DEMG609: Homework 1

Nick Graetz

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1.(a)

```
# Because the instantaneous growth rate
# is constant over the interval, it can
# be calculated as the mean annualized
# growth rate.
N_t1 <- 281421906
N_t2 <- 308745538
t1 <- 2000.25
t2 <- 2010.25
T <- t2 - t1
r_star <- log(N_t2/N_t1)/T
answer_1a <- r_star
```

Answer: 0.0092662

1.(b)

```
tripling_time <- log(3)/r_star
answer_1b <- tripling_time
```

Answer: 118.5606806

1.(c)

```
# Given our growth rate is constant...
py <- ((N_t2 - N_t1) * T)/log(N_t2/N_t1)
answer_1c <- py
```

Answer: 2948727626.35409

1.(d)

```
## What was the population size in the
## middle of the intercensual period?
## Given that growth rate is constant...
pop_at_t <- function(t) {
    new_pop <- N_t1 * exp(r_star * t)
    return(new_pop)
}
N_mid <- pop_at_t(T/2)
answer_1d <- N_mid
```

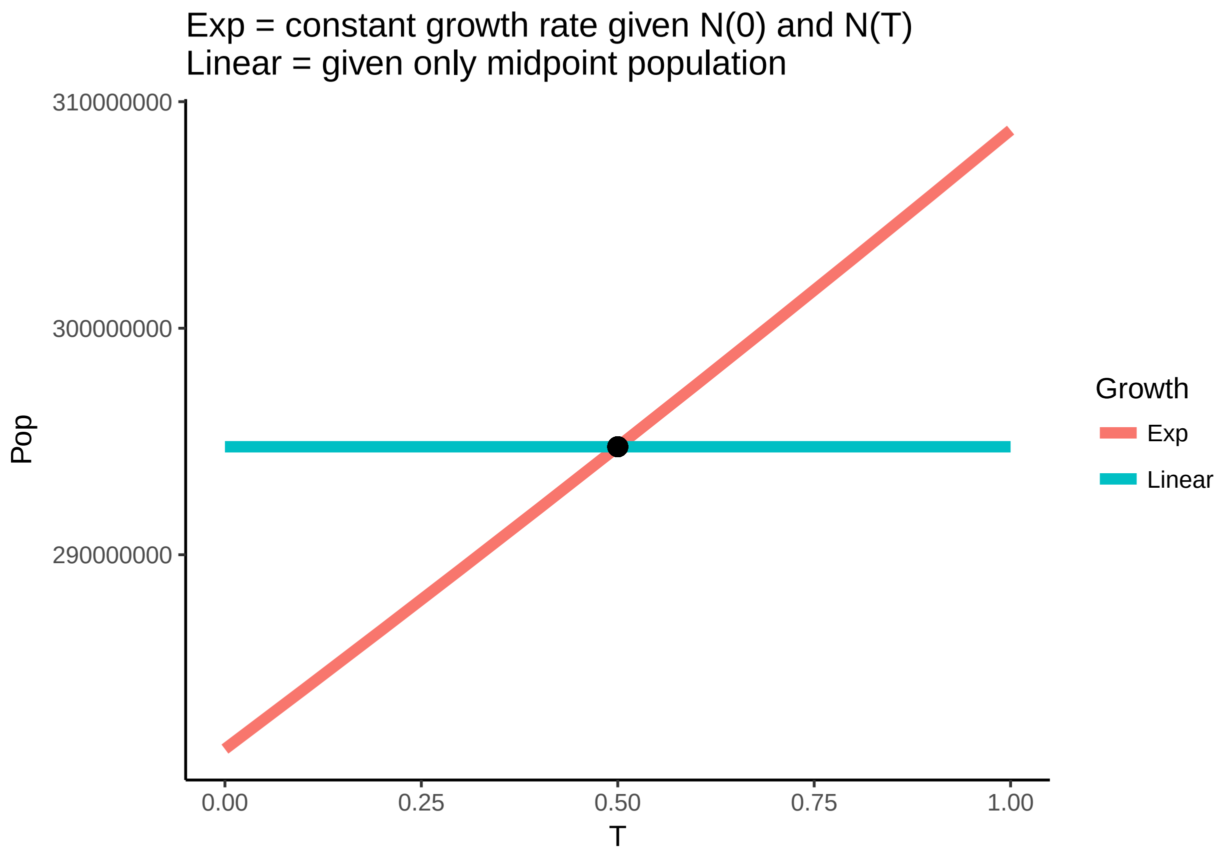
Answer: 294767294.272881

1.(e)

```
## Given only midpoint population, we have
## to assume linear growth
py_linear <- N_mid * T

growth <- data.table(T = seq(0, 1, 0.1),
    Pop = c(rep(N_mid, 11), pop_at_t(0:10)),
    Growth = c(rep("Linear", 11), rep("Exp",
        11)))

ggplot(data = growth) + geom_line(aes(x = T,
    y = Pop, color = Growth), size = 2) +
    geom_point(x = 0.5, y = N_mid, color = "black",
        size = 3) + ggtitle("Exp = constant growth rate given N(0) and N(T)\nLinear = given only midpoint populatio
    theme_classic()
```



Answer:
Linear PY: 2947672942.72881
Exp PY: 2948727626.35409
Difference: -1054683.6252751

2.(a)

Answer: 0.0092662

3.(a)

```
## Instead of 2000.25 to 2010.25, it's now
## 2000.25 - 2010.08333
N_t1 <- 281421906
N_t2 <- 308745538
t1 <- 2000.25
t2 <- 2010 + (1/12)
T <- t2 - t1
r_star <- log(N_t2/N_t1)/T
answer_3a <- r_star
```

Answer: 0.0094233

3.(b)

```
pop_at_t <- function(t, r_star, N_t1) {
    new_pop <- N_t1 * exp(r_star * t)
    return(new_pop)
}
N_mid <- pop_at_t(t = (T/2), r_star = r_star,
    N_t1 = N_t1)
answer_3b <- N_mid
## Bigger, because it has to get to the
## same end population faster
```

Answer: 294767294.272881
Answer to 1(d): 294767294.272881

3.(c)

Given that N_0 and N_t and the assumption of constant r between N_0 and N_t: $N_t = N_0 * e^{rt}$ and $r = ln\left(\frac{N_t}{N_0}\right)$

$$N_t = N_0 e^{\frac{ln\left(\frac{N_t}{N_0}\right)}{T}T}$$

$$N_t = N_0 e^{ln\left(\frac{N_t}{N_0}\right)}$$

$$N_t = N_0 * \frac{N_t}{N_0}$$

$$N_t = N_t$$

$$1 = 1$$

4.(a)

```
## Reset to initial pops and dates
N_t1 <- 281421906
N_t2 <- 308745538
t1 <- 2000.25
t2 <- 2010.25
T <- t2 - t1
## Inputs: j = number of times to compound
## per year year_interval = years in the
## period start_pop end_pop Outputs: r_j =
## annual geometric growth rate
calc_geo_r <- function(j, year_interval,
    start_pop, end_pop) {
    r_j <- j * ((end_pop/start_pop)^(1/(j *
        year_interval)) - 1)
    return(r_j)
}

## Apply function over j = 1, 12, 365
## (annually, monthly, daily,
## continuously)
geom_rates <- lapply(c(1, 12, 365, 1000000),
    calc_geo_r, year_interval = T, start_pop = N_t1,
    end_pop = N_t2)
```

Answer:
Annually: 0.0093093
Monthly: 0.0092698
Daily: 0.0092664
Cont: 0.0092662

4.(b)

```
calc_geo_r_2 <- function(j, annual_rate) {
    r_j <- j * ((1 + annual_rate)^(1/j) -
```

```r
        1)
    return(r_j)
}
r_j_cont <- calc_geo_r_2(j = 1000000, annual_rate = geom_rates[[1]])
answer_4b <- r_j_cont
```

Answer: 0.0092662

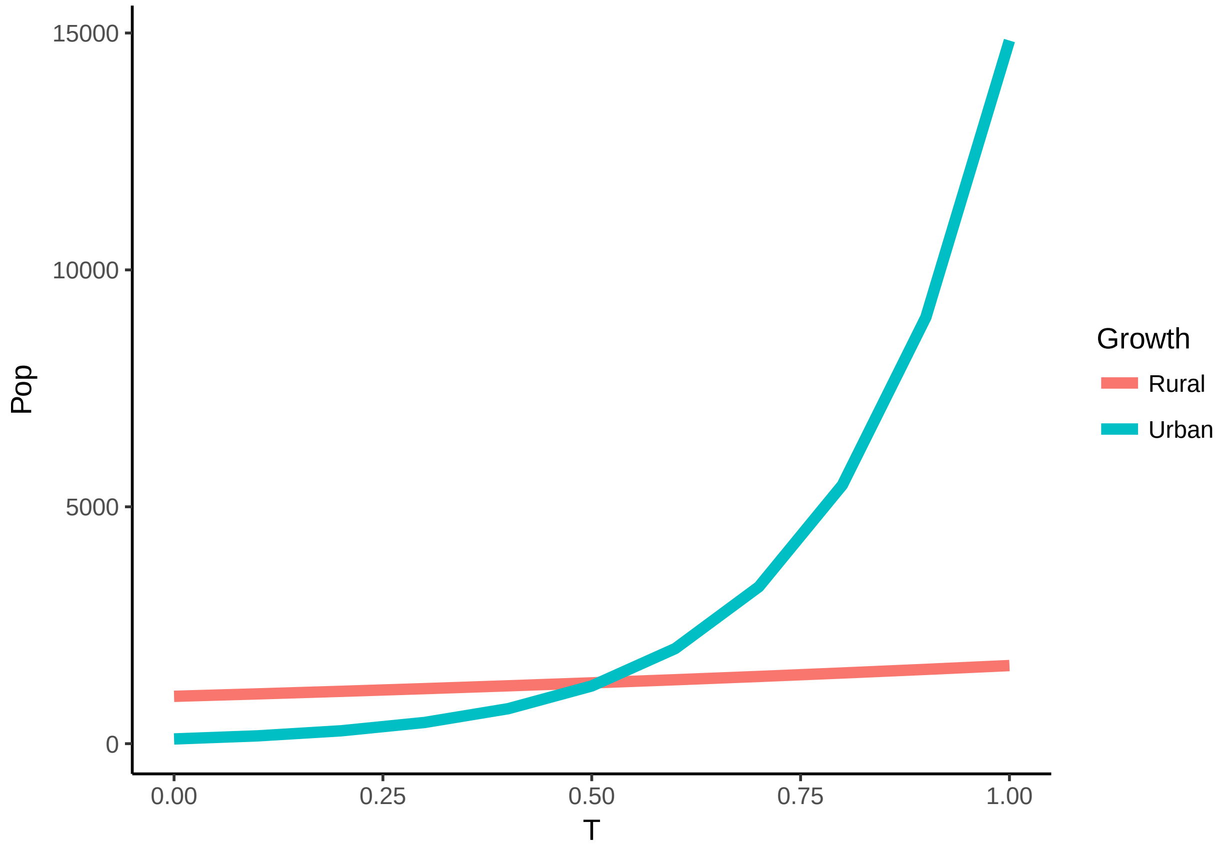
4.(B)

```r
r_star_ukraine <- -0.49/100
halving_time <- log(0.5)/r_star_ukraine
answer_4B <- halving_time
```

Answer: 141.4586083

4.(C)

```r
t1_urban <- 100
t1_rural <- 1000
r_star_urban <- 0.5
r_star_rural <- 0.05
growth_comparison <- data.table(T = seq(0,
    1, 0.1), Pop = c(pop_at_t(0:10, r_star = r_star_urban,
    N_t1 = t1_urban), pop_at_t(0:10, r_star = r_star_rural,
    N_t1 = t1_rural)), Growth = c(rep("Urban",
    11), rep("Rural", 11)))
ggplot() + geom_line(data = growth_comparison,
    aes(x = T, y = Pop, color = Growth),
    size = 2) + theme_classic()
```




```
growth_comparison <- dcast(growth_comparison,
    T ~ Growth, value.var = "Pop")
growth_comparison[, `:=`(pop_ratio, Urban/Rural)]
t1_ratio <- growth_comparison[T == 0, pop_ratio]
t2_ratio <- growth_comparison[T == 1, pop_ratio]
ratio_growth_rate <- log(t2_ratio/t1_ratio)/10
ratio_growth_rate == (r_star_urban - r_star_rural)
```

```
## [1] TRUE
```

Answer:
Urban growth rate: 0.5
Rural growth rate: 0.05
Difference in growth rates: 0.45
Growth rate of urban/rural population ratio: 0.45

Analytic Answer:

$$R = \frac{N_u}{N_r}$$

Prove that: $Growth_R = Growth_u - Growth_r$

$$\frac{ln\left(\frac{R_t}{R_0}\right)}{T} = Growth_u - Growth_r$$

$$\frac{ln\left(\frac{\frac{N_{t,u}}{N_{t,r}}}{\frac{N_{0,u}}{N_{0,r}}}\right)}{T} = \frac{ln\left(\frac{N_{t,u}}{N_{0,u}}\right)}{T} - \frac{ln\left(\frac{N_{t,r}}{N_{0,r}}\right)}{T}$$

$$ln\left(\frac{\frac{N_{t,u}}{N_{t,r}}}{\frac{N_{0,u}}{N_{0,r}}}\right) = ln\left(\frac{N_{t,u}}{N_{0,u}}\right) - ln\left(\frac{N_{t,r}}{N_{0,r}}\right)$$

Use this law to reduce all terms: $ln\left(\frac{a}{b}\right) = ln(a) - ln(b)$

$$ln\left(\frac{N_{t,u}}{N_{t,r}}\right) - ln\left(\frac{N_{0,u}}{N_{0,r}}\right) = ln\left(\frac{N_{t,u}}{N_{0,u}}\right) - ln\left(\frac{N_{t,r}}{N_{0,r}}\right)$$

$$ln(N_{t,u}) - ln(N_{t,r}) - ln(N_{0,u}) - ln(N_{0,r}) = ln(N_{t,u}) - ln(N_{0,u}) - ln(N_{t,r}) - ln(N_{0,r})$$

$$1 = 1$$