#9: Representation-based Search

Sankha Narayan Guria

EECS 700: Introduction to Program Synthesis

The problem statement

Behavioral constraints = examples

```
[1,4,7,2,0,6,9,2,5]  →  [1,2,4,7,0]
[0] → [0]
[5,1] → [1,5,0]
```

Structural constraints = grammar

```
L ::= sort(L)  |  L[N..N]
   |  L + L  |  [N]  |  x
N ::= find(L,N)  |  0
```

Search strategy?

Enumerative
Representation-based
Stochastic
Constraint-based

Representation-based search

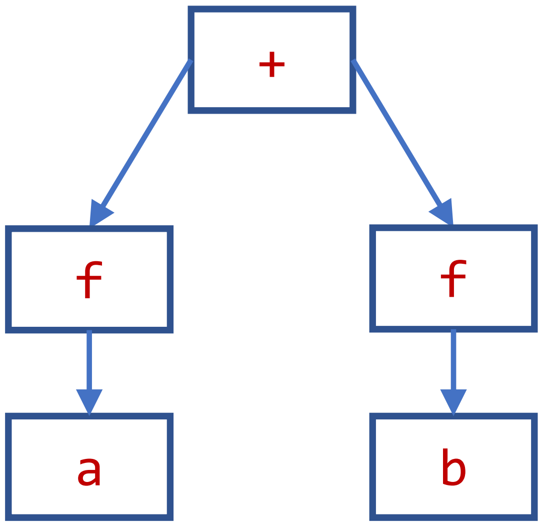
- **Idea:**
  1. build a data structure that compactly represents good parts of the program space
  2. extract solution from that data structure

Compact term representation

- Consider the space of 9 programs:

```
f(a) + f(a)   f(a) + f(b)   f(a) + f(c)
f(b) + f(a)   f(b) + f(b)   f(b) + f(c)
f(c) + f(a)   f(c) + f(b)   f(c) + f(c)
```

- Can we represent this compactly?
  - observation 1: same top level structure, independent subterms



Compact term representation

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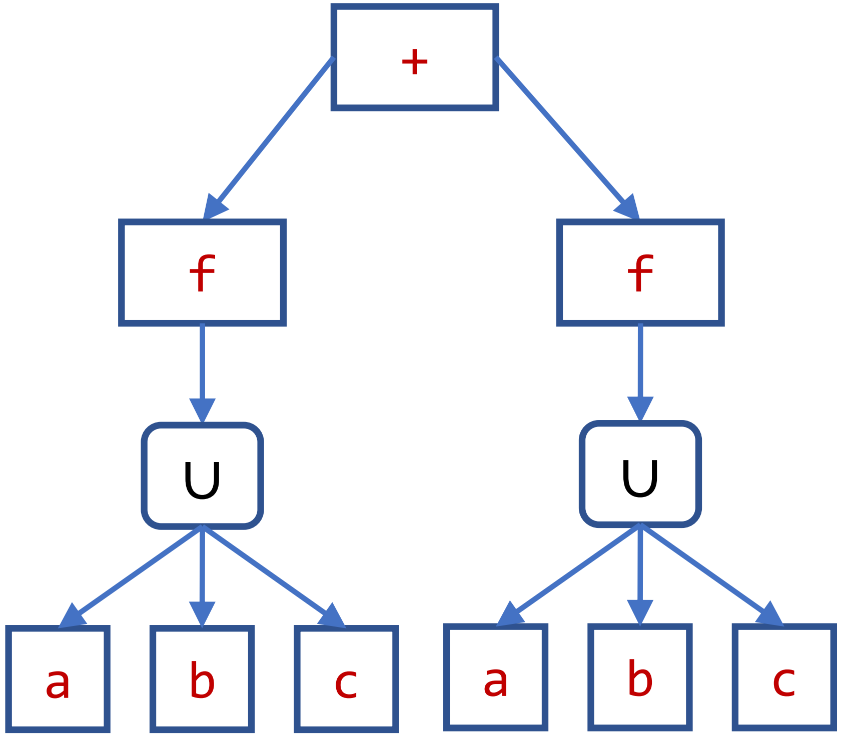
```

- Can we represent this compactly?
  - observation 1: same top level structure, independent subterms
  - observation 2: shared sub-spaces



Compact term representation

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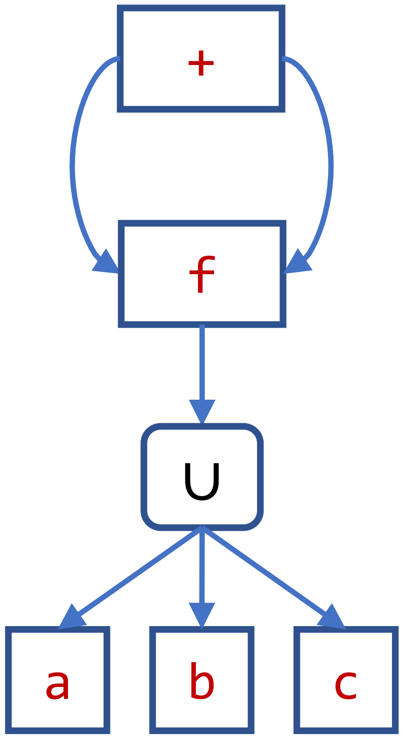
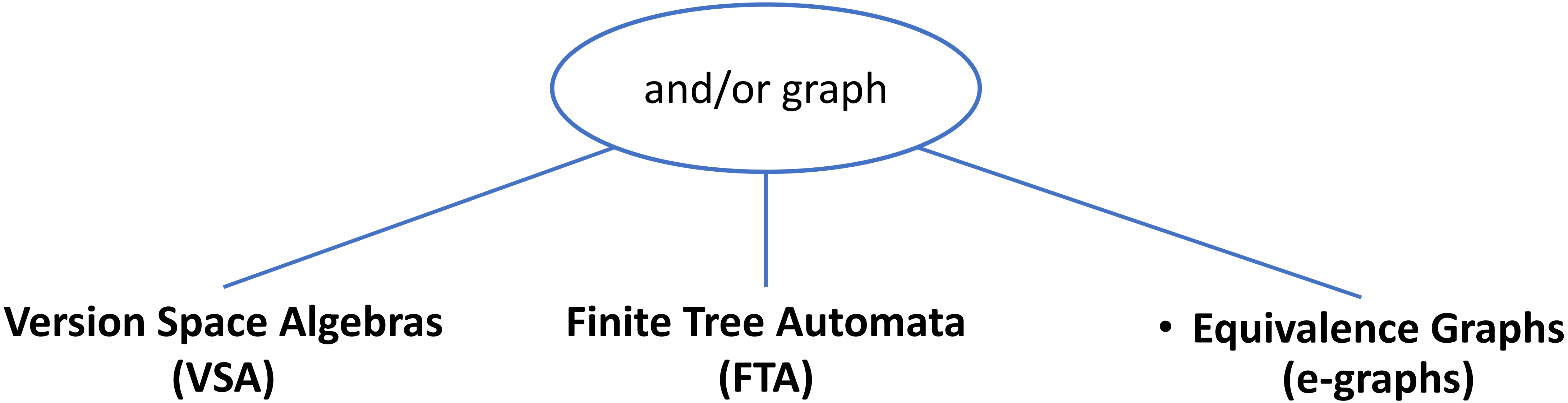
f(c) + f(a)   f(c) + f(b)   f(c) + f(c)
```

- Can we represent this compactly?
  - observation 1: same top level structure, independent subterms
  - observation 2: shared sub-spaces
- Key idea: use and-or graph!



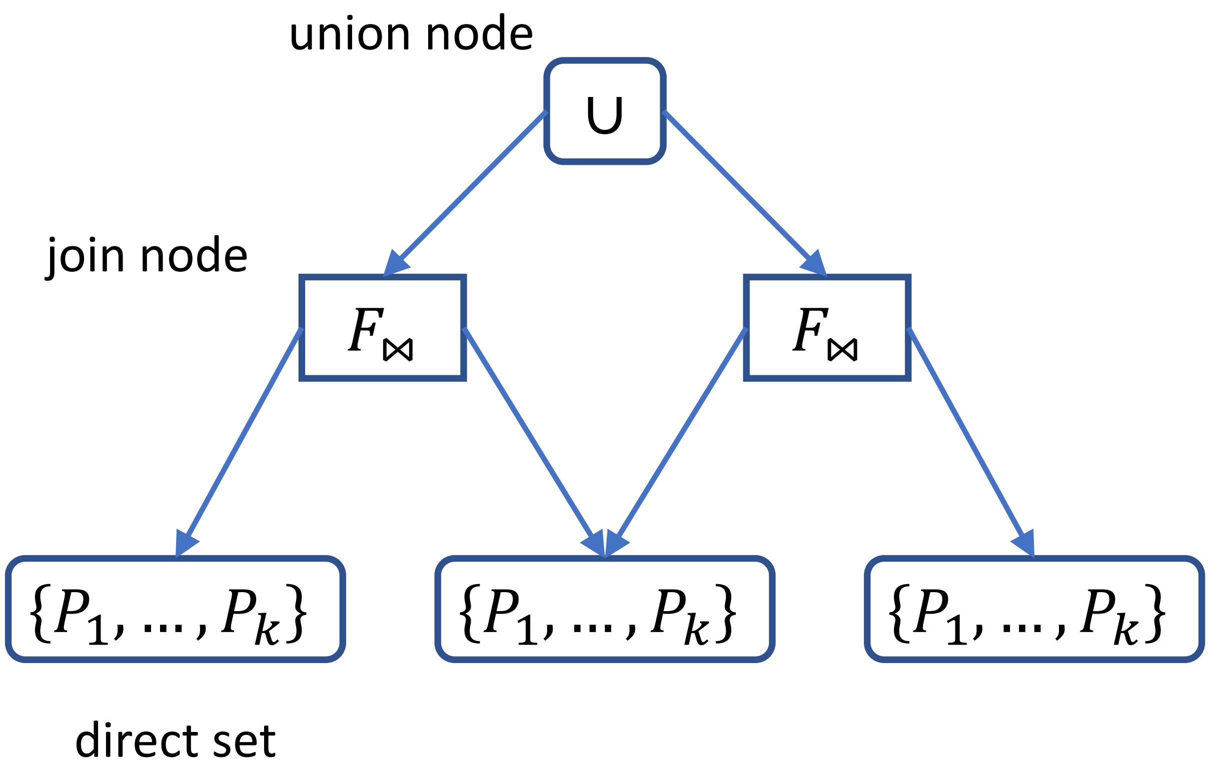
Representation-based search



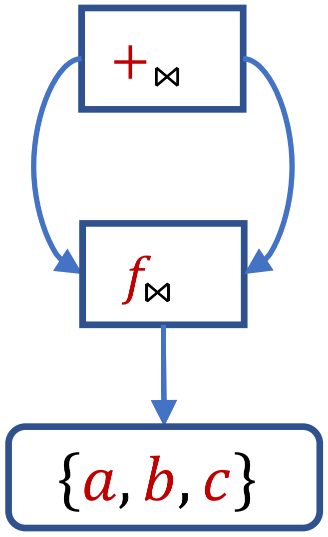
Version Space Algebra

- **Idea:** build a graph that succinctly represents the space of *all* programs consistent with examples
  - called a **version space**
- Operations on version spaces:
  - `learn <i, o> → VS`
  - $\mathrm{VS}_1 \cap \mathrm{VS}_2 \rightarrow \mathrm{VS}$
  - `extract VS → program`
- `Algorithm:`
  1. `learn a VS for each example`
  2. `intersect them all`
  3. `extract any (or best) program`

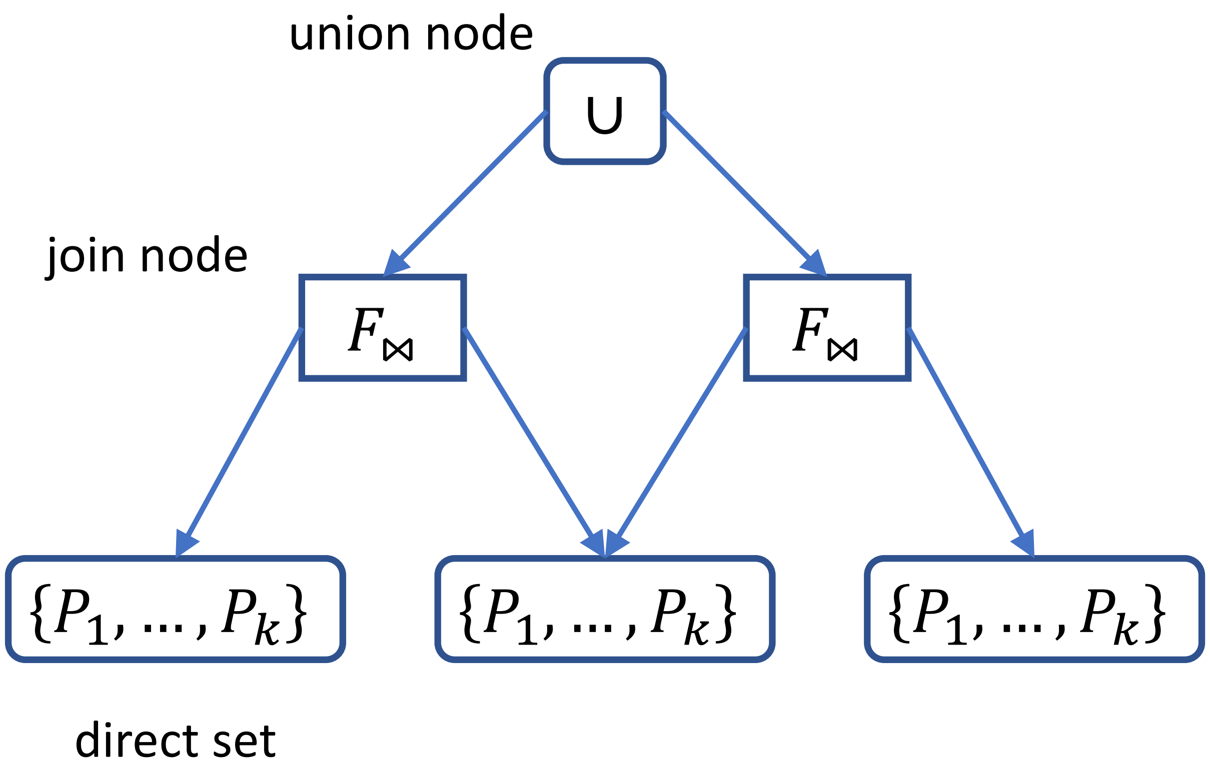
Version Space Algebra



example:



Version Space Algebra



Volume of a VSA
(the number of nodes) $V(VSA)$

Size of a VSA
(the number of programs) $|VSA|$

$$V(VSA) = O(\log|VSA|)$$

VSA-based search

- Mitchell: *Generalization as search*. AI 1982
- Lau, Domingos, Weld. *Version space algebra and its application to programming by example*. ICML 2000
- Gulwani: *Automating string processing in spreadsheets using input-output examples.* POPL 2011.
  - Follow-up work: BlinkFill, FlashExtract, FlashRelate, …
  - generalized in the PROSE framework

FlashFill

[Gulwani '11]

Simplified grammar:

E	::= F | concat(F, E)	"Trace" expression
F	::= cstr(*str*) | sub(P, P)	Atomic expression
P	::= cpos(*num*) | pos(R, R)	Position expression
R	::= tokens(T_1, ..., T_n)	Regular expression
T	::= C | C+	Token expression
C	::= ws | digit | alpha | Alpha | $ | ^ | …	

FlashFill: example

```
0 1 2 3 4 5 6 7 8 9 …
“Hello POPL 2023” → “POPL’2023”
“Goodbye PLDI 2021” → “PLDI’2021”
```

```
E ::= F | concat(F, E)
F ::= cstr(str) | sub(P, P)
P ::= cpos(num) | pos(R, R)
R ::= tokens(T1, ..., Tn)
T ::= C | C+
```

```
concat(
  sub(pos(ws, Alpha), pos(Alpha, ws)),
  concat(
    cstr(“’”),
    sub(pos(ws, digit), pos(digit, $))))
```

VSAs for Flashfill

- Recall operations on version spaces:
  - `learn <i, o> → VS`
  - $VS_1 \cap VS_2 \rightarrow VS$
  - `extract VS → program`
- How do we implement `learn`?
  - define $\texttt{learn}_N$ `<i, o>` for every non-terminal `N`
  - build VS top-down, propagating `<i, o>` the example

```
E ::= F | concat(F, E)
F ::= cstr(str) | sub(P, P)
P ::= cpos(num) | pos(R, R)
R ::= tokens(T1, ..., Tn)
T ::= C | C+
```

Learning atomic expressions

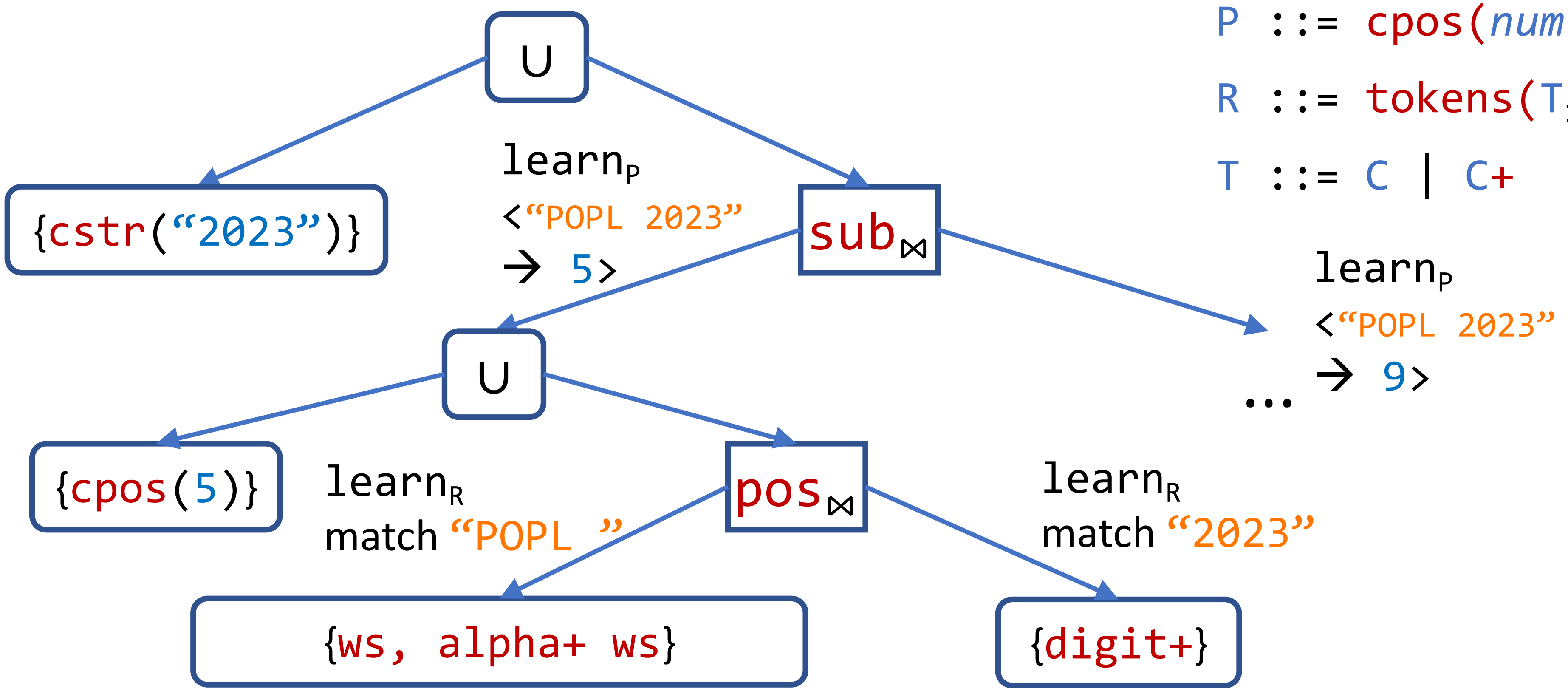
- learn$_F$ <“POPL 2023” → “2023”>

F ::= cstr(*str*) | sub(P$_1$, P$_2$)

P ::= cpos(*num*) | pos(R$_1$, R$_2$)

R ::= tokens(T$_1$, …, T$_n$)

T ::= C | C+

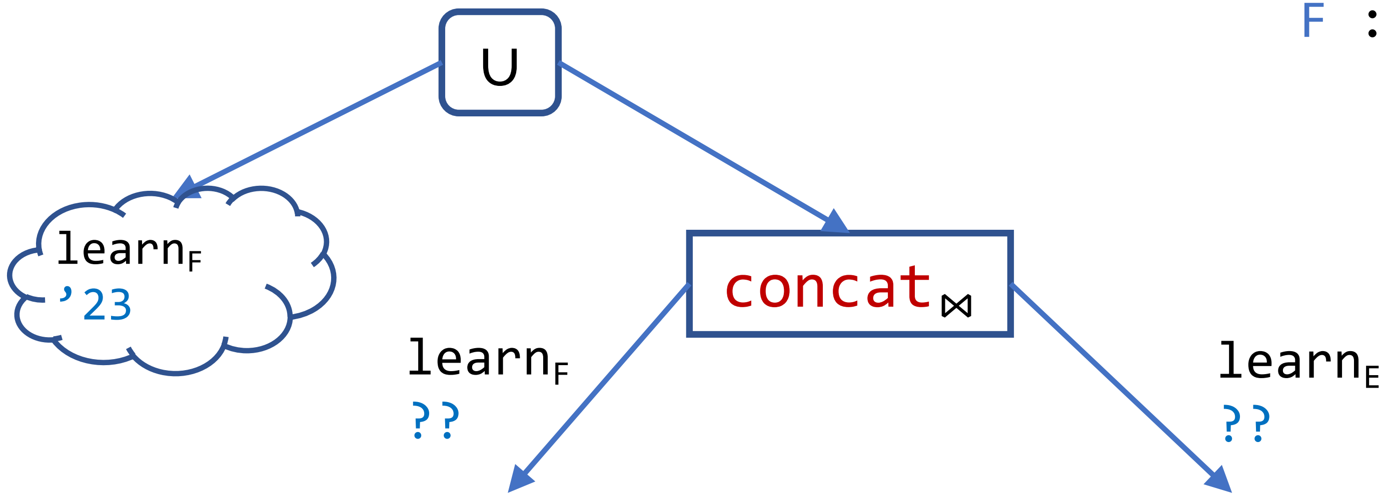


Learning trace expressions

- learn$_E$ <“POPL 2023” → “’23”>

E ::= F | concat(F, E)

F ::= ...



Learning trace expressions

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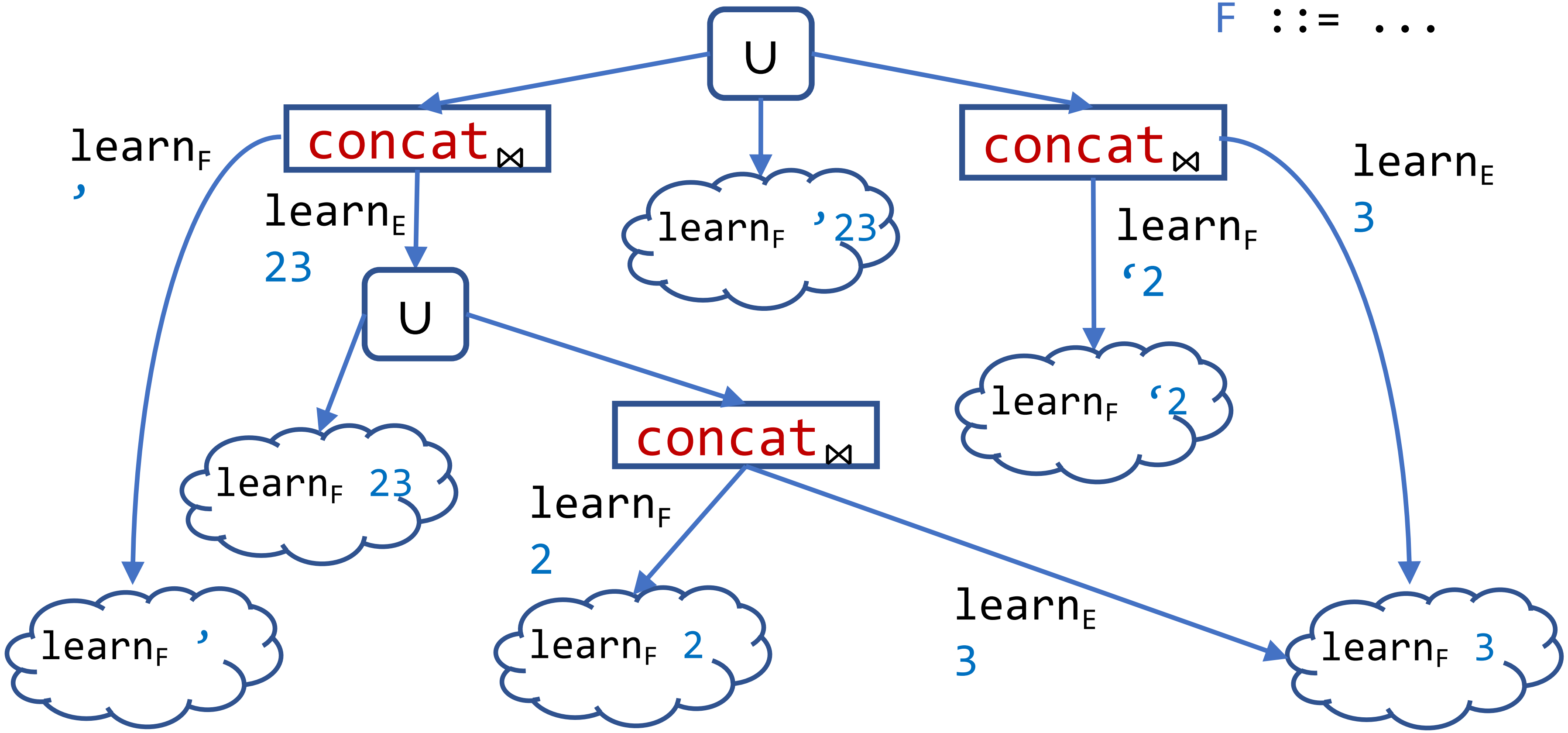
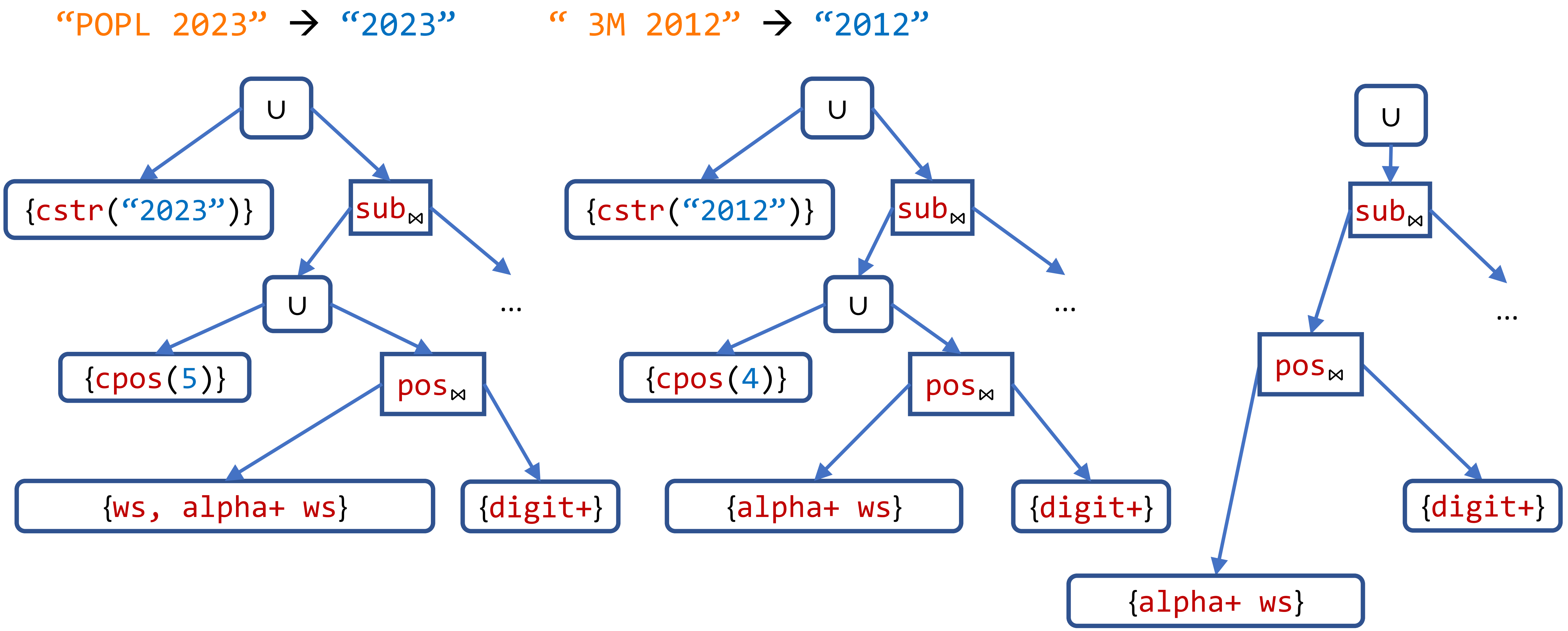


VSAs for Flashfill

- Recall operations on version spaces:
  - `learn <i, o>` → VS
  - $VS_1 \cap VS_2$ → VS
  - `extract VS` → `program`
- How do we implement intersection?
  - top-down
  - union: intersect all pairs of children
  - join: intersect children pairwise

```
E ::= F | concat(F, E)
F ::= cstr(str) | sub(P, P)
P ::= cpos(num) | pos(R, R)
R ::= tokens(T1, ..., Tn)
T ::= C | C+
```

Intersection



VSAs for Flashfill

- Recall operations on version spaces:
  - `learn <i, o> → VS`
  - $VS_1 \cap VS_2 \rightarrow VS$
  - `extract VS → program`
- How do we implement extract?
  - any program: just pick one child from every union
  - best program: shortest path in a DAG

```
E ::= F | concat(F, E)
F ::= cstr(str) | sub(P, P)
P ::= cpos(num) | pos(R, R)
R ::= tokens(T₁, ..., Tₙ)
T ::= C | C+
```

Discussion

- What do VSAs remind you of in the enumerative world?
  - VSA learning ~ top-down search with top-down propagation
- How are they different?
  - Caching of sub-problems (DAG!)
  - Can construct one per example and intersect
  - This allows it to guess arbitrary constants!

Discussion

- Why could we build a finite representation of all solutions?
  - Could we do it for this language?

$$\texttt{E} ::= \texttt{F} + \texttt{F}$$
$$\texttt{F} ::= k \mid \texttt{x}$$

$k \in \mathbb{Z}$ + is integer addition

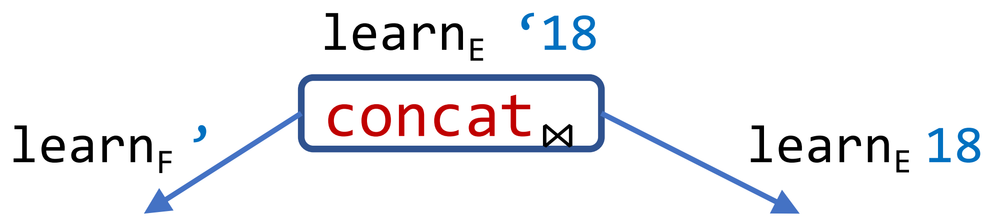
  - What about this language?

$$\texttt{E} ::= \texttt{E} + \texttt{1} \mid \texttt{x}$$

DSL restrictions: efficiently invertible

- Every operator has a small, easily computable inverse
  - Example when an inverse is small but hard to compute?
- The space of sub-specs is finite
  - either non-recursive grammar
  - or finite space of values for the recursive non-terminal (e.g. bit-vectors)
  - or every recursive production generates a strictly smaller spec

```
E ::= F | concat(F, E)
```



PROSE

[Polozov, Gulwani ‘15]

Ranking function

```
r :: Program -> Int
```

Grammar

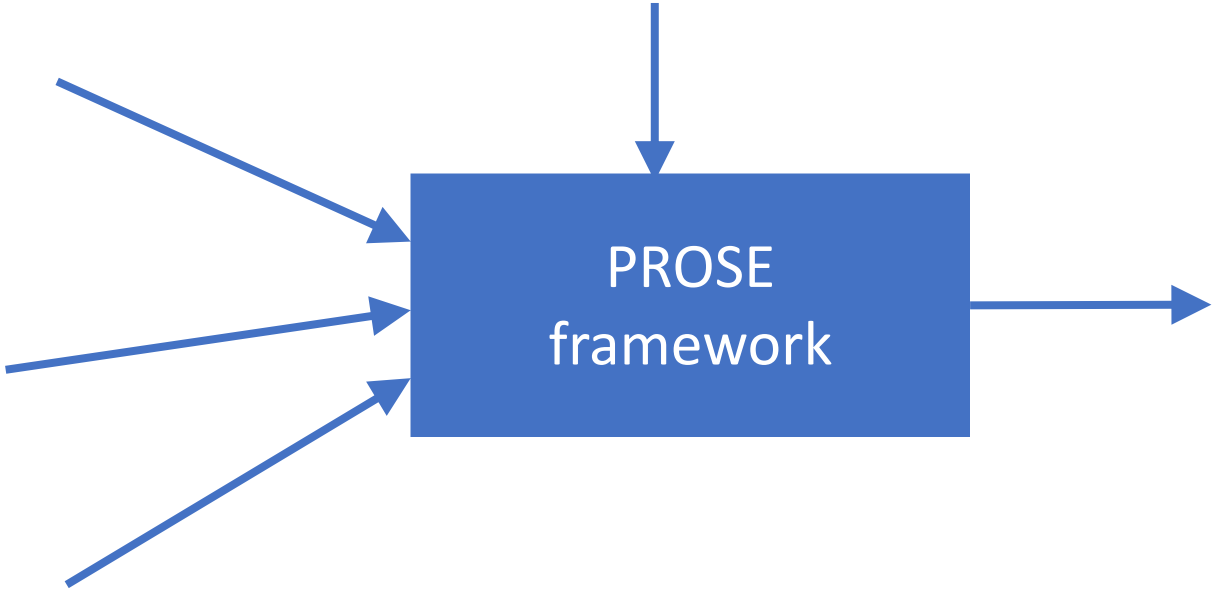
```
A ::= f(B, C) | ...
B ::= g(D, E) | ...
```

Semantics

```
f :: (B, C) -> A
f (b, c) = ...
```

Inverse semantics

```
f⁻¹ :: A -> {(B, C)}
f⁻¹ a = ...
```



VSA learning algorithm

https://microsoft.github.io/prose/

Representation-based search

and/or graph

Version Space Algebras (VSA)

Finite Tree Automata (FTA)

- **Equivalence Graphs (e-grpahs)**

ops: learn-1, intersect, extract
DSL: efficiently invertible
similar to: top-down prop, but can infer constants

Finite Tree Automata

states

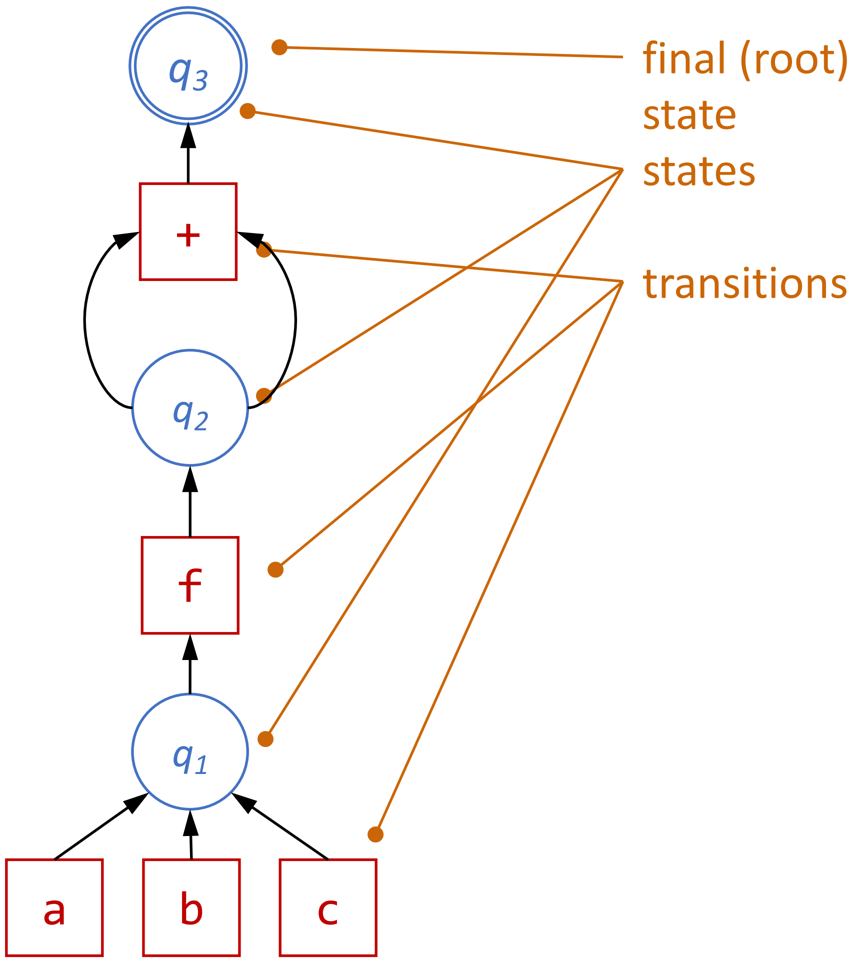
final state(s)

$$\mathcal{A} = \langle Q, F, Q_f, \Delta \rangle$$

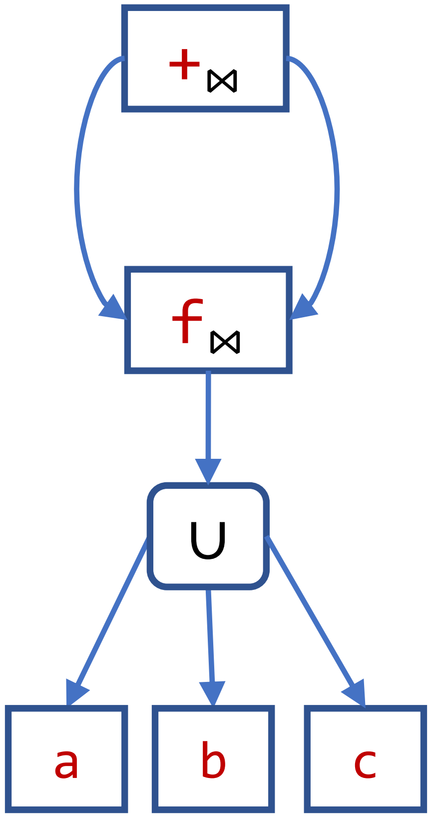
ranked alphabet

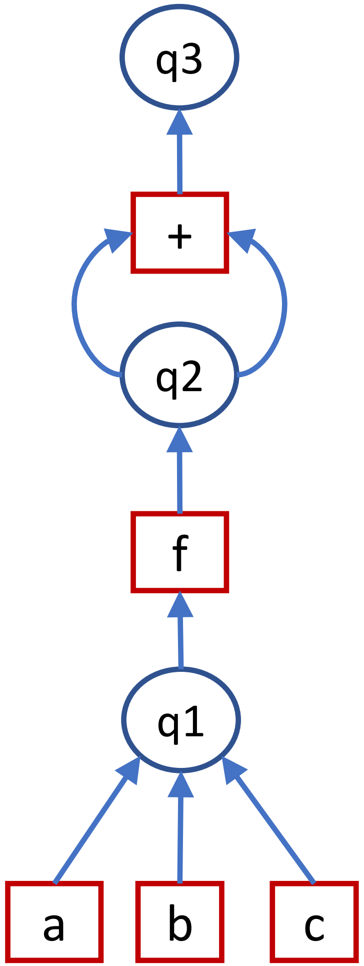
transitions

$$f(q_1, \ldots, q_n) \rightarrow q$$



VSA vs FTA





- Both are and-or graphs
- FTA state = VSA union node
  - in VSAs singleton joins are omitted
- FTA transition = VSA join node

FTA-based search

- **Synthesis of Data Completion Scripts using Finite Tree Automata**
  Xinyu Wang, Isil Dillig, Rishabh Singh, *OOPSLA’17*
- **Program Synthesis using Abstraction Refinement**
  Xinyu Wang, Isil Dillig, Rishabh Singh, *POPL’18*

- **Searching Entangled Program Spaces**
  James Koppel, Zheng Guo, Edsko de Vries, Armando Solar-Lezama, Nadia Polikarpova. *ICFP'22*

FTA-based search

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Example

Grammar

```
N ::= id(V) | N + T | N * T
T ::= 2 | 3
V ::= x
```

Spec

$1 \rightarrow 9$

PBE with Finite Tree Automata

$\langle A, \mathbb{Z} \rangle$

$A \in \{\mathsf{N}, \mathsf{T}, \mathsf{X}\}$

$\{\langle \mathsf{N}, 9 \rangle\}$

states

final states

$$\mathcal{A} = \langle Q, F, Q_f, \Delta \rangle$$

alphabet

transitions

`id, +, *`

$f(q_1, \ldots, q_n) \rightarrow q$

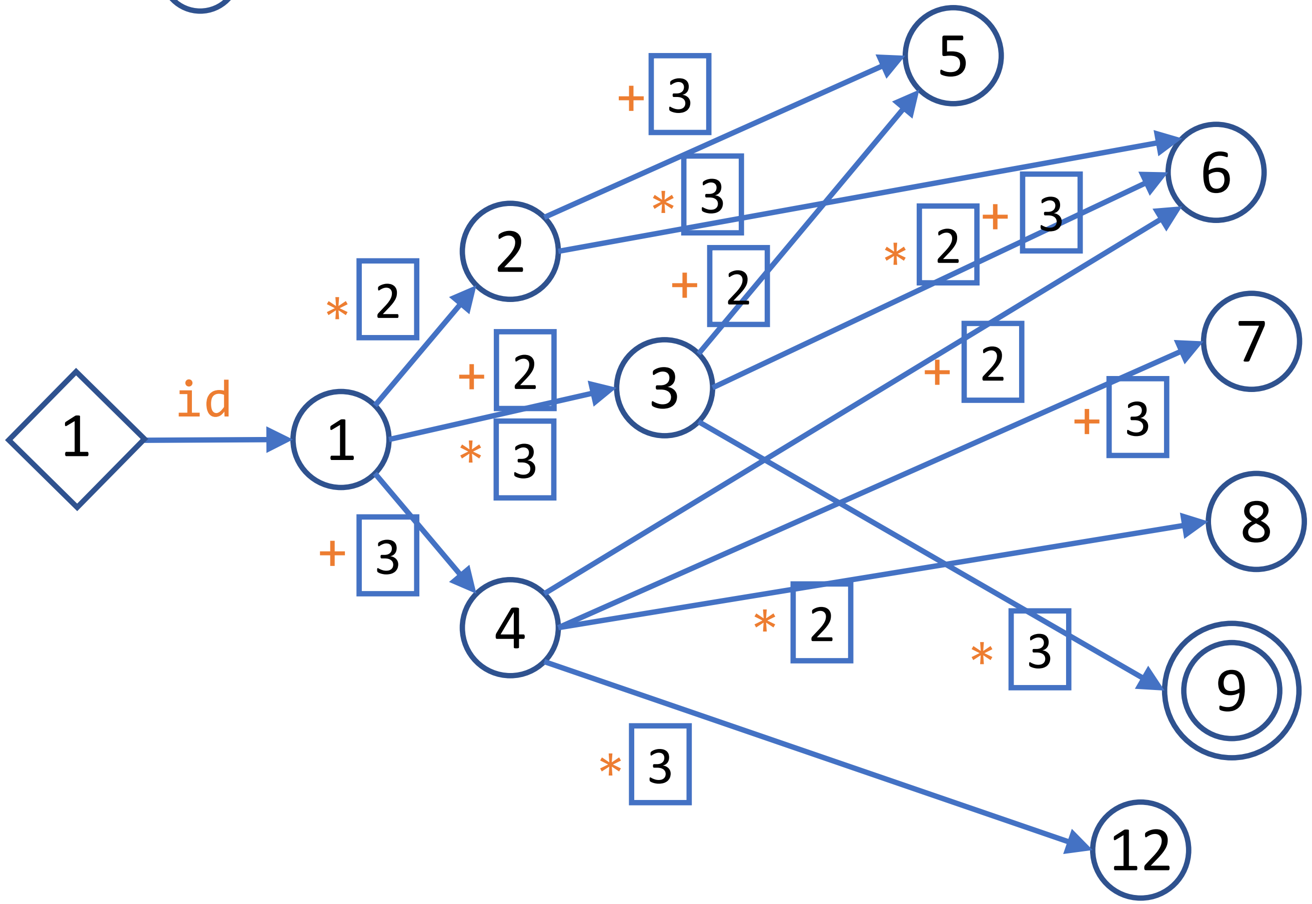
`+(<N,1>,<T,2>) → <N,3>`

`...`

PBE with Finite Tree Automata

```
N ::= id(V) | N + T | N * T
T ::= 2 | 3
V ::= x
```

1 → 9



Discussion

- What do FTAs remind you of in the enumerative world?
  - FTA ~ bottom-up search with OE
- How are they different?
  - More size-efficient: sub-terms in the bank are replicated, while in the FTA they are shared
  - Hence, can store all terms, not just one representative per class
  - Can construct one FTA per example and intersect
  - More incremental in the CEGIS context!

FTA-based search

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Abstract FTA

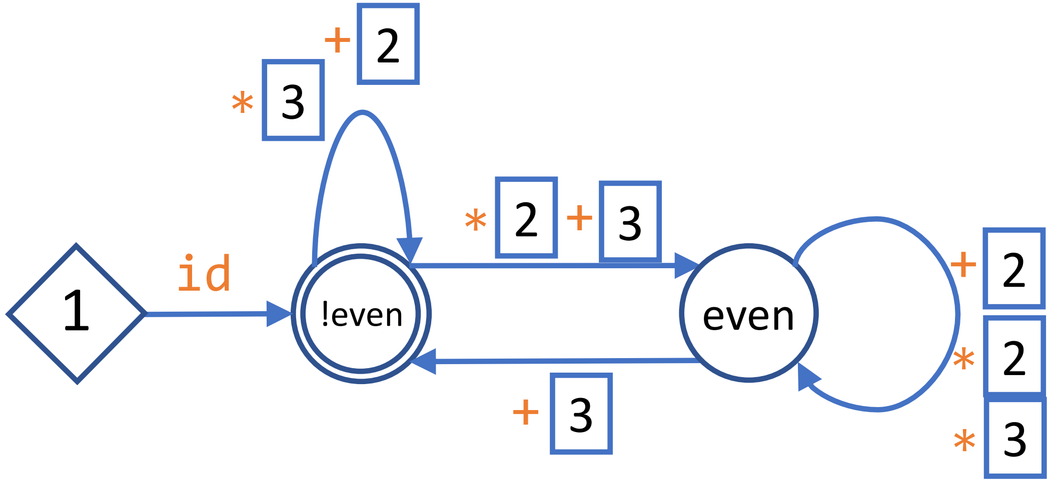
- **Challenge:** FTA still has too many states
- Idea:
  - instead of one state = one value
  - we can do one state = set of values (= abstract value)

Abstract FTA

[Wang, Dillig, Singh POPL'18]

```
N ::= id(V) | N + T | N * T
T ::= 2 | 3
V ::= x
```

1 → 9



- **What now?**
  - idea 1: enumerate from reduced space
  - idea 2: refine abstraction!

Abstract FTA

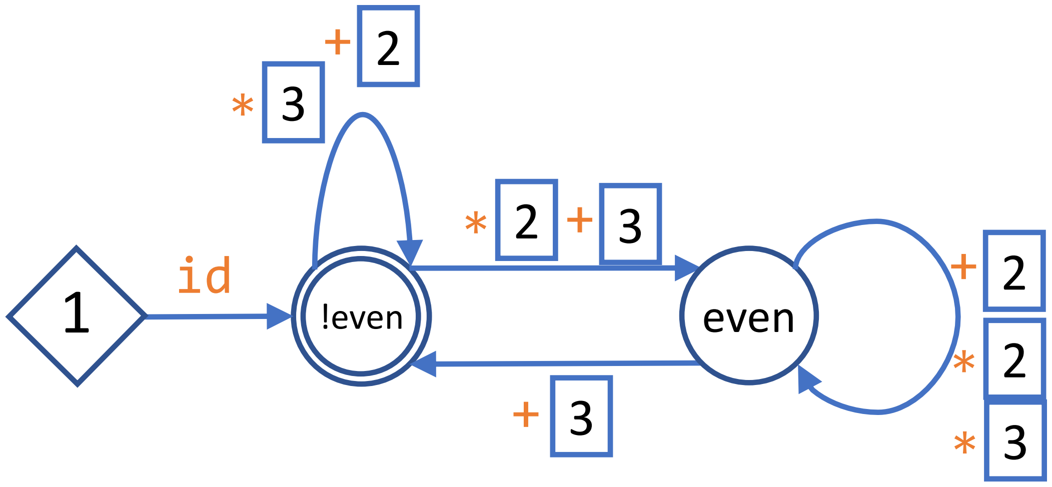
N ::= id(V) | N + T | N * T

T ::= 2 | 3

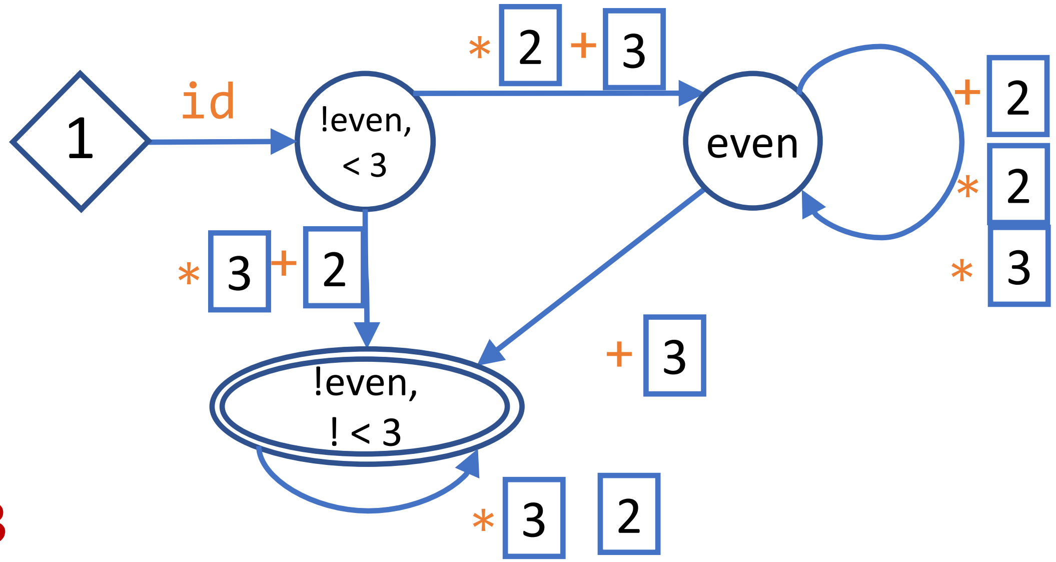
V ::= x

1 → 9

Predicates: {even, < 3, …}



solution: id(x)



solution: id(x)*3

Representation-based search

and/or graph

Version Space Algebras (VSA)

Finite Tree Automata (FTA)

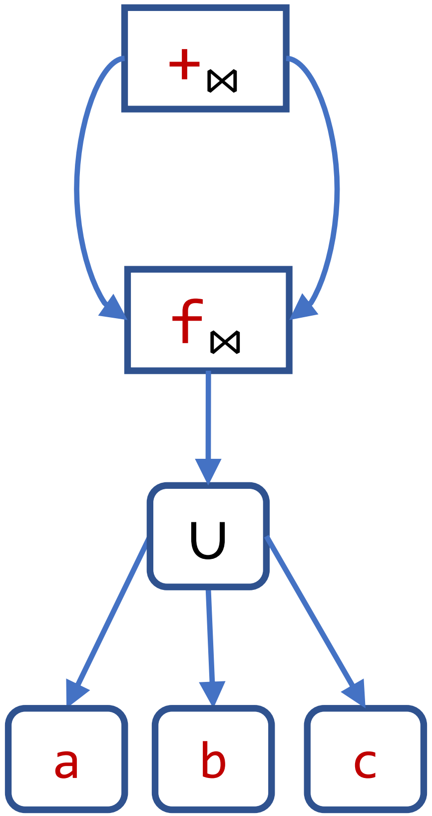
- **Equivalence Graphs (e-grpahs)**

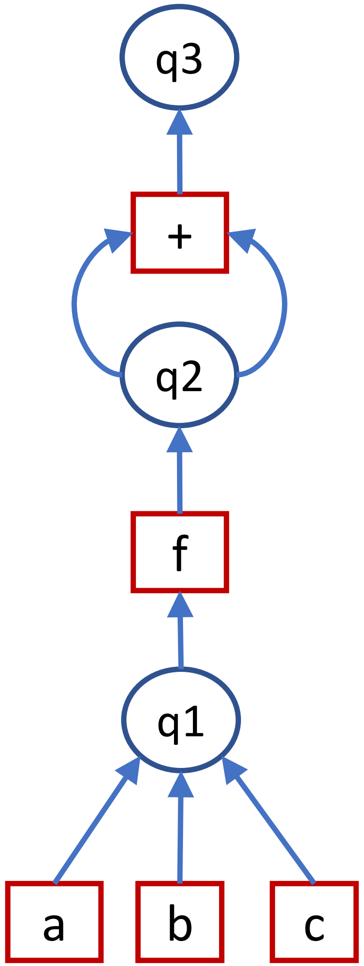
ops: learn-1, intersect, extract
DSL: efficiently invertible
similar to: top-down prop, but can infer constants

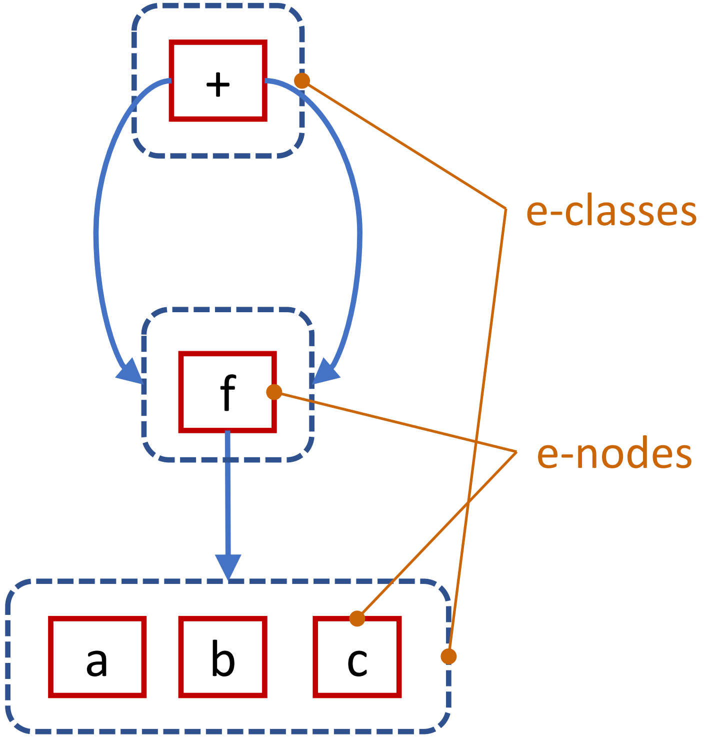
ops: learn-1, intersect, extract
DSL: efficiently enumerable
similar to: bottom-up with OE, but can store all programs (and add examples incrementally)

state: represents a set of observationally-equivalent programs

VSA vs FTA vs E-Graphs







Program search with e-grpahs

- **Equality saturation: a new approach to optimization**
  Ross Tate, Michael Stepp, Zachary Tatlock, Sorin Lerner, *POPL'09*
- **egg: Fast and Extensible Equality Saturation**
  Max Willsey, Chandrakana Nandi, Yisu Remy Wang, Oliver Flatt, Zachary Tatlock, Pavel Panchekha, *POPL'21*

- **Semantic Code Search via Equational Reasoning**
  Varot Premtoon, James Koppel, Armando Solar-Lezama. *PLDI'20*

Equality saturation

Program optimization via rewriting:

```
   (a * 2) / 2
⇒ a * (2 / 2)
⇒ a * 1
⇒ a
```

useful rules:

```
(x * y) / z = x * (y / z)
      x / x = 1
      x * 1 = x
```

not so useful:

```
x * 2 = x << 1
x * y = y * x
```

Challenge: which ones to apply and in what order?

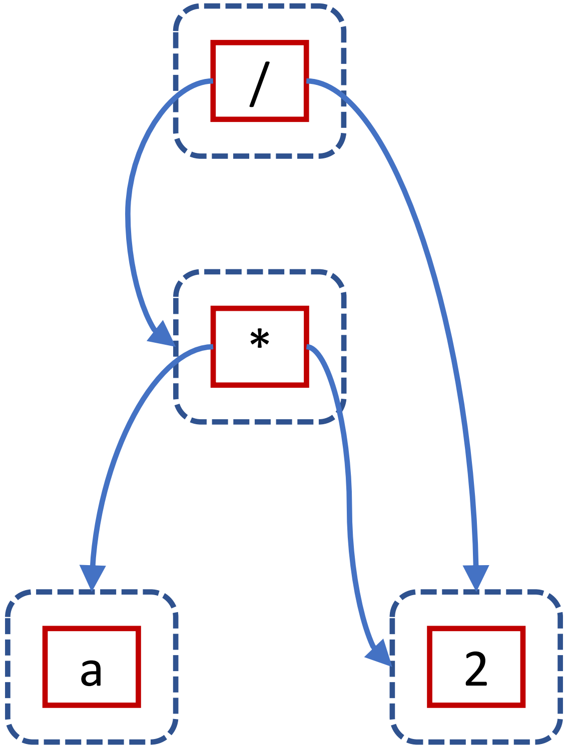
Idea: all of them all the time!

Equality saturation

Initial term: `(a * 2) / 2`

Rewrite rules:

```
(x * y) / z = x * (y / z)
          x / x = 1
          x * 1 = x
       x * 2 = x << 1
        x * y = y * x
```



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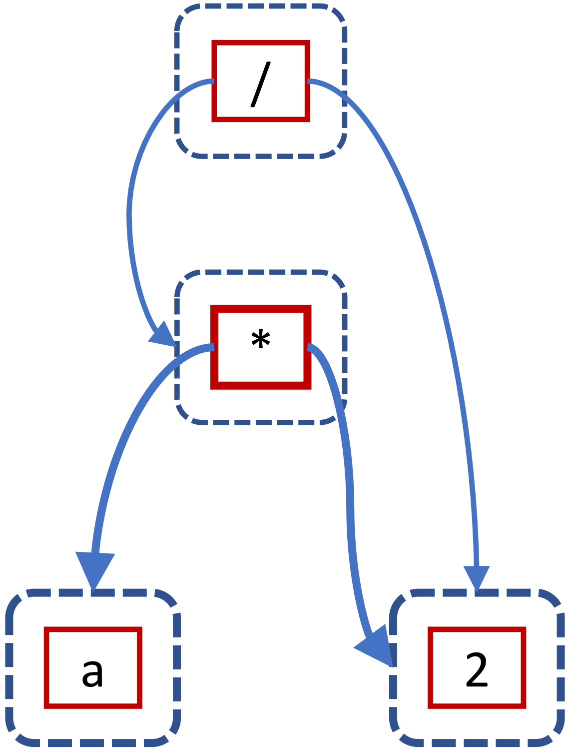

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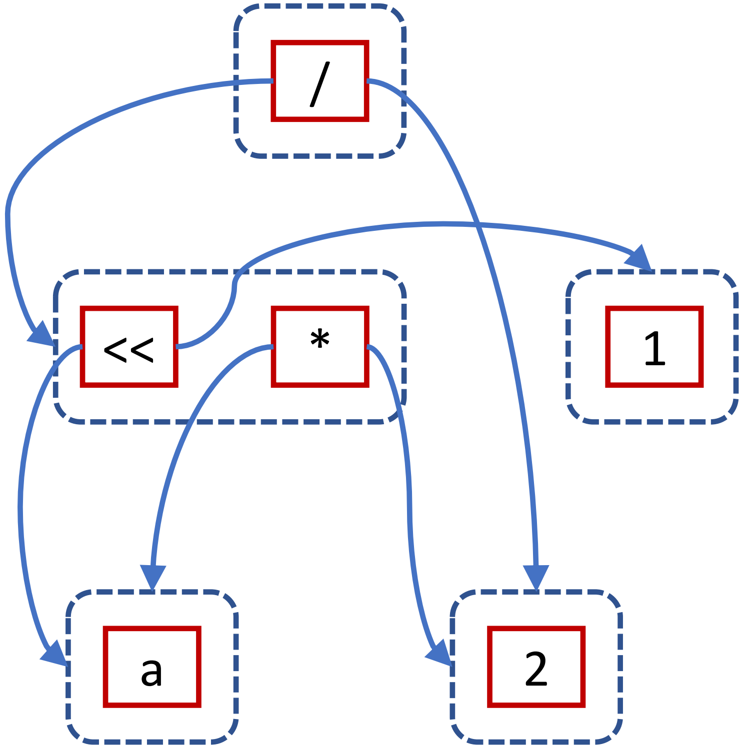

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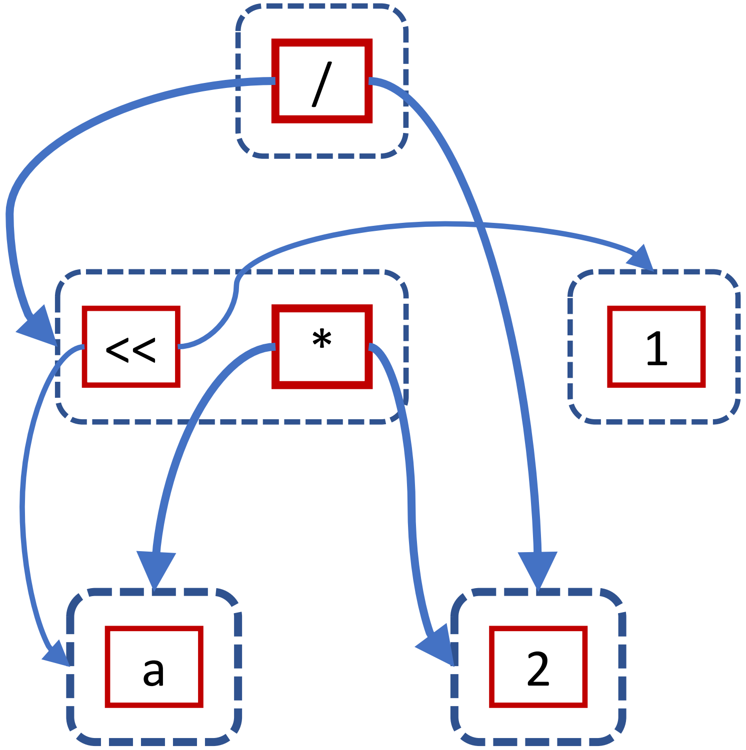

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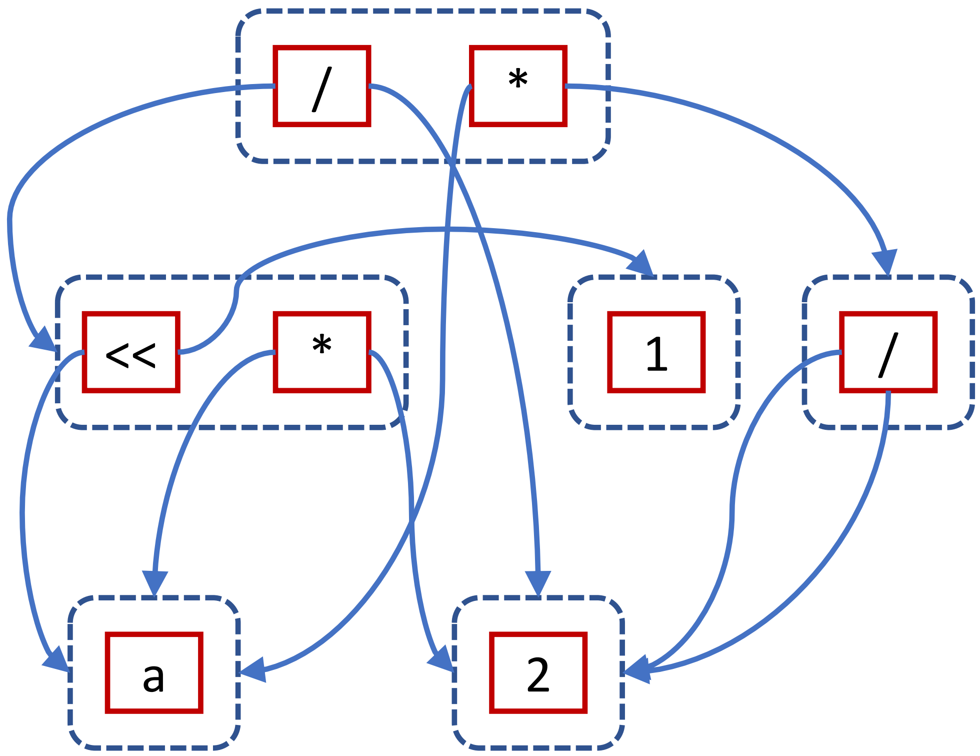

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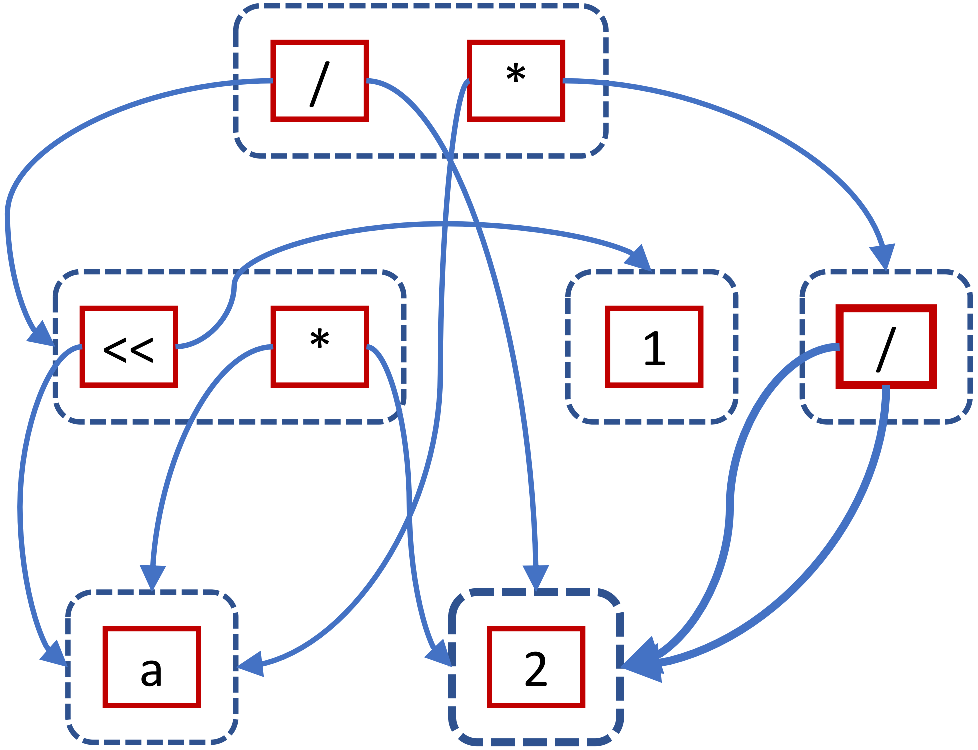

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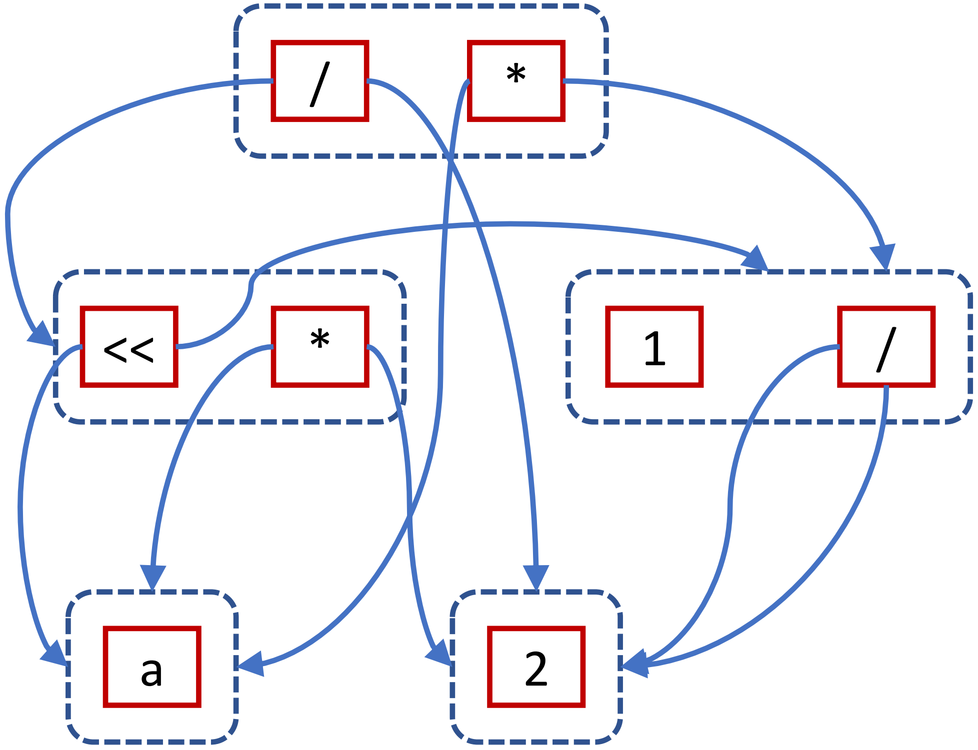

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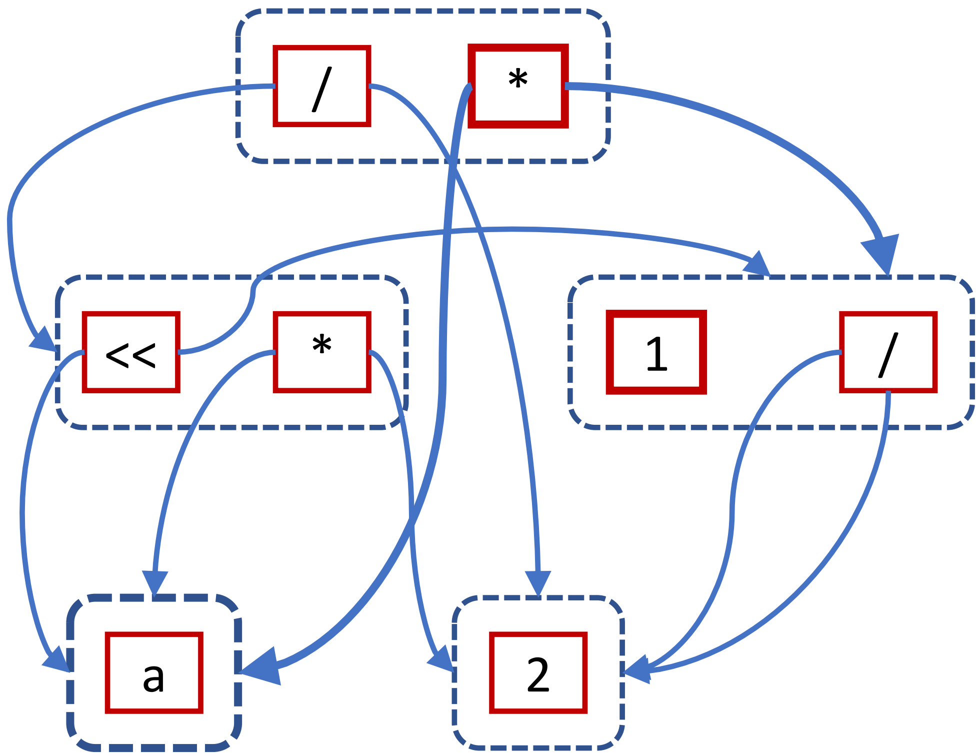

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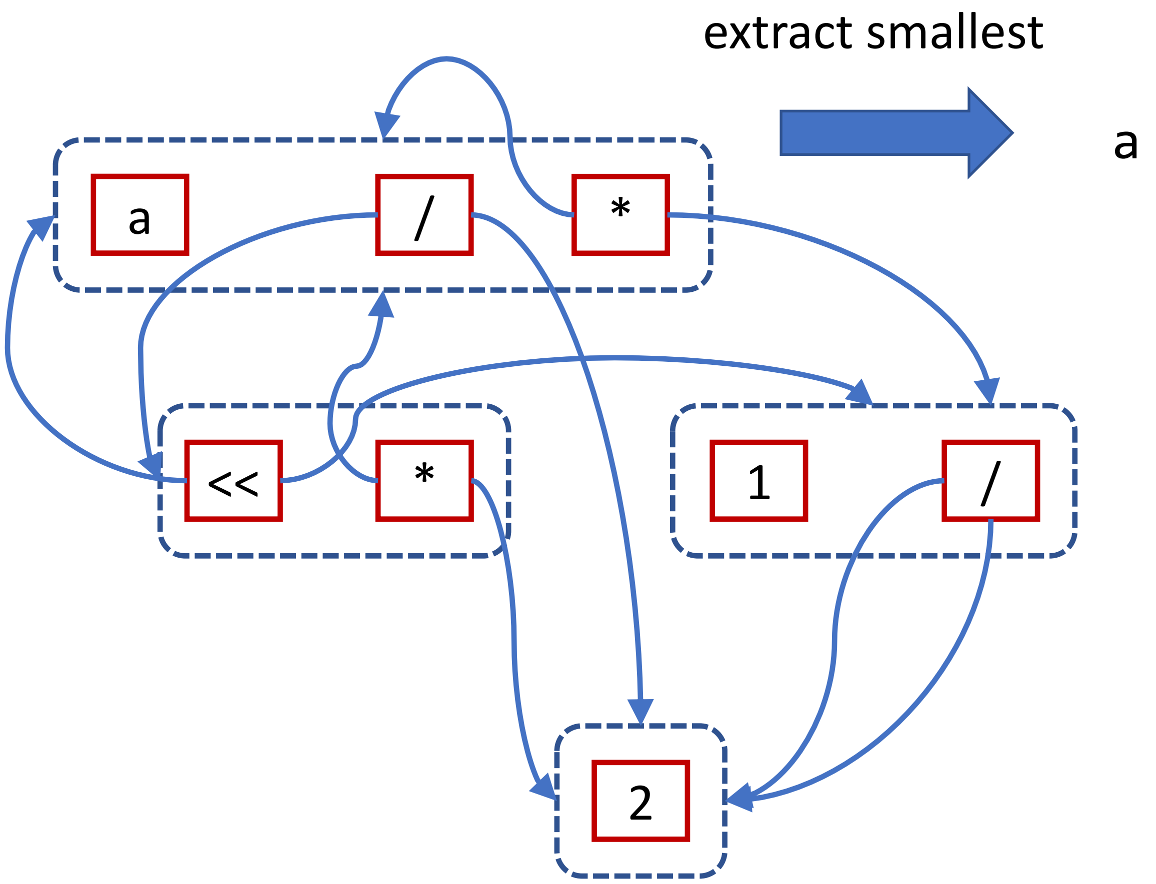
       x * y = y * x
```



Representation-based search

and/or graph

Version Space Algebras (VSA)

Finite Tree Automata (FTA)

- **Equivalence Graphs (e-grpahs)**

ops: learn-1, intersect, extract
DSL: efficiently invertible
similar to: top-down prop, but can infer constants

ops: learn-1, intersect, extract
DSL: efficiently enumerable
similar to: bottom-up with OE, but can store all programs (and add examples incrementally)

ops: rewrite, extract

similar to: term rewriting, but can store all programs

state: represents a set of observationally-equivalent programs

e-class: represents a set of programs equivalent up to rewrites

BlinkFill

- What does BlinkFill use as behavioral constraints? Structural constraints? Search strategy?
  - input-output examples (+ input examples); custom string DSL; VSA
- What is the main technical insight of BlinkFill wrt FlashFill?
  - BlinkFill uses the available inputs (with no outputs) to infer structure (segmentation) common to all inputs
  - it uses this structure to shrink the DAG and to rank substring expressions

Example

"Los Angeles, United States"

- learn_F <"Mumbai, India" → "India">