Midterm study guide

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Single decrement life table

Cohort

$x =$ age

$l_x =$ survivors at exact age x

${}_nd_x =$ deaths between age x and x + n

${}_nq_x =$ probability of dying between age x and x + n

${}_np_x =$ probability of surviving from age x to x + n

${}_nL_x =$ person-years lived between age x and x + n

$T_x =$ total person-years lived after exact age x

$e_x^0 =$ average years of life lived after exact age x (in this case, since birth)

${}_nm_x =$ mortality rate between ages x and x + n

${}_nm_x$ to ${}_nq_x$ conversion:

$${}_nq_x = \frac{n * {}_nm_x}{1 + (n - {}_na_x) * {}_nm_x}$$

- In a real cohort, ${}_na_x$ is known: average person-years lived by individuals who died between ages x and x + n

Period (synthetic cohort)

- Based on data for a specific period, ${}_nM_x = \frac{{}_nD_x}{{}_nN_x}$
- Only need to assume one paramter to make conversion from ${}_nm_x$ to ${}_nq_x$: ${}_na_x = \frac{n}{2}$, usually 2.5
- Other methods: borrow from another population, graduation/iteration techniques, assume rate constant

1. **Assumption 1**: ${}_nM_x =_n m_x$, assume that a hypothetical cohort is to experience an observed set of period age-specific deaths rates.
2. **Assumption 2**: we have to make an estimate of ${}_na_x$, usually $\frac{n}{2}$. At the open-ended age group, ${}_na_x = \frac{1}{{}_nm_x}$.
3. Conversion: ${}_nq_x = \frac{n*_nm_x}{1+(n-_na_x)*_nm_x}$

- If we make the assumption that the age-specific death rate is constant from x to x + n, no ${}_na_x$ is required.

- ${}_np_x = 1 - {}_nq_x = e^{-n*_nm_x}$

4. ${}_np_x = 1 - {}_nq_x$
5. $l_{x+n} = l_x * {}_np_x$
6. ${}_nd_x = l_x - l_{x+n}$

7. ${}_nL_x = n * l_{x+n} + {}_na_x * {}_nd_x$
8. $T_x = \sum_{a=x}^{\infty} {}_nL_a$
9. $e_x^0 = \frac{T_x}{l_x}$

Stationary population interpretation

Conditions (results of these conditions is that age structure is constant):

- Constant annual number of births
- Constant mortality
- Zero migration

${}_nL_x$ = PY lived between age x and x + n in a calendar year AND cohort AND number of people between age x and x + n at any given time
T_0 = total population size
CBR = CDR = $\frac{1}{e_0^0}$
e_0^0 = mean age at death
Death rate above age x = $\frac{l_x}{T_x} = \frac{1}{e_x^0}$

Mortality as a continuous process

The force of mortality is the derivative of the mortality rate: $\mu_x = \lim_{n \to 0} {}_nm_x$
If μ_x is constant, you don't have to make any assumption about ${}_na_x$:

1. $\mu_x = {}_nm_x = {}_nM_x$

2. $l_{x+n} = l_x * e^{-n*\mu_x}$

3. ${}_np_x = e^{-n*{}_nm_x}$

4. ${}_nq_x = 1 - {}_np_x$

Multiple decrement life tables

Cohort

${}_nd_x^i$ = number of decrements from cause i between ages x and x + n

${}_nq_x^i = \frac{{}_nd_x^i}{l_x}$ = probability of decrement from cause i between ages x and x + n

${}_nm_x^i = \frac{{}_nd_x^i}{{}_nL_x}$ = rate of decrement from cause i between ages x and x + n

$l_x^i = \sum_{a=x}^{\infty} {}_nd_a^i$

$\sum_i {}_nm_x^i =_n m_x$

$\sum_i {}_nq_x^i =_n q_x$

Period (synthetic cohort)

${}_nm_x^i$ to ${}_nq_x^i$ conversion:

$$ {}_nq_x^i = \frac{n * {}_nm_x^i}{1 + (n - {}_na_x) * {}_nm_x} $$

$$ {}_nq_x^i = \frac{n * {}_nm_x^i}{1 + (n - {}_na_x) * ({}_nm_x^i +_n m_x^{-i})} $$

${}_nq_x^i$ is referred to as a "dependent probability" because if the mortality rates increase from other causes besides i (${}_nm_x^{-i}$) between ages x and x + n, the probability of exiting from cause i will be lower (people who may have exited from cause i are exiting from other causes at higher rates).

Given the master life table ${}_nq_x^i$, total deaths (${}_nD_x$), and deaths from cause i (${}_nD_x^i$):

1. ${}_nq_x^i = {}_nq_x * \frac{{}_nD_x^i}{{}_nD_x}$ (if μ_x is given, use conversion above)
2. ${}_nd_x^i = {}_nq_x^i * l_x$
3. $l_x^i = \sum_{a=x}^{\infty} {}_nd_a^i$
4. $\frac{l_x^i}{l_x}$ = probability that someone aged x will eventually exit from cause i

Given master ${}_nd_x$, total deaths (${}_nD_x$), and deaths from cause i (${}_nD_x^i$):

Associated sinlge decrement life table (ASDLT)

${}_n^*\mathrm{p}_x^i$ = probability of surviving from age x to x + n where only cause i is operating. You must make an assumption about μ_i:

1. μ_i is constant between x and x + n:

$$ {}_n^*\mathrm{p}_x^i = e^{-n*_nM_x^i} $$

2. Assume μ_i is proportional to μ between x and x + n.

$$ {}_n^*\mathrm{p}_x^i = {}_np_x{}^{R^i} \quad \text{where } R^i = \frac{{}_nD_x^i}{{}_nD_x} $$

${}^*\mathrm{l}_{x+n}^i = {}^*\mathrm{l}_x^i * {}_n^*\mathrm{p}_x^i$