**Aitken Vector Sequence**

**1. Introduction**

Given a sequence is monotonous such that *x*(*i*) ≤ *x*(*j*) or *x*(*i*) ≥ *x*(*j*) for all *i* and *j*. Suppose approaches a limit *x\**.

We have the following approximate with *k* large enough (Lambers, 2009, p. 1):

We establish the following equation from the above approximation, as follows (Lambers, 2009, p. 1):

Hence, *x\** is approximated by (Lambers, 2009, p. 1)

We construct Aitken sequence such that (Wikipedia, Aitken's delta-squared process, 2017)

|  |  |
| --- | --- |
|  | (4.3.1) |

Where Δ is first-order forward difference operator,

And Δ2 is second-order forward difference operator,

According theorem of Aitken acceleration, Aitken sequence approaches *x\** faster than the sequence .

When *X* is vector as *X* = (*x*1, *x*2,…, *xr*)*T* such that then, Aitken sequence is re-defined. The first-order partial forward difference operator Δ*ixj*(*k*) is defined as a deviation between element *xi*(*k*+1) and *xj*(*k*) as follows:

The first-order forward difference operator Δ*X*(*k*) for vector is:

So Δ*X*(*k*) is a row vector. The second-order partial forward difference operator Δ*i*2*xj*(*k*) is defined as follows:

Of course, we have:

Let

So ΔΔ*X*(*k*) is a row vector. The second-order forward difference operator Δ2*X*(*k*) for vector is defined as follows:

So Δ2*X*(*k*) is a matrix. Aitken vector sequence is defined based on , Δ2*gX*(*k*), and Δ2*sX*(*k*) as follows:

Suppose *X\** is a limit of , we must prove that Aitken sequence approaches *X\** faster than the sequence . In other words, we must prove that

Where |.| denotes length or module of a vector in Euclidean space.