**Expectation Maximization Algorithm with Combinatorial Assumption**

**Abstract**

**Keywords:**

**1. Introduction**

**2. EM with combinatorial assumption**

Without loss of generality, suppose every random vector variable *Yi* degrades into random scalar variable *yi* and thus, the condition expectation *Q*(Θ | Θ’) becomes:

Suppose *X* = (*x*1, *x*2,…, *xn*) which is vector of size *n* distributes normally with mean vector *μ* and covariance matrix Σ as follows:

Where the superscript “*T*” denotes vector (matrix) transposition operator. Let *y* be random variable representing all sample random variable *yi*. Suppose there is an assumption that *y* is a combination of partial random variables (components) of as follows:

This implies the equation above is regression function in which *y* is called responsor and each *xi* is called regressor whereas *αj* are called regressive coefficients. As a convention, let

The regression function is re-written as follows:

Suppose *y* distributed normally with mean and variance *σ*2 as follows:

The density probability function (PDF) of *y* is now defined by support of regression model as follows:

The equation above is not really total probability rule but it implies that the conditional PDF *f*(*y* | *X*) is substituted by the regression model. Consequently, the condition expectation *Q*(Θ | Θ’) is becomes:

Where parameter Θ = (*μ*, Σ, *α*, *σ*2)*T*. It is necessary to specify the conditional PDF *f*(*X* | *y*, Θ). Indeed, we have:

Let *g*(*X*, *y* | *μ*, Σ, *α*, *σ*2) be the numerator of *f*(*X* | *y*, Θ):

Where,

The expression is approximated with *μ* as follows:

As a result, *g*0(*X*, *y* | *μ*, Σ, *α*, *σ*2) and *g*(*X* | *μ*, Σ, *α*, *σ*2) is approximated by:

Of course, we have:

Let *A* be the denominator of *f*(*X* | *y*, Θ):

Where *B* is the integral of *g*(*X*, *y* | *μ*, Σ, *α*, *σ*2) with regard to *X*,

It requires to calculate *B* to determine *f*(*X* | *y*, Θ). Due to:

We have:

Thus, *A* is approximated as follows:

As a result, the PDF *f*(*X* | *y*, Θ) is approximated as follows:

Let *k*(*y*|Θ) be the constant with subject to *X*, which is defined as follows:

Shortly, the conditional PDF *f*(*X* | *y*, Θ) is specified (approximated) at E-step of some *t*th iteration process as follows:

Consequently, the condition expectation *Q*(Θ | Θ(*t*)) at some *t*th iteration is totally determined:

At M-step of the current *t*th iteration, *Q*(Θ|Θ(*t*)) is maximized by setting its partial derivatives regarding Θ to be zero. The first-order partial derivative of *Q*(Θ | Θ(*t*)) with regard to *μ* with suppose that *Q*(Θ | Θ(*t*)) is analytic function is:

Due to:

We have:

Note, Σ is invertible and symmetric. The next parameter *μ*(*t*+1) at E-step of some *t*th iteration is solution of the equation as follows:

**3. Numerical simulation**

**4. Discussions and conclusions**

**References**