Lecture 35
Synthesis with Optimization

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with slides from
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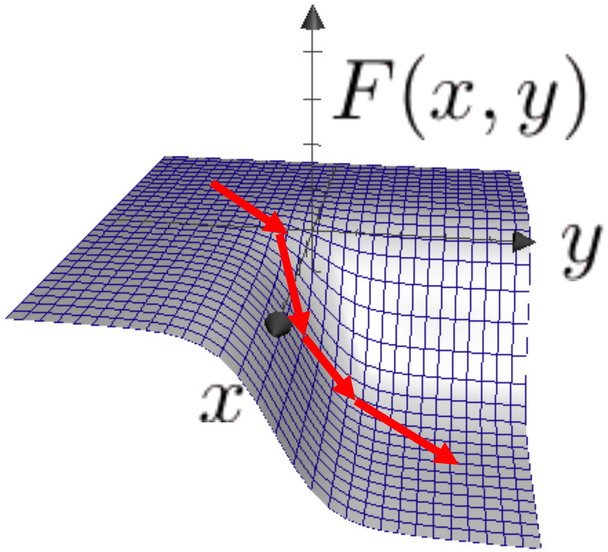
Numerical Optimization

Goal: Find a (hopefully global) minima of a function

- Sample the function on a small set of values
- Based on the result sample in a new set of locations



- Gradient descent
- Newton iteration

…

Numerical Optimization for Synthesis

Many synthesis problems are best framed as optimization problems

- Especially true for problems involving hybrid systems or probabilities

The Parameter Synthesis Problem

```
tOff := ??;    tOn := ??;  h := ??
forever{
   temp := readTemp();
   if (isOn() && temp > tOff)
        switchHeaterOff();
  elseif ( !isOn() && temp < tOn)
       switchHeaterOn(h);
}
```

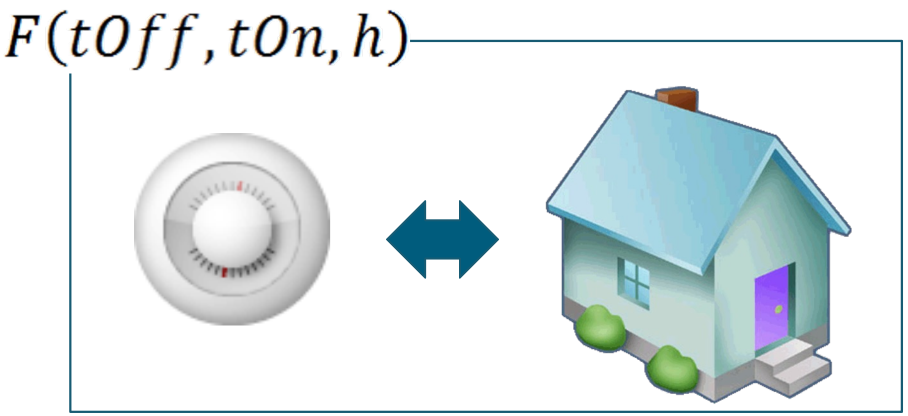
Warming:

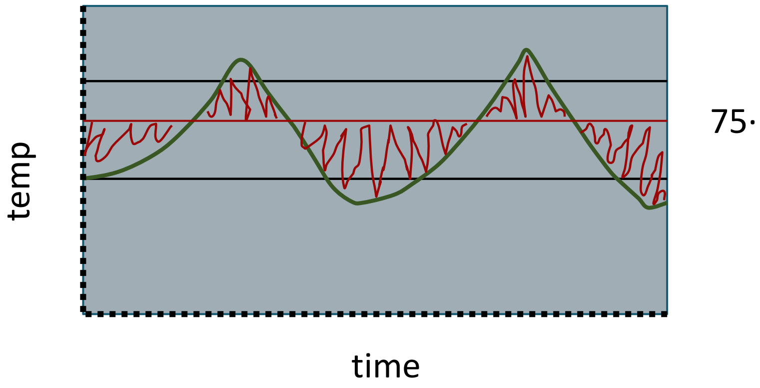
$$\frac{d}{dt} temp = -k \cdot temp + h$$

Cooling:

$$\frac{d}{dt} temp = -k \cdot temp$$

The Parameter Synthesis Problem



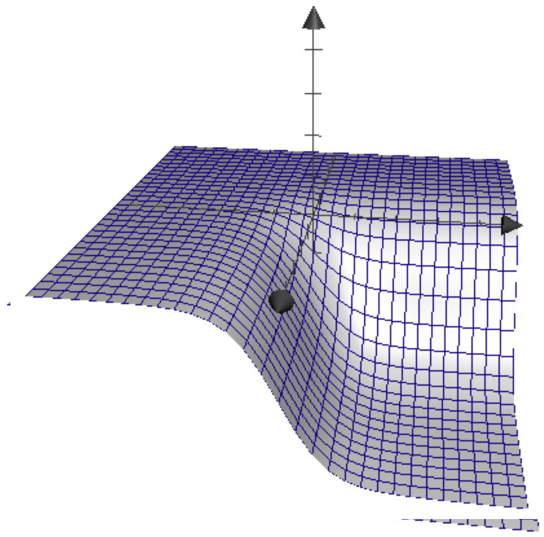


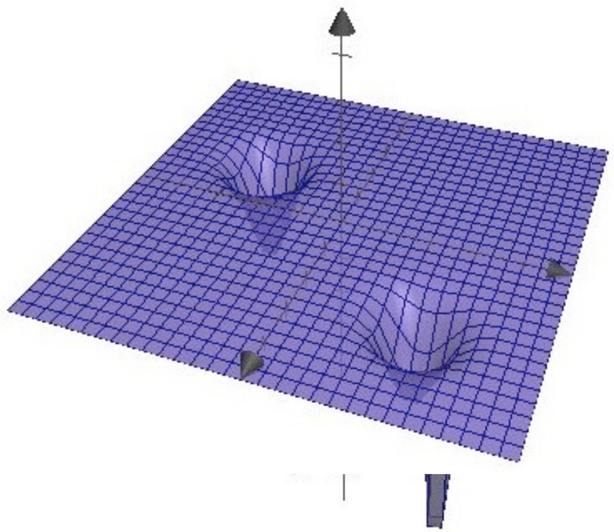
$$F(tOff, tOn, h) = [\, t_0, t_1, \dots, t_k]$$

$$Err(tOff, tOn, h) := \sum (t_i - 75)^2$$

Can we find values of (tOff, tOn, h) to minimize Err?

The challenge of optimizing programs





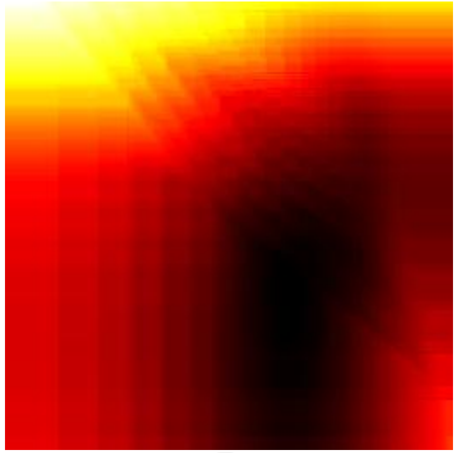
Plateaus and discontinuities thwart search

Branches introduce discontinuities

For the thermostat example

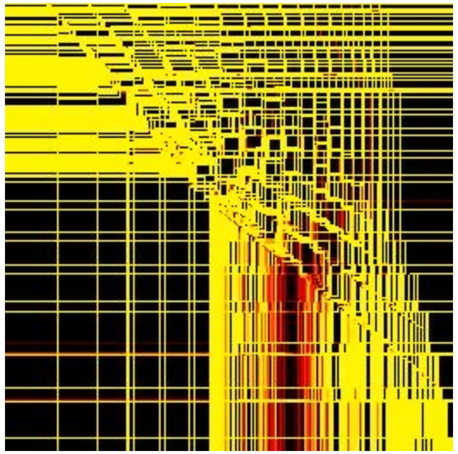
$Err(tOn, tOff, 150)$

$tOff$



tOn

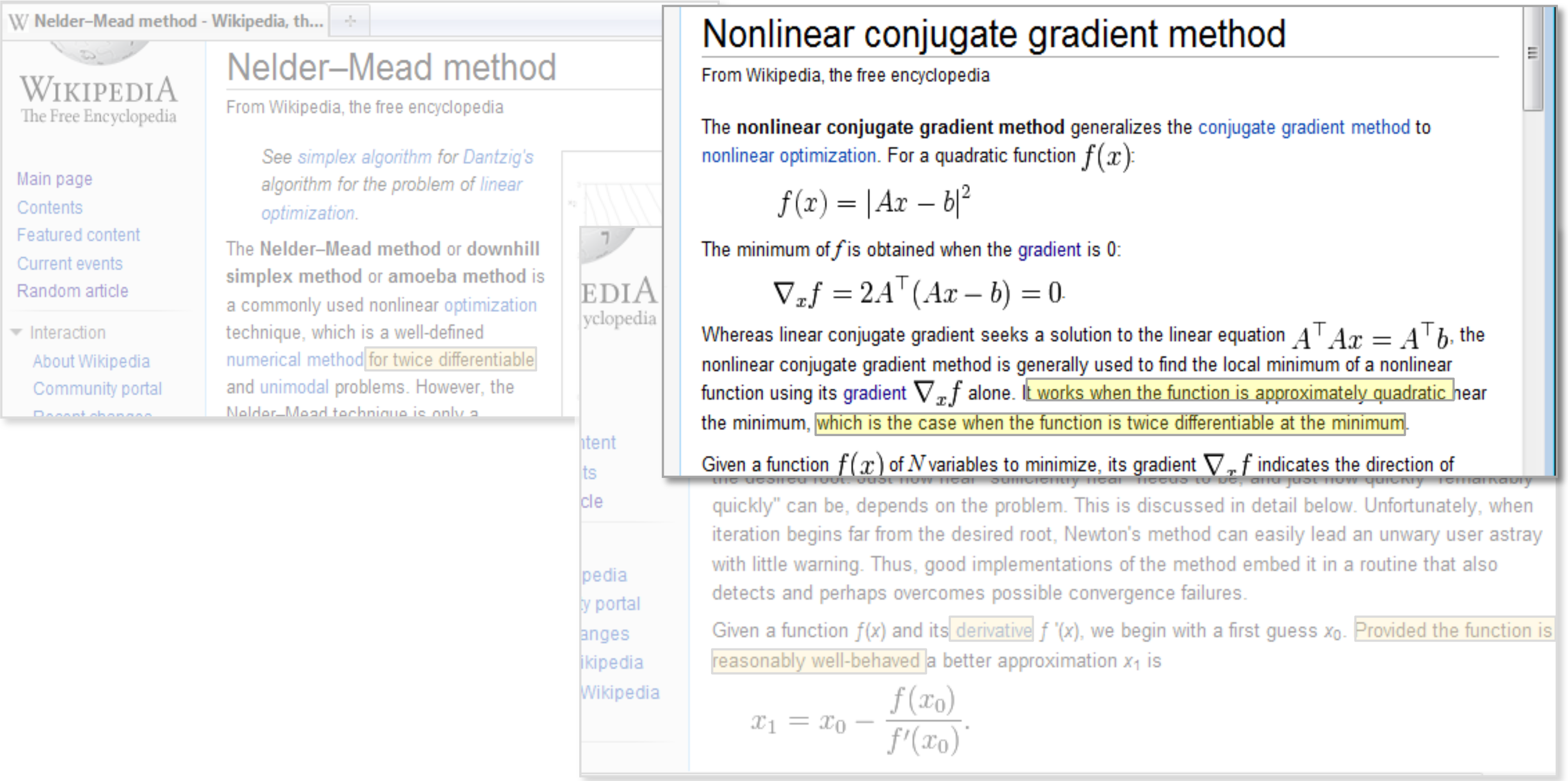
$\left|\vec{\nabla} Err(tOn, tOff, 150)\right|$



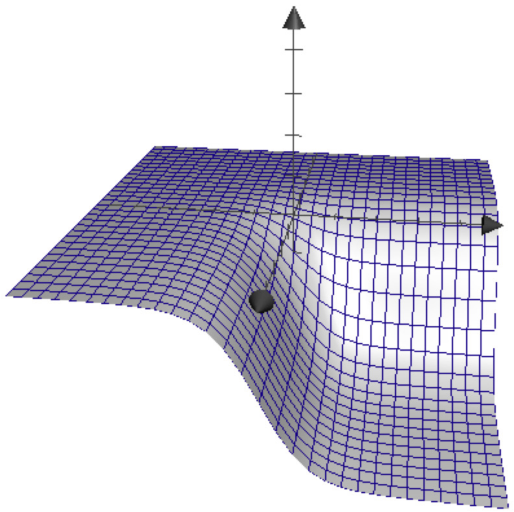
tOn

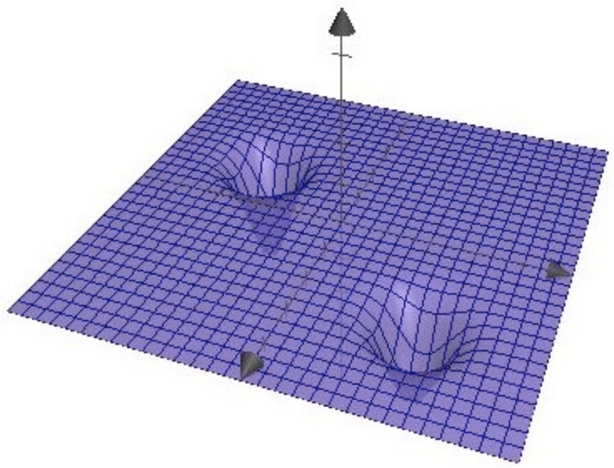
Minimization by numerical Search

Discontinuities affect even the best methods



Key Idea: Smooth the Function

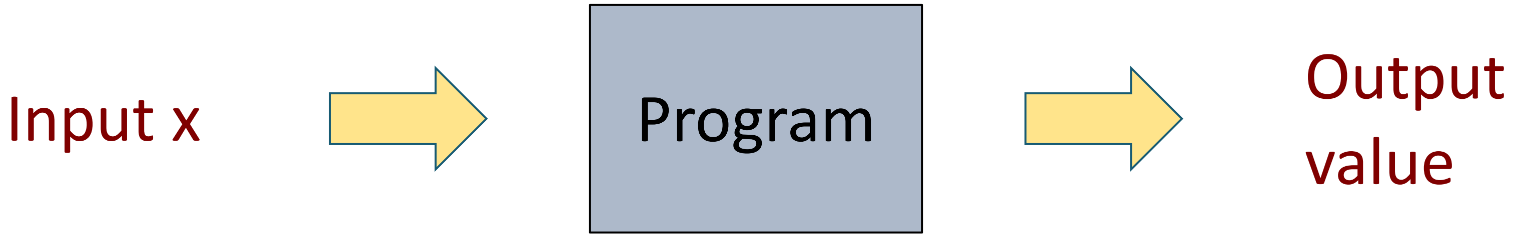




Approximate the problem by a smooth function

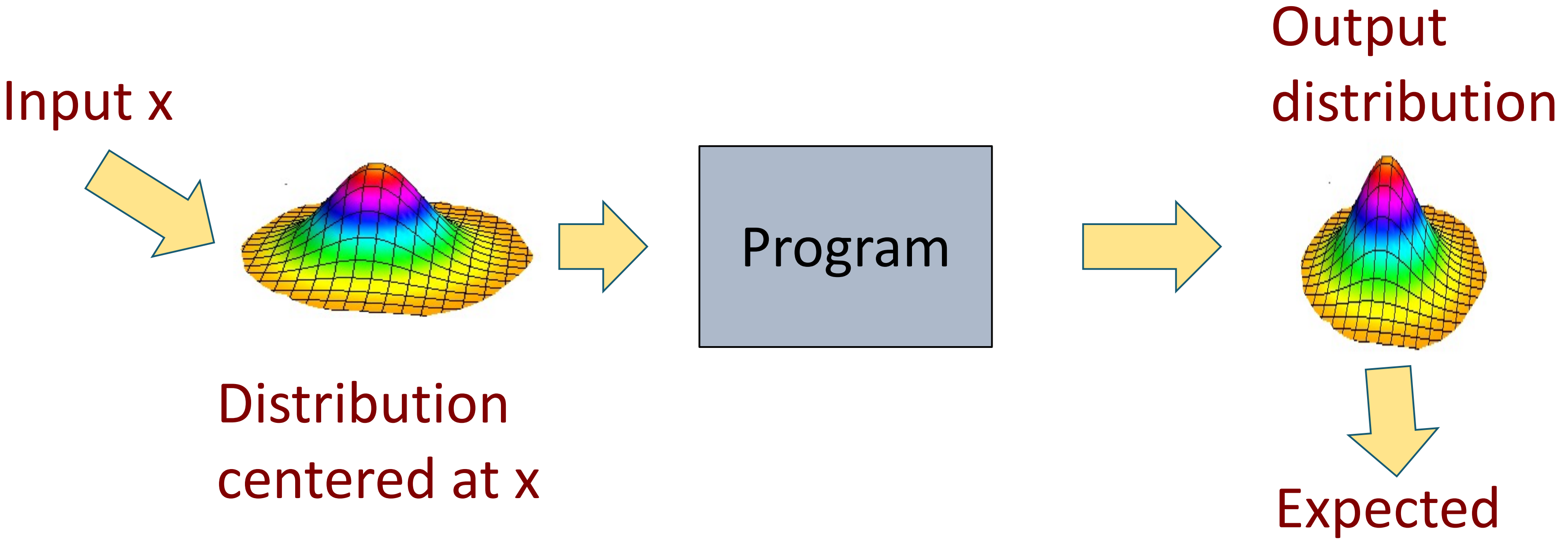
Example: Gaussian Smoothing

Execute on a distribution instead of a value



Example: Gaussian Smoothing

Execute on a distribution instead of a value

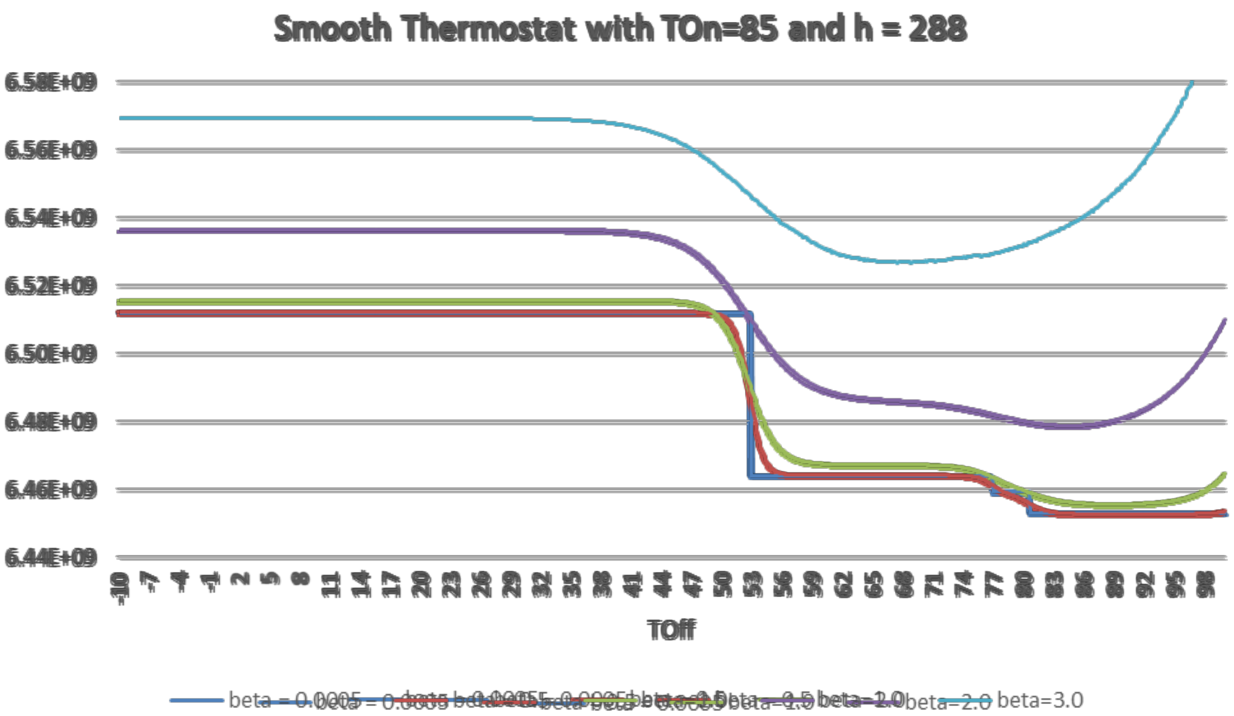


Expected value smooths discontinuities

Parameterized Smoothing

Parameterized by a value β

- Corresponds to std dev when smoothing with Gaussian noise

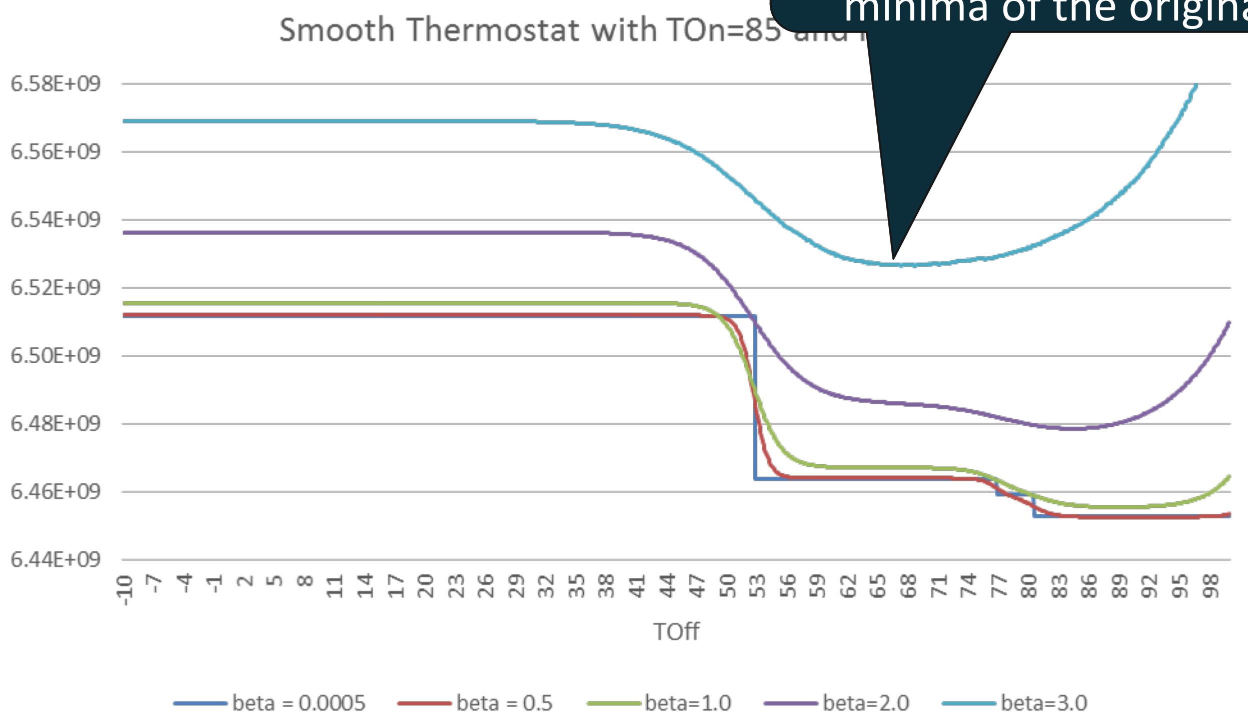


Parameterized Smoothing

$F_{smooth}(p, \beta_i)$ parameterized by a value β

- Corresponds to std dev when smoothing with Gaussian noise

Minima of smooth function is different from minima of the original



Smoothed Numerical Search

$p_0 = random\ starting\ point$

$\beta_0 = initial\ beta$

$While\ \beta_i > \epsilon$

$\quad p_{i+1} = optimize\ F_{smooth}(p, \beta_i) starting\ at\ p_i$

$\quad \beta_{i+1} = \beta_i * q$

$return\ p_i$

$0 \leq q < 1$

Smoothed Numerical Search

Very general procedure

Steps:

- Frame your problem as an optimization problem
- Define a parameterized smoothing procedure
- Run the algorithm

Smoothing doesn't have to be perfect

- But it should approximate the desired function in the limit as $\beta \to 0$

Euler:
SNS for programs

SNS in Euler

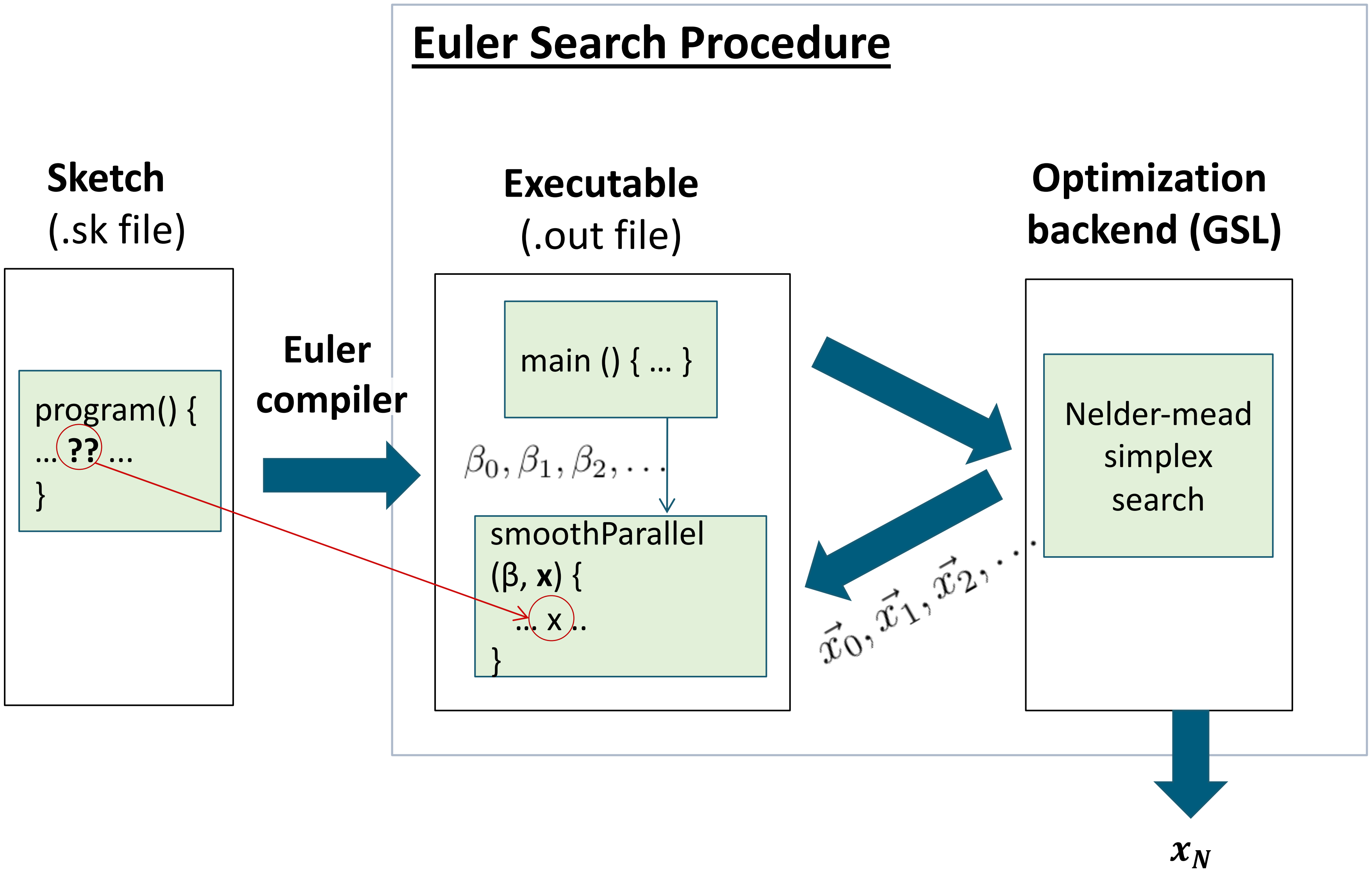
Use Symbolic Gaussian Smoothing

- β is simply the standard deviation

Quantifier alternation is handled through fixed test harnesses

- not ideal,
- but works well in practice

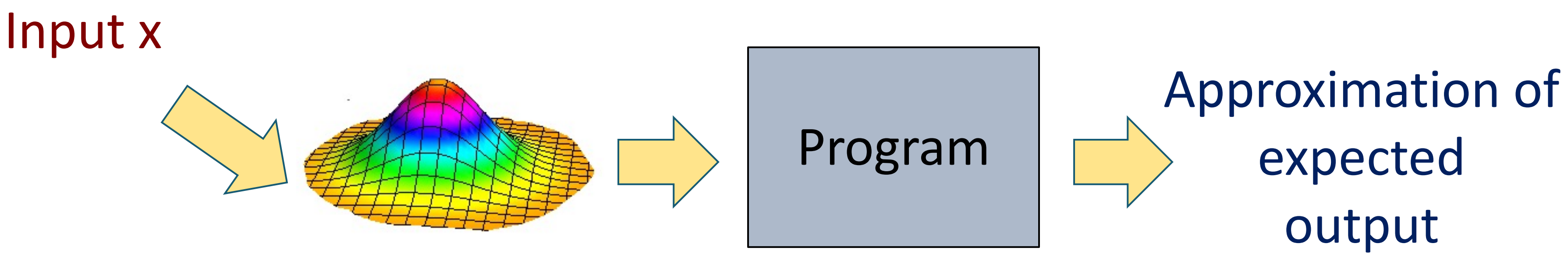
Smoothed Numerical Search in Euler



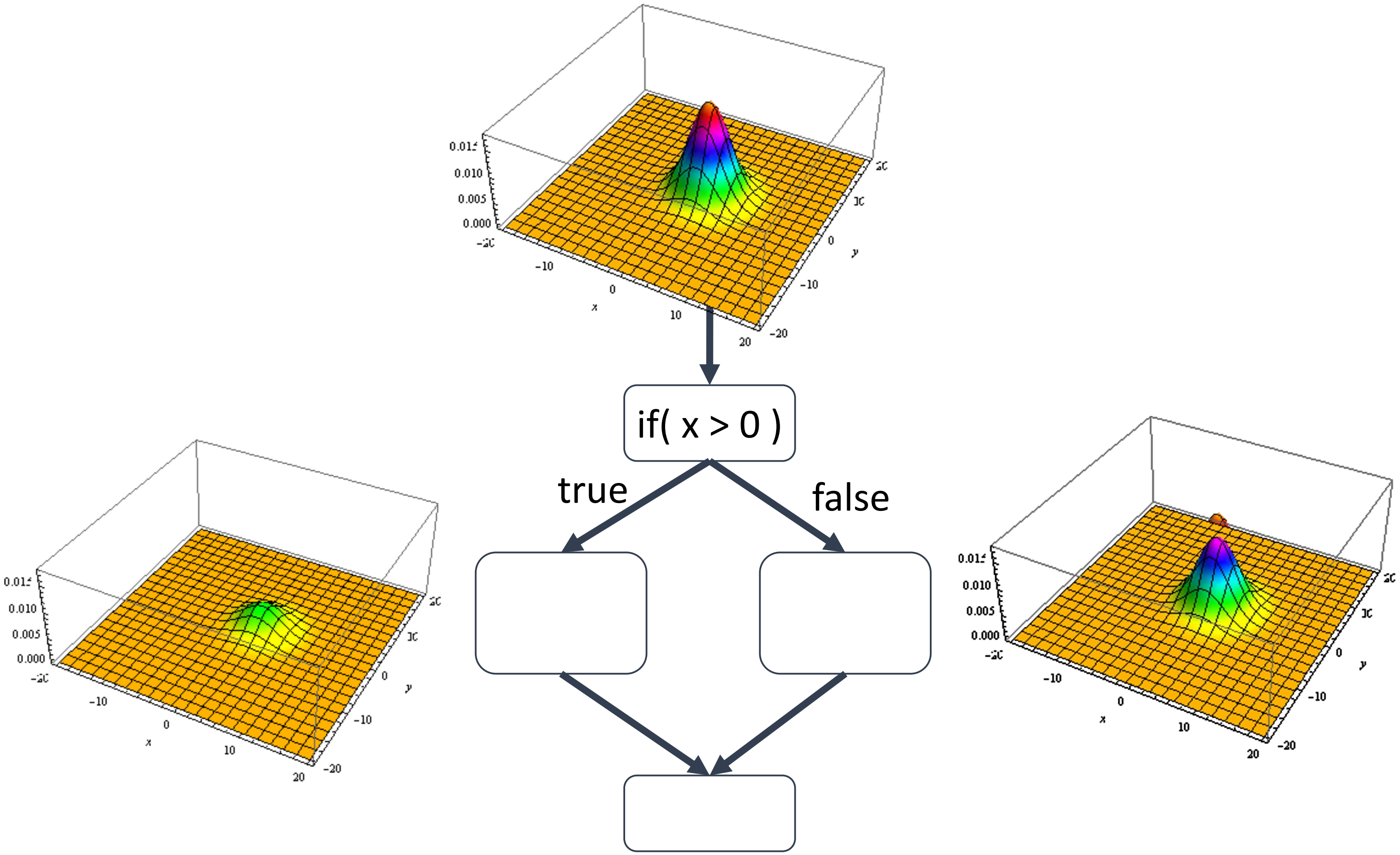
Smooth Interpretation

How do we execute a program on a distribution of inputs?

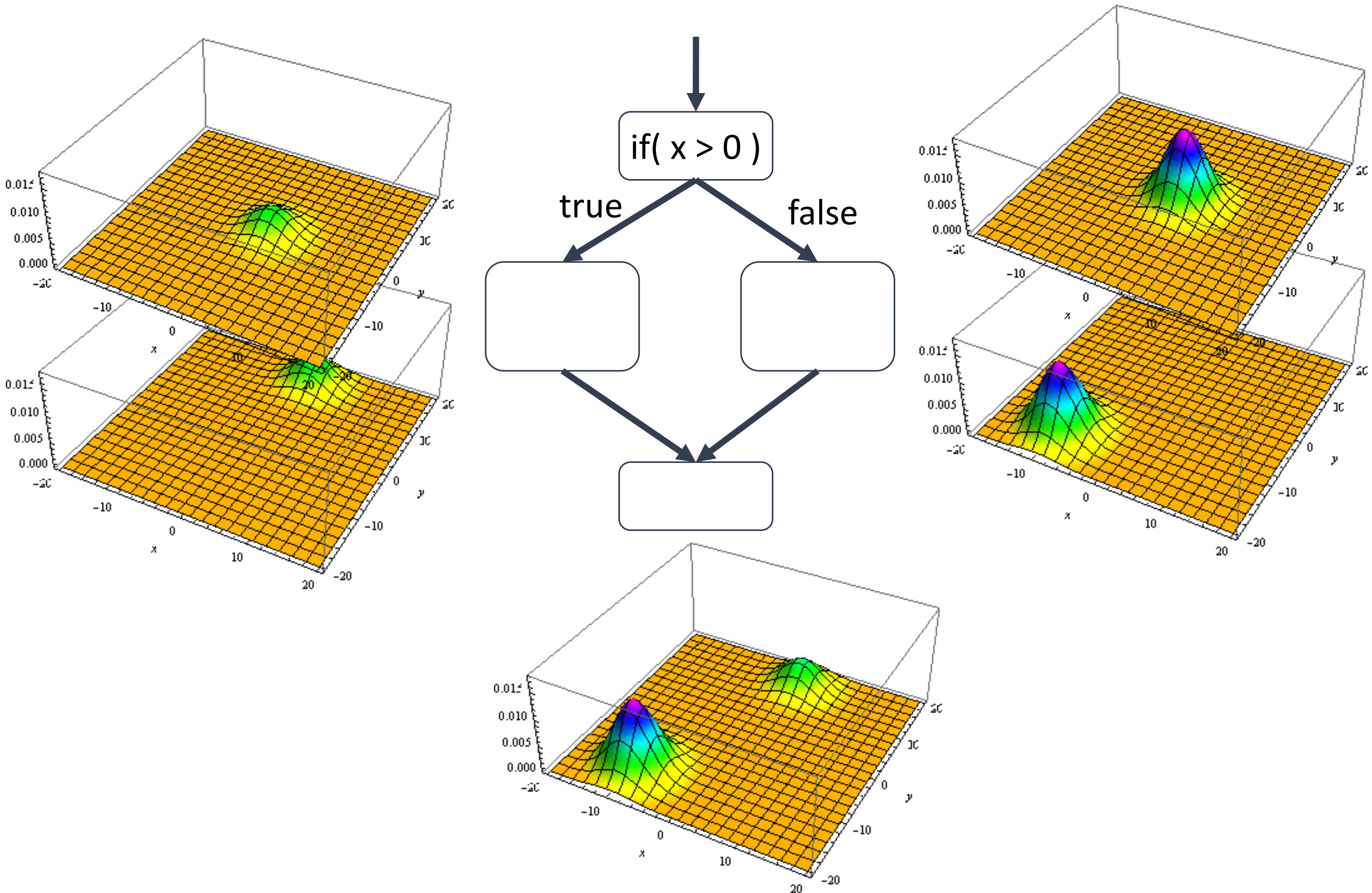
- Not by sampling
  - we would miss essential features of the search landscape.
- Symbolic execution:
  - propagate symbolic representations of distributions.



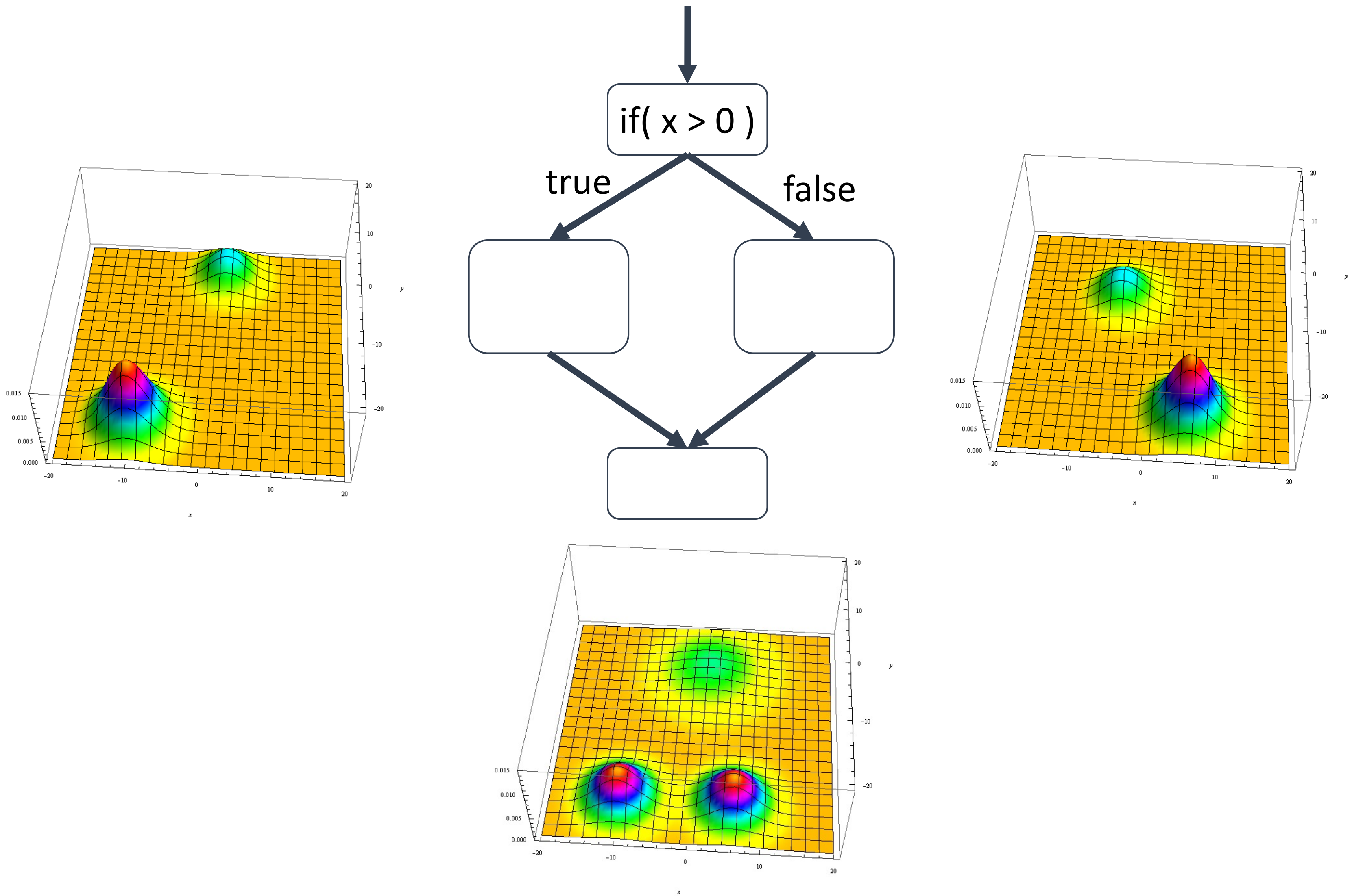
Smooth Interpretation: Branch



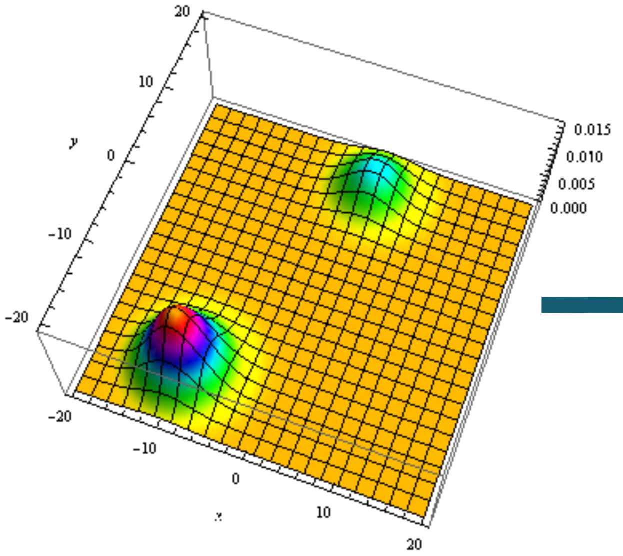
Smooth Interpretation: Join



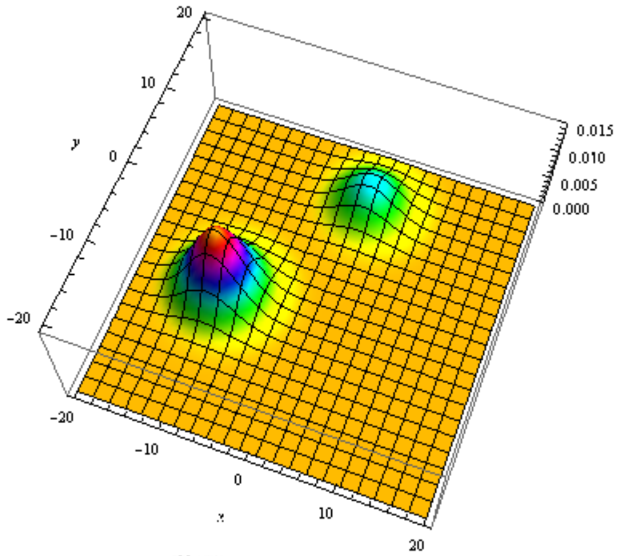
Smooth Interpretation: Join

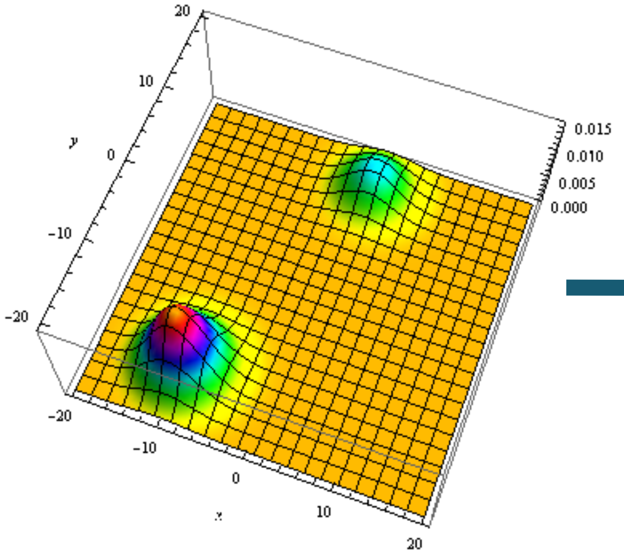


Smooth Interpretation: Assignment

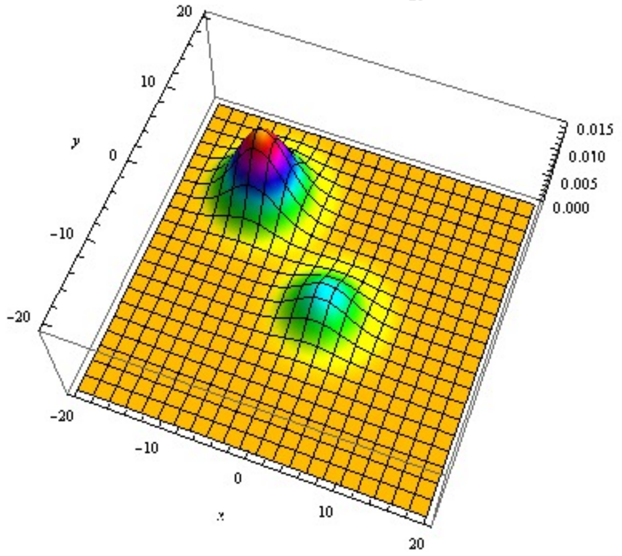


`x = x - y;`





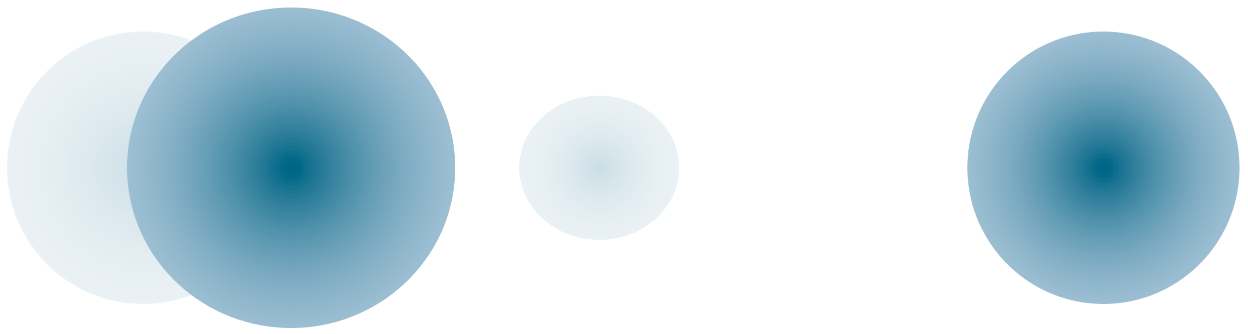
`x = -y;`



Approximations

Trick Question:

- Which of the previous approximations is problematic?



Approximations

Trick Question:

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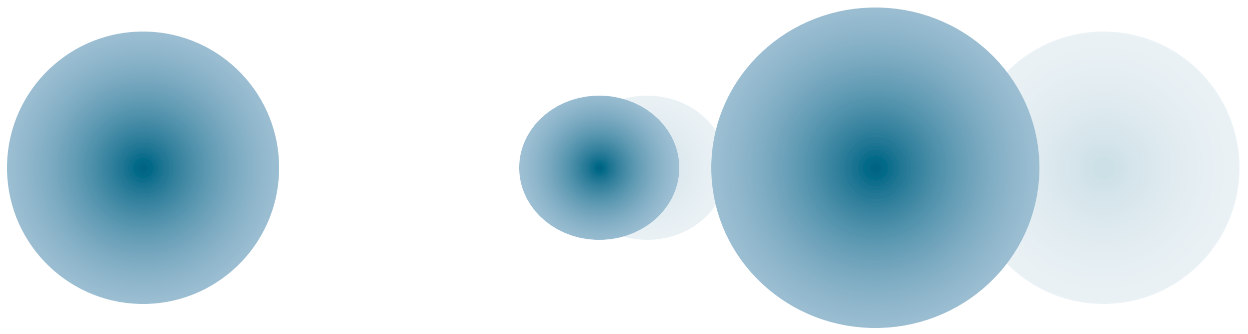
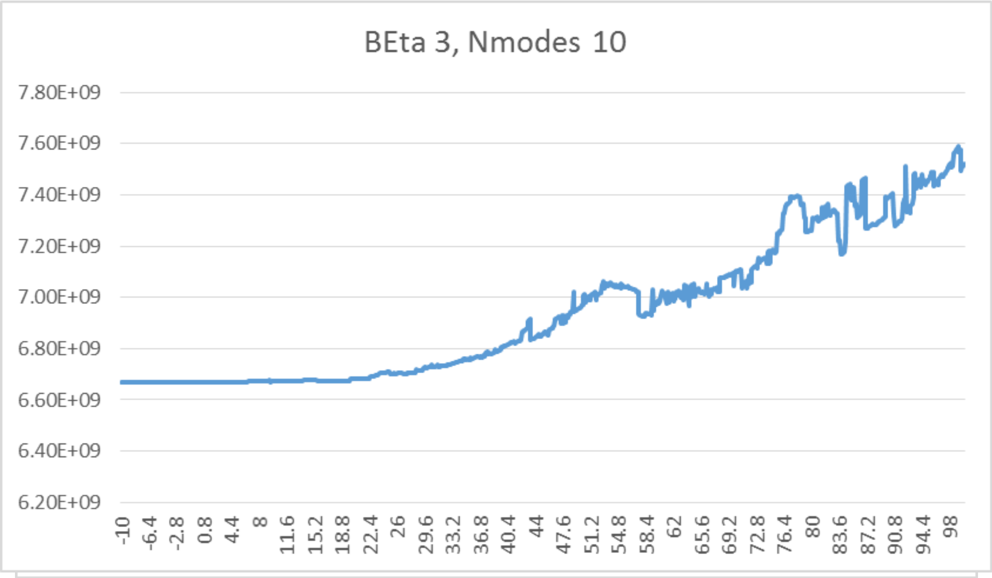


Merging can introduce discontinuities!

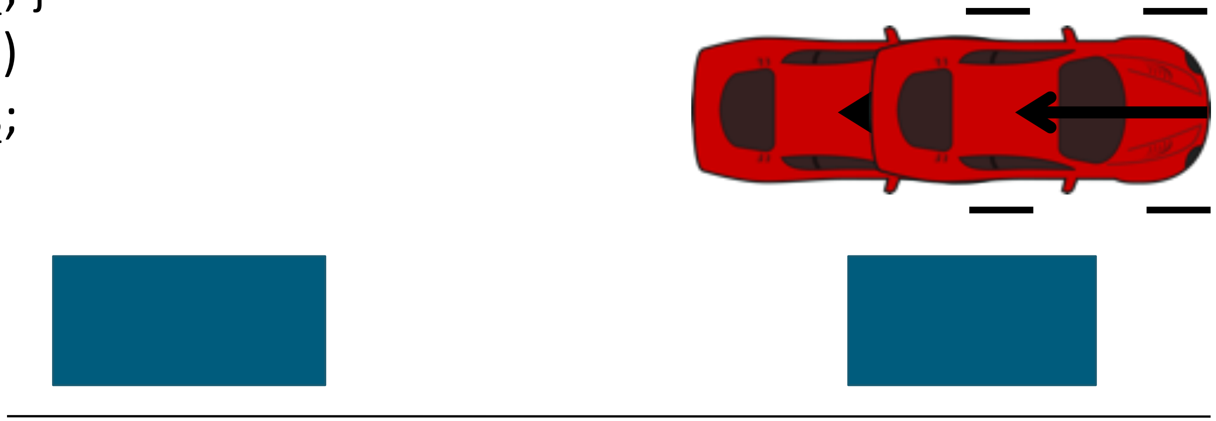
Excessive merging can cause problems



Examples

Parameter synthesis

```
t1 := ??;    t2 := ??;  A1 := ??;
A2 := ??;
repeat  (every ΔT)  {
    if (stage = BACK && time > t1)
        stage = INTURN;
    if (stage = INTURN) {
        if (time > t2) then stage = OUTTURN;
        angle = angle – A1; }
    if (stage = OUTTURN)
        angle = angle + A2;
  moveCar();
}
```



Parameter synthesis

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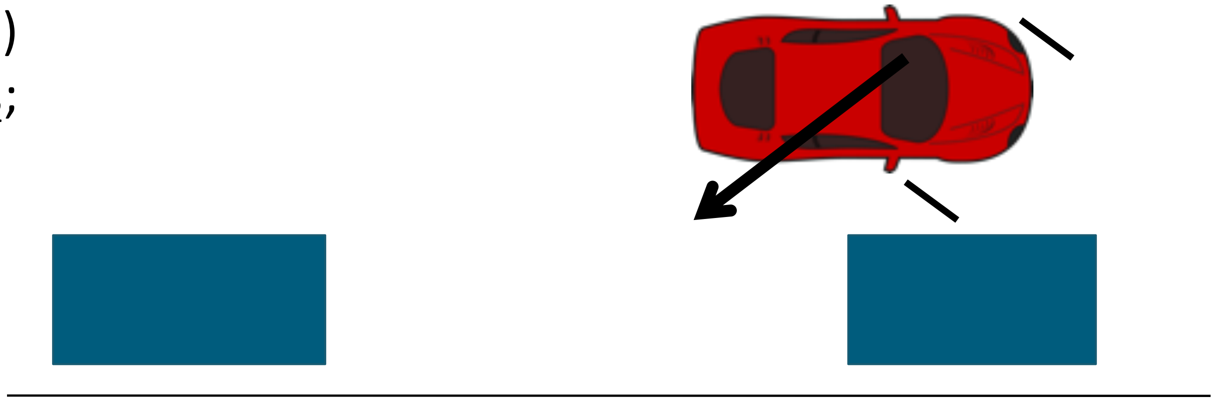

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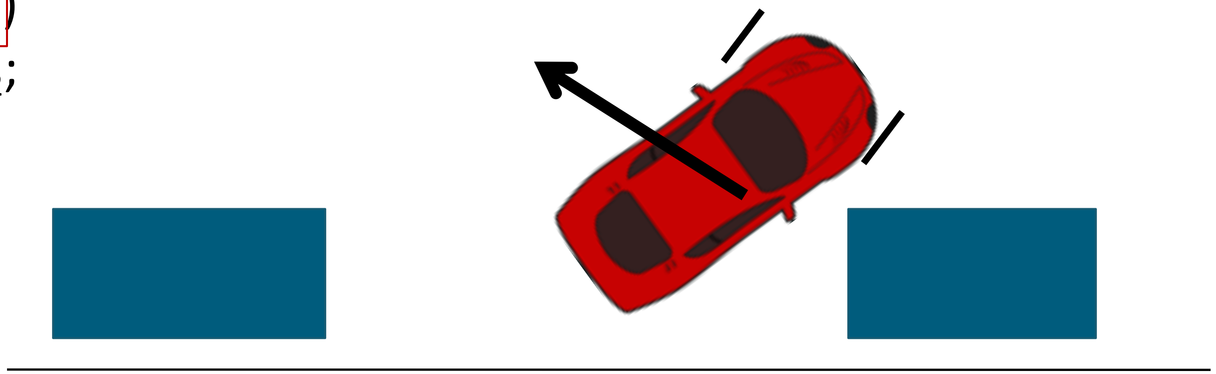
  moveCar();
}
```

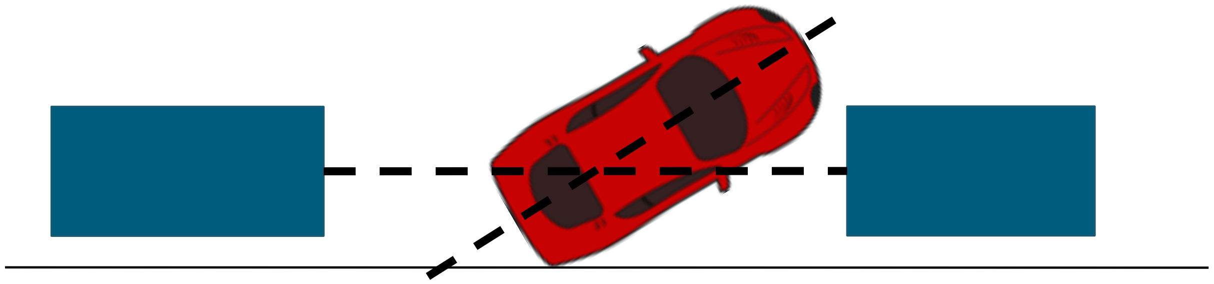


Application : Parameter synthesis

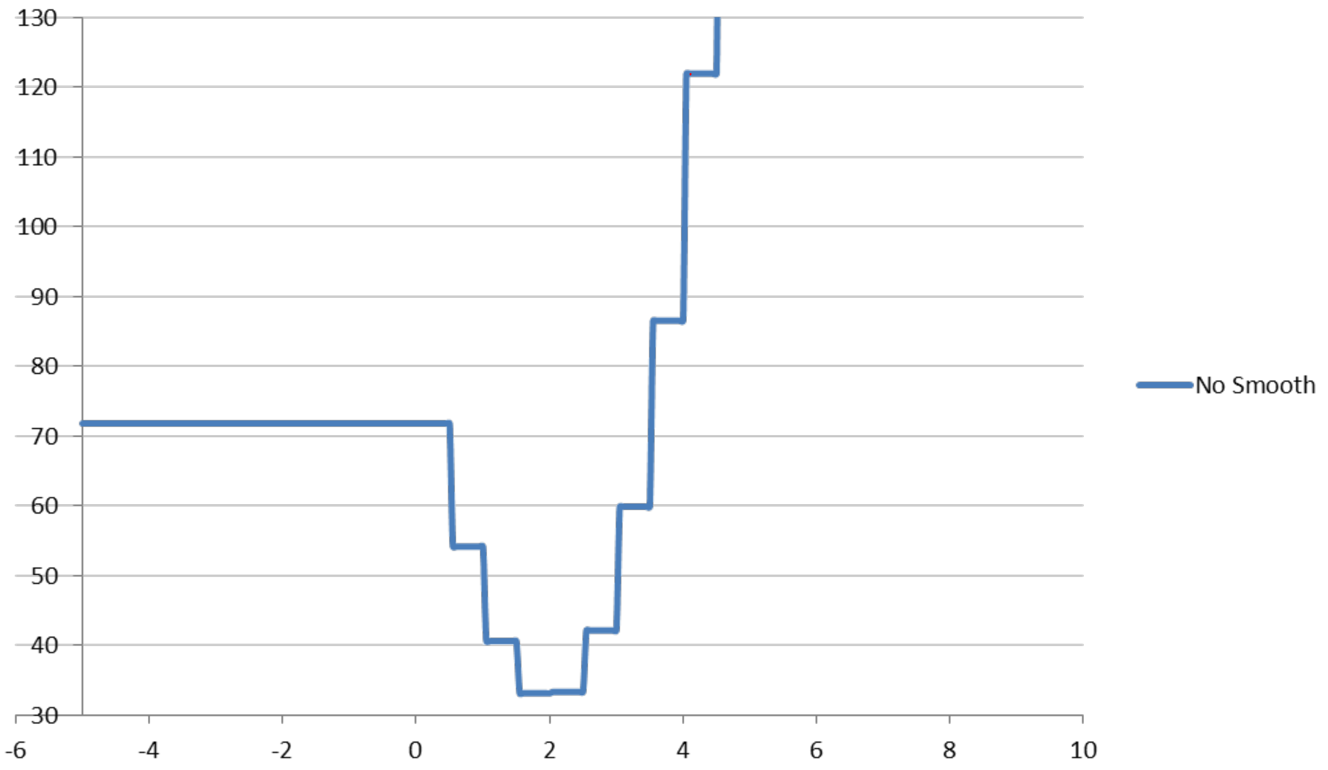
```
t1 := ??;    t2 := ??;  A1 := ??;
A2 := ??;
repeat  (every ΔT)   {
    if (stage = BACK && time > t1)
         stage = INTURN;
    if (stage = INTURN) {
        if (time > t2) then stage = OUTTURN;
        angle = angle – A1; }
    if (stage = OUTTURN)
        angle = angle + A2;
   moveCar();
}
```

Error = distance between ideal and actual final position

Goal: minimize Error.



Slice of Error function WRT T2



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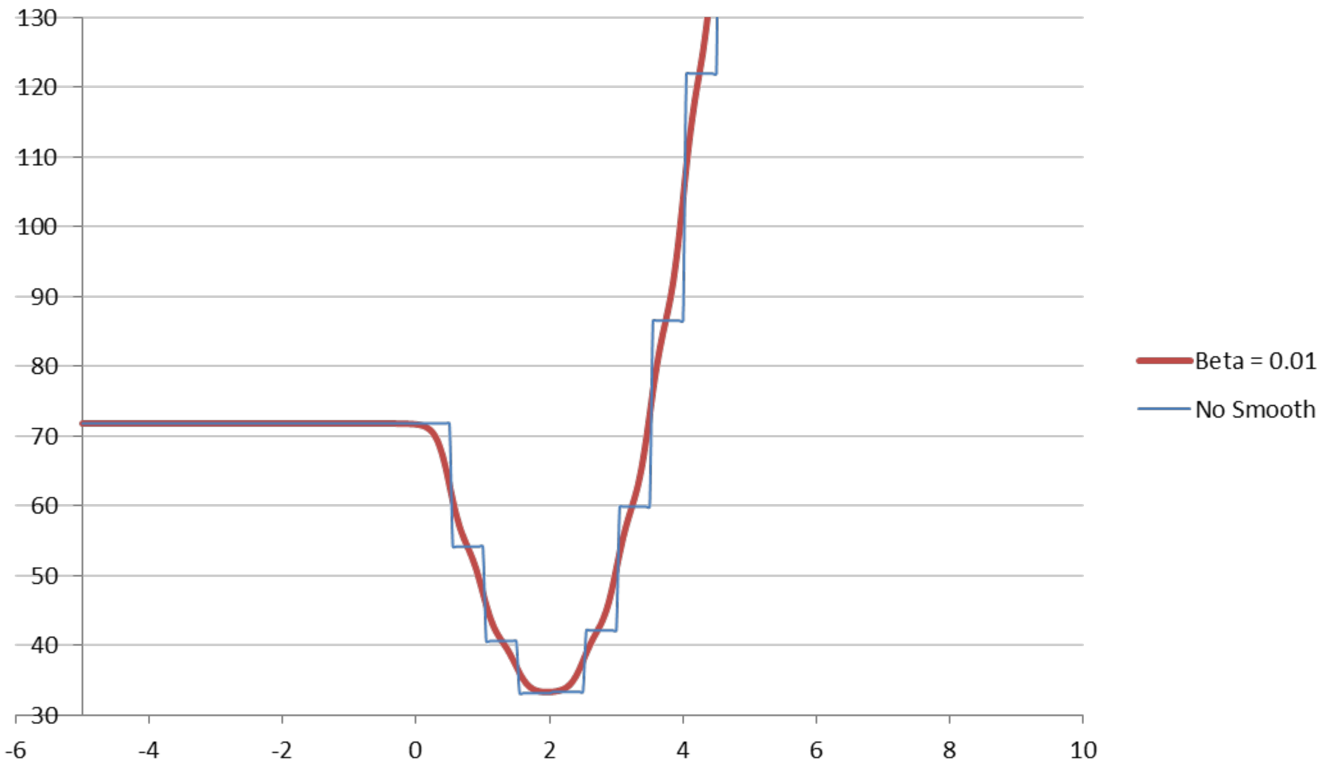
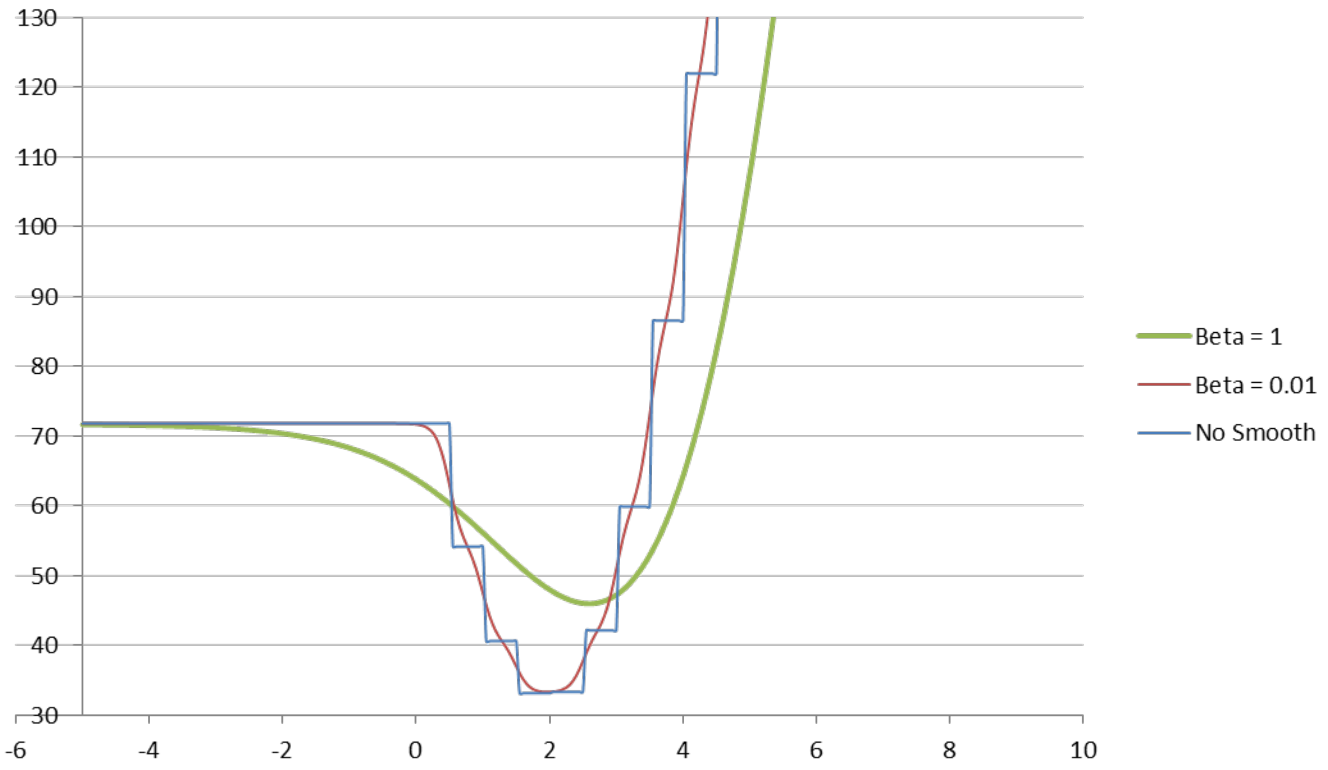


Slice of Error function WRT T2



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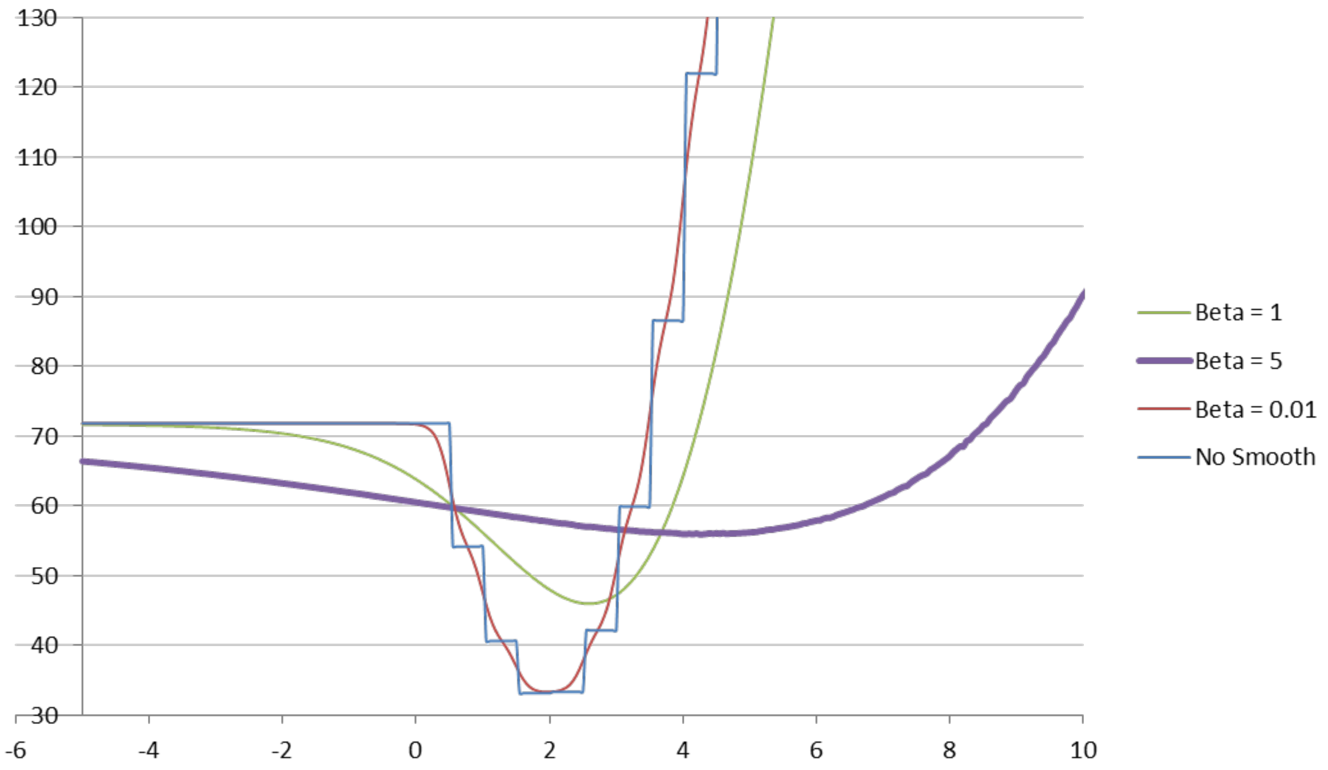
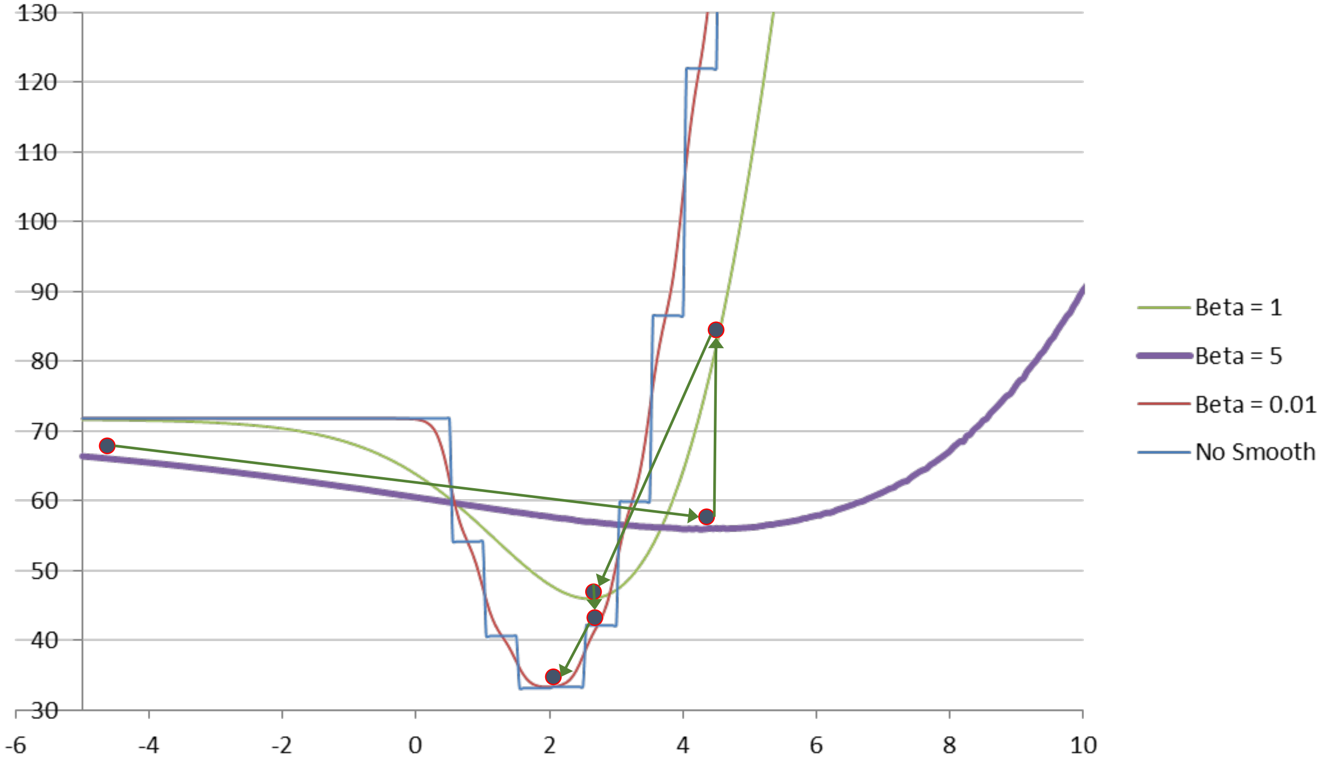


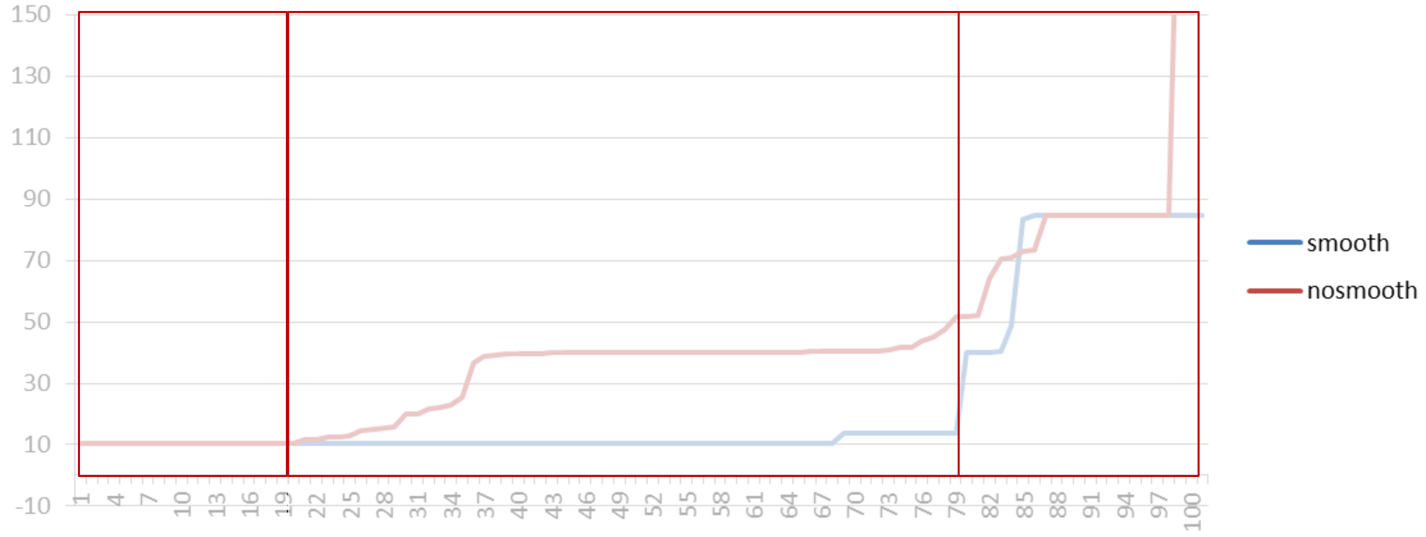
Numerical Search with Smoothing



Errors from different starting points

Both are quite good

Both are quite bad



Result of smoothing is much better