#2: Syntax-Guided Synthesis

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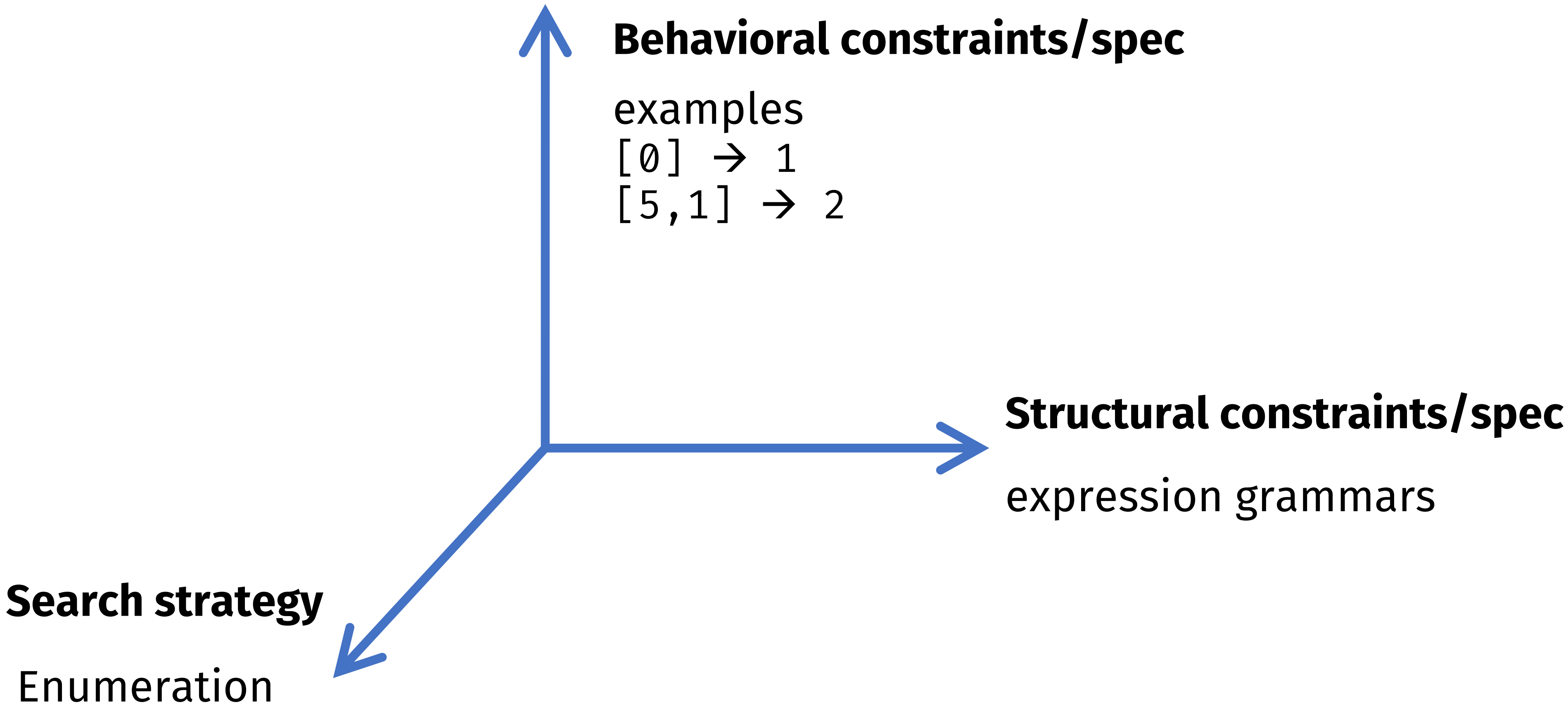
EECS 700: Introduction to Program Synthesis

Logistics

- **Discord**
  - Has everyone signed up?
  - `#seeking-teammates` channel
- **Office hours time changed**
  - Monday: 4pm – 5pm
- **Sep 1 Friday class on Zoom**
  - Will share link later
- **Other questions?**

This week


Behavioral constraints/spec
examples
[0] → 1
[5,1] → 2
Structural constraints/spec
expression grammars
Search strategy
Enumeration

Today

- **Synthesis from examples**
- **Syntax-guided synthesis**
    - expression grammars as structural constraints
    - the SyGuS project

Synthesis from examples

Synthesis from Examples

=

Programming by Example

=

Inductive Programming /
Inductive Learning

Inductive learning: History

MIT/LCS/TR-76

LEARNING STRUCTURAL DESCRIPTIONS FROM EXAMPLES

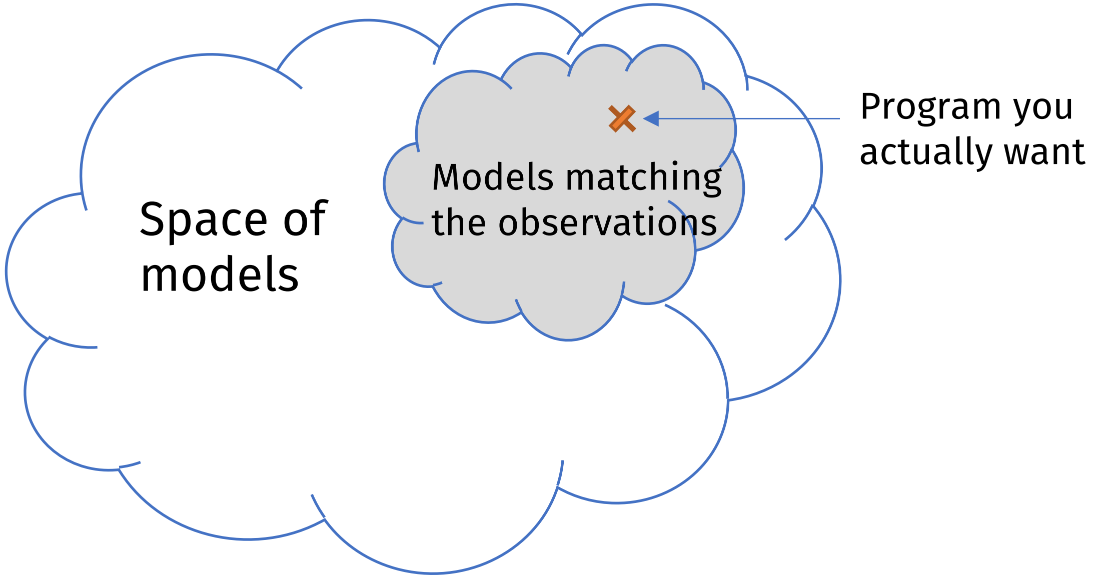
Patrick H. Winston

September 1970

Patrick
Winston

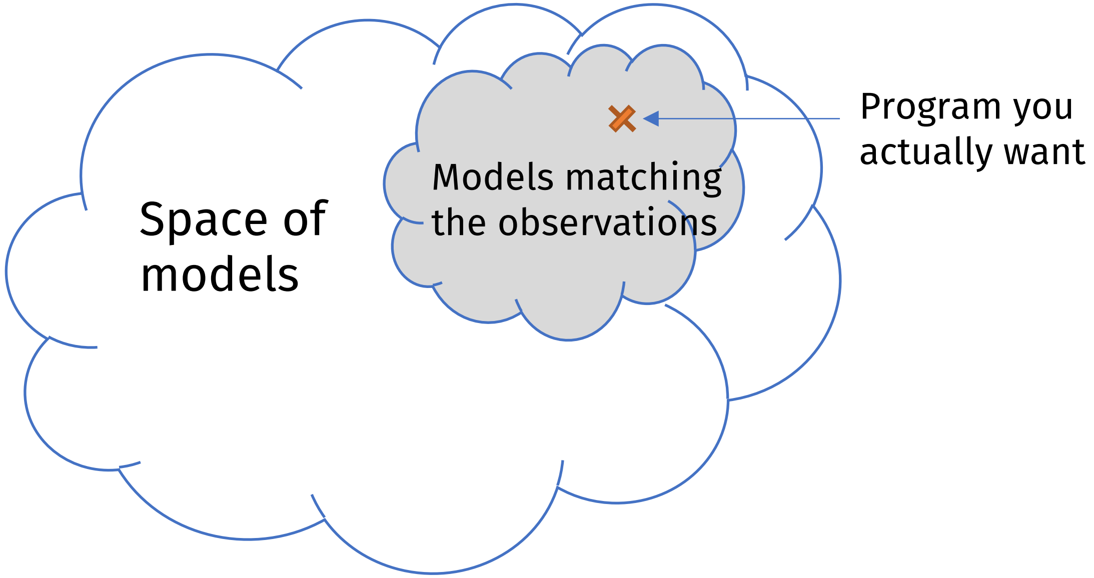
- Explored the question of generalizing from a set of observations
- Became the foundation of machine learning

Key issues in inductive learning



- (1) How do you find a model that matches the observations?
- (2) How do you know it is the model you are looking for?

Key issues in inductive learning



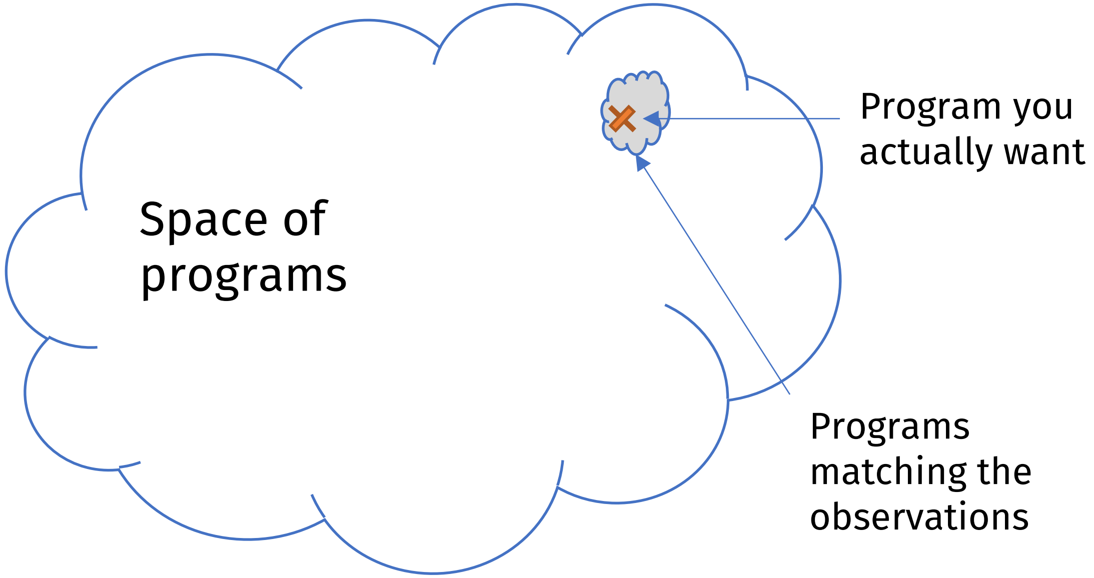
Traditional ML:

- Fix the space so that (1) is easy
- (2) becomes the main challenge

- (1) How do you find a model that matches the observations?

- (2) How do you know it is the model you are looking for?

The synthesis approach

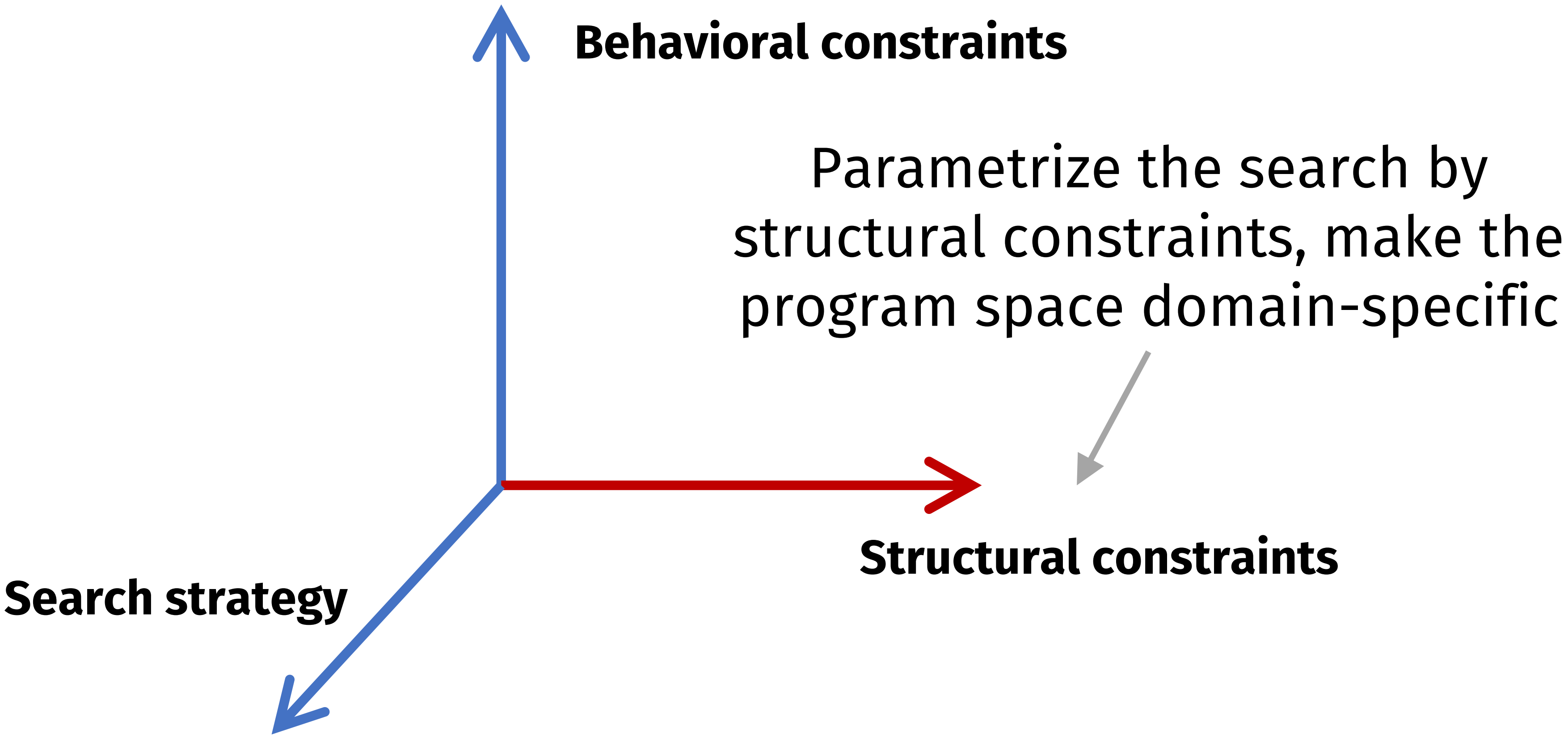


Program synthesis:

- Customize the space so that (2) becomes easier
- (1) is now the main challenge

- (1) How do you find a program that matches the observations?
- (2) How do you know it is the program you are looking for?

Key idea


Behavioral constraints
Parametrize the search by structural constraints, make the program space domain-specific
Structural constraints
Search strategy

Syntax-Guided Synthesis

Example

```
[1,4,7,2,0,6,9,2,5,0]  →  [1,2,4,7,0]

L ::= x               |      the input
      single(N)       |      single(1) = [1]
      sort(L)         |      sort([6,9,2,5]) = [2,5,6,9]
      slice(L,N,N)    |      slice([6,9,2,5],0,2) = [6,9]
      concat(L,L)            concat([6,9],[2,5]) = [6,9,2,5]
N ::= find(L,N)       |      find([6,9],9) = 1
      0                      0

f(x) := concat( sort(slice(x,0,find(x,0))),  single(0))
```

Regular tree grammars (RTGs)

starting nonterminal

ranked alphabet (terminals)

nonterminals

```
L ::= x              |
      single(N)      |
      sort(L)        |
      slice(L,N,N)   |
      concat(L,L)
N ::= find(L,N)      |
      0
```

productions

Regular tree grammars (RTGs)

nonterminals rules (productions)
alphabet starting nonterminal

<Σ, N, R, S>

- Trees: $\tau \in T_\Sigma(N)$ = all trees made from $N \cup \Sigma$
- Rules are of the form: $A \to \sigma(A_1, \ldots, A_n)$
- Derives in one step: $\mathcal{C}[A] \to \mathcal{C}[t]$ if $(A \to t) \in R$
  A is the leftmost non-terminal in $\mathcal{C}[A]$
- Incomplete terms/programs: $\{\tau \in T_\Sigma(N) | A \to^* \tau\}$
- Complete terms/programs: $\{t \in T_\Sigma | A \to^* t\}$
  = programs without holes
- Whole programs: $\{t \in T_\Sigma | S \to^* t\}$
  = roughly, programs of the right type

```
concat(L,0)
```

$L \to concat(L, L)$

```
concat(L,L) -> concat(x,L)

find(concat(L,L),N)

find(concat(x,x),0)

sort(concat(L,L))
```

RTGs as structural constraints

Space of programs
= the *language* of an RTG $L(G)$
= all complete, whole programs

```
L ::= x              |
      single(N)      |
      sort(L)        |
      slice(L,N,N)   |
      concat(L,L)    |
N ::= find(L,N)      |
      0
```

→

`x` `sort(x)` `concat(x, x)` `slice(x,0,0)`

...

`slice(x,0,find(x,0))`

...

`concat(sort(slice(x,0,find(x,0))), single(0))`

...

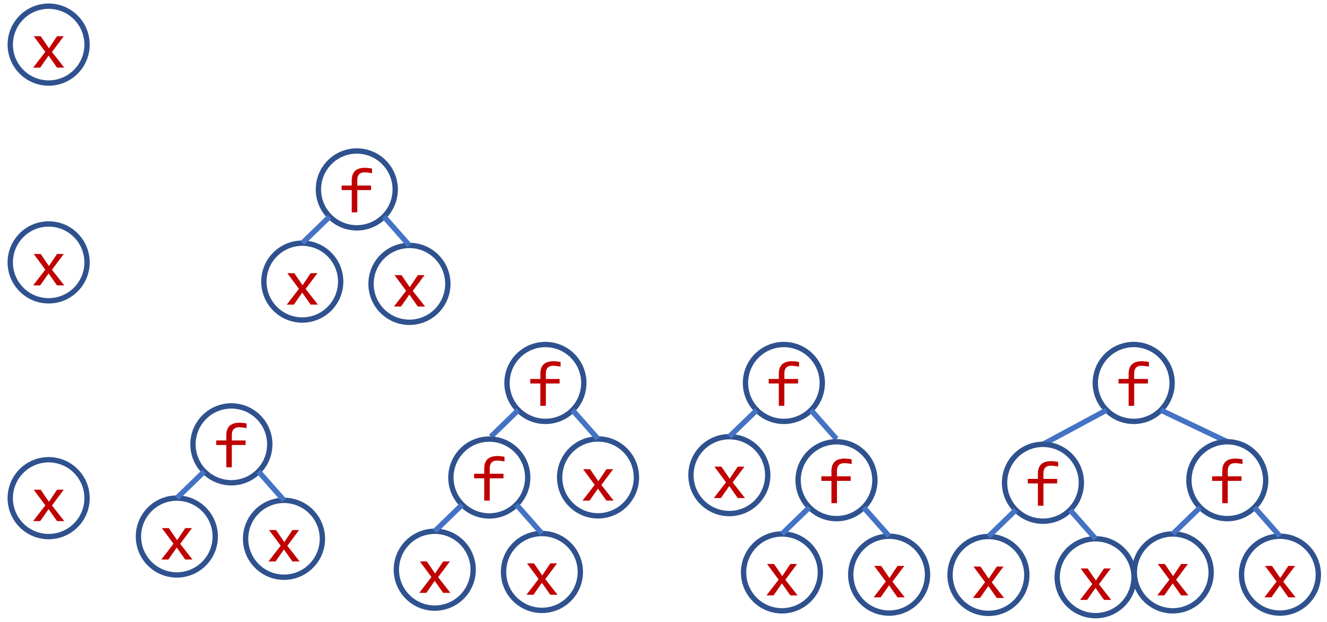
How big is the space?

`E ::= x | f(E,E)`

depth <= 0 — N(0) = 1

depth <= 1 — N(1) = 2

depth <= 2 — N(2) = 5



$$N(d) = 1 + N(d - 1)^2$$

How big is the space?

```
E ::= x | f(E,E)
```

$$N(d) = 1 + N(d-1)^2 \qquad N(d) \sim c^{2^d} \qquad (c > 1)$$

N(1) = 1
N(2) = 2
N(3) = 5
N(4) = 26
N(5) = 677
N(6) = 458330
N(7) = 210066388901
N(8) = 44127887745906175987802
N(9) = 1947270476915296449559703445493848930452791205
N(10) = 3791862310265926082868235028027893277370233152247388584761734150717768254410341175325352026

How big is the space?

$$E ::= x_1 \mid \ldots \mid x_k \mid f_1(E,E) \mid \ldots \mid f_m(E,E)$$

$$N(0) = k$$

$$N(d) = k + m * N(d - 1)^2$$

N(1) = 3
N(2) = 30
N(3) = 2703
N(4) = 21918630
N(5) = 1441279023230703
N(6) = 6231855668414547953818685622630
N(7) = 116508075215851596766492219468227024724121520304443212304350703

$k = m = 3$

Syntactic sugar

Instead of this:

```
L ::= x              |
      single(N)      |
      sort(L)        |
      slice(L,N,N)   |
      concat(L,L)
N ::= find(L,N)      |
      0
```

We will often write this:

```
L ::= x         |
      [N]       |
      sort(L)   |
      L[N..N]   |
      L + L
N ::= find(L,N) |
      0
```

- allow custom syntax for terminal symbols
- not an RTG strictly speaking, but you know what we mean...

Syntactic sugar

```
L ::= sort(L)     |
      L[N..N]     |
      L + L       |
      [N]         |
      x
N ::= find(L,N)   |
      0
```

→

```
x    sort(x)    x + x    x[0..0]
...
x[0..find(x, 0)]
...
sort(x[0..find(x, 0)]) + [0]
...
```

The SyGuS project

https://sygus.org/

- Goal: Unify different syntax-guided approaches
- Collection of synthesis benchmarks + yearly competition
  - 6 competitions since 2013
  - consider writing a SyGuS solver for your project!
- Common input format + supporting tools
  - parser, baseline synthesizers

[Alur et al. 2013]

SyGuS problems

- SyGuS problem = < theory, spec, grammar >

A "library" of types and function symbols

Example: Linear Integer Arithmetic (LIA)

```
True, False
0,1,2,...
∧, ∨, ¬, +, ≤, ite
```

RTG with terminals in the theory
(+ input variables)

Example: Conditional LIA expressions w/o sums

```
E ::= x | 0 | ite C E E
C ::= E ≤ E | C ∧ C | ¬C
```

SyGuS problems

SyGuS problem = < theory, spec, grammar >

A first-order logic formula over the theory

Examples:

```
f(0, 1) = 1 ∧
f(1, 0) = 1 ∧
f(1, 1) = 1 ∧
f(2, 0) = 2
```

SyGuS demo

SyGuS problems

- SyGuS problem = < theory, spec, grammar >

can inductive synthesis handle these?

A first-order logic formula over the theory

Examples:

```
f(0, 1) = 1 ∧
f(1, 0) = 1 ∧
f(1, 1) = 1 ∧
f(2, 0) = 2
```

Formula with free variables:

```
x ≤ f(x, y) ∧
y ≤ f(x, y) ∧
(f(x, y) = x ∨ f(x, y) = y)
```