DEMG609: Problem Set 5

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October 18, 2017

A.(1)

```r
data <- fread("C:/Users/ngraetz/Documents/repos/demg609/hw4_data_clean.csv")
setnames(data, 'nDx i', 'nDx_i')

# 1. Calculate multiple decrement life table for period.

# Calculate npx from lx
  for(r in 1:length(data[, lx])) {
    lx_n <- data[r+1, lx]
    data[r, npx := lx_n / lx]
    if(r == length(data[, lx])) data[r, npx := 0]
  }
  data[, nqx := 1 - npx]

# Calculate nqx_i
  data[, nqx_i := nqx * (nDx_i / nDx)]

# Calculate ndx_i
  data[, ndx_i := nqx_i * lx]

# Calculate lx_i
  for(r in 1:length(data[, ndx_i])) {
    data[r:length(data[, ndx_i]), lx_i := sum(ndx_i)]
  }

# Calculate ndx
  for(r in 1:length(data[, lx])) {
    lx_n <- data[r+1, lx]
    data[r, ndx := lx - lx_n]
    if(r == length(data[, lx])) data[r, ndx := lx]
  }

# Calculate the probability that someone aged x will eventually exit from cause i
  data[, p_i := lx_i / lx]

  lt <- data
```

Answer:

x	nDx	nDx_i	lx	npx	nqx	nqx_i	ndx_i	lx_i	ndx	p_i
0	15646	494	100000	0.99228	0.00772	0.00024	24	4018	772	0.04018
1	2975	1127	99228	0.99846	0.00154	0.00058	58	3994	153	0.04025
5	1964	897	99075	0.99904	0.00096	0.00044	43	3936	95	0.03973
10	2528	1077	98980	0.99874	0.00126	0.00054	53	3893	125	0.03933
15	9780	4624	98855	0.99518	0.00482	0.00228	225	3839	476	0.03884
20	12634	5416	98379	0.99314	0.00686	0.00294	289	3614	675	0.03674
25	12542	4555	97704	0.99310	0.00690	0.00251	245	3325	674	0.03403
30	15734	4542	97030	0.99199	0.00801	0.00231	224	3080	777	0.03174

x	nDx	nDx_i	lx	npx	nqx	nqx_i	ndx_i	lx_i	ndx	p_i
35	23681	5411	96253	0.98952	0.01048	0.00240	231	2856	1009	0.02967
40	33437	5837	95244	0.98492	0.01508	0.00263	251	2625	1436	0.02756
45	43303	4996	93808	0.97735	0.02265	0.00261	245	2375	2125	0.02531
50	52356	3606	91683	0.96757	0.03243	0.00223	205	2129	2973	0.02323
55	63441	2753	88710	0.94952	0.05048	0.00219	194	1925	4478	0.02170
60	79283	2262	84232	0.92281	0.07719	0.00220	186	1730	6502	0.02054
65	107529	2336	77730	0.88295	0.11705	0.00254	198	1545	9098	0.01987
70	147391	2656	68632	0.82543	0.17457	0.00315	216	1347	11981	0.01963
75	174853	3096	56651	0.74855	0.25145	0.00445	252	1131	14245	0.01997
80	166117	3276	42406	0.62461	0.37539	0.00740	314	879	15919	0.02073
85	209989	4480	26487	0.00000	1.00000	0.02133	565	565	26487	0.02133

A.(2)

Answer:
Fraction of newborns who will die from accidents: 0.0401826 Fraction of those who survive to 50 who will die from accidents: 0.0232256

A.(3)

```
# 3.
  lt[, R_i := (nDx - nDx_i) / nDx]
  lt[, npx_minus_i := npx^R_i]
```

Answer:

x	nDx	nDx_i	lx	npx	nqx	nqx_i	ndx_i	lx_i	ndx	p_i	R_i	npx_minus_i
0	15646	494	100000	0.99228	0.00772	0.00024	24	4018	772	0.04018	0.97	0.99252
1	2975	1127	99228	0.99846	0.00154	0.00058	58	3994	153	0.04025	0.62	0.99904
5	1964	897	99075	0.99904	0.00096	0.00044	43	3936	95	0.03973	0.54	0.99948
10	2528	1077	98980	0.99874	0.00126	0.00054	53	3893	125	0.03933	0.57	0.99927
15	9780	4624	98855	0.99518	0.00482	0.00228	225	3839	476	0.03884	0.53	0.99746
20	12634	5416	98379	0.99314	0.00686	0.00294	289	3614	675	0.03674	0.57	0.99607
25	12542	4555	97704	0.99310	0.00690	0.00251	245	3325	674	0.03403	0.64	0.99560
30	15734	4542	97030	0.99199	0.00801	0.00231	224	3080	777	0.03174	0.71	0.99430
35	23681	5411	96253	0.98952	0.01048	0.00240	231	2856	1009	0.02967	0.77	0.99190
40	33437	5837	95244	0.98492	0.01508	0.00263	251	2625	1436	0.02756	0.83	0.98754
45	43303	4996	93808	0.97735	0.02265	0.00261	245	2375	2125	0.02531	0.88	0.97993
50	52356	3606	91683	0.96757	0.03243	0.00223	205	2129	2973	0.02323	0.93	0.96977
55	63441	2753	88710	0.94952	0.05048	0.00219	194	1925	4478	0.02170	0.96	0.95166
60	79283	2262	84232	0.92281	0.07719	0.00220	186	1730	6502	0.02054	0.97	0.92493
65	107529	2336	77730	0.88295	0.11705	0.00254	198	1545	9098	0.01987	0.98	0.88534
70	147391	2656	68632	0.82543	0.17457	0.00315	216	1347	11981	0.01963	0.98	0.82829
75	174853	3096	56651	0.74855	0.25145	0.00445	252	1131	14245	0.01997	0.98	0.75240
80	166117	3276	42406	0.62461	0.37539	0.00740	314	879	15919	0.02073	0.98	0.63043
85	209989	4480	26487	0.00000	1.00000	0.02133	565	565	26487	0.02133	0.98	0.00000

A.(4)

Survivorship has increased by a factor of 1.0480789 by deleting mortality from accidents. An individual is ~5% more likely to survive to 85 if there were no accidents, relative to the master life table that includes accidents (absolute difference in mortality rates is only ~1%). This can't be interpreted in terms of surviving to 85 if the only cause of death was accidents. We would need to calculate ${}^{*}\mathrm{l}_x^{\text{accidents}}$ (mortality from only accidents) vs. mortality from everything except accidents.

B.(1)

Given dependent probabilities (marriage or death are the only ways individuals may exit the cohort),

$$ {}_n\mathrm{q}_x^i = \frac{n * {}_n\mathrm{M}_x^i}{1 + (n - {}_n\mathrm{a}_x) * ({}_n\mathrm{M}_x^i + {}_n\mathrm{M}_x^{-i})} $$

$$ P(\text{Never married at 50}) = \prod_{x=0}^{45} (1 - {}_5\mathrm{q}_x^M) \text{ where x = 0,5,10,...,45} $$

$$ \text{And where } {}_5\mathrm{q}_x^M = \frac{5 * {}_5\mathrm{M}_x^M}{1 + (5 - 2.5) * ({}_5\mathrm{M}_x^M + {}_5\mathrm{M}_x^D)} $$

B.(2)

$$ {}_n^*\mathrm{p}_x^i = e^{-n * {}_n\mathrm{M}_x^i} $$

$$ P(\text{Never married at 50, net of D}) = \prod_{x=0}^{45} e^{-5 * {}_5\mathrm{M}_x^M} \qquad \text{where x = 0,5,10,...,45} $$

B.(3)

Reducing the mortality rate by 20% only decreases our denominator for the first expression, implying a higher probability of first marriage. This means that the probability of never married at 50 will be smaller than the first cohort.

$$ P(\text{Never married at 50}) = \prod_{x=0}^{45} (1 - {}_5\mathrm{q}_x^M) \text{ where x = 0,5,10,...,45} $$

$$ \text{And where } {}_5\mathrm{q}_x^M = \frac{5 * {}_5\mathrm{M}_x^M}{1 + (5 - 2.5) * ({}_5\mathrm{M}_x^M + 0.8({}_5\mathrm{M}_x^D))} $$

Our second expression, the probability of being never married at 50 net of mortality, will remain unchanged.

$$P(\text{Never married at 50, net of D}) = \prod_{x=0}^{45} e^{-5 * {}_5\mathrm{M}_x^M} \quad \text{where x = 0,5,10,...,45}$$

B.(4)

Examined in aggregate, one would conclude that rates of first marriage are stable over time and perhaps further conclude that the probability of first marriage is also stable. However, there is a trend of decreasing mortality rates of never-married individuals over time as well. This trend coupled with stable rates of first marriage implies that the probability of first marriage is actually increasing over age (given expression (1) above).