#12: Constraint Solving

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EECS 700: Introduction to Program Synthesis

Why do we care?

- 1. Synthesis is combinatorial search, and so is SAT/SMT
- 2. SAT/SMT solvers are really good these days
- 3. ??? ⟵ this week
- 4. Profit!!!

Boolean SATisfiability

gin ∨ tonic

Solution:

minor ↦ T

gin ↦ F

tonic ↦ T

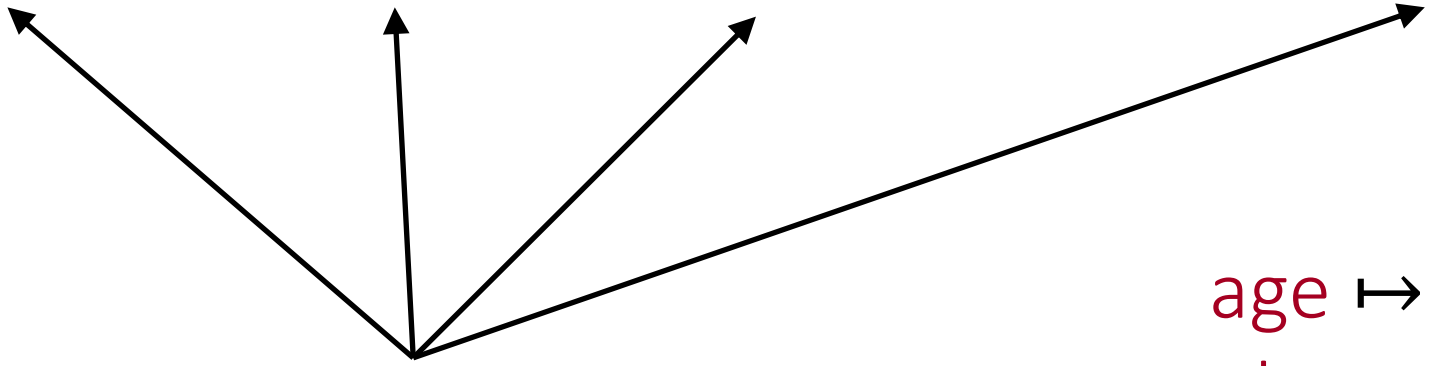
Satisfiability Modulo Theories

$$(\text{gin} \vee \text{tonic}) \wedge (\text{age} < 21 \Rightarrow \text{abv} = 0) \wedge (\text{age} = 20)$$

> In the United States, "gin" is defined as an alcoholic beverage of no less than 40% ABV…
> Wikipedia

Satisfiability Modulo Theories

$$(\text{gin} \vee \text{tonic}) \wedge (\text{age} < 21 \Rightarrow \text{abv} = 0) \wedge (\text{age} = 20) \wedge (\text{gin} \Rightarrow \text{abv} \geq 40)$$



theory of Linear Integer Arithmetic

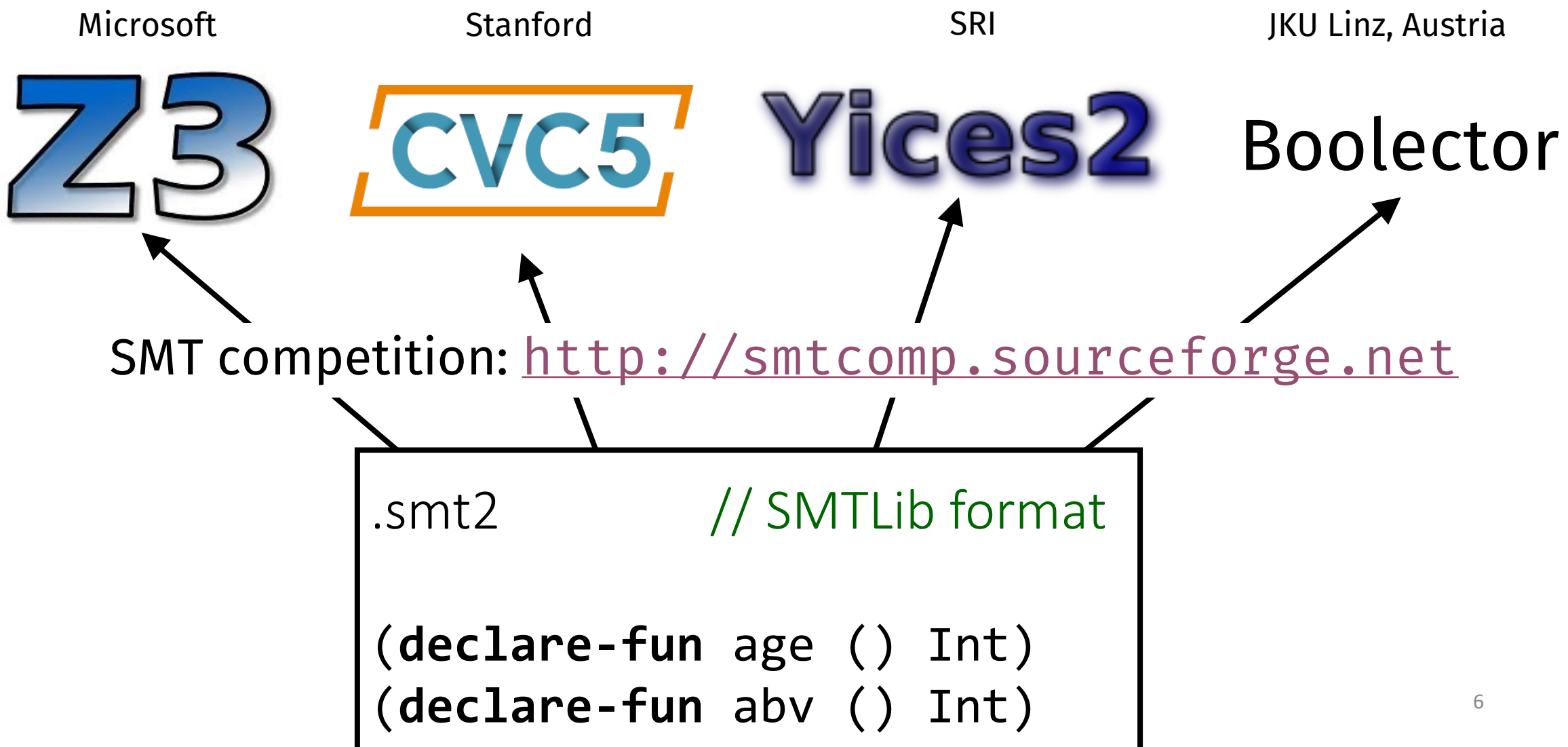
$\text{age} \mapsto 20$

$\text{abv} \mapsto 0$

$\text{gin} \mapsto \text{F}$

$\text{tonic} \mapsto \text{T}$

Popular Solvers



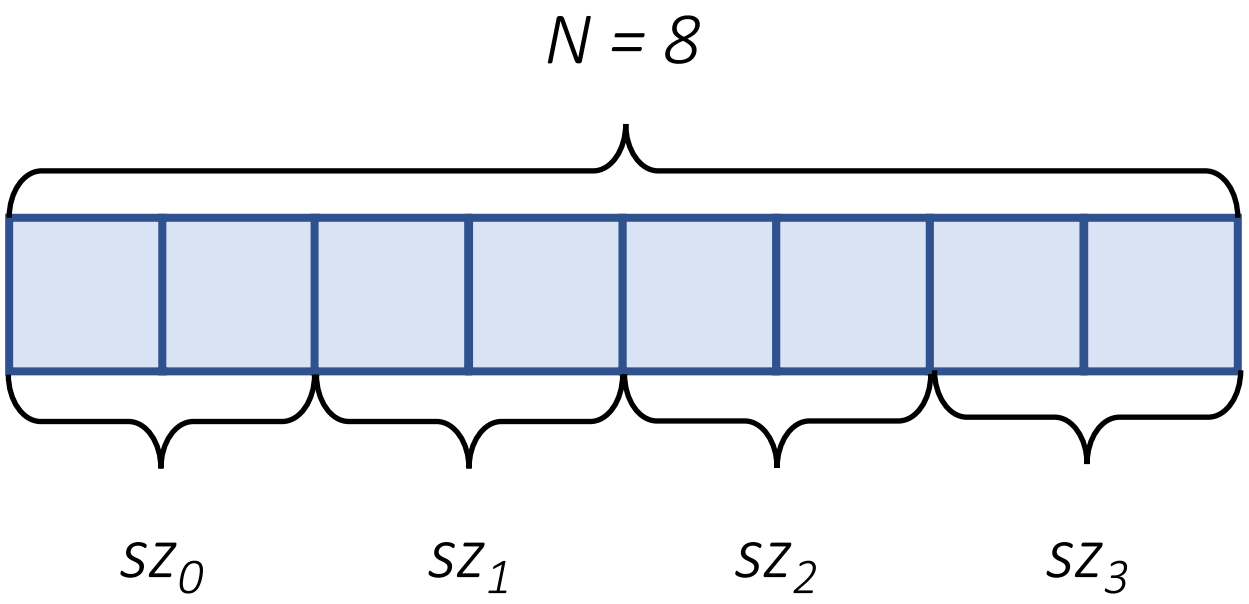
SMT-LIB

Uniform format for SMT problems understood by all solvers

```
(declare-fun age () Int)
(declare-fun abv () Int)
(declare-fun gin () Bool)
(declare-fun tonic () Bool)
(assert (or gin tonic))
(assert (implies (< age 21) (= abv 0)))
(assert (= age 20))
(assert (implies gin (>= abv 40)))
(check-sat)
(get-model)
```

Problem: Array Partitioning

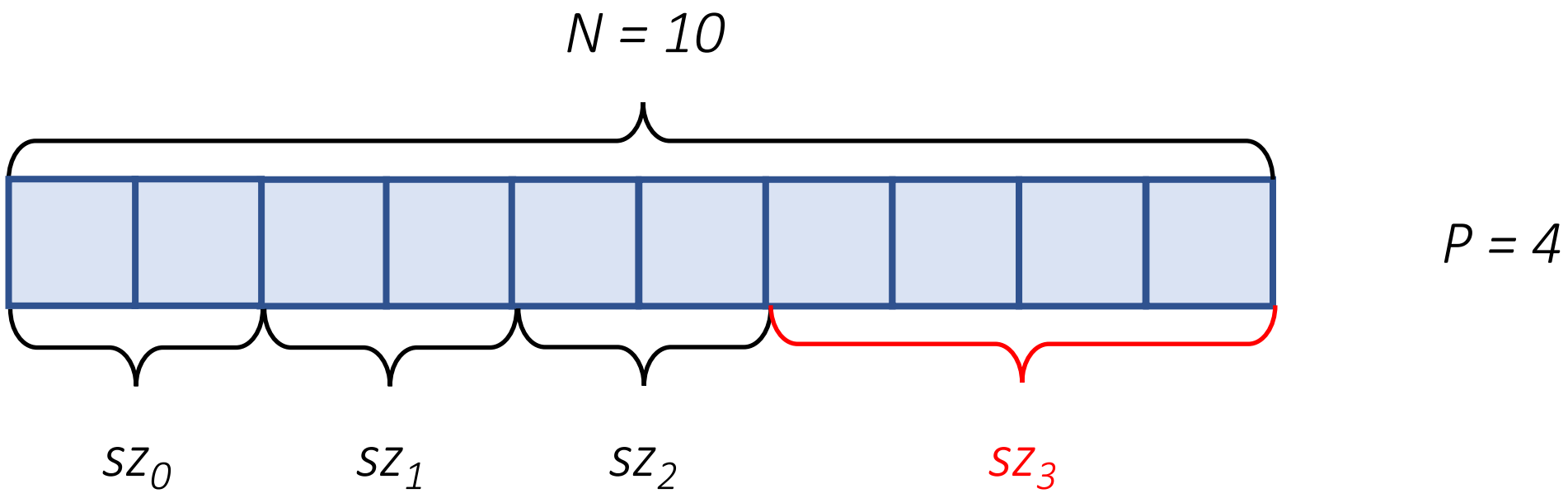
Partition an array of size *N* evenly into *P* sub-ranges



P = 4

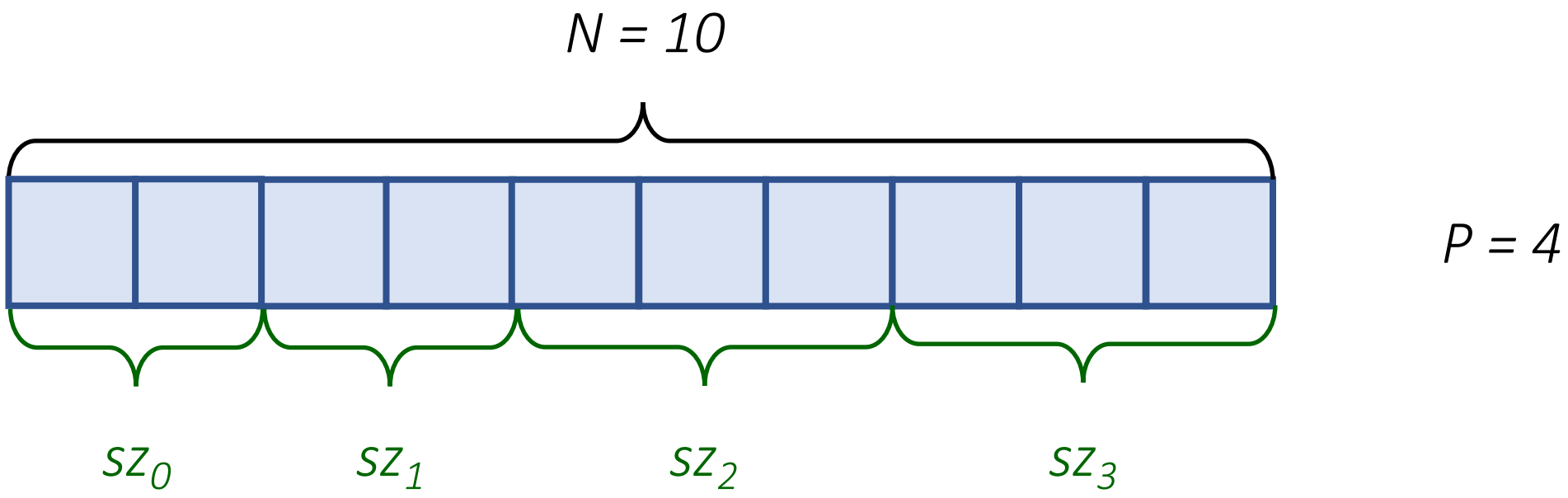
Problem: Array Partitioning

Partition an array of size *N* evenly into *P* sub-ranges



Problem: Array Partitioning

Partition an array of size *N* evenly into *P* sub-ranges

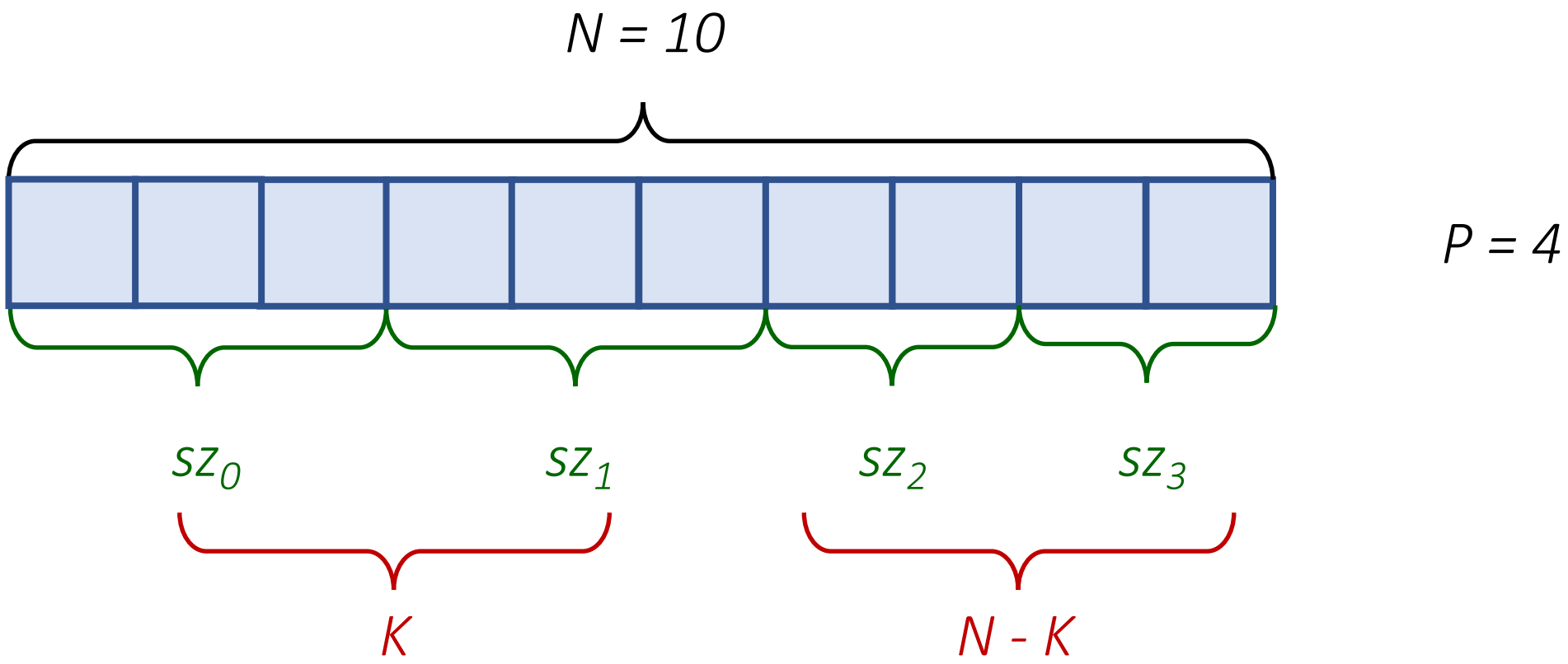


Can we always make them differ by at most 1?

to the rescue!

code: https://github.com/nadia-polikarpova/smt-talk

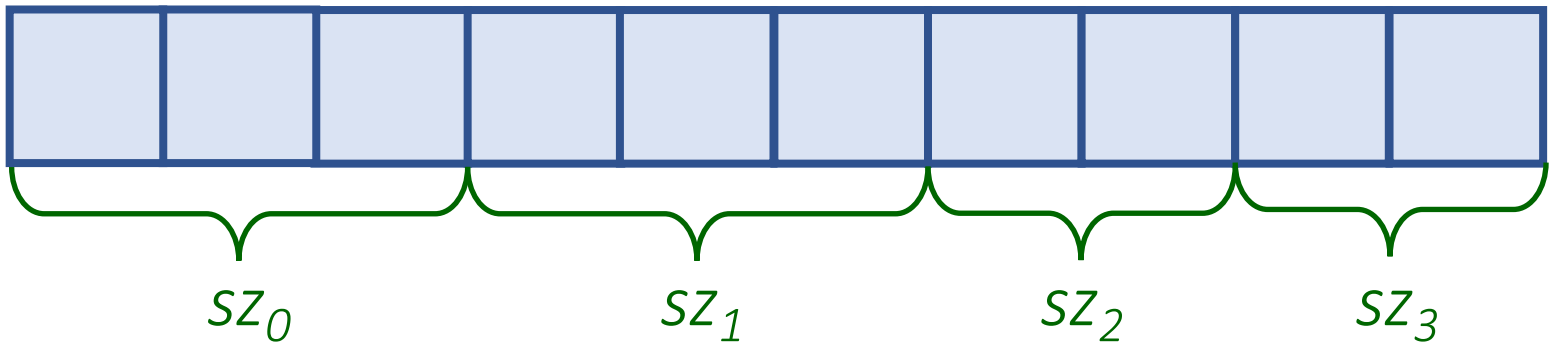
Let’s generalize this into a program!



A program for partitioning

```
for i in range(P):
  if i < K:
    sz[i] = n/P + 1
  else:
    sz[i] = n/P
```

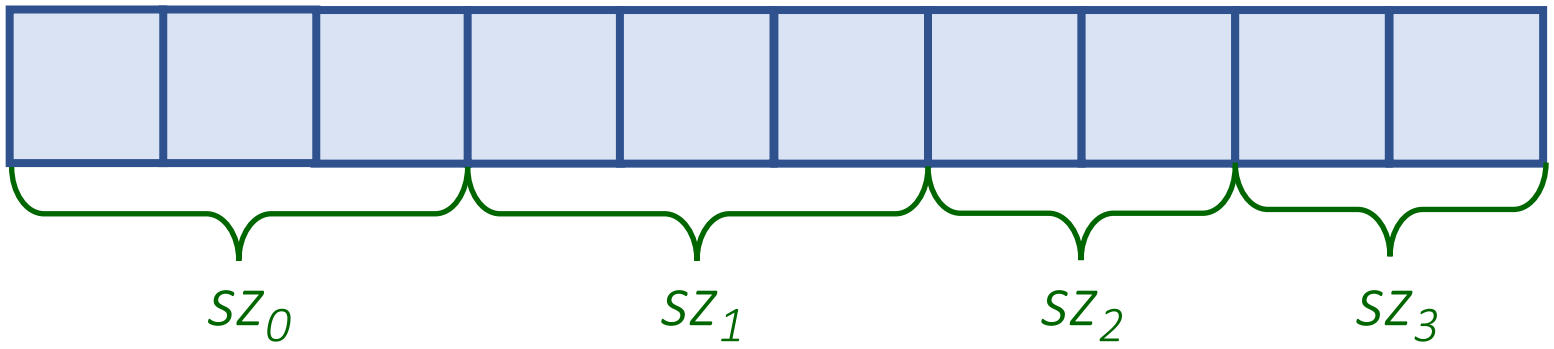
I want this program to work for all *n*!
What should my *K* be?



A program for partitioning

```
for i in range(P):
  if i < K:
    sz[i] = n/P + 1
  else:
    sz[i] = n/P
```

How do I prove that this program works for all *n*?



Verification with SMT

```
for i in range(P):
  if i < K:
    sz[i] = n/P + 1
  else:
    sz[i] = n/P
```

want: prove $\forall n. spec(n)$

have: solve $\exists n. prop\ (n)$

idea: solve for counter-examples!

$\forall n. \sum sz = n$

$\exists n. \neg spec(n)$

$\exists n. \sum sz \neq n$