**Complex Diagnostic Bayesian Network**

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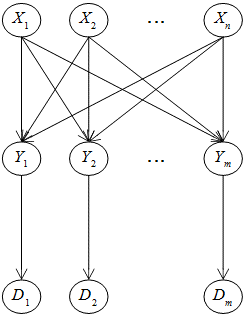
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**Abstract**

**Keywords**: diagnostic relationship, Bayesian network

The joint probability of M-HE-D network is:

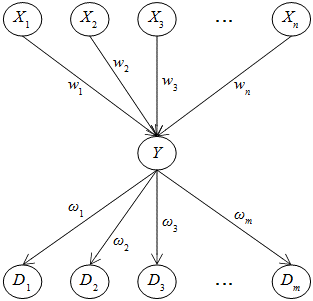
Following is the extended M-HE-D network



The joint probability of extended M-HE-D network is:

It is implied that extended M-HE-D network is equivalent to M-HE-D network in probabilistic inference with regard to Ω and Φ.

Following is the grouped M-HE-D network.



The joint probability of grouped M-HE-D network is:

It is implied that grouped M-HE-D network is equivalent to extended M-HE-D network and M-HE-D network in probabilistic inference with regard to Ω and Φ.

Without loss of generality, suppose the index *i* of variable *Xi* is odd. Let *C*1, *C*2, and *C*3 be the number of arrangements *a*(*Xi*=1), *a*(*Xi*=0), and *a*(*D*), respectively. We have:

Because there are many cases in formula 3.11, it is useful to count them. Formula 3.11 varies according to arrangements of *X*1, *X*2,…, *Xn*. Let *c*1, *c*2, and *c*3 be the number of such arrangements with regard to fixed *Xi*=1, fixed *Xi*=0, and non-fixed cases. Table 3.1 shows these counters.

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**Table 3.1.** Counters relevant to XOR-gate inference

Later on, we will research deeply these arrangements.

Let *U* be a set of indices such that *Ai*=*ON* and |*U*|=*n*/2. Let be the set of all possible *U* (s) extracted from {1, 2,…, *n*}. We have:

If then,

This implies all sets *U* must be subsets of *K* and |*K*| ≥ *n*/2 and |*U*|=*n*/2. So is the set of all subsets of *K* and its cardinality is:

The XOR2-gate probability is rewritten as follows:

The cardinality of *K* given |*K*| ≥ *n*/2 is:

Let *V* be a set of indices such that *Ai*=*ON* and |*U*|=*n*/2. Let be the set of all possible *U* (s) extracted from {1, 2,…, *n*}. Cardinality of is:

Formula 3.3 specifies the positive strength of relationship between *Xi* and *Y*. Event *Xi*=1 causes event *Ai*=*ON* with positive weight *wi*. There is a question “how likely the event *Ai*=*OFF* is given *Xi*=0”. In order to solve this problem, I define the negative strength of relationship between *Xi* and *Y* denoted *ωi*. Event *Xi*=0 causes event *Ai*=*OFF* with negative weight *ωi*. In other words, each arc in simple graph is associated with a positive weight *wi* and a negative weight *ωi*. Such graph is called *bi-weight simple graph* shown in figure 3.3.

**Figure 3.3.** Bi-weight simple graph

With bi-weight simple graph, all X-gate inferences are extended as so-called X-gate bi-inferences. Derived from formula 3.3, formula 3.20 specifies conditional probability of accountable variables with regard to bi-weight graph.

**Formula 3.20.** Conditional probability of accountable variables with regard to bi-weight graph

The probabilities *P*(*Ai*=*ON* | *Xi*=0) and *P*(*Ai*=*OFF* | *Xi*=1) are called positive adder *di* and negative adder *δi*. As usual, *di* and *δi* are smaller than *wi* and *ωi*. When *di*=0, bi-weight graph becomes normal simple graph.

The total positive (negative) weight is defined as sum of positive (negative) weight and respective adder. Formula 3.21 specifies such total weights *Wi* and . These weights are also called relationship powers.

Where,

**Formula 3.21.** Total positive and negative weights

By extending aforementioned X-gate inferences, we get bi-inferences for AND-gate, OR-gate, NAND-gate, NOR-gate, XOR-gate, XNOR-gate, and U-gate as shown in table 3.3. Arrangement counters are calculated by the same way.

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| There are four conditions of *U*: |*U*|=*α*, |*U*|≥*α*, |*U*|≤*β*, and *α*≤|*U*|≤*β*. Note that is the complement of *U*,  The largest cardinality of is: |

**Table 3.3.** Bi-inferences for AND-gate, OR-gate, NAND-gate, NOR-gate, XOR-gate, XNOR-gate, and U-gate

As a convention, the product of probabilities is 1 if indices set is empty.

With regard to SIGMA-gate bi-inference, the sum of all of total positive weights must be 1 as follows:

Derived from formula 3.19, the SIGMA-gate probability for bi-weight graph is:

Shortly, formula 3.22 specifies SIGMA-gate bi-inference.

Where,

**Formula 3.22.** SIGMA-gate bi-inference

It is necessary to validate SIGMA-D network and direct SIGMA-D network with SIGMA-gate bi-inference. By applying formula 3.22 into SIGMA-D network, we have:

According to diagnostic theorem, SIGMA-D network with regard to bi-inference satisfies diagnostic condition due to *s*(Ω)=2*n*–1.

By applying formula 3.22 into direct SIGMA-D network, we have:

There is a question: Does an X-D network which is different from SIGMA-D network and not aforementioned exist such that it satisfies diagnostic condition? The most general form of X-gate inference except SIGMA-gate inference is represented by U-gate inference given condition on *U* is arbitrary.

Let *f* be the arrangement sum of U-gate inference.

According to diagnostic theorem, U-gate network satisfies diagnostic condition if and only if *f(pi*, *ρi*) = 2*n*–1 for all abstract variables *pi* and *ρi*. Such variables are not concrete numbers although they can have some constraints. If degree of *f* is less than 5, we cannot conclude whether a given U-gate network satisfies diagnostic condition. This degree can be always less than 5 if U-gate inference establishes some advanced constraints. The degree of *f* will be larger than or equal to 5 for large enough *n* without advanced constraints. Without loss of generality, each *pi* or *ρi* is sum of variable *x* and a variable *ai* or *bi*, respectively. Note that all *pi*, *ρi*, *ai* are *bi* are abstract variables.

The equation *f*–2*n*–1 = 0 becomes equation *g*(*x*) = 0. Given large enough *n*, its degree is *m* ≥ 5.

Where coefficients *Ci* (s) are functions of *ai* and *bi* (s). According to Abel-Ruffini theorem (Wikipedia, Abel-Ruffini theorem, 2016), equation *g*(*x*) = 0 has no algebraic solution when *m* ≥ 5. Thus, abstract variables *pi* and *ρi* cannot be eliminated entirely from *g*(*x*)=0, which causes that there is no specification of U-gate inference *P*(*X*1x*X*2x…x*Xn*) so that diagnostic condition is satisfied. Consequently, it is more likely that an unknown X-gate network will satisfy diagnostic condition if it follows specification of linear combination like SIGMA-D network because the degree of *g*(*x*) which is linear combination of *pi* and *ρi* is always 1.

which causes that equation *f*–2*n*–1 = 0 is irreducible in general case, according to Abel-Ruffini theorem (Wikipedia, Abel-Ruffini theorem, 2016). Consequently, it is more likely that an unknown X-gate network will satisfy diagnostic condition if it follows specification of linear combination like SIGMA-D network. The linear combination asserts that the degree of *f*–2*n*–1 = 0 is always 1 for all *n*.

It is concluded that there is no nonlinear X-D network satisfying diagnostic condition but a new question is raised: Does there exist the general linear X-D network satisfying diagnostic condition? Let GL-D network denote such linear network. Note that SIGMA-D network is specific case of GL-D network. The GL-gate probability must be linear combination of weights because *s*(Ω) is linear combination of weights.

Where *C* is arbitrary constant.

The GL-gate inference is singular if *αi* and *βi* are functions of only *Xi* as follows:

The functions *hi* and *gi* are not relevant to *Ai* because the final formula of GL-gate inference is only relevant to *Xi* and weights (s). The singular GL-gate inference is fair if every function *hi* or *gi* acts on *Xi* by the same way. In other words, we have:

Because GL-D network is a pattern, we only survey singular and fair GL-gate. Mentioned GL-gate is singular and fair by default and it is dependent on how to define functions *h* and *g*. The arrangement sum with regard to GL-gate is:

Where *α* and *β* are constants because functions *h* and *g* act over all possible values of *Xi*.

GL-D network satisfies diagnostic condition if

Shortly, formula specifies the singular and fair GL-gate inference so that GL-D network satisfies diagnostic condition.

The additional constrain of weights is:

Where,

**Formula.** Singular and fair GL-gate inference

Functions *h*(*X*) and *g*(*X*) are always linear due to *Xm*=*X* for all *m*≥1 when *X* is binary. It is easy to infer that SIGMA-D network is GL-D network with *C*=0 and following definition of functions *h* and *g*.