

PHYS 272: Pre-Lecture Notes and Functions

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1 Electricity

BEGIN TEST 1

1.1 Lecture 1: Coulomb's law

1.1.1 Pre-Lecture

Coulomb's Law:

$$k = 9 * 10^9 \frac{Nm^2}{C^2}$$

$$\vec{F}_{12} = k \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12}$$

REMEMBER, convert from μC to C! $1C = 1,000,000 \mu$

$$\vec{F} = G \frac{m_1 m_2}{r^2} \hat{r}$$

Superposition principle

$$\vec{F} = q\vec{E} + q\vec{v} * \vec{B}$$

1.2 Lecture 2: Electric Fields

1.2.1 Pre-Lecture

$$\vec{F} = k * \frac{Qq}{r^2} \hat{r}$$

Discrete Distribution

$$\vec{E} \equiv \frac{\vec{F}}{q} = k \frac{Q}{r^2} \hat{r}$$

$$\vec{E} \equiv \frac{\vec{F}}{q} = \frac{1}{q} \sum_i \vec{F}_{iq} \quad \vec{E} = k \sum_i \frac{Q_i}{r_{iq}^2} \hat{r}_{iq}$$

Continuous Distribution

$$\vec{E} \equiv \frac{\vec{F}}{q} = \frac{1}{q} \int d\vec{F} = k \int \frac{dQ}{r^2} \hat{r}$$

1.2.2 Homework

I can't find my work for Question 1, sorry...

Question 2-1: $E_x(P)$

$$E_x(P) = k \frac{q_1}{r^2} \cos(45)$$

Question 2-2: $E_y(P)$
 $E_y(P) = E_x(P) + k \frac{q_2}{d^2}$

Question 2-3: A third charge is added
 $E_x(P) = k(\frac{q_1}{2d^2} \cos(45) + \frac{q_3}{d^2})$

Question 2-4
 I guessed C, I don't have the work for it sorry...

Question 2-5
 I guessed E, I don't have the work for it sorry...

1.3 Lecture 3: Electric Flux and Field Lines

1.3.1 Pre-Lecture

Electric Flux
 $\phi \equiv \int \vec{E} \cdot d\vec{A}$
 $\epsilon_0 = 8.85 \times 10^{-12}$

1.3.2 Homework

Question 1-1: What is $E_x(P)$
 $E = 2k \frac{\lambda_1}{a}$
 Conversion for λ is $\lambda \times 10^{-4}$
 For mine, that means $\lambda_1 = -2.2$ is used as -2.2×10^{-4}
 That converts from $\frac{\mu C}{cm}$ to $\frac{C}{m}$
 I looked it up...

Question 1-2: $E_y(P)$
 There is no Y movement, so the answer is 0.

Question 1-3: Total Flux pt.1
 $\phi = h \frac{\lambda_1}{\epsilon_0}$

Question 1-4: What is the new value for $E_x(P)$?
 $E_x(P) = \frac{2}{4\pi\epsilon_0} (\frac{\lambda_1}{a} + \frac{\lambda_2}{x})$
 $x = \frac{a}{2}$

Question 1-5: total flux ϕ pt.1
 I fucked this one up too many times with typos, but I'm pretty sure it's
 $h(\frac{\lambda_1 + \lambda_2}{\epsilon_0})$
 This is also the answer for 7

Question 1-6: Total Flux pt.2
 $2k(\frac{\lambda_2}{a} + \frac{\lambda_1}{\frac{3a}{2}})$

Question 1-7: Total flux ϕ pt.2
 Same function as 1-5, hopefully...

1.4 Lecture 4: Gauss' Law

1.4.1 Pre-Lecture

Gauss' Law

$$\phi_{Net} = \oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\epsilon_0}$$

$$\phi = \frac{q_1}{\epsilon_0} + \frac{q_2}{\epsilon_0}$$

On conducting shell

$$Q_{inner} = -q_o$$

Induced inner charge density

$$\sigma_i = \frac{-q_o}{4\pi R_i^2}$$

Out charge density

$$\sigma_o = \frac{Q+q_o}{4\pi R_o^2}$$

Gauss' Law on a Sphere

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

Gauss' Law on a Cylinder

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

Infinite sheet of charge

$$E = \frac{\sigma}{2\epsilon_0}$$

Spherical (3d) Field line density

$$\frac{1}{A_{sphere}} \left(\frac{1}{r^2} \right)$$

Cylindrical (2d) Field line density

$$\frac{1}{A_{cylinder}} \left(\frac{1}{r} \right)$$

Planar (1d) Field line density

Constant

1.4.2 Homework

Question 1

$$E = \frac{q_{enclosed}}{A_{sphere}\epsilon_0}$$

$$q_{enclosed} = \frac{Q}{4\pi 100(b^2 - a^2)}$$

$$A_{sphere} = 4\pi r^2$$

Since the answer seems to be the same no matter what

$$\text{Answer} = -1.28798 \times 10^7 \frac{N}{C}$$

Question 2

$$E = \frac{x}{2\pi\epsilon_0 r}$$

$$E_x = \frac{Q_{inner}}{2\pi\epsilon_0 r}$$

Again, answer seems to be the same

$$\text{Answer} = 311.485 \frac{N}{C}$$

Question 3

3-1

$$E_x(P) = \frac{q_1 + q_2}{r^2} \quad 3-2$$

$$E_y(P) = 0$$

3-3

$$E_x(R) = 0$$

3-4

$$E_y(P) = k \frac{q_1}{r^2}$$

3-5

$$\sigma_b = \frac{q_1 + q_2}{4\pi b^2}$$

3-6

$$\sigma_a = \frac{q_1}{4\pi a^2}$$

3-7

A, none

Field is treated as if it's a single point.

Really though, we have 3 choices and 5 guesses

Guess until it's right!

3-8

B, $E_2 = E_0$

Fields are equal as the charge on the outer shell has no effect on field in shell.

Again though, 3 choices 5 chances, throw a dart!

Question 4

4-1

$$\lambda_2 = \rho * \pi(b^2 - a^2)$$

4-2

Answer = 0

4-3

$$E_y(P) = \frac{(\lambda_1 * 10^6 + \lambda_2 * 10^6)}{2\pi\epsilon_0 r}$$

4-4

$$E = \left(\frac{\lambda_1 * 10^6}{2\pi\epsilon_0 r} \right)$$

4-5

$$E = \left(\frac{\lambda_1 * 10^6}{2\pi\epsilon_0 r} \right)$$

4-6 through 4-8

a, b, and d

Question 5

5-1

$$E = \frac{\lambda_1 * 10^6 + \lambda_2 * 10^6}{2\pi\epsilon_0 P}$$

5-2

Answer is 0

5-3

$$E = \left(\frac{\lambda_1 * 10^6}{2\pi\epsilon_0 r} \right) \sin(30)$$

5-4

$$E = \left(\frac{\lambda_1 * 10^6}{2\pi\epsilon_0 r} \right) \cos(30)$$

5-5

$$\lambda_1 + \lambda_2$$

5-6

$$- \lambda_1$$

5-7

$$\lambda_1$$

Question 6

6-1

$$E = \frac{\sigma_1 + \sigma_2}{2\epsilon_0}$$

6-2

Answer = 0

6-3

$$E = \frac{\sigma_1 - \sigma_2}{2\epsilon_0}$$

6-4

Answer = 0

6-5

$$\sigma_b = \frac{\sigma_1 + \sigma_2}{2}$$

6-6

Answer = 0

6-7

$$\sigma_a = \frac{\sigma_2 - \sigma_1}{2}$$

6-8

None

1.5 Lecture 5: Electric Potential Energy

1.5.1 Pre-Lecture

Coulomb Force, conservative force

$$\vec{F}_E = \frac{1}{4\pi\epsilon_0} \frac{Qq}{r^2} \hat{r}$$

Work done by Coulomb Force

$$\vec{F}_E = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}$$

$$W_{A \rightarrow B} = \int_{r_A}^{r_B} \vec{F}_E \cdot d\vec{r}$$

$$W_{A \rightarrow B} = \frac{q_1 q_2}{4\pi\epsilon_0} \int_{r_A}^{r_B} \frac{1}{r^2} dr$$

$$W_{A \rightarrow B} = \frac{q_1 q_2}{4\pi\epsilon_0} \left(\frac{1}{r_A} - \frac{1}{r_B} \right)$$

Electric Potential Energy

$$\Delta U_{AB} = W_{A \rightarrow B}$$

Often use $r_A = \infty$

End up with

$$U_r \equiv \Delta U_{\infty r} = \frac{q_1 q_2}{4\pi\epsilon_0 r}$$

Calculate Speed

$$v = \sqrt{\frac{q_1 q_2}{2\pi\epsilon_0 m_2} \left(\frac{1}{d} - \frac{1}{x} \right)}$$

$$v_{max} = \sqrt{\frac{q_1 q_2}{2\pi\epsilon_0 m_2 d}}$$

System of Three Particles

$$\Delta U_1 = 0$$

$$\Delta U_2 = k \frac{q_1 q_2}{d}$$

$$\Delta U_3 = k \frac{q_1 q_3}{d} + k \frac{q_2 q_3}{d}$$

$$U_{System} = \Delta U_1 + \Delta U_2 + \Delta U_3$$

System of N charged Particles

$$U_{System} = k \sum_{i < j} \frac{q_i q_j}{r_{ij}}$$

Lecture slides

$$\Delta U = + \frac{1}{4\pi\epsilon_0} \frac{Qq}{r} - \frac{1}{4\pi\epsilon_0} \frac{2Qq}{r+d}$$

For charges $Q \rightarrow d \rightarrow Q \rightarrow r$ Q

$$U = U_i + kqxq\left(\frac{1}{r} - \frac{2}{d+r}\right) = 0$$

For charges $Q_1 \rightarrow r$ Q_2 , with distance d between charges Q_1 and Q_2

$$U = U_i + k \frac{qxq}{r} - k \frac{qx2x}{d-r}$$

I don't remember what the x's represent in this case...

1.5.2 Homework

5-1

$$\Delta PE = \frac{q_1 q_2}{4\pi\epsilon_0} \left(\frac{1}{d_2} - \frac{1}{d_1} \right)$$

5-2

$$\Delta U = 2 \left(k \frac{q_1 q_3}{r_2} - k \frac{q_1 q_2}{r_1} \right)$$

$$r_1 = \sqrt{a^2 + d_1^2}$$

$$r_2 = \sqrt{a^2 + d_2^2}$$

5-3

$$U = k \frac{q_a q_b}{r^2} + k \frac{q_c q_d}{2a}$$

5-4

$$U = k \frac{q_3 q_5}{2a}$$

5-5

Answer is 0

1.6 Pre-Lecture 6: Electric Potential

1.6.1 Pre-Lecture

Electric Potential

$$V \equiv \frac{U}{q}$$

$$\vec{E} \equiv \frac{\vec{F}}{q} = k \frac{Q}{r^2} \hat{r}$$

Electric Potential Energy

$$W_{A \rightarrow B} = \int_A^B \vec{F} \cdot d\vec{l}$$

$$\Delta U_{A \rightarrow B} = -W_{A \rightarrow B}$$

Electric Potential Difference

$$\Delta V_{A \rightarrow B} \equiv \frac{\Delta U_{A \rightarrow B}}{q} = \int_A^B \vec{E} \cdot d\vec{l}$$

$$\Delta V_{A \rightarrow B} = kQ \left(\frac{1}{r_B} - \frac{1}{r_A} \right)$$

Electric Potential

$$V(r) \equiv \Delta V_{r_0 \rightarrow r} = kQ \left(\frac{1}{r} - \frac{1}{r_0} \right)$$

Electric Potential for Point Charge

$$V(r) = \frac{kQ}{r}$$

The Gradient in different coordinate systems

Cartesian

$$\vec{\nabla} V = \frac{\partial V}{\partial x} \hat{i} + \frac{\partial V}{\partial y} \hat{j} + \frac{\partial V}{\partial z} \hat{k}$$

Spherical

$$\vec{\nabla} V = \frac{\partial V}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial V}{\partial \theta} \hat{\theta} + \frac{1}{r \sin(\theta)} \frac{\partial V}{\partial \phi} \hat{\phi}$$

Cylindrical

$$\vec{\nabla} V = \frac{\partial V}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial V}{\partial \theta} \hat{\theta} + \frac{\partial V}{\partial z} \hat{k}$$

$$V_{Total} = \sum_i V_i$$

$$V_p = k \frac{q}{a} \left(2 - \frac{\sqrt{2}}{2} \right)$$

if $(r < a)$

$$E = k \frac{Q}{a^3} r$$

if $(r > a)$

$$E = k \frac{Q}{r^2}$$

Find $V(r)$

For $r > a$

$$V(r) = k \frac{Q}{r}$$

For $r < a$

$$V(r) = k \frac{Q}{2a^3} (3a^2 - r^2)$$

1.6.2 Homework

1-1

Spheres V

$$\int_{\infty}^0 E dr = -V(0)$$

$$\int_{\infty}^0 E dr = \int_{\infty}^{9cm} E dr + \int_{2.5cm}^{6cm} E dr$$

$$\int E dr = \frac{-1}{4\pi r \epsilon_0}$$

$$99.86 \text{ V} + 629.13 \text{ V} = 728.99 \text{ V}$$

Factor in direction, answer $\approx -729 \text{ V}$

2-1

I made a power of 10 error here

Answer is something along

$$Q = \rho * \pi * \frac{4}{3} * a^3$$

$$E_x = \frac{kQ}{r^2}$$

I fucked up the unit conversion somewhere...

2-2

$$V = \frac{KQ}{c}$$

Remember to convert $K * 10^{-6}$

2-3

$$V_b + kQ \left(\frac{1}{r_a} - \frac{1}{r_b} \right)$$

2-4

Absolute value of $V(a) - V(b)$

2-5

I did it wrong, but essentially you redo part 2

$$V_b = \frac{kQ_{new}}{c}$$

Then add it to the value for 4

I kept getting minor errors.

3-1

$$\lambda_{inner} = \rho \pi a^2$$

$$\lambda_{enclosed} = \lambda_{inner} + \lambda_{outer}$$

$$E = 2k \frac{\lambda_{enclosed}}{d}$$

3-2

$$-\left(\frac{\lambda_{enclosed}}{2\pi\epsilon_0} \ln(P) - \frac{\lambda_{enclosed}}{2\pi\epsilon_0} \ln(R)\right)$$

3-3

$$\frac{\rho a^2}{2\epsilon_0} \ln\left(\frac{b}{a}\right)$$

I have no idea why that's correct, but it is...

3-4

$$A, V(a) < 0$$

3-5

$$\rho = -\frac{\lambda_{outer}}{\pi a^2}$$

4-1

$$E = \frac{\sigma_1}{2\epsilon_0} + \frac{|\sigma_2|}{2\epsilon_0}$$

4-2

$$-\frac{|\sigma_i + \sigma_2|}{2} = \sigma_a$$

4-3

0

4-4

Bear with me on this one

$$E^*((S_x - R_x) - (b_x - a_x))$$

Where S_x etc. is the x value for each point

4-5

$$E = \frac{\sigma_1}{2\epsilon_0} - \frac{|\sigma_2|}{2\epsilon_0}$$

4-6

B

1.7 Pre-Lecture 7: Conductors and Capacitance

1.7.1 Pre-Lecture

Capacitance

$$C \equiv \frac{Q}{\Delta V}$$

$$\Delta V = \frac{Q}{\epsilon_0 A} d$$

For parallel-plates

$$C = \frac{\epsilon_0 A}{d}$$

$$dU = V dq$$

Stored Energy difference

$$U = \frac{1}{2} QV = \frac{1}{2} 2 \frac{Q^2}{C} = \frac{1}{2} CV^2$$

$$U = \frac{1}{2} \epsilon_0 E^2 Ad$$

Energy density in area between plates

$$u \equiv \frac{U}{Ad} = \frac{1}{2} \epsilon_0 E^2 \text{ General energy density}$$

$$u = \frac{1}{2} \epsilon_0 E^2$$

1.7.2 Homework

Please note when converting cm^2 to m^2 the conversion rate is $cm^2 \times 10^{-4} = m^2$

1-1

$$C = \frac{A\epsilon_0}{d}$$

$$Q = CV_b$$

My answer was on the order of $10^{-9} C$

1-2

$$U = \frac{QV}{2}$$

My answer was on the order of $10^{-8} J$

1-3

$$U = QV$$

Moving the plates apart does work and changes (increases) the voltage. Double distance = double voltage.

1-4

$$\sigma = \frac{Q}{A}$$

$$E = \frac{\sigma}{\epsilon_0}$$

My answer was on the order of $10^3 \frac{N}{C}$

1-5

$$V > V_b$$

1-6

Both E and V decrease

2 DC Circuits

2.1 Section 8: Capacitors

2.1.1 Pre-Lecture

Capacitance of two parallel-plates = $\frac{a*b}{d}$

Dielectric increases Capacitance and reduces electric field

Dielectric constant κ

$$C_{Dielectric} = \kappa C_0$$

$$U = \frac{1}{2} Q V_C$$

Parallel processing is basically a sum of the capacitors

$$C_{equivalent} = \frac{\varepsilon_0 A_{equivalent}}{d} = \frac{\varepsilon_0 (A_1 + A_2)}{d}$$

$$C = C_1 + C_2$$

Series

$$\frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{C_{equivalent}}$$

$$\frac{1}{C_{equivalent}} = \frac{d_1 + d_2}{\varepsilon_0 A}$$

2.1.2 Homework

1-1

$$1 \text{ NC} = 10^9 C$$

One cap is 10 nF, the other is 10x2.6nF. Total of 36nF

$$36\text{nF} * 12 \text{ V} = 432 \text{ nC} = 4.32 * 10^{-7}$$

2-1

$$C_3 + C_4 = \frac{2}{3}$$

$$\text{In parallel with } C_5 \text{ and } C_6 = \frac{20}{3}$$

$$\text{In series with } C_1 = \frac{3 * (\frac{20}{3})}{(3 + \frac{20}{3})} = \frac{60}{29} = 2.069$$

$$Q = CV = 12 * 2.069 = 24.8 \mu C$$

$$V_{AB} = \frac{24.8}{\frac{20}{3}} = 3.72 \text{ V}$$

voltage across C_3 and C_4 together

$$\text{charge on these two is } Q = 3.72 * \frac{2}{3} = 2.48 \mu C$$

$$\text{Voltage across } C_4 = \frac{Q}{C} = \frac{2.48}{1} = 2.48 \text{ V}$$

3-1

$$C_{23} = (\frac{1}{C_2} + \frac{1}{C_3})^{-1}$$

$$C_{ab} = C_{23} + C_4$$

3-2

$$C_{ab} = (\frac{1}{C_1} + \frac{1}{C_{ab}})^{-1}$$

3-3

$$Q_5 = C_{total} V$$

3-4

$$V_{ab} = V - 2 \frac{Q_5}{C_5}$$

$$Q_2 = V_{ab} * \frac{C_2 * C_3}{C_2 + C_3}$$

3-5

$$Q_5 = C_{total}V$$

3-6

$$V_4 = V_{ab} = V - 2 \frac{Q_5}{C_5}$$

4-1

MIND YOUR UNITS HERE! I shit the bed SO bad on this section!

$$C = \epsilon_0 \frac{A}{d}$$

Convert cm^2 to mm^2 and cm to mm

cm^2 to mm^2 , add two zeroes!

4-2

$$Q = C * V_b$$

4-3

$$Q_{new} = Q * \frac{2\kappa}{1+\kappa}$$

4-4

Convert μC to C

$$1 \mu C = 10^{-6} C$$

$$U_{new} = \frac{Q_{new} V_b}{2}$$

4-5

$$V = \frac{Q_{new}}{C}$$

5-1

$$E_x(P) = 2k \frac{\lambda_{inner}}{d}$$

5-2

Answer should be positive! Mind the negative!!!

$$- \frac{-\lambda_{inner} * 10^{-6}}{2\pi\epsilon_0} \ln\left(\frac{b}{a}\right)$$

5-3

I fucked up the unit conversion, should be answer⁻⁵

In my case it's $6.55679 * 10^{-5}$

$$\frac{1}{2 * k * \ln\left(\frac{b}{a}\right)}$$

5-4

B, just guess

5-5

$$\lambda_{outer,new} = \lambda_{outer} * 2$$

2.2 Section 9: Electric Current

2.2.1 Pre-Lecture

Electric Current

$$I \equiv \frac{dq}{dt}$$

Ohm's Law

$$J = \sigma E$$

$$\text{Ampere } A = \frac{\text{Coulomb}(C)}{\text{second}(S)}$$

$$\text{Current Density } J \equiv \frac{I}{A} = n_e e v_{drift}$$

$$n_e = N_A \frac{\rho_{mass}}{M}$$

$$J \propto E$$

σ evidently means conductivity now

$$v_{drift} = \frac{\sigma}{n_e e} E$$

Resistance

$$R \equiv \frac{1}{\sigma} \frac{L}{A}$$

$$J \equiv \sigma \frac{V}{L}$$

$$R = \rho \frac{L}{A}$$

$$V = IR$$

$$V_{AB} = V_1 + V_2$$

$$R_{equivalent} = R_1 + R_2$$

$$\text{Power} = IV = I^2 R$$

2.2.2 Homework

1-1

$$I_1 = \frac{V}{R_1 + R_3}$$

1-2

$$V_2 = V_1 = I_1 R_1 = V * \frac{R_1}{R_1 + R_3}$$

1-3

$$I_2 = \frac{V_2}{R_2}$$

1-4

$$R_x = R_2 \frac{R_3}{R_1}$$

1-5

$$V_1 = V_2$$

1-6

B

2-1

$$R_{23} = R_{ab} = \frac{R_4(R_2 + R_3)}{R_4 + R_2 + R_3}$$

2-2

$$R_{ac} = R_1 + R_{ab}$$

2-3

$$R_{equiv} = R_5 + R_{ac}$$

$$I_5 = \frac{V}{R_{equiv}}$$

2-4

$$V_{ab} = V \left(\frac{R_{ac} - R_5}{R_{ac} + R_5} \right)$$

$$I_2 = \frac{V_{ab}}{R_2 + R_3}$$

2-5

$$I_1 = I_5$$

2-6

$$V_4 = V_{ab}$$

2.3 Section 10: Kirchoff's Rules

2.3.1 Pre-Lecture

Voltage Rule

$$\sum \Delta V_n = 0$$

Current Rule

$$\sum I_{in} = \sum I_{out}$$

$$V_c = \frac{Q}{C}$$

$$V_b = V_0 \frac{\frac{R}{r}}{1 + \frac{R}{r}}$$

2.3.2 Homework

1-1: Current Div

You only have to write i_2 in terms of i_1

In this case it's $i_2 = \frac{1}{3}i_1$

Flipit requires it in the form $(1/3)*i_1$

2-1: Multiloop

$$I_1 + I_2 + I_3 = 0$$

$$I_2 = I_2 + I_3$$

$$V_1 - I_1 * R_1 - I_3 * R_3 = 0$$

$$v_2 + I_3 * R_3 - I_2 * R_2 = 0$$

If you're too lazy, just put .058...

Section 3

You first must solve this matrix (matrix solver), it encodes the circuit using Kirchhoff's Laws and allows us to solve for all of the currents simultaneously.

analyzed circuit

$$\left[\begin{array}{ccc|c} -R_1 & R_3 + R_1 & 0 & -V_{s1} \\ R_2 + R_6 & R_3 & 0 & V_{s2} - V_{s1} \\ 0 & 0 & R_5 + R_4 & V_{s2} \end{array} \right] = \begin{bmatrix} I_2 \\ I_3 \\ I_4 \end{bmatrix}$$

3-1

$$V_4 = I_4 R_4$$

3-2

Solved via the matrix.

3-3

Solved via the matrix.

3-4

Kirchoff's node Law

$$I_1 = I_2 - I_3$$

3-5

$$V_{ab} = I_2 R_6$$

Same deal as problem 3.

analyzed circuit

$$\left[\begin{array}{cc|c} R_1 + R_2 & -R_2 & V \\ -R_2 & R_2 + R_3 + R_4 + R_5 & 0 \end{array} \right] = \left[\begin{array}{c} I_1 \\ I_3 \end{array} \right]$$

4-1

Solved via the matrix.

$I_1 \times 10^3$ (milliampere conversion)

4-2

$$r = \frac{V - V_b}{I_1}$$

4-3

Solved via the matrix.

$I_3 \times 10^3$ (milliampere conversion)

4-4

$$P_2 = (I_1 - I_3)^2 R_2$$

4-5

$$V_2 = (I_1 - I_3) R_2$$

2.4 Section 11: RC Circuits

BEGIN TEST 2!!

2.4.1 Pre-Lecture

If $t = 0$

$$V_C(0) = 0$$

$$I(0) = \frac{V}{R}$$

As t increases

q , V_C increases

I , V_R decreases

As $t \rightarrow \infty$

$$q \rightarrow CV_b$$

$$I \rightarrow 0$$

Question 1

Charge flows into the top of the capacitor and out of the bottom of the capacitor
but no charge actually crosses the gap between the plates

Kirchoff's Voltage Rule

$$IR + \frac{q}{C} - V_b = 0$$

$$R \frac{dq}{dt} + \frac{1}{C} q - V_b = 0$$

$$q(t) = CV_b(1 - e^{-\frac{t}{RC}})$$

$$I(t) = \frac{V_b}{R} e^{-\frac{t}{RC}}$$

Boundary Conditions

$$q(0) = 0$$

$$I(0) = \frac{V_b}{R}$$

$$q(\infty) = CV_b$$

$$I(\infty) = 0$$

Discharging a Capacitor

$$t = 0$$

$$V_C(0) = \frac{q_0}{C}$$

$$q(t) = q_0 e^{-\frac{t}{RC}} \text{ Boundary Conditions}$$

$$q(0) = q_0$$

$$q(\infty) = 0$$

$$I(t) = -\frac{q_0}{RC} e^{-\frac{t}{RC}} \text{ Boundary Conditions}$$

$$|I(0)| = \frac{q_0}{RC}$$

$$I(\infty) = 0$$

Time Constant

$$\tau = RC$$

$$I(\tau) = I_0 e^{-1} \approx I_0(0.37)$$

Question 2

$$t=2, V = \frac{V}{2}$$

$$t=6, V = \frac{V}{8}$$

Power in an RC Circuit

Fuck this section for making me type this much...

$$P_{Battery}(t) = V_b I_0 e^{-\frac{t}{RC}}$$

$$P_R(t) = R I_0^2 e^{-\frac{2t}{RC}}$$

$$P_C(t) = \left(\frac{q_0}{C}(1 - e^{-\frac{t}{RC}})\right)(I_0 e^{-\frac{t}{RC}})$$

2.4.2 Homework

Section 1

Convert μC to C!

$$Q_{2final} = \frac{\frac{R_2 V}{R_1 + R_2}}{\frac{1}{C_1} + \frac{1}{C_2}}$$

If your come down with a case of fuckit, Interactive example = .000213

Section 2

2-1

$$I_1(0) = \frac{V}{R_1 + R_4}$$

2-2

$$I_1(\infty) = \frac{V}{R_1 + R_2 + R_3 + R_4}$$

2-3

$$I = \frac{V}{R_1 + R_2 + R_3 + R_4}$$

$$V = I * R_{23}$$

$$Q = VC$$

2-4

$$I_1(0) = \frac{V}{R_1 + R_{523}}$$

$$R_{23} = R_2 + R_3$$

$$R_{235} = \frac{1}{\frac{1}{R_{23} + R_5}}$$

$$I_1 = \frac{V}{R_1 + R_{523} + R_4}$$

2-5

Same as 2-3

Section 3

3-1

$$I_4(0) = \frac{V}{R_{equiv}}$$

$$R_{equiv} = R_1 + R_4 + \frac{R_2 R_3}{R_2 + R_3}$$

3-2

$$Q(\infty) = I(\infty) R_3 C$$

$$Q(\infty) = CV \frac{R_3}{R_1 + R_3 + R_4}$$

3-3

$$\tau = (R_2 + R_3)C$$

t_{open} is provided

$$Q(t_{open}) = Q(\infty) e^{\frac{-t_{open}}{\tau}}$$

3-4

$$I_{c,max}(closed) = \frac{V - I_4(0)(R_1 + R_4)}{R_2}$$

3-5

$$I_{c,max}(open) = \frac{Q(\infty)}{(R_2 + R_3)C}$$

3 Magnetism

3.1 Section 12: Magnetism

3.1.1 Pre-lecture

Lorentz Force

$$\vec{F} = q\vec{E} + q\vec{v} * \vec{B}$$

Fucking magnets man! How do they work?

Cross Product

$$\vec{F} = q\vec{v} * \vec{B}$$

Force perpendicular to Current and magnetic field direction

$$\vec{F} \perp \vec{I}$$

$$\vec{F} \perp \vec{B}$$

Cross Product

$$|\vec{A} * \vec{B}| = AB \sin \theta$$

$$\vec{F}_{Electric} = -\vec{F}_{Magnetic}$$

$$q\vec{E} = -q\vec{v} * \vec{B}$$

$$F_B = qvB$$

$$F_e = qE$$

$$v = \frac{E}{B} \text{ when } \vec{F}_E = -\vec{F}_B$$

$$a_c = \frac{v^2}{R}$$

$$R = \frac{mv}{qB}$$

3.1.2 Homework

1-1

$$r = \frac{mv}{qB}$$

$$t = \frac{\pi r}{v}$$

It's an Interactive example, answer = $2.73 \cdot 10^{-8}$ s

2-1

$$\text{path} = \frac{1}{2} \pi d$$

$$v = \frac{\text{path}}{t}$$

Remember to convert μs to s

$$t \cdot 10^{-6}$$

2-2

$$F = \frac{mv^2}{r}$$

$$\theta = \pi \left(\frac{t_1}{t} + 1 \right)$$

$$F_x = F \sin(\theta)$$

2-3

$$F = \frac{mv^2}{r}$$

$$\theta = \pi \left(\frac{t_1}{t} + 1 \right)$$

$$F_y = F \cos(\theta)$$

2-4

Sign must be ascertained from the cross product of the direction of the \vec{B} and \vec{v} .

$$q = \frac{F}{vB}$$

2-5

A

3-1

$$v = \sqrt{v_x^2 + v_y^2}$$

3.2 Section 13: Forces and Torques on Currents

3.2.1 Pre-lecture

Net force acting on current

$$\vec{F} = \sum_i \vec{F}_i$$

$$\vec{F} = q * \sum_i \vec{v}_i * \vec{B}$$

$$\vec{F} = q(N\vec{v}_{avg}) * \vec{B}$$

N = number of charge carriers n*AL

$$I = nAqv_{avg}$$

$$\vec{F} = qnAL\vec{v}_{avg} * \vec{B}$$

Force on Current-Carrying Wire

$$\vec{F} = I\vec{L} * \vec{B}$$

$$\vec{F}_{wire} = I\vec{L} * \vec{B}$$

Force on a closed loop is 0

$$\vec{F}_{Loop} = I(0) * \vec{B} = 0$$

$$\vec{F}_{ClosedLoop} = 0$$

Torque stuff

$$\vec{\tau} = \vec{r} * \vec{F}$$

Magnitude

$$\tau = rF\sin(\theta)$$

$$\tau_{total} = \frac{h}{2}(2IwB)\sin(\theta)$$

$$\tau_{loop} = IwhB\sin(\theta)$$

Generalized form

$$\tau_{loop} = IAB\sin\theta$$

Magnetic Dipole Moment

$$\vec{\mu} = I\vec{A}$$

If there are many turns, like a coil $\vec{\mu} = NIA\vec{A}$

Torque on a loop

$$\vec{\tau} = \vec{\mu} * \vec{B}$$

$$W = \int_{\theta_1}^{\theta_2} (-\mu B \sin\theta) d\theta$$

$$\Delta U = \int_{\theta_1}^{\theta_2} \mu B \sin\theta d\theta$$

$$U(\theta) = -\vec{\mu} * \vec{B}$$

3.2.2 Homework

1-1, IE with consistent answer

$$\tau = p_B * B * \sin(\theta)$$

$$p_B = I * (\frac{\sqrt{3}}{4})d^2$$

$$W = -p_B B (\cos\theta_2^0 - \cos\theta_1)$$

$$\theta_2 = 180$$

$$\theta_1 = 0$$

$$W = -0.25 * \frac{\sqrt{3}}{4} (0.008)^2 * 1.3(-1 - 1)$$

$$W = -1.8 * 10^{-3} \text{ J}$$

2-1, IE with consistent answer

$$\frac{2IBdN}{g}$$

$$g = 9.82$$

convert d from cm to m

$$\text{answer} \approx 0.74786$$

3-1

$$\mu = IA = IWH$$

$$\mu_x = -\mu \sin\theta = -IWH \sin\theta$$

3-2

$$\mu_y = \mu \cos \theta = IWH \cos \theta$$

3-3

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

$$\tau_z = -\mu B \sin \theta = -IWHB \sin \theta = \mu_x B$$

3-4

$$F_{bc} = IHB$$

$$\tau = IWHB \sin \theta$$

$$\tau = WF \sin \theta$$

$$F = IHB$$

3-5

C, throw a dart...

4-1

$$F_{ac} = IL_{ac}B = IB\sqrt{L_{ab}^2 + L_{bc}^2}$$

$$F_{ac,x} = -IBL_{bc}$$

Convert L to m

Convert I to A ($I \cdot 10^{-3}$)

4-2

$$F_{ac,y} = F_{ac} \cos \theta = IBL_{ab}$$

4-3

$$\Delta U_{12} = IL_{ab}L_{bc}B$$

4-4

A

4-5

Same as 4-3

3.3 Section 14: Biot-Savart Law

3.3.1 Pre-lecture

Biot-Savart Law

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times \hat{r}}{r^2}$$

$$B = \frac{\mu_0 I}{2\pi R}$$

$$\vec{F}_1 = -\vec{F}_2$$

$$F_2 = F_1 = \frac{\mu_0}{2\pi d} I_1 I_2 L$$

$$B_{center} = \frac{\mu_0 I}{2R}$$

$$B = \frac{\mu_0 I}{2} \frac{R^2}{(R^2 + z^2)^{\frac{3}{2}}}$$

3.3.2 Homework

Preamble

My work assigns x_i values based on order of appearance

The first x,y coordinates are x_1, y_1 etc.

Convert distances to m, and when d is referred to it's $|x_1| + x_2$

It too is in meters.

The constant $\mu_0 = 1.25663706 * 10^{-6} \text{ m kg s}^{-2} \text{ A}^{-2}$

I don't know where it's from, but every example I found used it.

1-1

$$B_x(0,0) = \frac{\mu I_3}{2\pi y_3}$$

1-2

$$B_y(0,0) = \frac{-\mu}{\pi d} (I_1 + I_2)$$

1-3

$$B_y(1) = \frac{-\mu I_1}{2\pi d} (I_3 \sin(30)) + I_2$$

DEGREES

1-4

$$F_y(1) = \frac{-\mu I_1 I_3}{2\pi d} \cos(30)$$

1-5

$$F_x(2) = \frac{\mu I_2}{2\pi d} (I_1 - I_3 \sin(30))$$

1-6

C

2-1

$$F = IL_x B$$

$$F_{ad,x} = I_2 H \frac{\mu I_1}{2\pi L}$$

2-2

$$F_{bc,x} = -I_2 H \frac{\mu I_1}{2\pi(L+W)}$$

2-3

$$F_{net,y} = 0$$

2-4

B

2-5

$$I_3 = 2I_1 \frac{2L+W}{L+W}$$

3.4 Section 15: Ampere's Law

3.4.1 Pre-lecture

Ampere's Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enclosed}$$

$$= \frac{\mu_0 I}{2\pi R} (2\pi R)$$

$$= \mu_0 I$$

Magnetic Field inside of a wire

Ampere's Law, cylindrical case

$$B(2\pi R) = \mu_0 I$$

$$B(2\pi R) = \mu_0 I * \frac{\pi r^2}{\pi a^2}$$

$$B = \frac{\mu_0 I}{2\pi a^2} r \text{ for } r < a$$

$$B = \frac{\mu_0 I}{2\pi r} \text{ for } r > a$$

Infinite sheets of charge

y components cancel, x components add

$$\oint \vec{B} \cdot d\vec{l} = 2BL$$

$$I_{enclosed} = (\text{number of wires}) * I = nLI$$

$$B = \frac{1}{2} \mu_0 n I$$

3.4.2 Homework

Interactive Example

Assume B(r)

Remember that $\mu_0 = 1.25663706 * 10^{-6}$

$$B = \mu_0 I_{enclosed}$$

$$B = \mu_0 (I_1 - I_2 * \frac{\pi(r^2 - b^2)}{\pi(c^2 - b^2)})$$

Factor π out

$$B = 4.8x10^{-7} \text{ T}$$

$$B_x = -B \sin 30 = -2.4x10^{-7} \text{ T}$$

Section 1

Found a better definition of μ_0 as $4\pi * 10^{-7}$

Mind your directions, and remember the right hand rule is your friend

I'll include what directions I had since it's probably the same for you

2-1

R hand rule, B is only in y direction

$$B_y = |B|$$

$$B = \frac{\mu_0(I_1 + I_2)}{2\pi d}$$

2-2

Section R - S = 0, $\vec{B} * d\vec{\ell} = 0$

Section P - R is $\frac{1}{8}$ of the loop so $\frac{1}{8} \int \vec{B} * d\vec{\ell} = \frac{1}{8} \mu_0 I$

I think it's the same for all, but mine is

$$\frac{1}{8} (4 * \pi * 10^{-7}) * (I_1 - I_2) = 3.30 * 10^{-7} \text{ Tm}$$

2-3

$$-B_T(2\pi r) = \mu_0 I_{enc} + I_1$$

Negative due to right hand rule

I don't know why the two I values get added.

My initial attempt didn't work without it, and I found an example that used it that worked.

$$I_{enc} = I_2 \frac{r^2 - a^2}{b^2 - a^2}$$

$$B_T = \frac{-\mu_0(I_1 + I_{enc})}{2\pi r}$$

2-4

Negative of 2-2, same end points

2-5

C

Section 2

3-1

Both fields are y direction only, so $B_x = 0$

3-2

Convert n to wires per meter, mine went from 18 to 1800.

$B(2L) = \mu_0 n L I$, no x field so B becomes

$$B = \frac{1}{2}\mu(nI_1 + nI_2)$$

3-3

$$B_r = \frac{1}{2}\mu_0 n(I_1 - I_2)$$

3-4

$$I_{enc} = nhI_1$$

$$B = \mu_0 I_{enc}$$

3-5

Positive answer from 3-3

My 3-3 was -0.001357, so my 3-5 is 0.001357

3-6

$$B \cdot h = (\text{answer from 3-2}) \cdot (h \text{ from 3-4})$$

3.5 Section 16: Motional EMF

3.5.1 Pre-lecture

Electrodynamics

$$\vec{E}(\vec{r}) \rightarrow \vec{E}(\vec{r}, t)$$

$$\vec{B}(\vec{r}) \rightarrow \vec{B}(\vec{r}, t)$$

Faraday's Law

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A}$$

Maxwell's Displacement Current

$$I = \varepsilon_0 \frac{d}{dt} \int \vec{E} \cdot d\vec{A}$$

Ampere-Maxwell Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d}{dt} \int \vec{E} \cdot d\vec{A}$$

At Equilibrium

$$E = vB$$

Potential Difference

$$\varepsilon = vBL$$

Current

$$I = \frac{vBL}{R}$$

Power Dissipated

$$P_R = \frac{(vBL)^2}{R}$$

Power from External Agent

$$P_{external} = \frac{v^2 B^2 L^2}{R}$$

EMF

$$\varepsilon_{loop} = vL(B_{bottom} - B_{top})$$

$$B_{top} = \frac{\mu_0 I_0}{2\pi(R+w)}$$

$$B_{bottom} = \frac{\mu_0 I_0}{2\pi R}$$

At Equilibrium

$$E = v B \cos \theta$$

Motional EMF

$$\varepsilon = \omega AB \cos(\omega t)$$

$$\Phi = \int \vec{B} * d\vec{A}$$

3.5.2 Homework

1-1: Interactive example

At $t = 0$

$R = 2W$

$W = 3 \text{ cm} = .03 \text{ m}$

$L = 8 \text{ cm} = .08 \text{ m}$

$B = 1.6 \text{ T}$

$L_B = 15 \text{ cm} = .15 \text{ m}$

constant $v = 5 \frac{\text{cm}}{\text{s}} = .05 \frac{\text{m}}{\text{s}}$

Use $\varepsilon = vBL$, or W in this case

Then $I = \frac{\varepsilon}{R}$

Finally, use $F = ILB = IWM$

$F(0.8) = 5.76 * 10^{-5} \text{ N}$

2-1

$\varepsilon = vBL = vB(S_1)$

Use $vB(S_1)$ for this part

$I = \frac{V}{R} = \frac{\varepsilon}{R}$

Mind current direction, I got a sign error!

2-2

$\varepsilon = vBL = vB(S_2)$

$I = \frac{\varepsilon}{R}$

Mind your direction, this one was positive on mine

2-3

Power = $F*v = \frac{V^2}{R}$

$F = \frac{vB^2S_2^2}{R}$

2-4

Opposite sign answer from 2-1

My 2-1 was -0.00646, so my 2-4 is 0.00646

2-5

A

3-1

$B = \frac{\mu_0 I}{2\pi d}$

$\varepsilon = v B W$

Mind your directions here!

3-2

$d_1 \text{ at } t_1 = d - (v*t_1)$

$B(d_1) = \frac{\mu_0 I}{2\pi d_1}$

$\varepsilon = v B(d_1) W$

3-3

$d_2 = L + d$

$B_1 = \frac{\mu_0 I}{2\pi d}$

$$B_2 = \frac{\mu_0 I}{2\pi d_2}$$

$$\varepsilon = vW(B_1 - B_2)$$

$$I = \frac{\varepsilon}{R}$$

3-4

C

3-5

$$d_2 = d + W$$

Use the I from 3-3!

For those who care, the final function is

$$I_2 = \frac{2\pi(RI)}{vL\mu_0(\frac{1}{d_1} - \frac{1}{d_2})}$$

Working backwards

$$I = \frac{vL\mu_0}{2\pi R} I_2 (\frac{1}{d_1} - \frac{1}{d_2})$$

$$I = \frac{vL}{R} B_1 - B_2$$

$$B_1 = \frac{\mu_0 I_2}{2\pi d_1}$$

$$B_2 = \frac{\mu_0 I_2}{2\pi d_2}$$

3.6 Section 17: Faraday's Law

3.6.1 Pre-lecture

Faraday's Law

$$\varepsilon_{induced} = -\frac{d\Phi_B}{dt}$$

Motional EMF

$$|\varepsilon| = \frac{d\Phi_B}{dt}$$

$$|\varepsilon| = vl(B_{bottom} - B_{top})$$

$$|\varepsilon| = \frac{\Delta\Phi_B}{\Delta t}$$

$$|\varepsilon| = \omega B A \cos(\omega t)$$

$$\Phi_B = B A \sin(\omega t)$$

$$\varepsilon = -\omega N B A \cos(\omega t)$$

3.6.2 Homework

Section 1

1-1

$$\Phi = \frac{\mu_0 I_1}{2\pi} * \ln(\frac{L+d}{d})$$

1-2

$$\varepsilon_1 = \frac{\Phi}{t_1}$$

1-3

No change, it's constant

Answer = 0

1-4

A, clockwise

1-5

$$\varepsilon_4 = \frac{\Phi}{\Delta t}$$

$$\Delta t = t_4 - t_3$$

Section 2

2-1

$$\omega = \frac{2\pi}{t}$$

2-2

$$I_{max} = \frac{\omega BA}{R}$$

$$A = .5 * b * h$$

2-3

$$\theta = \omega t$$

Convert to degrees!

$$|\Phi| = |B * A \cos \theta|$$

2-4

$$I = \frac{\omega BA}{R} \sin(\theta)$$

2-5

$$B, \phi_0 = 0 \text{ and } I_0 = I_{max}$$

2-6

C, ϕ_{max} remains the same and I_{max} doubles

4 AC Circuits

4.1 Section 18: Induction and RL Circuits

4.1.1 Pre-Lecture

Self-Inductance

$$L \equiv \frac{\Phi_B}{I}$$

SI Unit

$$\text{H (Henry)} = \frac{T \cdot m^2}{A}$$

Inductor Voltage

$$\varepsilon = -L \frac{dI}{dt}$$

The Solenoid

Magnetic field of a Solenoid

$$B = \mu_0 n I$$

Magnetic flux of a Solenoid

$$\Phi_B = \mu_0 n^2 z \pi r^2 I$$

Self-Inductance of a Solenoid

$$L = \mu_0 n^2 z \pi r^2$$

Where n is the wire density, z is the length, r is the radius and I is the current.

RL Circuit

With battery

Closing Switch, at $t = 0$

$$V_R(0) = 0$$

$$V_L(0) = V_b$$

$$I_1(0) = 0$$

Switch is closed for a long time, $t \rightarrow \infty$

$$V_R(\infty) = V_b$$

$$\begin{aligned}
V_L(\infty) &= 0 \\
I_1(\infty) &= \frac{V_b}{R} \text{ Without battery} \\
\text{Opening switch, } t &= 0 \\
V_R(0) &= V_b \\
V_L(0) &= -2V_b \\
I(0) &= \frac{V_b}{R} \\
\text{Switch is closed, } t &\rightarrow \infty \\
V_R(\infty) &= 0 \\
V_L(\infty) &= 0 \\
I(\infty) &= 0
\end{aligned}$$

RL Circuit: Quantitive Description

Kirchoff's Voltage Rule for RL circuits

$$L \frac{dI_1}{dt} + I_1 R - V_b = 0$$

redefine $\tau = \frac{L}{R}$

Where L is the length of the inductor and R is the resistance of the resistor

$$I_1(t) = \frac{V_b}{R} (1 - e^{-\frac{Rt}{L}})$$

Yes that's t not τ

$$V_L = V_b e^{-\frac{Rt}{L}}$$

Energy in an inductor

Inductor energy

$$U_L = \frac{1}{2} L I^2$$

Capacitor energy

$$U_C = \frac{1}{2} \frac{Q^2}{C} \text{ Magnetic Energy Density}$$

$$u_B = \frac{B^2}{2\mu_0}$$

4.1.2 Homework

Interactive Example

$$4.88 * 10^{-6}$$

noindent 1-1

$$I = \frac{V}{\sum R}$$

1-2

$$I = \frac{V}{R_1 + R_4}$$

1-3

$$V_L = V_{R23} = I * (R_2 + R_3)$$

1-4

Yours might be different, but try $\frac{V}{R_2 + R_4}$, if nothing else try .139A since it might be the same

1-5

Same as 1-1

2-1

$$I_1 = \frac{V}{R_1 + R_3 + R_4}$$

2-2

Mind your rounding here!

$$R_{23} = \left(\frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$$

$$R_{1234} = R_1 + R_{23} + R_4$$

$$I = \frac{V}{R_{1234}}$$

2-3

$$V_2 = V_{23} = IR_{23}$$

$$I_2 = \frac{V_2}{R_2}$$

2-4

Mind your directions here! Mine was a negative.

Also convert your L from mH to H, ex 369 to .369

$$R = R_2 + R_3$$

$$I(t) = I_2 e^{\frac{-t_{open} R}{L}}$$

2-5

Use I_1 from 2-1!

$$V_L = V_{R3} = I_1 R_3$$

2-6

$$V_L = V_R = I_2(R_2 + R_3)$$

4.2 Section 19: LC and RLC Circuits

4.2.1 Pre-Lecture

LC Circuits: A qualitative description

Oscillator made by a capacitor and inductor

LC Circuits: A quantitative Description

$$Q(t) = Q_{max} \cos(\omega t + \phi)$$

Defined by initial conditions

$$Q_{max} = CV$$

ϕ also determined, explanation given doesn't fit all

LC Circuits: Part 3

$$\omega = \frac{1}{\sqrt{LC}}$$

LC Circuits and Energy

$$I = -\omega Q_{max} \sin(\omega t + \phi)$$

RLC Circuits

Rate of Energy Loss

$$P = I^2 R$$

$$\beta = \frac{R}{2L}$$

β can also equal ω_0

$$\omega'^2 = \omega_0^2 - \beta^2$$

$$Q(t) = Ae^{-\beta t} \cos(\omega' t + \phi)$$

4.2.2 Homework

IE 1

UNITS UNITS UNITS MY DUDE!!!

Convert uF to F (C)

Convert mH to H (L)

Convert mA to A (I)

$$Q_{max} = \sqrt{2CLI^2}$$

If you come down with Whogivesashit, answer is $1.54 * 10^{-5}$

IE 2

$$U = \frac{1}{2}LI_L^2 + \frac{1}{2}CV_C^2$$

$$U = \frac{1}{2}((.008)(.0293)^2 + (250 * 10^{-6})(6.44^2))$$

$$U = 0.005188$$

Mind your rounding and units here!

Convert μF to F

Convert mA to A

Convert mH to H

Convert mS to S

1-1

$$C = (\frac{1}{C_1} + \frac{1}{C_2})^{-1}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

1-2

Mind your rounding on this one, also whitespace

$$Q_{max} = \frac{I_L}{\omega_0}$$

1-3

$$V_{bc} = -L\omega_0 I_L \sin(\omega t)$$

1-4

$$Q_{max} = \frac{I_L}{\omega_0}$$

1-5

$$D, Q_1 = 0 \text{ and } V_L = 0$$

Convert MH to H

Convert μF to F

2-1

$$I = \frac{V}{R_1}$$

$$U_1 = \frac{1}{2}L_1 I^2$$

2-2

$$L = L_1 + L_2$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

2-3

$$Q_{max} = \frac{I}{\omega_0}$$

2-4

$$-Q_{max} \sin(\omega t)$$

2-5

Convert answer to mS from S

$$t_2 = \frac{\pi}{2\omega_0}$$

2-6

$$U_{total} = \frac{1}{2}LI^2$$

4.3 Section 20: AC Circuits

4.3.1 Pre-Lecture

Induced Voltage

$$\varepsilon(t) = \varepsilon_m \sin(\omega t)$$

$$I = I_m \sin(\omega t - \phi)$$

$$\text{KVR} = V_c - \varepsilon = 0$$

$$V_R = I_R R = \varepsilon_m \sin(\omega t)$$

$$I_R = \frac{\varepsilon_m}{R} \sin(\omega t)$$

Question 1, The voltage is positive and decreasing

$$\frac{Q}{C} = \varepsilon_m \sin(\omega t)$$

$$Q = C \varepsilon_m \sin(\omega t)$$

$$I_C = \frac{dQ}{dt} = \omega C \varepsilon_m \cos(\omega t)$$

Reactance of the Capacitor

$$X_C \equiv \frac{1}{\omega C}$$

$$I_L = -\frac{\varepsilon_m}{X_L} \cos(\omega t)$$

Reactance of the inductor

$$X_L \equiv \omega L \quad \tan \phi = \frac{X_L - X_C}{R}$$

$$I_m = \frac{\varepsilon_m}{\sqrt{R^2 + (X_L - X_C)^2}}$$

4.3.2 Lecture notes

$$\omega = \frac{2\pi}{T}$$

$$\text{Amplitude} = \frac{V_{max}}{R}$$

$$\varepsilon(t) = V_{max} \sin(\omega t)$$

$$V_{max} = I_{max} * \frac{1}{\omega C}$$

$$V_C = I_C X_C$$

$$X_C = \frac{1}{\omega C} \text{ Inductor}$$

$$\text{KVR: } V_L - \varepsilon = 0$$

$$\int L = \int \varepsilon(t) = \int V_{max} \sin(\omega t) dt$$

$$V_L = I_L X_L = I_L \omega L$$

Voltage lead

L, V_L leads I

Voltage Lags

C, V_C lags I

$$V_L + V_R = \varepsilon$$

$$V_L = I \omega L + I R = V_{max} \sin(\omega t)$$

Currents I_L and I_R are the same

4.3.3 Homework

IE1

$$\phi = \tan^{-1} \left(\frac{X_L - X_C}{\frac{\varepsilon^2}{I^2} - (X_L - X_C)} \right)$$

$$\phi = 23.30575$$

Convert C from μF to F

Convert L from mH to H

1-1

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$$

1-2

$$I_{max} = \frac{\varepsilon_m}{Z}$$

1-3

$$\phi = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

MIND YOUR SIGNS!!!

1-4

Convert ω to degrees per second

$$\frac{90^\circ + \phi}{\omega} = t$$

1-5

B

1-6

$$V_{C,max} = \frac{I_{max}}{\omega C}$$

$$V_C(t) = V_{C,max} \cos(\phi)$$

2-1

In these examples, use ϕ instead of the φ

Convert L from mH to H

Convert C from μF to F

$$t = \frac{\phi}{\omega}$$

2-2

$$Z = \frac{R}{|\cos \phi|}$$

2-3

$$L = \frac{R \tan(\phi) + \frac{1}{\omega C}}{\omega}$$

Convert resulting value to mH

2-4

$$I_{L,max} = \frac{\varepsilon_m}{Z}$$

$$V_{L,max} = I_{L,max}(\omega L) = \frac{\varepsilon_m \omega L}{Z}$$

2-5

$$V_{L,max} = V_{L,max} \cos(\phi)$$

2-6

A

4.4 Section 21: AC Circuits: Resonance and Power

4.4.1 Pre-Lecture

Reactance

Inductor: $X_L = \omega L$

Capacitor: $X_C = \frac{1}{\omega C}$

Maximum Voltages

Inductor: $V_L = I_m X_L$

Capacitor: $V_C = I_m X_C$

Resistor: $V_R = I_m R$

Impedance

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Phase

$$\tan \phi = \frac{X_L - X_C}{R}$$

Maximum Current

$$I_m = \frac{\varepsilon_m}{Z}$$

$$I_m = \frac{\varepsilon_m}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

$$\omega_0^2 = \frac{1}{LC} \text{ Question 1}$$

C, it will decrease

$$I_m = \frac{\varepsilon_m}{R} \cos(\phi)$$

$$\cos(\phi) = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$x \equiv \frac{\omega}{\omega_0}$$

$$Q^2 \equiv \frac{L}{R^2 C}$$

$$I_m = \frac{\varepsilon_m}{R} \frac{1}{\sqrt{1 + Q^2 \frac{(x^2 - 1)^2}{x^2}}}$$

btw fuck these symbols...

$$\langle P_{\text{Generator}} \rangle = \langle P_{\text{Resistor}} \rangle = \frac{1}{2} I_m \varepsilon_m \cos \phi$$

Average Power per cycle

$$\langle P_{\text{Generator}} \rangle = \varepsilon_{rms} I_{rms} \cos \phi = \frac{\varepsilon_{rms}^2}{R} \cos^2 \phi$$

$$\langle P_{\text{Generator}} \rangle = \frac{\varepsilon_{rms}^2}{R} \frac{x^2}{x^2 + Q^2(x^2 - 1)^2}$$

$$\varepsilon_{rms} = \frac{\varepsilon_m}{\sqrt{2}}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

Quality Factor

$$Q \equiv 2\pi \left[\frac{U_{max}}{\Delta U} \right]_{\text{cycle}}$$

$$Q^2 = \frac{L}{R^2 C}$$

$$V_{L_{max}}|_{\omega=\omega_0} = V_{C_{max}}|_{\omega=\omega_0} = Q \varepsilon_m$$

Question 2

A, 0 Volts

Ideal Transformer

$$V_S = \frac{N_S}{N_P} V_P$$

$$I_P = \frac{N_S}{N_P} I_S$$

4.4.2 Lecture notes

Cannot operate at different frequencies at once

90 degrees for all lines is always the same and do not change relative

$$I(t) = I_m \sin(\omega t - \phi)$$

$$Z = \frac{\varepsilon_{max}}{I_{max}}$$

Always out of phase by 90 degrees

Current remains constant

In phase with voltage drop of the resistor

$$I_\epsilon(t) = I_R(t) = I_L(t) = I_C(t)$$

All out of phase with the current

Slides, "Ohms" Law for each element

ONLY for maximum values

RMS = Root mean square

$$I_{peak} = I_{max}\sqrt{2}$$

$$\langle I^2 R \rangle = I_{max}^2 R$$

There will be a Phasor question in the exam

Redefinition of $\epsilon = I_m Z$ Wanna deliver 1500 W, power. 100 Volts over transmission lines, resistance $R = 5$ ohms.

How much power is lost in the lines?

Power = 1500 W

Calculating power lost in the lines

Power dissipated = $P_R = IV = 20 * 100V = 2000W$, first attempt not right

$$I = \frac{V}{R} = \frac{100V}{5\Omega} = 20A$$

$$P_R = IV = I(IR) = I^2 R = 15^2 A 5\Omega = 1125W$$

$$I_{10kv} = \frac{1500}{10000} = 0.15A$$

$$(.15A)^2 * (5\Omega) = 0$$

High voltage low current prevents loss

4.4.3 Homework

IE

I'm not going to show the functions for this, since the functions aren't used later

If you want the functions, let me know and I'll add them from now on!

0.000839

I use ϕ instead of φ for consistency in notes.

1-1

$$I_m = \frac{2P_{avg}}{\epsilon_m \cos\phi}$$

1-2

$$R = \frac{2P_{avg}}{I_m^2}$$

1-3

$$C = (\omega^2 L - \omega R \tan(-\phi))^{-1}$$

1-4

B

1-5

$$P_{avg} = \frac{1}{2} \frac{\epsilon_m^2}{R}$$

2-1

$$L = \frac{1}{\omega^2 C}$$

Answer in H, convert H \rightarrow mH!!!

2-2

$$U_{max} = \frac{1}{2} L I_m^2$$

Multiply answer by 10^{-3} to convert to Joules!

My answer went from 49.46 \rightarrow 0.0495

2-3
 $\Delta U = \frac{\pi \varepsilon_m I_m}{\omega}$
 2-4
 $Q = \frac{U_{max}}{\Delta U} 2\pi$
 2-5
 $R = \frac{\varepsilon_m}{I_m}$
 2-6
 B

5 Light and Optics

5.1 Section 22: Displacement Current and Electromagnetic Waves

5.1.1 Pre-Lecture Notes

Modified Ampere's Law
 $\oint \vec{B} * d\vec{l} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$

$$Q = \varepsilon_0 \phi_E$$

Displacement Current

$$I_D = \varepsilon_0 \frac{d\phi_E}{dt}$$

$$h(x,t) = A \cos(kx - \omega t)$$

Harmonic Plane Wave

Amplitude A

$$\text{Wave number } k = \frac{2\pi}{\lambda}$$

$$\text{Angular Frequency } \omega = \frac{2\pi}{T}$$

Period T

$$\text{Frequency } f = \frac{1}{T}$$

$$\text{Velocity } v = \lambda f = \frac{\omega}{k}$$

Wave Equation

$$\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_x}{\partial t^2}$$

Speed of Electromagnetic Wave

$$v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \text{ A harmonic solution}$$

$$E_x = E_0 \cos(kz - \omega t)$$

$$B_y = \frac{k}{\omega} E_0 \cos(kz - \omega t)$$

B_y is in phase with E_x

$$B_0 = \frac{E_0}{c}$$

5.1.2 Homework

1-1

Convert λ from nm \rightarrow m

$$k = \frac{2\pi}{\lambda}$$

1-2

$$Z_{max} = \frac{\pi}{2k}$$

Convert answer from m \rightarrow nm

1-3

c is the speed of light

$$c = 3 * 10^8 \frac{m}{s}$$

$$E_{max} = \sqrt{2}cB_1$$

1-4

Mind your directions here! Mine was negative!

$$E_y = cB_1$$

1-5

B

1-6

$$t_{max} = \frac{\lambda}{4c}$$

Make sure λ is in m

1-7

B

END TEST 2!

5.2 Section 23: Properties of Electromagnetic Waves

BEGIN FINAL!!!

5.2.1 Pre-Lecture Notes

$$E_x = E_0 \sin(kz - \omega t)$$

$$B_y = B_0 \sin(kz - \omega t)$$

$$\text{Velocity } c = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \frac{E_0}{B_0}$$

$$\text{E-M Wave Speed } c = f\lambda$$

$$\text{Doppler Shift } f' = f \frac{1 \pm \beta}{1 \mp \beta}$$

Energy Densities

$$\text{Electric Fields } u_E = \frac{1}{2} \epsilon_0 E^2$$

$$\text{Magnetic Fields } u_B = \frac{1}{2} \frac{B^2}{\mu_0}$$

$$E = cB$$

$$u = \epsilon_0 E^2$$

Average Energy Density

$$\langle u \rangle = \frac{1}{2} \epsilon_0 E_0^2 \text{ Intensity}$$

$$I \equiv \frac{\langle \text{Power} \rangle}{\text{Area}} \text{ Poynting Vector}$$

$$S = c \epsilon_0 E^2$$

Sunlight Electric Field Strength

$$E_{rms}^2 = \mu_0 c I$$

$$I = 100 \frac{mW}{cm^2}$$

Impedance of Free Space $Z_0 \equiv \mu_0 c$

$$\text{Energy} = E = hf$$

$$\text{Momentum} = p = \frac{E}{c} = \frac{h}{\lambda}$$

5.2.2 Homework

IE

$$B_Z = -61.36 * 10^{-8}$$

c is the speed of light, $3 * 10^8$

1-1

$$f = \frac{c}{\lambda}$$

1-2

$$\varepsilon_0 = 8.854E - 12$$

$$I = \frac{1}{2} \varepsilon_0 c^3 (B_x^2 + B_y^2)$$

1-3

$$\mu_0 = 4\pi * 10^{-7} \quad S_Z = -\frac{c}{\mu_0} (B_x^2 + B_y^2)$$

1-4

$$E_x = -cB_y \quad 1-5$$

D

5.3 Section 24: Polarization

5.3.1 Pre-Lecture Notes

$$E_x = E_0 \cos \theta \sin(kz - \omega t + \phi)$$

$$E_y = E_0 \sin \theta \sin(kz - \omega t + \phi)$$

Incident Light

$$\text{Unpolarized: } I_{final} = \frac{1}{2} I_0$$

Law of Malus

$$\text{Polarized: } I_{final} = I_0 \cos^2 \theta$$

$$I_1 = \frac{1}{2} I_0$$

$$I_2 = I_1 \cos^2 30$$

$$I_3 = I_2 \cos^2 60$$

Linear Polarization

$$\text{Relative Phase } \phi \equiv \phi_x - \phi_y = 0$$

Circular Polarization

$$\text{Relative Phase } \phi \equiv \phi_x - \phi_y = \pm \frac{\pi}{2}$$

+ is right rotation

- is left rotation

$$\Delta\phi = \phi_y - \phi_x = \omega d \left(\frac{1}{v_{fast}} - \frac{1}{v_{slow}} \right)$$

$$\Delta\phi = \frac{\pi}{2}$$

5.3.2 Homework

IE

$$75.78^\circ$$

1-1

$$I_1 = \frac{I_0}{2}$$

$$I_2 = I_1 \cos^2(\theta_1 - \theta_2)$$

1-2

$$I_{final} = I_2 \cos^2(\theta_3 - \theta_2)$$

1-3

$$I_{final,new} = \frac{1}{2} I_0 (\cos(\theta_1 - \theta_3) \cos(\theta_2 - \theta_3))^2$$

1-4

B

1-5

$$I'_{final} = \frac{1}{2} I_0 (\cos(\theta_2 - \theta_3) \cos \theta_3)^2$$

2-1

$$I_{mid} = \frac{I_0}{2}$$

2-2

$$I_{final} = I_{mid} \cos^2 \theta_1$$

2-3

$$\frac{E_{y,final}}{E_0} = \sqrt{\frac{I_{final}}{I_0}} \sin \theta_1$$

2-4

$$I_{final,new} = I_0 \cos^2(90^\circ - \theta_1) \cos^2 \theta_1$$

2-5

B

2-6

C

5.4 Section 25: Reflection and Refraction

5.4.1 Pre-Lecture

Law of Reflection

$$\theta_i = \theta_r$$

$$ct_{ab} = ct_{dc}$$

$$v = \frac{1}{\sqrt{\mu\epsilon}}$$

Index of refraction

$$n \equiv \frac{c}{v} = \frac{\sqrt{\mu\epsilon}}{\sqrt{\mu_0\epsilon_0}}$$

$$n \approx \sqrt{\frac{\epsilon}{\epsilon_0}}$$

Snell's Law

$$n_2 \sin \theta_2 = n_1 \sin \theta_1$$

Glancing incidence

$$\theta = 90^\circ$$

Normal incidence

$$\theta = 0^\circ$$

$$R = \left(\frac{n_2 - n_1}{n_2 + n_1} \right)^2$$

Critical Angle

$$\theta_c = \sin^{-1} \left(\frac{n_2}{n_1} \right)$$

Brewster's Angle

$$\tan \theta_1 = \frac{n_2}{n_1}$$

5.4.2 Homework

5.5 Section 26: Lenses

5.5.1 Pre-Lecture

Converging lenses are convex

Diverging lenses are concave

Lense Equation

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$s' = \frac{fs}{s-f}$$

Magnification

$$M \equiv \frac{h'}{h} = -\frac{s'}{s}$$

$$M = \frac{-f}{s-f}$$

Lensmaker's Formula

$$\frac{1}{f} = (n-1)\frac{1}{R}$$

5.5.2 Homework

5.6 Section 27: Mirrors

5.6.1 Pre-Lecture

5.6.2 Homework

5.7 Section 28: Optical Instruments

5.7.1 Pre-Lecture

5.7.2 Homework