Transform definitions:

$$c_{kk'} = \frac{1}{N_k}\sum_n e^{-ik'r_n} iB_{i_+}\langle a_k \hat{\lambda}_n^{i_+}\rangle \qquad \bar{c}^T_{kk'} = \frac{1}{N_k}\sum_n e^{ikr_n}\left(-iB_{i_-}\langle a^\dagger_{k'}\hat{\lambda}_n^{i_-}\rangle\right) \tag{1}$$

$$d^{j_+}_{mn} = \sum_k e^{-ikr_m}\langle a_k \hat{\lambda}_n^{i_+}\rangle \qquad \left(\text{we have } d^{j_+}_{nn}, \bar{d}^{j_+}_{nn}\right) \tag{2}$$

$$\beta^{i_+}_{nk} = \frac{1}{N_k}\sum_m e^{ikr_m}\left(-iB_{j_-}\langle \hat{\lambda}_n^{i_+}\hat{\lambda}_m^{j_-}\rangle\right) \qquad \alpha_{nk} = \sum_{k'} e^{ik'r_n}\langle a^\dagger_{k'} a_k\rangle \tag{3}$$

$$d^{i_+}_{mn} = \sum_k e^{-ikr_m}\langle a_k \hat{\lambda}_n^{i_+}\rangle \qquad \bar{d}^{j_+}_{nm} = \sum_k \overline{e^{-ikr_n}\langle a_k \hat{\lambda}_m^{j_+}\rangle}. \tag{4}$$

The fast and final form of the equations:

$$\partial_t\langle a^\dagger a\rangle_{k'k} = C^{(1,1)}_{\underline{k'}\,\underline{k}}\langle a^\dagger a\rangle_{\underline{k'}\,\underline{k}} + c_{kk'} + T[\bar{c}_{kk'}] \tag{5}$$

$$\langle\lambda\rangle^{i_0}_n = C^{(2,1)}_{i_0 j_0}\langle\lambda\rangle^{j_0}_n + C^{(2,2)}_{i_0 n} + \mathrm{Re}[C^{(2,3)}_{i_0 j_+} d^{j_+}_{\underline{nn}}] \tag{6}$$

$$\partial_t\langle a\hat{\lambda}\rangle^{i_+}_{kn} = C^{(3,1)}_{i_+ j_+ \underline{k}}\langle a\hat{\lambda}\rangle^{j_+}_{\underline{kn}} + \beta^{i_+}_{nk} + C^{(3,2)}_{j_+ k\underline{n}}\langle\hat{\lambda}\hat{\lambda}\rangle^{i_+ j_-}_{\underline{nn}} \tag{7}$$

$$+ C^{(3,3)}_{i_+ k\underline{n} i_0}\langle\lambda\rangle^{i_0}_{\underline{n}} + C^{(3,4)}_{i_+ kn} + C^{(3,5)}_{i_0 i_+}\langle\lambda\rangle^{i_0}_n \alpha_{nk} \tag{8}$$

$$\begin{aligned}\partial_t\langle\hat{\lambda}\hat{\lambda}\rangle^{i_+ j_-}_{nm} &= C^{(4,1)}_{i_+ p_+}\langle\hat{\lambda}\hat{\lambda}\rangle^{p_+ j_-}_{nm} + C^{(4,2)}_{j_+ p_+}\langle\hat{\lambda}\hat{\lambda}\rangle^{i_+ p_-}_{nm} \\ &+ C^{(4,3)}_{i_0 j_+}\langle\lambda\rangle^{i_0}_{\underline{m}} d^{i_+}_{\underline{m}n} + C^{(4,4)}_{i_0 i_+}\langle\lambda\rangle^{i_0}_{\underline{n}}\bar{d}^{j_+}_{\underline{n}m}\end{aligned} \tag{9}$$

Here we took the opportunity to group all factors that do not vary between timesteps into constants and moved indices outside of angled brackets for readability. Indices with an underline indicate *no* summation is to be made (e.g. in Eq. (5) $C^{(1,1)}$ multiplies $\langle\tilde{a}^\dagger\tilde{a}\rangle$ element-wise); otherwise, the Einstein summation convention applies. The indices are placed in the order they will appear in the code, i.e., axes of multidimensional `numpy` arrays, with upper indices always being considered before lower ones. The constants are

$C^{(1,1)}_{k'k}$	$i(\omega_{k'}-\omega_k)-(\kappa_{k'}+\kappa_k)/2$	$C^{(2,1)}_{i_0 j_0}$	$\xi_{i_0 j_0}$
$C^{(2,2)}_{i_0 n}$	ϕ_{i_0}	$C^{(2,3)}_{i_0 j_+}$	$4B_{i_+} f^+_{i_0 i_+ j_+}/N_\mathrm{m}$
$C^{(3,1)}_{i_+ j_+ k}$	$\xi^+_{i_+ j_+} - \left(i\omega_k + \kappa_k/2\right)\delta_{i_+ j_+}$	$C^{(3,2)}_{j_+ kn}$	$iB_{j_+} e^{ikr_n}/N_\mathrm{m}$
$C^{(3,3)}_{i_+ kn i_0}$	$-C^{(3,3)}_{j_+ kn} Z^+_{i_0 i_+ j_+}$	$C^{(3,4)}_{i_+ kn}$	$-C^{(3,3)}_{i_+ kn}(N_\mathrm{m}/N_\nu)$
$C^{(3,5)}_{i_0 i_+}$	$2B_{j_+} N_\mathrm{m} f^+_{i_0 i_+ j_+}$	$C^{(4,1)}_{i_+ p_+}$	$\xi^+_{i_+ p_+}$
$C^{(4,2)}_{j_+ p_+}$	$\xi^-_{j_+ p_+}$	$C^{(4,3)}_{i_0 j_+}$	$\overline{C}^{(3,6)}_{i_0 j_+}$
$C^{(4,4)}_{i_0 i_+}$	$C^{(3,6)}_{i_0 j_+}$		