).

The first notation

To "make instructions so someone else can play the tune on the bells the way they are set up," Brad now makes use of the labels that he has attached to the bells.
He copies each number labeling a bell onto his paper in the order in which he plays them.

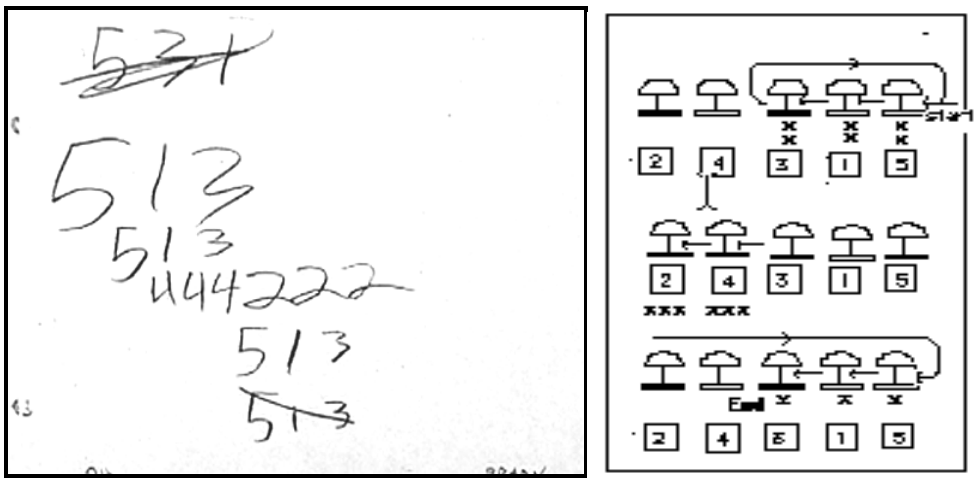


Figure 3:
Notation 1

Brad's notation strategy bears a similarity to map-makers' notation strategy in that he uses the symbols, the number-labels, as a guide rather than iconic stick-figures, but again there are obvious differences in just how the numbers refer. (Figure 14).

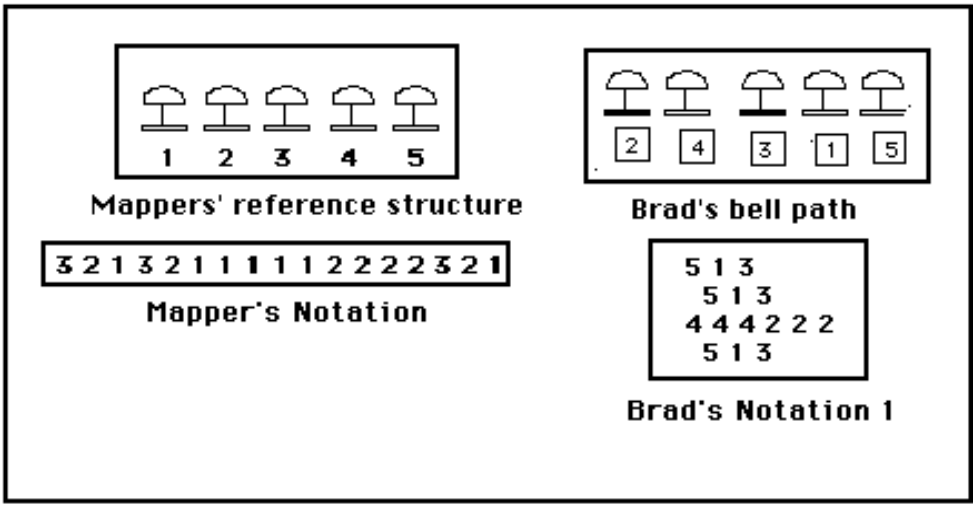


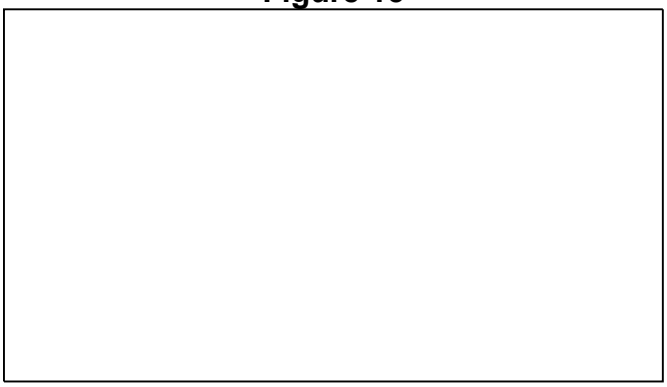
Figure 14: Comparing notations

Brad's numbers tell the player which bells to play and in which order but the numbers tell the player nothing about the pitch properties or the pitch relationships that create the tune. In contrast, the numbers in map-makers' notation not only tell the player which bells to play and in which order but the numbers refer specifically to the properties and relations among the bells. Thus, the reader who is familiar with the structure of this all-purpose "instrument," on seeing 3-2-1 will know she is going "down stepwise" from high (3) to low (1) moving right-->left along a familiar, fixed reference series. The reader of Brad's instructions must simply follow the swquence of the numbers he has written on the page and match them directly to the sequence of labels as they occur on the bells.

And there is another more significant difference: Brad's notation is unique in that he spatially groups his numbers on the page (see Figure 13). In contrast, both path-makers and map-makers write out numbers or pictures in a single row straight across the page. Brad's spatial groupings are significant because they mark the boundaries of figures and in this sense they also provide an interpretive analysis of the tune. The notated boundaries most noticeably coincide with changes in direction in his action path: There is the switchback in the prevailing right-left direction for the immediate repetition of the opening figure **(5 1 3)** and also its return at the end. The changes in direction "bundle up" these events helping to generate, along with the repetitions themselves, the figural or motivic grouping boundaries of the tune. The middle group which Brad notates as **4 4 4 2 2 2,** is initially bounded by the move onward to new bells and the repeated events played on these single bells, and bounded again by the subsequent return to the beginning figure. The spatial grouping boundaries in Brad's notation thus mark landmarks that, in turn, mark the boundaries of melodic figures. In short, Brad's notation is a kind of structural analysis of the tune, reflecting aspects that are not shown at all in map-makers' notations nor, indeed, in conventional staff notation.

Finally, it is interesting that in putting pencil to paper, Brad abandons the prevailing right-to-left direction of his action-path and spontaneously invokes the left-to-right directional convention associated with writing for his notation-path. Apparently the learned left-->right spatial convention associated with writing does not apply to Brad's action-space in moving on the bells (Figure 15).[8]

Figure 15



Action space *vs* notation space

III.B
Phase II: A Surprising Discovery

In this phase Brad makes a surprising discovery that gives us some insight into the kinds of unexpected situations that lead to conceptual transformations and to the awareness of new kinds of entities and their relations.

The transformations in this case are triggered by an accidental discovery made by another child, Celia. Working on the same tune-building task using bells with the same set of pitches, Celia has set up her bells in a different configuration from Brad's--3 bells arranged vertically on the left side for the first part of the tune and two bells on the right side for the middle part.

After Celia built and played the tune, she experimented a bit and discovered to her surprise that she could, "…play the beginning of Hot Cross Buns in two different ways so it sounds just the same" (Figure 16).

Celia's "two ways" seen from above

[8] This is another nice example of meta-representational competence as it applies to making a representation but not to actions in real time/space.

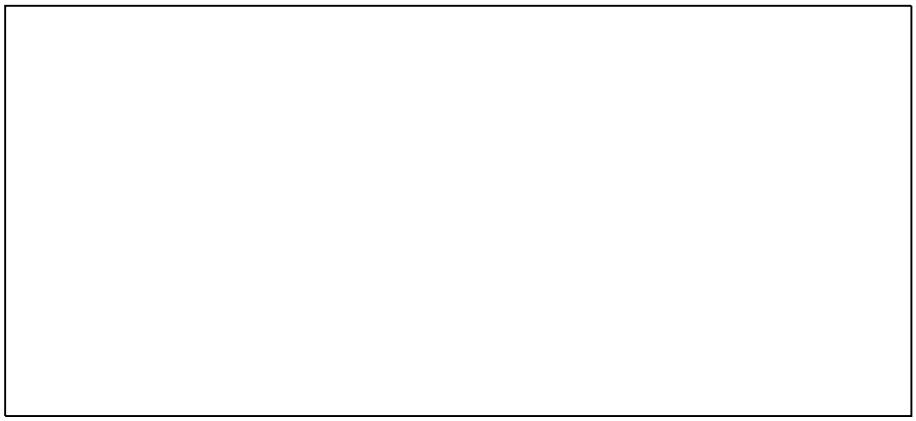
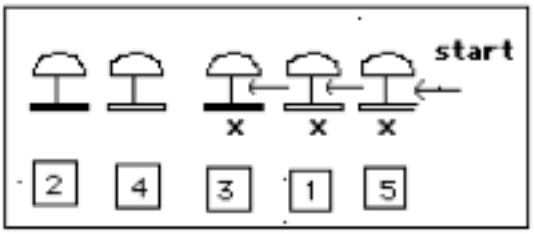


Figure 16: ***"…it sounds just the same"***

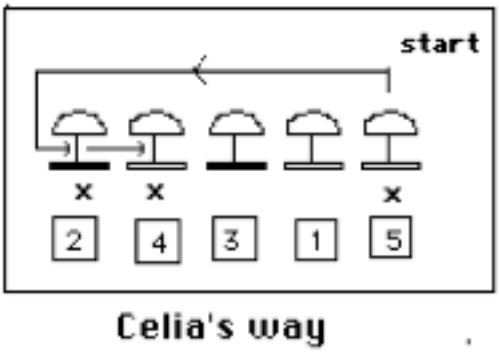
Adapting Celia's route

The discovery remained simply a mystery for Celia. But in the spirit of collaborative learning in the Lab, I showed Celia's new way to Brad and asked, "How do you explain this? See if it will work on your bells?"

My proposal to Brad, entirely unintentionally on my part, resulted in a chain of events that quite transforms Brad's view of the his bells, the tune, and its structure. His moves were as follows: He starts out by playing the beginning figure of the tune in his usual way, [5-1-3]. But to play the repeat of the same opening figure he adapts Celia's route to the shape of his bells jumping to the end of his bell path to play bells [5 2 4].



Brad's way



Celia’s way on Brad’s bells

Pausing for just a moment, he goes on to play the middle part of the tune in a new way, as well: Using the two bells labeled **3 and 1**, Brad played the middle part of the tune using bells that he had previously used only to play the beginning and ending of the tune. To complete the tune, he played the return to first part in his usual way,

I (Figure 19).

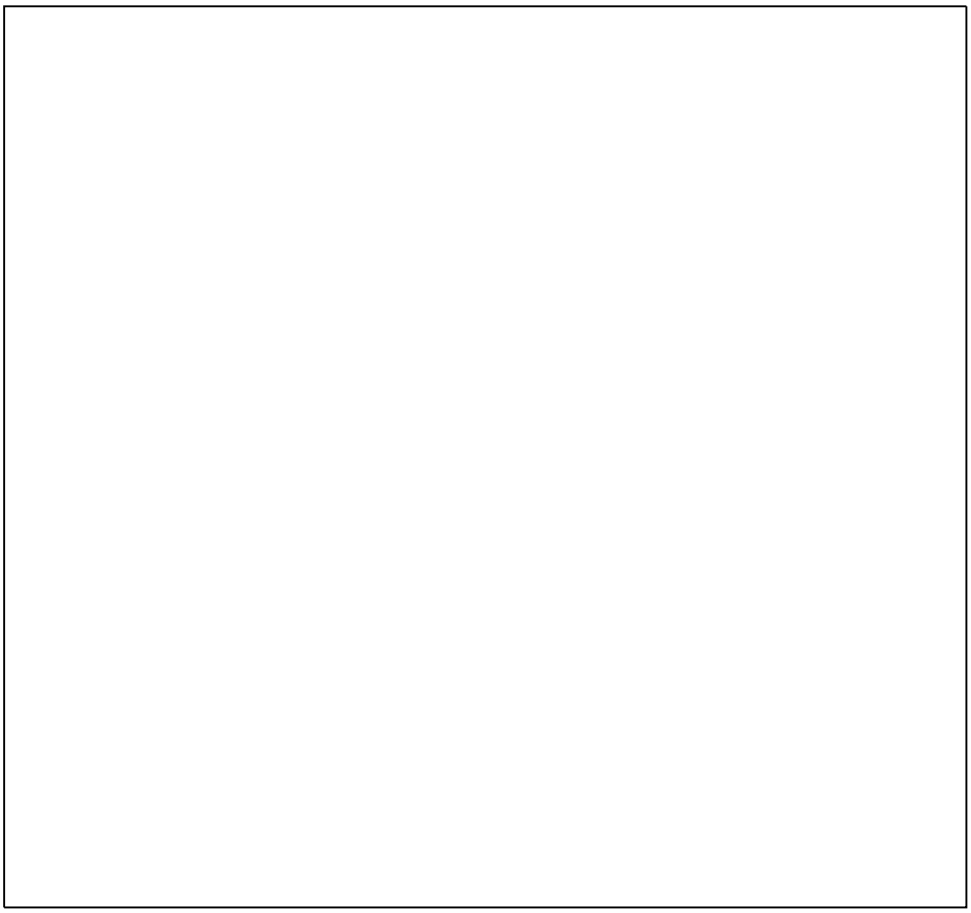


Figure 19: Brad's new action path

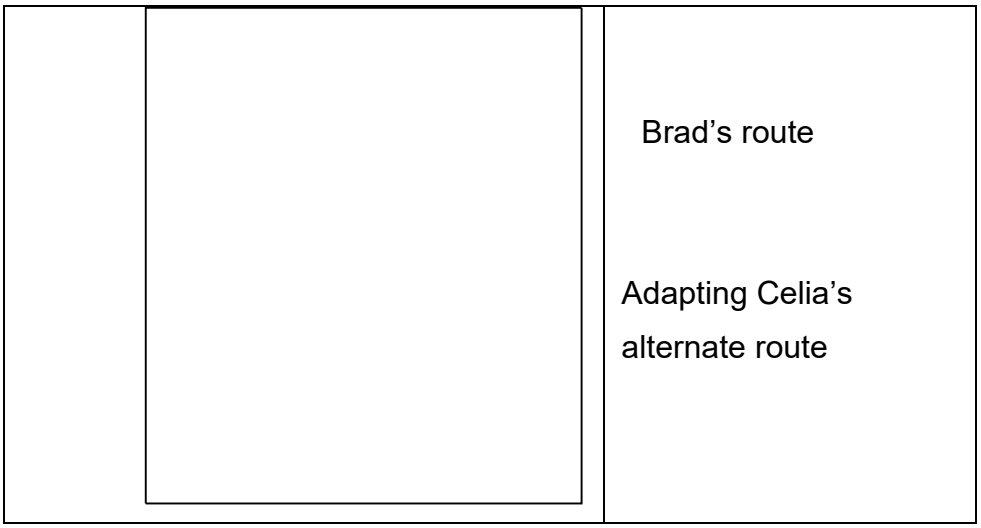
At this point Brad stopped, looked up with an expression of puzzlement and surprise, and said: **this is weird! I can play it with just three bells**!" And he pushed aside the bell-pair labeled [**4-2].**

This was clearly a surprise and as it turns out, a surprising instance of noticing that leads on to a significant cognitive leap. What were the moves and why did they result in such an unexpected insight?

> *To adapt and follow Celia's alternate route, Brad must break up his original action path. That is, he can play the beginning of Hot Cross Buns along either of two alternate routes--5-1-3 or the "detour," 5-2-4. In doing so, Brad breaks up the chronological order, crosses the boundaries of two previously distinct and bounded figures, and displaces their constituent members. The bells have, so to speak, changed who they are.*

The process results in major disruptions in the structure and functions of his bells and actions on them. Considering this from Brad's view, by adopting Celia's alternate route, Brad has disrupted and broken up the previous neatly parallel chronological orders--of tune events, of bells, and of his familiar felt path. He has broken up the integrity of his beginning figure [5-1-3], crossed over and violated its boundary, leaped (rather than stepped) from bell-5 over the

remaining members of his initial bell group [1-3] and arriving at the pair of bells at the other end of the bell-path [2-4] to make a whole new spatial and action configuration (see Figure 20)



.

But the critical moment occurs as Brad goes on. Continuing on in the same direction, L→R, as he when playing bells 2-->4, he plays bells [3-1], each 3 times, for the middle figure of the tune. Again transforming an earlier function, this pair previously functioned as members of the first figure, they also work as members of the middle figure. In turn, bells [4-2] are displaced as members of the middle figure. Finally, Brad plays the return, returning to bells [5 1 3] as before. Thus, bells [1 3] are also returned to their initial function, but now played in the opposite direction from just previously when they were members of the middle figure.

I argue that these events which lead to the cognitive leap, "Oh this is weird; I can play it with just t bells," are an embodiment of what Bartlett described as "a crucial step in organic development." Brad found a way to "break up the chronological order [of his bell and action paths] and rove more or less will in any order over the events" (Bartlett, op cit: 206).

Piaget also comments on the important effect of children taking "detours. He says, for instance, of children's alternate paths in getting from home to school:

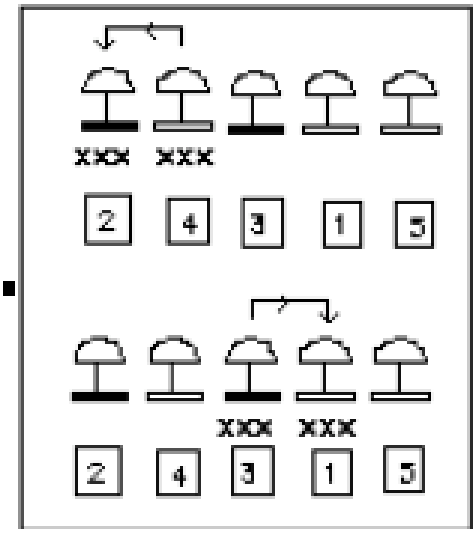
"Operations...are found formed by a kind of thawing out of intuitive structures, by the sudden mobility which animates and co-ordinates the configurations that were hitherto more or less rigid despite their progressive articulation. Each detour leads to interactions which supplement the various points of view (Piaget, 1960: 38)

Piaget's insights including the conceptual leaps that detours portend, and the logic implicit in them, bear an eerie similarity to Brad's "detours" in traveling in the very small space of his bell terrain.

Figure 21

Brad's three-bell theory suggests a logical leap that grows from these previous actions but goes beyond them. His reasoning might have gone something like this:

I can use either the [1-3] pair or the [2-4] pair to play both the beginning and middle figures of the tune. Now, if these two bell-pairs can be used equally well as members of two different figures, they must be <u>functionally equivalent pairs</u> So I don't need both pairs; just the 5-bell and one or the other pair will do it. That makes "just three bells" in all.



Functionally equivalent pairs

Looking back, now, at the sequence of events in this phase of Brad's work, I will argue that we can recognize in Brad's insights not only the importance that Piaget gives to "detours" but also an instance of Bartlett's related view which he describes as "a crucial step in organic development." Specifically, Brad found a way to "break up the chronological order [of his bell

and action paths] and rove more or less at will in any order over the events" (Bartlett, op cit: 206).

But I want to emphasize that, as in most on-the-spot learning, Brad's reasoning was emergent in real time and as such was almost entirely embedded in his actions. Indeed, judging from the way he expresses his discovery, "Oh, that's weird; I can play it with just three bells," his insight apparently feels to him, at the moment, more like magic than a series of logical steps such as I have proposed.

Of course, what is a surprising discovery for Brad, his three-bell theory, might seem to those of us who read and play from standard music notation, that Brad must have recognized that the bell-pairs 4-2 and 3-1 were simply matched pitch-pairs—as we know, both pairs of bells play pitches C and D. However, subsequent events towards the end of the session make clear that for Brad this was not the case. Brad's work provides evidence that constructing a class of functionally equivalent objects/events is perhaps a necessary intermediary step towards, but is not the same as, recognizing matched, de-contextualized pitch properties.

III.C
Phase III
Making the 3-bell theory work

Phase III marks the working out of transformations that were imminent in Phase II. In the spirit of the Lab culture, Mary Briggs, the teacher who worked regularly with the children in the Lab and was their mentor and advocate, helped Brad to reflect on his work, and together they came to better understand Brad's thinking.

> Pushing aside the two "extra" bells (2 & 4), Brad successfully plays the whole tune using just the three remaining bells (see Figure 22).

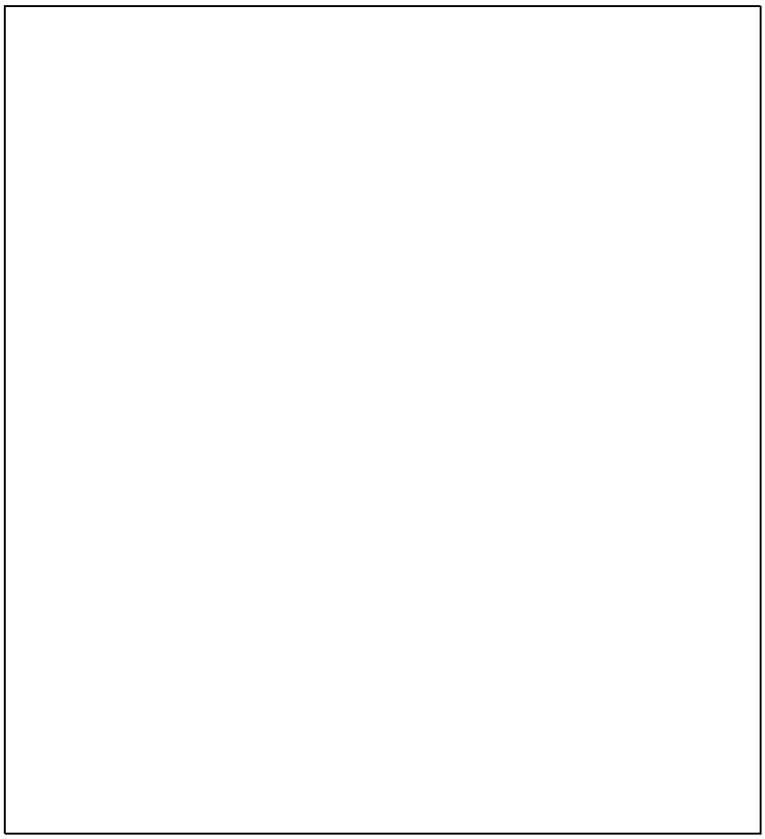


Figure 22: Just Three Bells

Pushing aside the two extra bells

Notation 2

Brad's second notation for playing the tune with just 3 bells is shown in Figure 23.

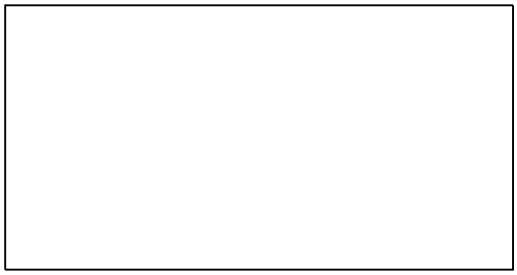


Figure 23: Brad's Notation 2

Brad's 3-bell notation might again seem in some ways to resemble a map-maker's notation. However, there are important differences. The number-labels Brad uses, [**5 1 3**], are still the arbitrary labels he attached to the bells at the outset; as such, they do not refer at all to pitch property, fixed reference numbers. And perhaps more important, Brad's spatial grouping of his numbers continues to reflect the figural/motivic structure of the tune—something not represented at all in the typical map-maker's notation.

Looking with Brad at the finished notation, Mary's probing question leaves no doubt about these groupings. Circling the middle row of numbers, Mary asks:

M: "Now, Brad, how come you put all those together?"

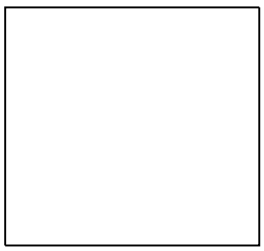


Figure 24

B: [rather haltingly] Because they're kinda together...cause it's kinda the same...it's the same as these three [Brad points to the previous three numbers, **[5 1 3]**].

To use the children's expression, Brad's groupings show the events that constitute the "things" of this small universe-- what we would call the structural entities of the tune. The

importance of invented notations looms large as Mary reflects together with Brad on the meanings inherent in his evolving notations.

III.D

Phase IV

Brad's Reflections Produce a Third Notation

A notation lends itself to insight in part because it is a kind of spatial version of a temporal sequence, but it has the advantage of "holding still" so that it can be scanned in any order and even out-of-order. Holding still to be looked at as a whole, the notation becomes a conduit towards the emergence of what Brad describes as "...seeing a pattern." As pointed out earlier, invented notations, like conventional notations associated with a community of professional users, are partial in being incomplete, and also partial to certain features while ignoring others. In turn, the maker's choice of aspects and the related names given to them is revealing of how the maker is mentally representing the phenomena—what he/she is choosing to notice and the assumptions inherent in the larger framework in which these aspects are embedded. Turning that idea back on itself, Brad's third notation not only reveals how he is re-thinking the constituents and relations of the materials. The new notation also reveals the surprising potential that an invented notation has to uncover assumptions hiding in our conventional notations, as well.

Brad's next moves seem clear evidence for the significance Bartlett gives not only to breaking up previous chronologies but to "turning back" on one's own 'schemata' and constructing new ones:

> [An organism] has somehow to acquire the capacity to turn round upon its own 'schemata' and to construct them afresh. This is a crucial step in organic development (Bartlett, op cit, P. 206).

Mary helps Brad to "turn round" by referring to a conversation that had occurred just a moment before: Mary says, "Now, Brad, you told me you saw a pattern. What was the pattern you saw?" In quick response to Mary's question, Brad says:

Well you could really number them one-two-three.

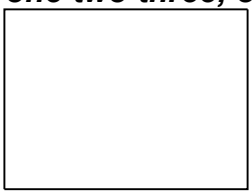
This surprising turn heralds a whole new notation. This third notation is a further transformation of Brad's previous insight, "I can play [the tune] with just three bells." With just the three bells still lined up on the table, still going right-to-left, Brad "sees a pattern." Giving up his *ad hoc* number-names, they are replaced with numbers *qua* numbers. That is, ordinal numbers, 1-2-3, represent and coincide with the order in which bells are lined up (first, second, third) and the sequence in which, as tune-events, they enter the song. In giving up the arbitrary number-names, Brad creates a whole new reference system. These are

numbers that refer unambiguously to an apprehended world—a row of objects on the table and events as they happen in real time.

This third notation is not written on paper but rather is implicit in the new numbering system, itself, together with Brad's verbal and gestural "instructions" which I report verbatim, here.

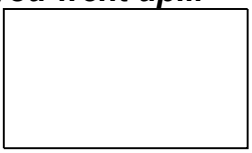
Pointing to each of the three bells on the table as he gestures, still going right--> left, Brad says:

Well you could really number them one-two-three; one-two-three.



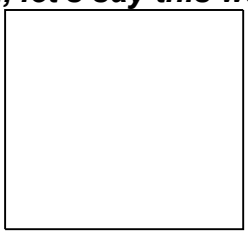
But then, suddenly, the new sequence of numbers takes on a specifically "directional" meaning. Gesturing right-to-left over the bells, Brad says,

You kinda say....you went up...



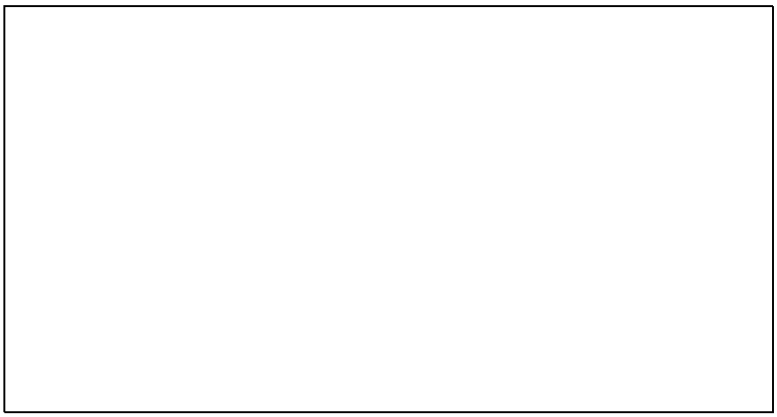
But interrupting this momentary thought Brad stops to explicitly label the bells. Repositioning his numbered paper squares in front of the three bells, he says:

I mean, let's say this was "1."

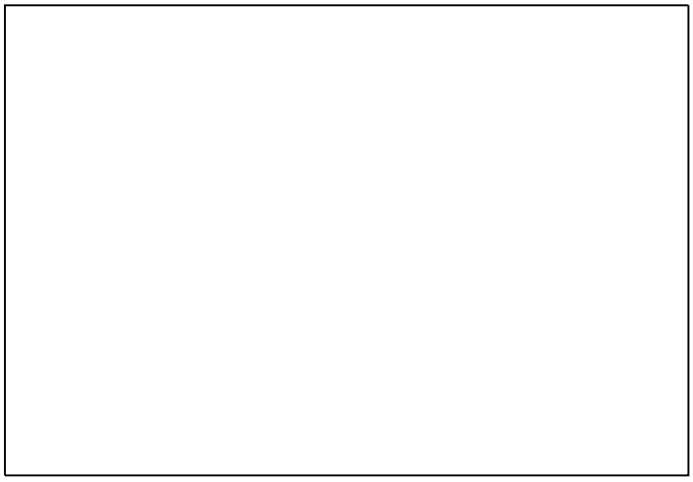


Completing the labeling, he gives directions for playing the whole tune in terms of his new number sequence. Gesturing to show the directionality, he says:

Then you would go like 1-2-3. Then you would go 3-3-3 which is high, 2-2-2 which is a little lower, then 1-2-3.



So you go up, and then up again, then you go down, and up again.



This episode and its evolution raise new intriguing puzzles and new issues. For instance, Brad speaks as if asking "you," the receiver of his instructions, to walk along his numbers and bells, "going up" and "going down." But numbers do not literally go up or down, and we are not literally "going up" or "going down" either as we follow Brad's directions for playing "Hot Cross Buns" on the bells. Notice that Brad's description, “So you go up...” corresponds exactly to our conventional usage when we say of the number line, the numbers “go up.”[9]

Animating numbers, putting them into motion we are, of course, invoking a metaphor: a static list, a chronology stuck in space, comes alive as if acting in time and motion. But these metaphors are so deeply embedded in our language that we have forgotten that the terms, "up/down", "high/low" literally only refer to visible, tangible objects that can move or can be

[9] This convention seems to imply some kind of quantitative meaning and with this meaning, the numbers would be considered cardinals instead of ordinals. That is, instead of next-next-next as with ordinal numbers, the expressions "going up" and "higher" could be understood as literally implying more-more-more. Wittgenstein describes a similar situation in the Brown Book in his discussion of "language games" (Wittgenstein, 1960. pp. 79-84). Indeed, we could see Brad as participating in language games of his own invention.

moved up and down through time in space.

Further, in adopting this new notation, Brad wipes out a central feature of his previous "instructions"—the notation as a physical embodiment of the tune's figural structure. Recall that Brad's initial bell-path was constructed in synchrony with the chronology of events in the tune. In turn, his initial notation-path was spatially grouped to reflect the tune's motivic structure. Over the last set of moves, Brad has gradually broken this synchrony apart. And now, with his ordinal numbering of the bells and his focus on directionality, he almost entirely abandons any reflection of this figural grouping structure.

Metaphors, Meanings, and Notations

Brad's newly invented number scheme and his use of metaphoric spatial/temporal language reveals a paradox and the paradox, in turn, helps to reveal aspects of notational conventions that ordinarily can remain comfortably hidden in their common usage and practice. Dead metaphors can come alive under conditions of uncertainty and confusion.

The paradox arises because similar spatial, directional, and motion metaphors are embedded in the terms we use to refer to pitch relations, as well as numeric relations. Just as we speak of numbers "going up" or "going down," so we speak of pitch "going up" or "going down." In referring to pitches that "go up," map-makers and conventional music reference systems analogously attach numbers that "go up." Thus, numerals 1-2-3-4-5 are assigned to the ascending fixed reference structure, the scale, starting with the key-tone or tonic pitch.

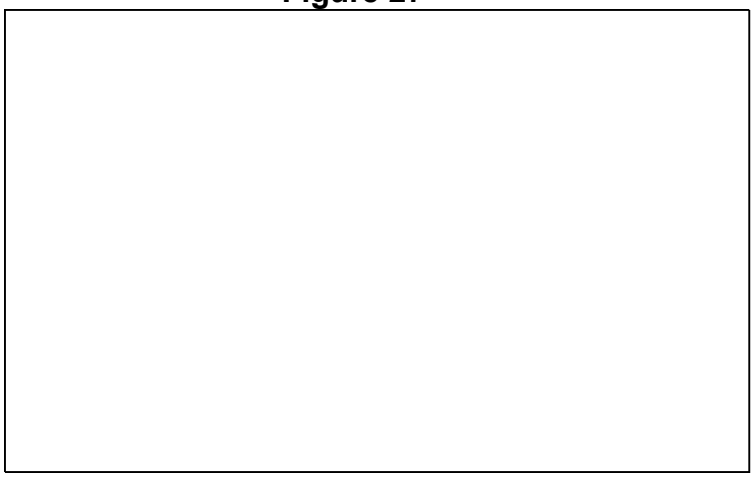
But consider that pitch, as a sounding entity, does not occupy space nor does it move, any more than numbers do. The sense we have of a melody "moving" is a mental construction: a melody (as perceived) is a unique sequence of discrete pitch/duration events following one another through time. And like the frames in a moving picture, these give the impression of movement—what we call "going up" and "going down." But it is performers who move, not pitches. Brad's controlled movements on his built bell-path produce the melody, "Hot Cross Buns," but once built, neither the bells, their pitches, nor the notation on paper literally move anywhere. It is Brad who moves.

Carrying this sense of apprehended movement into our language, we come to believe it--we attribute movement to melody, itself, as if pitch and melody were self-animated. And in similar ways, we attribute self-animation to numbers when we encourage children to say as they move along a number line, "the numbers are going up."[10]

[10] K. S. Lashley in his classic paper of 1951, "The problem of Serial Order in Behavior," speaks of the relation between syntax and action: "...the syntax of the act which can be described as the habitual order or mode of relating the expressive [symbolic] elements...to the generalized schema of action,... determine the sequence of specific acts, acts which in themselves or in their associations seem to have no temporal valence." (in Pribram, 1969, p. 525).

It is the self-animation we attribute to numbers that Brad is focusing on, not the direction of pitch motion as you sing or play the tune. And here the potential for confusion in metaphoric meanings becomes intense. Looking back at the map-maker's notation based on the "motion" of the pitches within the fixed reference structure, the beginning of Hot Cross Buns in fact goes down, not up. The first two figures are numbered 3 2 1; 3 2 1. To make Brad's notation which he consistently writes right-to-left match conventional notation, which is, of course, written left-right, the sequence of numbers under the bells, as well as the direction of motion in his notation would have to be exactly reversed (see Figure 27)[11]

Figure 27



Crossed metaphors

So the paradox that seems inherent in Brad's comment that the tune "goes up" when in fact it “goes down," is really not a paradox at all. Rather we see a beautiful example of a difference in focus of attention. Brad has numbered the bells according to their order of occurrence in the tune—the numbers are ordinals—1-2-3 are the first, second, third events in the tune. But once applying the numbers to the bells, the numbers change who they are—they become elements in a number-line and Brad is moving “up” along that line. But with a focus on pitch direction and assigning numbers according to music notation conventions, the sequence of pitches at the beginning of Hot Cross Buns, is "going down" [3-2-1]. Both designations are right; it just depends on what aspects you are "partial to."

IIIF

Phase V:

[11] Evidence that Brad was quite capable of distinguishing "up and down" in pitch was clear when, on hearing the beginning of the same tune played by the computer synthesizer, he said quite spontaneously, "Oh, it goes down."

Pitch, An Emergent Phenomenon

Thus far I have focused on Brad's actions, his notations, and his words as evidence for his changing understanding of the tune structure. Through these, I have proposed analogies with movement through space, specifically with making and following paths—"bell-path," "action-paths," "notation paths," and alternate "routes" traversed. Moreover, I have attributed Brad's insights to inferences he has drawn from observing and mentally coordinating his actions as he both made paths and followed them. Most of all, I have given causal importance to the moments in which these paths have been interrupted and their chronologies, their contiguous actions/events broken up. Returning to Bartlett and my primary proposal, Brad, with the help of Mary and others, has been able to "find some way in which [he] can break up this chronological order and rove more or less at will in any order over the events which have built up [his] present momentary 'schemata." It seems clear that Brad's invention of notations that hold still and that he can look at "out there," contributed to the possibilities for this "roving at will."

I have also emphasized the importance Brad gives to figures—these are the "things," the units of perception reflected in his written notations. But with his mostly verbal, gestural third notation, these figures as units of description have essentially disappeared.

Watching Brad's work, I asked myself what sorts of on-the-spot interventions might help him differentiate among and hold steady the multiple different meanings inherent in his notational inventions? In the light of my previous experiences with other children and adults on similar tasks, it was clear that to achieve this last step would require helping Brad focus his attention on de-contextualized pitch properties of the bells, themselves. Recall that with the bells all looking alike, pitch remains a hidden property of the bells. Indeed, at the outset of his work, the bells were simply objects that occupied space. But in order to build the tune Brad had to play the bells and listen for a match between the bells he heard and the tune in his head which he had sung and at times continued to sing as he went along. In building the tune and playing it, Brad necessarily did this pitch-recognizing entirely "by ear," in action, and in the local context of the tune in its becoming.

It is not surprising, then, that none of Brad's three notations referred to pitch or pitch relations, as such. The notations refer, in one way or another, to the ordering of bells as he has set them up in a row, to the sequence of tune events as coordinated with his actions, and, except for the last, to the grouping of these tune events into the figures of which they are members.

Thus, it was my hunch, that if Brad were to account for his insights and for the (often implicit) inferences that led to them, he would need to shift his focus of attention to pitch as an inherent and invariant property of a bell, independent of the functional role of that pitch

within figures. Such an intervention might also help him account for the "weirdness" of his three-bell theory. This, in turn, could move him towards conceptual map-making, hopefully without losing the relevant functional attributes of his previous representations?

As I argued in my initial proposal, the critical step in becoming a map-maker is to be able to remove oneself from action-paths so as to construct classes of objects/events. To do so, we must selectively interrupt the flow, the continuous succession of events, to pick out (by comparing backwards and forwards in time-space) a new conceptual succession inhabited by just those objects/events that share properties to which we are at some moment partial—i.e., that are congruent with our current field of attention. For instance the class of all the C's" or of all the D's in a tune, irrespective of where they occur or with what function. In Phase V, through a series of interventions, I begin the process of trying to carry out this program with Brad.

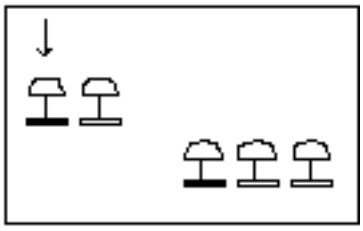
A Pitch for a Pitch

I began by asking Brad:

"How come you don't need to use those other two bells? Do you know why it works? Shaking his head, Brad said rather hesitantly, "No... I don't." Brad's response at least tentatively confirmed my hunch that he was unaware of the pitch matches. And the quality of his answer—pensive, reflecting some puzzlement, suggested that this was, indeed, something new for him to think about.

I now made an intervention of a more directly instructive kind. To help Brad isolate the pitch properties of the bells, take them out from his intuitive sense of their structural functions when embedded in the tune, I pointed to one of the "extra" bells standing apart from the three-bell tune-path, and said, "Can you find one (another bell) that sounds the same as this one?" This would be another version of stepping off a well-trodden path of actions, objects and events; instead of tune events, the bells would just play matching sounds.

Figure 29



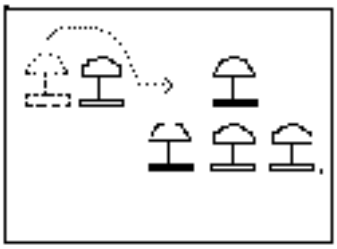
...find one that sounds the same?

Playing the "extra" bell and testing each of the others, Brad immediately found a match for the designated bell. This was important proof that Brad had no problem actually

recognizing matched pitches. However, he was visibly surprised to discover that matches were to be had—good evidence that this was a whole new view of the situation.

Quite spontaneously, Brad moved the matching “extra” bell over to position it together with its mate.

Figure 30

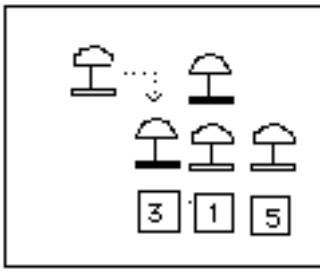


a match

And with this move, the new spatial arrangement became the beginning of a new embodied notation. Coupled with its mate, the “extra” bell no longer is (represents) an object that plays a tune-event but rather an object that “sounds the same” as another one. The bells were once more changing who they are.

Having found one pair of bells that matched, Brad pushed the remaining “extra“ bell over towards the bell labeled “3”. And without even playing it, he said aloud, “And these probably do too.”[12]

Figure 31

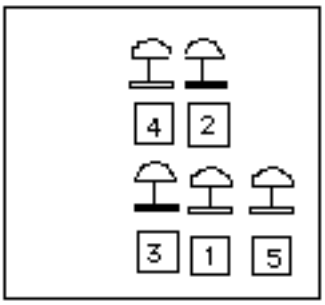


“And these probably do, too”

Testing the remaining bell with its hypothesized match, Brad positioned the new matches together to form a pair. With the matches completed he had also completed a new kind of embodied notation: the couples represent bells that “sound the same.”

Figure 34

[12] I use Brad's original labels for the bells, here, so as to make it easier to describe the inferences he now makes.



bells that sound the same

Leaving no doubt as to his new understanding, Brad pushed the two extra bells away again, and said:

“So you really only need.....that’s cool!”

My intervention has clearly shifted Brad’s focus to the pitch property of bells separate from their position or function in the tune. And with it, Brad has taken a critical step (perhaps the critical step) in his evolving conceptual change from path-making toward map-making. That is, he has made a first move from situation-dependent, functional meanings toward classification according to de-contexualized properties. Specifically, bell-pairs labeled (2 4] and [1-3] respectfully, were initially heard as pairs that could substitute for one, what I called “functionally equivalent pairs.” With this latest move these same bells are now heard as matched pitch pairs. Stepping off his built bell-path, interrupting the sequence of contiguous tune events, Brad has plucked out from their functional context just those bell/pitches that share the same hidden pitch property. And having done so, he has reason to be convinced that his three-bell discovery is not so “weird,” after all: you only do need three bells—“...that’s cool!”

III.G

Phase VI:

Brad Explains

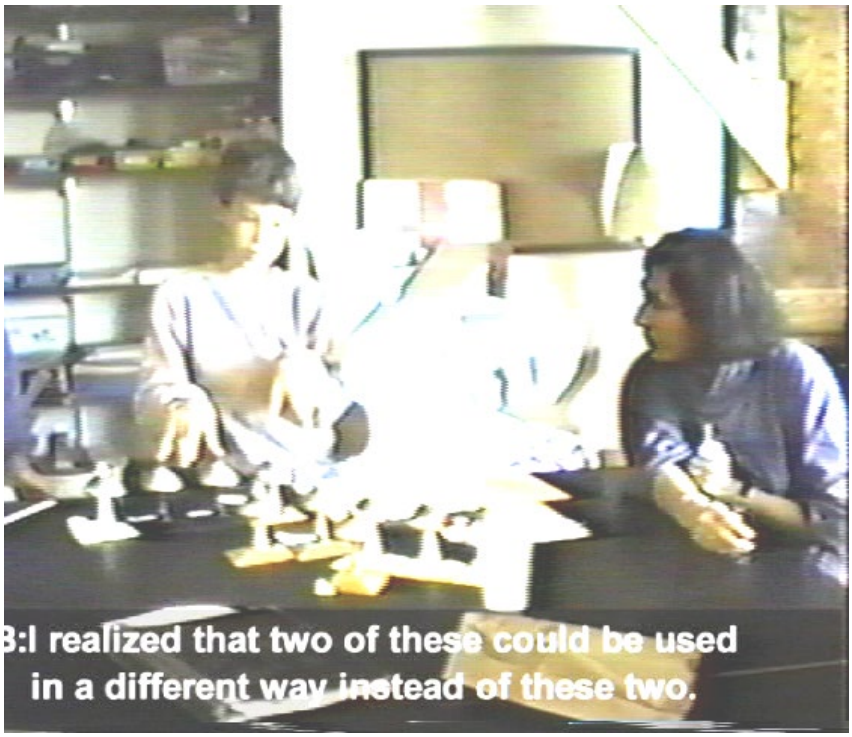
Finally in this last phase, Mary engages Brad in thinking back about the previous moments when he first discovered his three-bell theory. Mary asks:

> How'd you discover it? I want to know what happened. All of a sudden you said, 'Wait a minute, you can do it with three.

It is particularly fascinating that Brad’s explanation comes after the bell-matching and after he seemed to understand his 3-bell theory in terms of matched pitches. He starts back at the point when he had adapted Celia's alternate route to his own bells. I add comments drawn from my repeatedly watching the video.

Brad’s explanation Comments

"...I was realizing that if I could play it one way--like 5 1 3..."	He makes a gesture along the bells to show how he had "play (ed) it one way."
"...then I realized (pause) that if you could do all of them one way..."	He starts again adding to what he had just said. The addition of "do all of them" must refer elliptically to the fact that he was, at that point, using all five bells.
"Then I realized that two of these (circling the pair [1 3]) could be used in a different way instead of these two (points to the pair [4 2].)	Brad describes, in much condensed form, his actual process of inference in making the alternative path for the middle figure.



Brad's tells us in his own way the discovery of his alternative path for the middle figure and how that led to his 3-bell theory. Most remarkable is his ability and willingness to reflect back on his own actions and thought. The Lab culture and Mary in particular played a large role in making that possible. The critical moments were Brad's "realizing" that there were sets of bell-pairs (1-3 and 4-2) that could substitute for one another to play the same figure—i.e., bell pairs that were “functionally equivalent.” Brad articulates that principle in his expression "...could be used in a different way instead of..."

Finally, Mary does ask Brad the missing question: "So did you know that two had to be the same?" Brad's response is equivocal: "Yah, I realized something like that." Still left unanswered is the critical question: just what did Brad understand by "the same"—e.g., same pitch or same function? So much happened in the 45 minutes or so that Brad worked on this task, it is not surprising that at this point, Brad, too, was not quite clear about what he did know for sure.

VI

CONCLUSIONS

Paths, Maps, and Educational Implications

Looking back, now, at the evolution of Brad's work, what does it tell us about the evolution from path-maker to map-maker? I recapitulate the distinction, borrowing from both Piaget and Lynch. A map-maker, constructs a grid-like mental representation of an area, a situation-independent reference structure, such that objects and relations remain in consistent and invariant relation to one another. The musical map-maker extracts pitches as properties, independent of their sequence or function in a tune, making a grid-like pitch-series in which pitch properties are ordered according only to “pitch height,” or as we say, lower to higher. In either case, map-makers can place an object or event and measure distances among them in relation to these fixed reference structures rather than in relation to particular, personally constructed figures along a spatial or musical path.

Coming back to my earlier proposal and to Bartlett, map-making often develops, as it did with Brad, through the "break up of chronological order [so as to] rove more or less at will in any order over the events...” This "break up" becomes for Brad the discovery of functionally interchangeable bell-pairs. I propose that if these bell-pairs can be mentally coordinated in one representational space, common features are more likely be recognized and picked out. This, in turn, will lead to building a new collection, a class, that includes only the objects/events that share a selected property—e.g., Brad’s matching pairs. Once this is achieved, there may follow the construction of a fixed reference structure upon which to traverse the whole terrain in multiple ways, along multiple routes while still arriving at the same desired goal.

However, it also became clear at the beginning of Phase V before I intervene, that Brad is still accounting for his inferences in terms of figures and functions as the constituent elements. That is, Brad can map figures onto one another but the properties of the individual bells within figures, i.e., the pitches, remain embedded in their figural functions. This is the critical transitional phase which anticipated the matching task in which Brad did isolate bell-pitches from their functional role as members of a figure. Pitch property. as such, did become a differentiated "thing," an object of attention in itself, at least for the moment.

While Brad at the end of the session was making forays into becoming a musical map-maker, he was not yet there. For instance, he had not yet constructed a new sequence based only on pitch properties--i.e. a functioning fixed reference (the seriated pitches) to which he could relate any de-contextualized pitch event by placing it within the series, and measure its distance from others. But, I would hope that, with this new focus and more practice, he would not lose his important sense of figures and functions as they evolve in the unique context of an unfolding melody or even larger musical composition. For as Hasty as said, in learning conventions of music notation along with the necessary construction of fixed reference structures we can "take measurements and compare patterns, but to do so we must:

> arrest the flow of music... But music as experienced is never so arrested.... What we can hold onto are spatial representations (scores, diagrams, time lines) and concepts or ideas of order. Such ideas can usefully be drawn from musical organization presented as something completed and fully formed. However, a piece of music or any of its parts, while it is going on. is open, indeterminate, and in the process of becoming a piece of music (C. Hasty, 1999: 3).

Finally, what are the more general educational implications of the distinction between map-making and path-making? Consider that it is traditionally the case in schools, for instance, that symbolic conventions serve as the "spectacles" through which we see and judge a student's work. We look for either a match or a mis-match with convention and a match with conventional practice is judged correct. However, conventional symbolic expressions most often refer to the de-contextualized elements associated with map-making.

On this traditional basis for evaluating student work, Brad's inventions would run a serious risk of being seen as simply wrong. And if that were the case, the result would be to fatally miss the power of Brad's evolving notational inventions. Most important, such evaluation would miss seeing Brad's notations as a vehicle for revealing to himself and his teachers the cognitive work involved in his reasoning, his logical inferences, and the transformations they entail--in short, his learning.

What we are witnessing in Brad's multiple descriptions/notations is a stunning example of the multiplicity of criss-crossed intersections between notational conventions and inventions. And most importantly, through these intersections and confrontations, we witness the possibility that invention can illuminate convention. However, in accepting this possibility, implications and assumptions may be revealed that, once seen, can put into question the very tools with which we think and carry out our work. Questioning our notational conventions is a risky business because notational conventions shape our perceptions like

eyeglasses that we look through. And as with eye glasses, it is only when something happens to blur or confound an object of attention, that we look at the glass, focusing on it rather than passively permitting it to focus, to shape an image of phenomena in some particular way. Reversing this habit, looking at our notational conventions through the glass of a child's inventions, we can begin to see aspects inherent in our conventional symbol systems that otherwise remain hidden from view.

Learning how to do learn from the children we teach, we will also help children (and ourselves) towards the central task of education: being able to choose among possible vantage points, to coordinate them so as to make and appreciate alternate routes depending on when and where we are, and on what we want to do in and with the world around us. Perhaps this requires stepping off our well-trodden, well-learned symbolic paths to participate in and value the "felt paths" that we know best from moving about and being alive and well in the world of sensory experience. "Out of that tense multiplicity of vision {comes} the possibility of insight" (Bateson, 1994).

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