Fetal Weight Estimation in case of Missing Data

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**Abstract:**

The abstract gives you a chance to describe your article with concise sentences in about 300 words or less. It should summarize the problem or objective of your research and the significant section of the article with enough details to attract the readers should be mentioned in this part. Usually, an abstract doesn’t include references, figures, tables, undefined abbreviations or unspecified references.

**Keywords:**

Fetal Weight Estimation, Regression Model, Ultrasound Measures, Expectation Maximization Algorithm, Missing Data.

1. **Introduction**

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|  | (99) |

1. **Methodology**

Suppose we estimate the linear regression model *Z* = *α*0 *+ α*1*X*1 *+ α*2*X*2 *+ … + αnXn* where *Z* is fetal weight and *Y* is fetal age whereas *Xi* (s) are gestational ultrasound measures such as *bpd*, *hc*, *ac*, and *fl*. Suppose the random variable *Z* conforms normal distribution, according to equation 1 (Lindsten, Schön, Svensson, & Wahlström, 2017, pp. 8-9). Note, *Z* is random variable whereas *X* is data in equation 1.

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| --- | --- |
|  | (1) |

Where *α* = (*α*0, *α*1,…, *αn*)*T* is parameter vector and *X* = (1, *X*1, *X*2,…, *Xn*)*T* is data vector. The mean and variance of *Z* with regard to *P*(*Z* | *X*, *α*) are *αTX* and *σ*2, respectively. The superscript “*T*” denotes transposition operator in vector and matrix. Suppose each has an inverse linear regression model *Xj* = *βj*0 *+ βj*1*Z.* In other words, *Zj* now is considered as the random variable conforming normal distribution according to equation 2.

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|  | (2) |

Where *βj* = (*βj*0, *βj*1)*T* is a partial parameter vector and (1, *Z*)*T* is a partial data vector. The mean and variance of each *Xj* with regard to the inverse distribution *Pj*(*Xj* | *Z*, *βj*) are *βjT*(1, *Z*)*T* and *σj*2, respectively. Of course, there are *n* inverse linear regression models.

Let ***D*** = (***X***, ***z***) be collected sample in which ***X*** is a set of sample measures and ***z*** is a set of fetal weights with note that both ***X*** and ***z*** are incomplete. In other words, ***X*** and ***z*** have missing values. Now we focus on estimating *α* and *βj* based on ***D***. As a convention let *α\** and *βj\** be estimates of *α* and *βj*, respectively (Lindsten, Schön, Svensson, & Wahlström, 2017, p. 8).

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|  | (3) |

The expectation of sufficient statistic *Z* regard to the entire linear model *P*(*Z* | *X*, *α*) is specified by equation 4.

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|  | (4) |

The expectation of each sufficient statistic *Xj* with regard to each inverse linear model *Pj*(*Xj* | *Z*, *βj*) is specified by equation 5.

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|  | (5) |

Please pay attention to equations 4 and 5 because *Z* and *Xj* will be estimated by these expectations later.

By applying sample ***D*** into equations 1 and 2 and using maximum likelihood estimation (MLE) method, we retrieve equation 6 to estimate *α\** and *βj\** (Lindsten, Schön, Svensson, & Wahlström, 2017, pp. 8-9).

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|  | (6) |

Where ***X***, ***z***, ***Z***, and ***x****j* are specified in equation 3. Because ***X*** and ***Z*** are incomplete, we apply expectation maximization (EM) algorithm into estimating (*α\**, *βj\**)*T*. EM algorithm has many iterations and each iteration has expectation step (E-step) and maximization step (M-step) for estimating parameters. Given current parameter Θ(*t*) = (*α*(*t*), *βj*(*t*))*T* at the *t*th iteration, missing values *zi* and *xij* are calculated in E-step so that ***X*** and ***Z*** become complete. In M-step, the next parameter Θ(*t*+1) = (*α*(*t*+1), *βj*(*t*+1))*T* is determined by equation 6 and the complete data ***X*** and ***Z***.

The most important problem in our research is how to estimate missing values *zi* and *xij*. Recall that every missing value *zi* is estimated as the expectation based on the current parameter *α*(*t*), according to equation 4.

Let *Ui* be a set of indices of missing values *xij*. In other words, if then, *xij* is missing. The set *Ui* can be empty. The equation 5 is written:

Note, *xi*0 = 1. According to equation 5, missing value *xij* is estimated by:

Combining equation 4 and equation 5, we have:

It implies:

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|  | (7) |

Missing values *zi* and *xij* are estimated by the balanced estimation process shown in table 1.

**Table 1.** Balanced estimation process of missing values

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| --- |
| 1. Step 1: Missing values *zi* are estimated by equation 7, based on the current parameter Θ(*t*) = (*α*(*t*), *βj*(*t*))*T*.   Missing values *xij* where are estimated by equation 5 and the estimated values *zi* above, based on the current parameter Θ(*t*) = (*α*(*t*), *βj*(*t*))*T*.   1. Step 2: For balancing both *P*(*Z* | *X*, *α*) and *Pj*(*Xj* | *Z*, *βj*) in estimation, values *zi* and *xij* are re-estimated by equations 4 and 5 as new *zi*’ and *xij*’, based on the current parameter Θ(*t*) = (*α*(*t*), *βj*(*t*))*T*.   If the deviation between (*zi*’, *xij*’) and (*zi*, *xij*) is smaller than a pre-defined threshold, the estimation process stops; at that time *zi*’ and *xij*’ are final estimated values. Otherwise, going back step 1 with *xij* = *xij*’. |

In fact, the balanced estimation process is an iterative process which is a combination of equations 4, 5, and 7. As a result, EM algorithm associated with the balanced estimation process for regression model is shown in table 2 (Dempster, Laird, & Rubin, 1977, p. 4). This is our so-called Regression Expectation Maximization (REM) algorithm.

**Table 2.** Regression Expectation Maximization (REM) Algorithm

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| --- |
| 1. E-step: Missing values *zi* and *xij* are estimated by the balanced estimation process shown in table 1. 2. The next parameter Θ(*t*+1) = (*α*(*t*+1), *βj*(*t*+1))*T* is determined by equation 6 and the complete data ***X*** and ***Z*** fulfilled in E-step. |

The REM algorithm stops if at some *t*th iteration, we have Θ(*t*) = Θ(*t*+1) = Θ*\**. At that time, Θ*\** = (*α\**, *β\**)*T* is the optimal estimate of EM algorithm. In practice, the algorithm can stop if the deviation between Θ(*t*) and Θ(*t*+1) is smaller than a small enough terminated threshold. In this research such *terminated threshold* is *ε* = 0.1% = 0.001. The smaller the terminated threshold is, the more accurate the algorithm is.

An technique to improve the convergence of REM is to initialize the parameter Θ(1) = (*α*(1), *β*(1))*T* at the first iteration of EM process in proper way instead of initializing Θ(1) in arbitrary way. Note, by default, Θ(1) is initialized as zero vector. Let ***X***’ be the complete matrix of ultrasound measures, which is created by removing all rows whose values are missing from ***X***. Similarly, let ***Z***’ be the complete matrix of fetal weights, which is created by removing rows whose weights are missing from ***Z***. The advanced Θ(1) = (*α*(1), *β*(1))*T* is initialized by equation 8.

|  |  |
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|  | (8) |

Where ***z***’ is the complete vector of non-missing weights and ***x****j*’ is the complete vector of non-missing measures. Equation 8 is variant of equation 6 where ***X***, ***Z***, ***x****j*, and ***z*** are replaced by ***X***’, ***Z***’, ***x****j*’, and ***z***’.

1. **Results and Discussion**
2. **Conclusions**

**Conflicts of Interest**

The authors declare that there is no conflict of interest regarding the publication of this article.

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**References**

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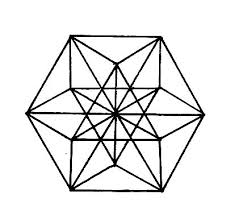
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| data | data | data |
| data | data | data 1 |

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