

# #30: Synthesis with Abstract Interpretation

**Sankha Narayan Guria**

EECS 700: Introduction to Program Synthesis



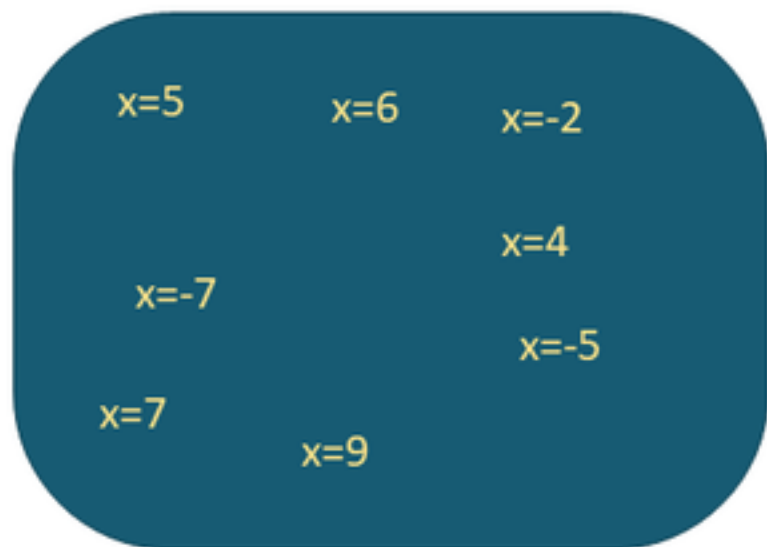
# Today

- Synthesizing data-structure manipulation from storyboards
  - Rishabh Singh, Armando Solar-Lezama
- Absynthe: Abstract Interpretation-Guided Synthesis
  - Sankha Narayan Guria, Jeff Foster, David Van Horn

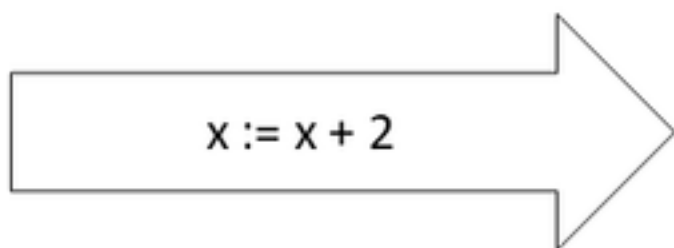
# Key idea 1: Abstract domain

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Concrete states



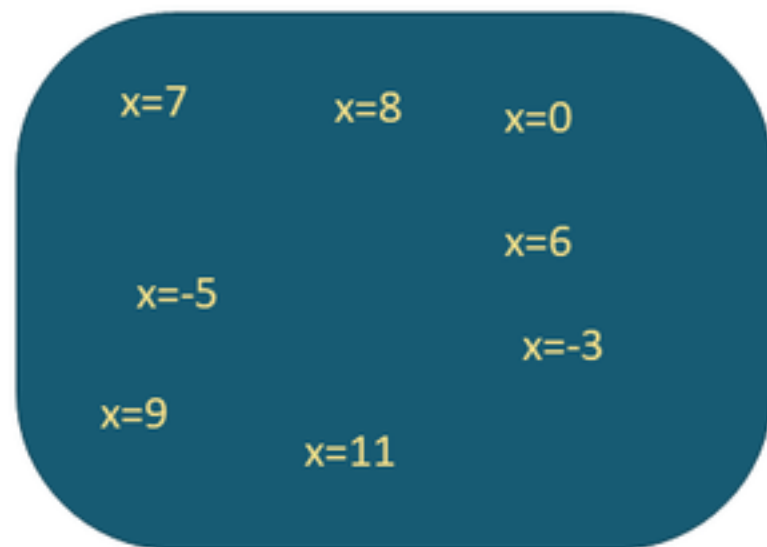
Concrete semantics



$x := x + 2$

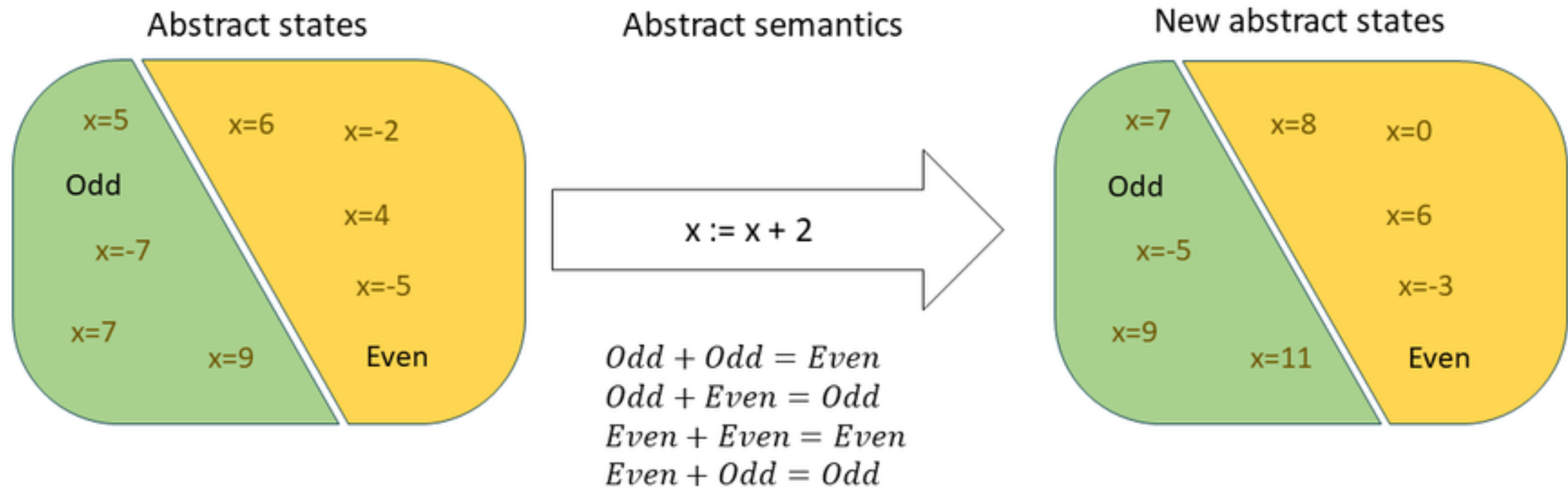
$\{E[x \mapsto x + 2]\} x := x + 2 \{E\}$

New concrete states



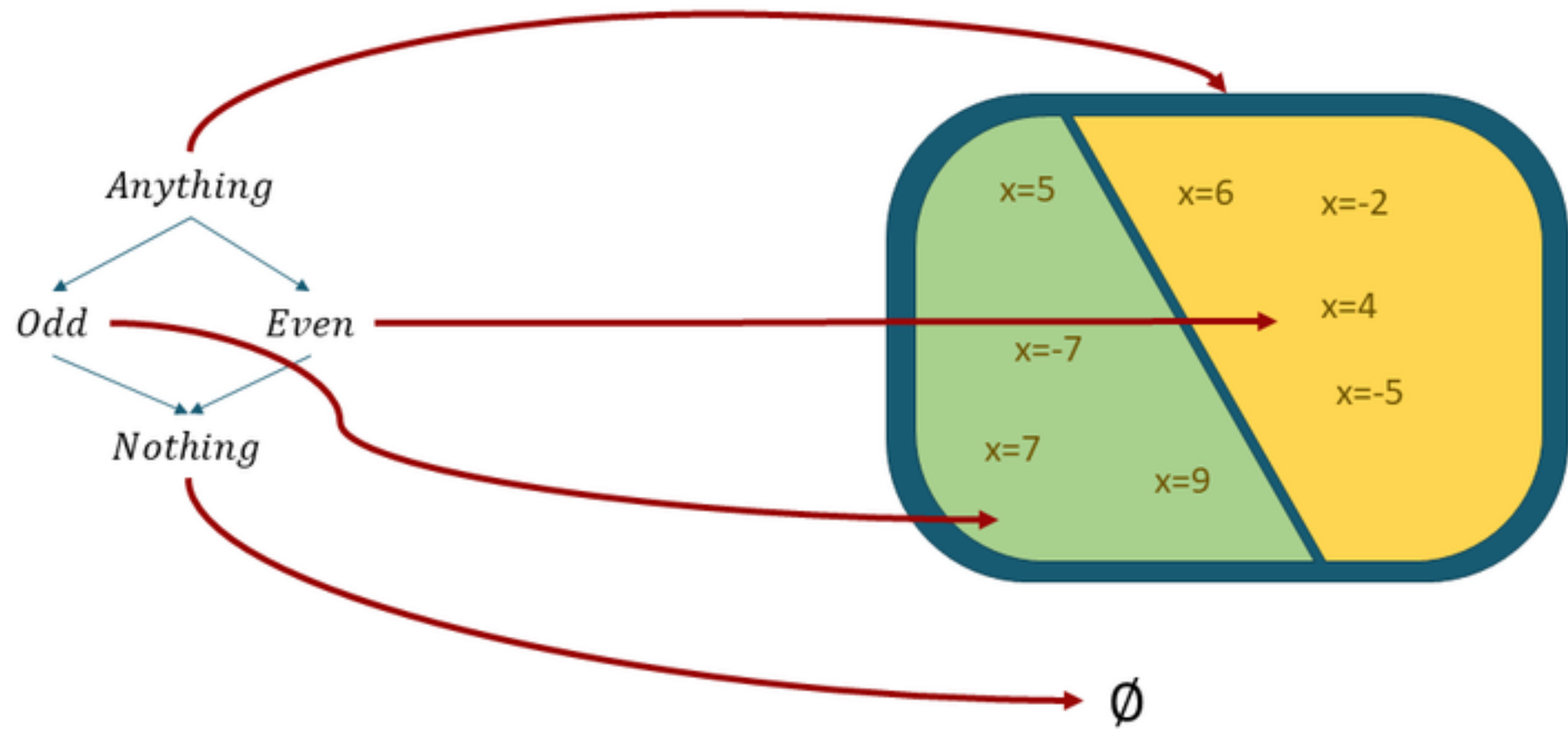
# Key idea 1: Abstract domain

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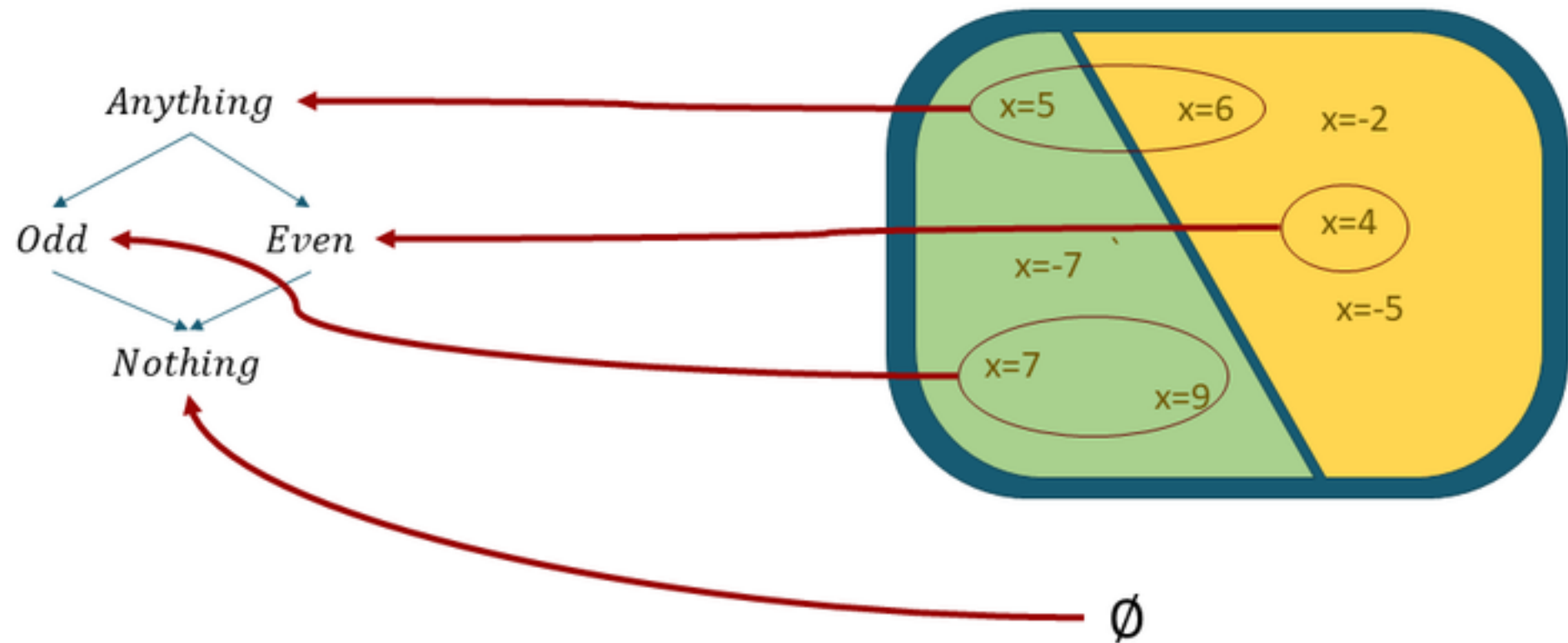
# Concretization

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# Abstraction

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# Key idea 2: Abstract Interpretation

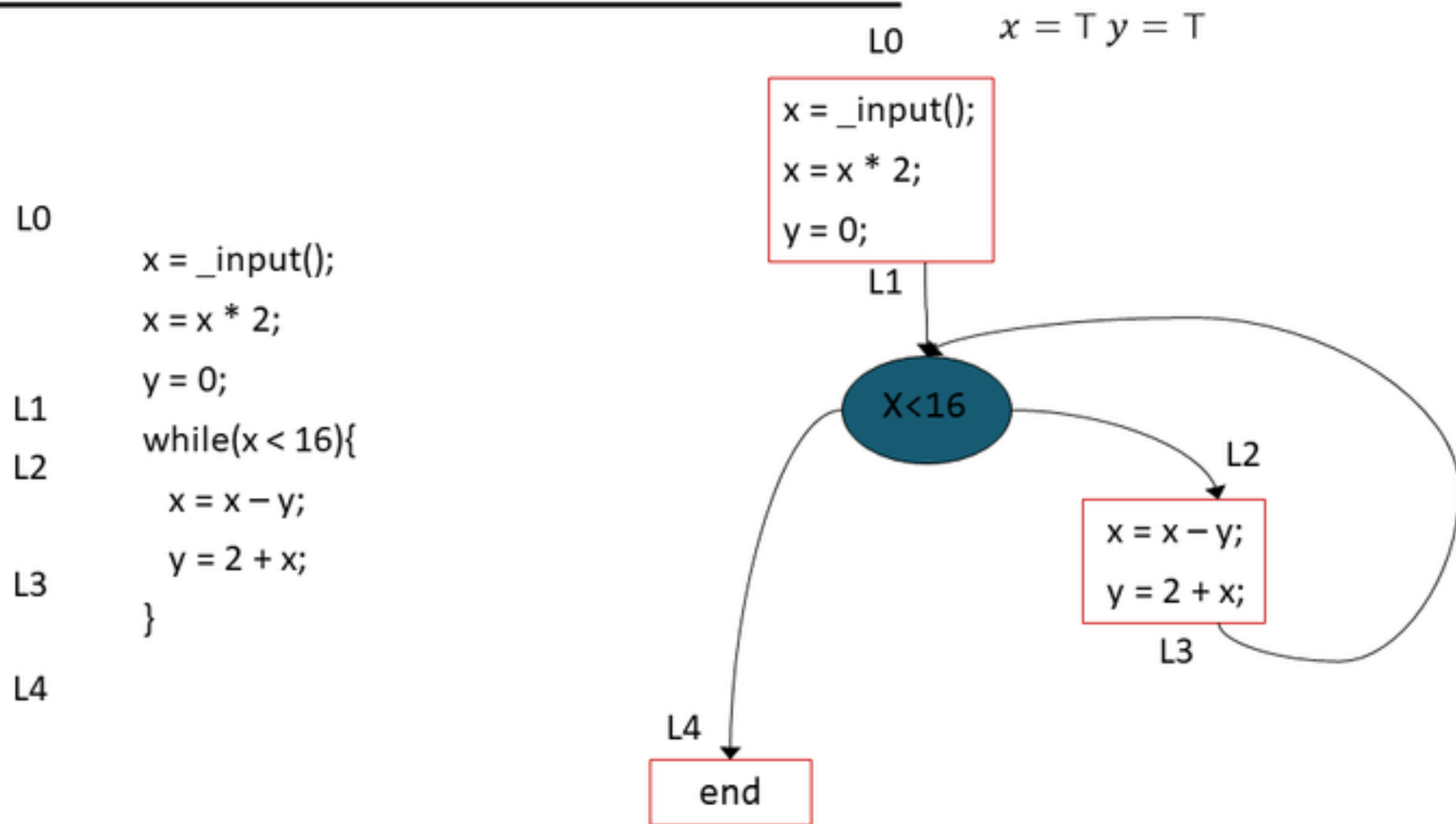
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Compute an abstract value for every program point

- Abstraction of the set of states possible at that point

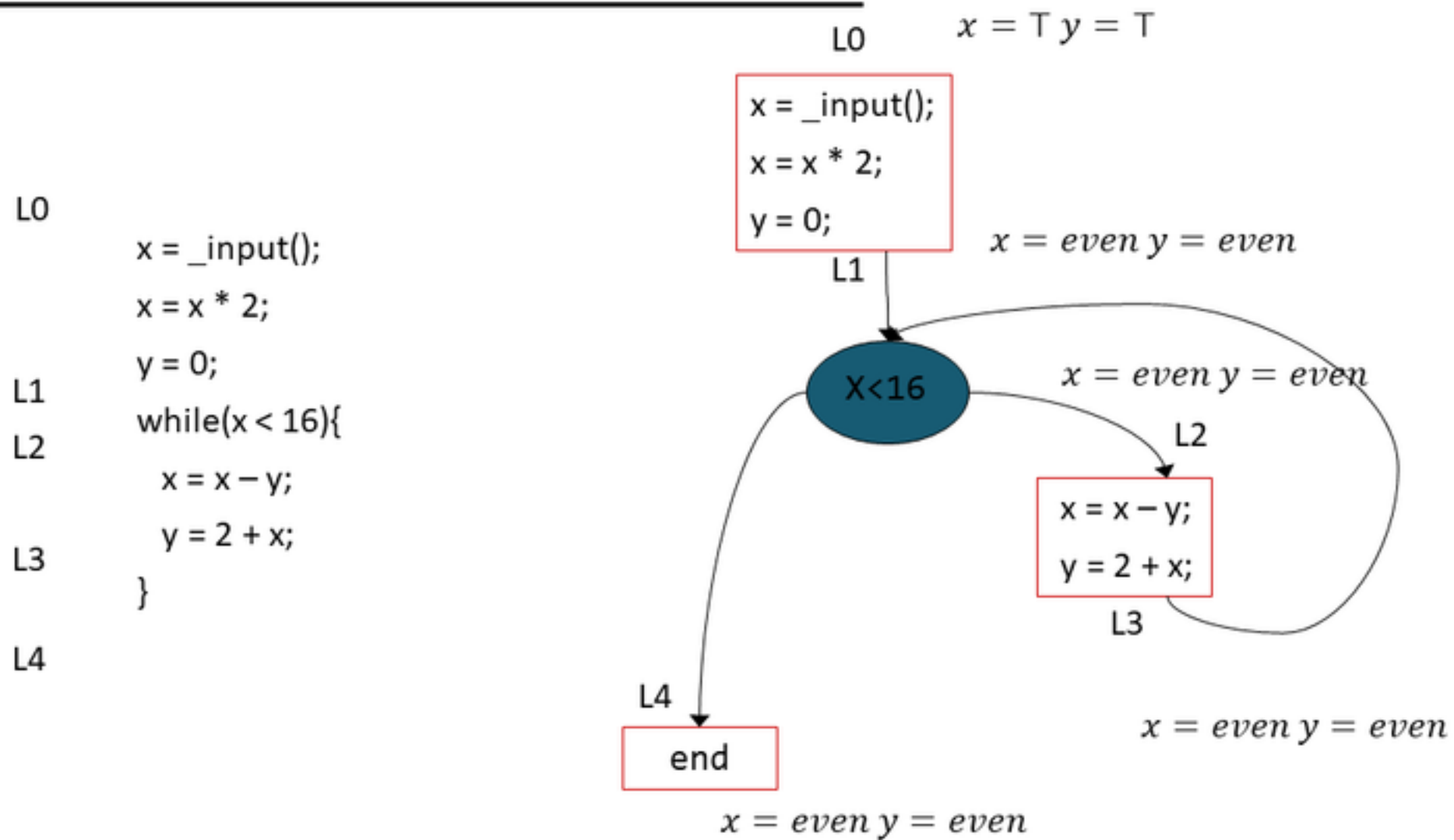
Iterate until computation converges

# Example



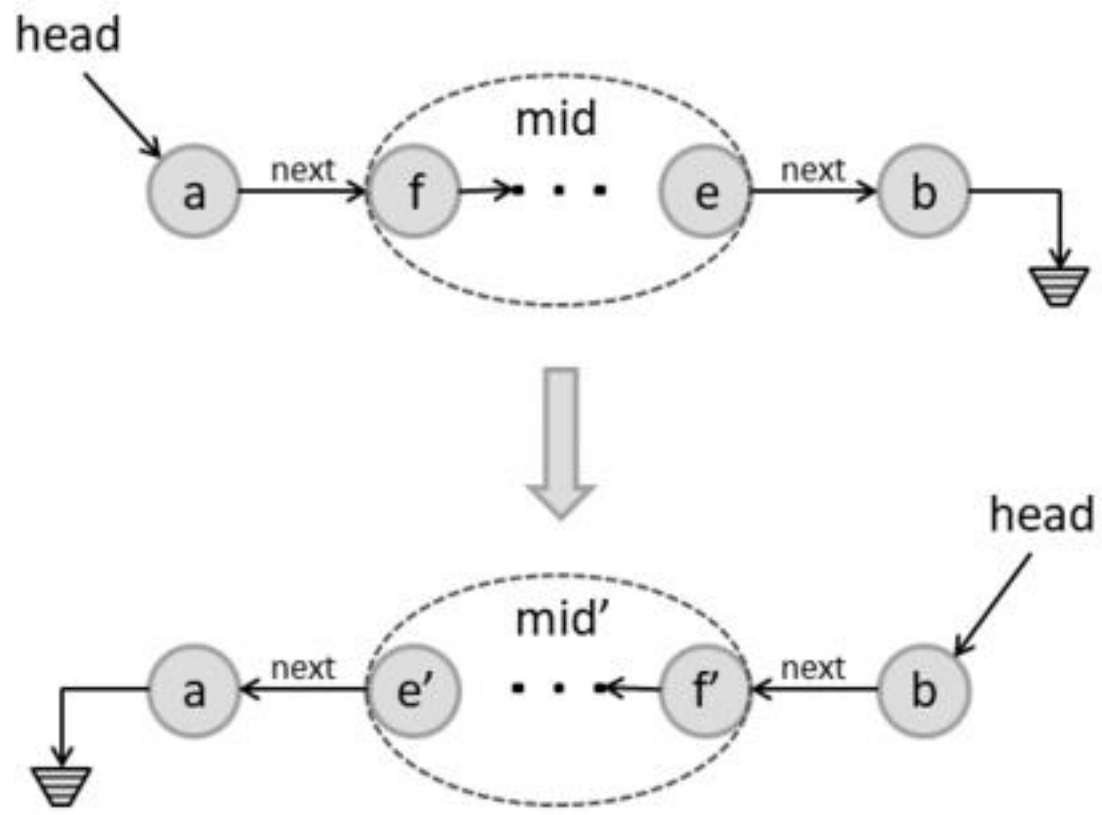


# Example



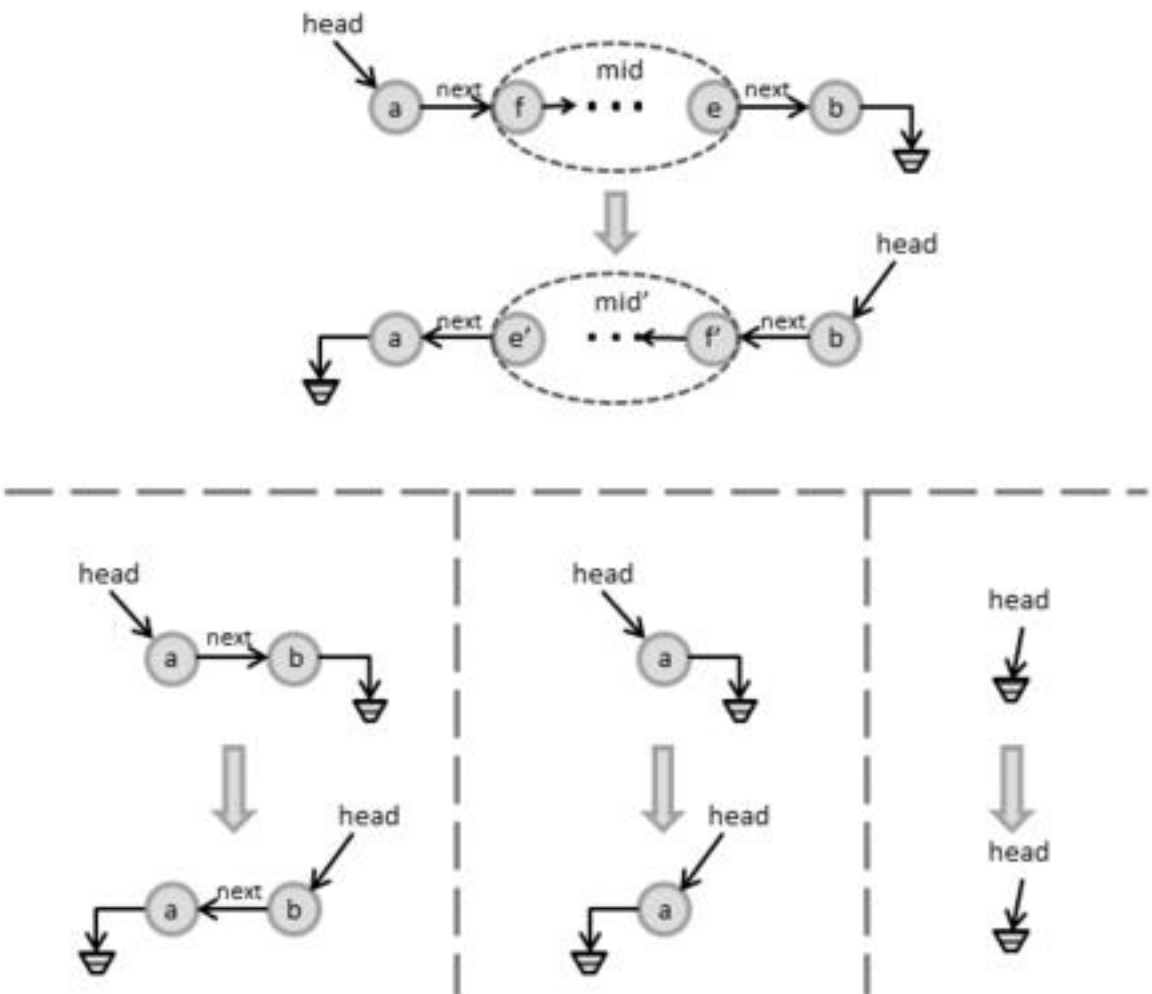
# Storyboard Programming

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# Scenarios for LL-reversal

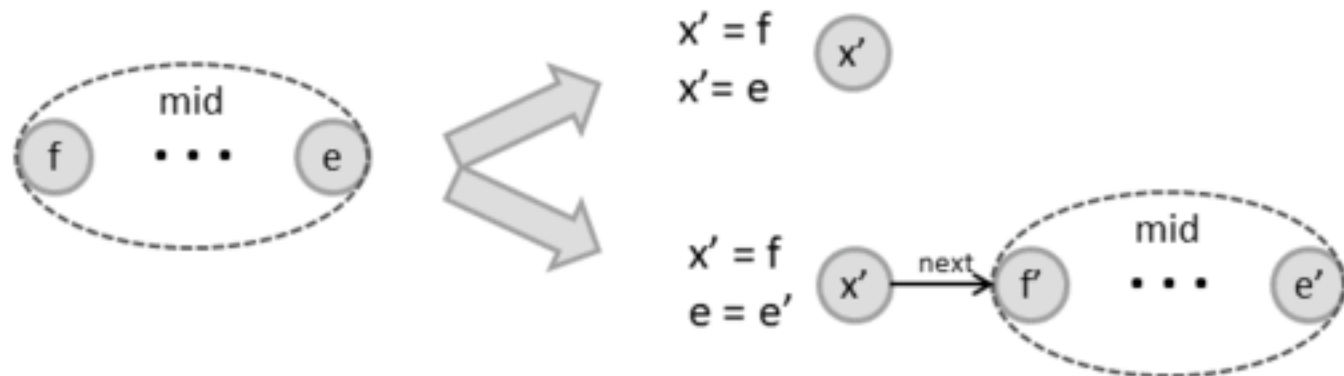
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# Inductive insights with fold/unfold

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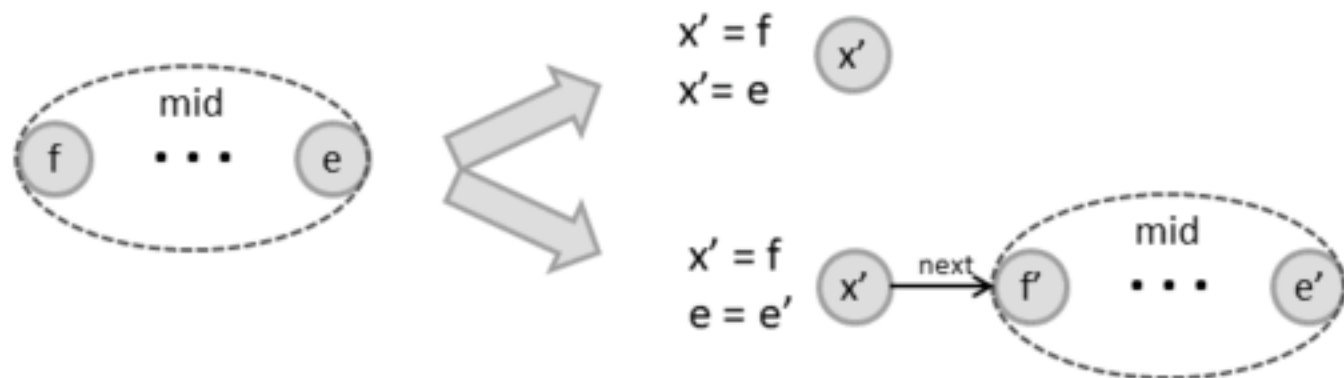
Unfold:



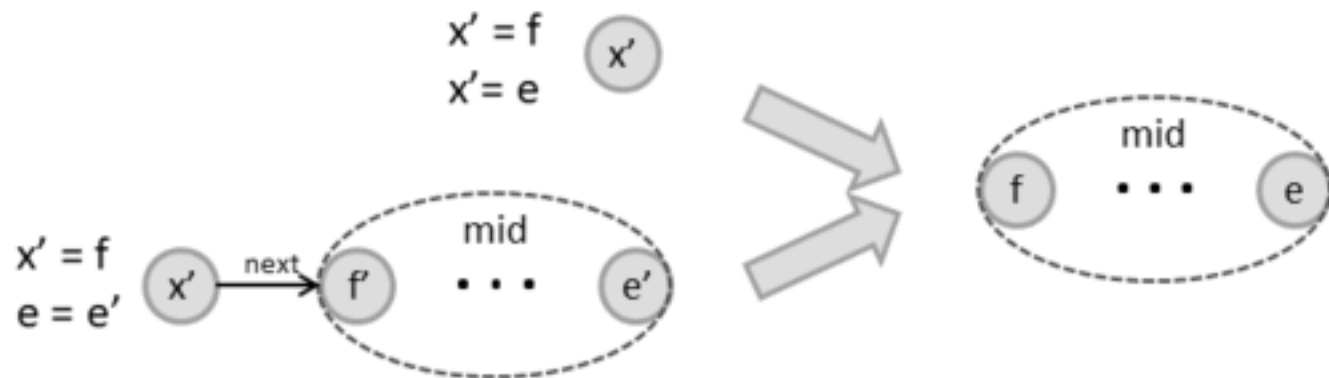
# Inductive insights with fold/unfold

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Unfold:



Fold:



# Concrete Domain

---

Memory locations:  $\mathcal{L}^\#$

Variables:  $v_0, v_1, \dots, v_k$

Variable predicates:  $v_i: \mathcal{L}^\# \rightarrow \text{Bool}$   $v_i(l)$  indicates that variable  $v_i$  points to loc  $l$

Fields:  $sel_0, sel_1, \dots, sel_k$

Field predicates:  $sel_0: \mathcal{L}^\# \times \mathcal{L}^\# \rightarrow \text{Bool}$

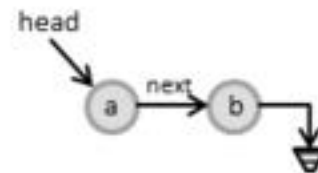
$sel_i(l_1, l_2)$  indicates that there is a field  $sel_i$  from object  $l_1$  to object  $l_2$

$\mathcal{L}^\# = \{a, b\}$

$head(a) = \text{true}$

$head(b) = \text{false}$

<i>Next</i>	<i>a</i>	<i>b</i>
<i>a</i>	false	true
<i>b</i>	false	false

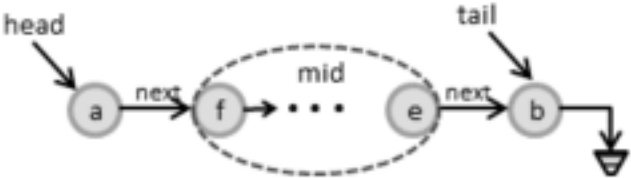


# Abstract Domain

- Abstract memory locations:  $\mathcal{L}$ 
  - represents a set of concrete locations
- Summary location indicator:  $sm: \mathcal{L} \rightarrow \text{Tree Valued Logic (TVL)}$ 
  - indicates if a location represents more than one concrete loc
- Attachment Points:  $\mathcal{A}: \mathcal{L} \rightarrow \{\mathcal{L}\}$ 
  - maps a summary node to a set of locations that serve as attachment points
- Variable predicates:  $v_i: \mathcal{L} \rightarrow \text{TVL}$   $v_i(l)$  indicates that variable  $v_i$  points to loc  $l$
- Field predicates :  $sel_0: \mathcal{L} \times \mathcal{L} \rightarrow \text{TVL}$ 
  - $sel_i(l_1, l_2)$  indicates that there is a field  $sel_i$  from object  $l_1$  to object  $l_2$

$$\mathcal{L} = \{a, f, e, mid, b\}$$

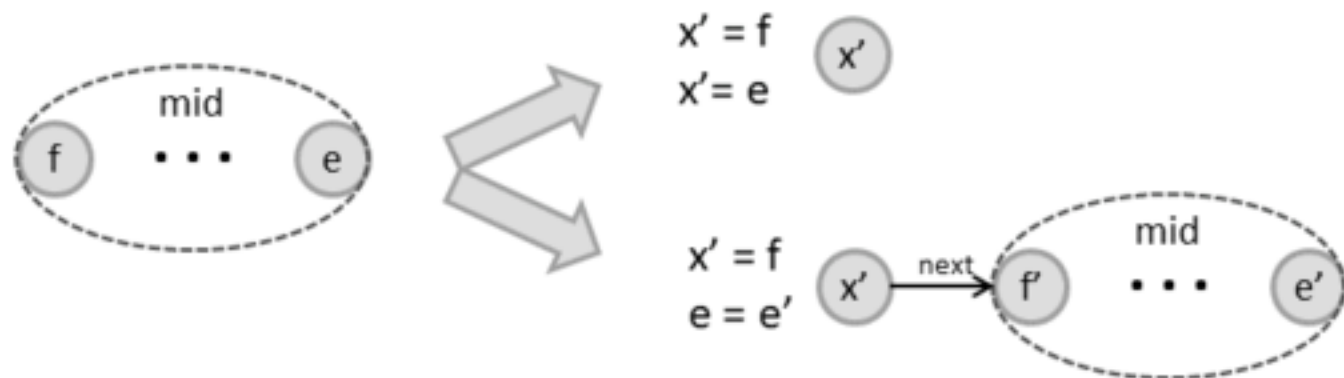
<i>sm</i>		<i>A</i>		<i>head</i>		<i>tail</i>		<i>next</i>	<i>a</i>	<i>f</i>	<i>e</i>	<i>mid</i>	<i>b</i>
<i>a</i>	false			<i>a</i>	true	<i>a</i>	false	<i>a</i>	F	T	/	/	F
<i>f</i>	false			<i>f</i>	false	<i>f</i>	false	<i>f</i>	F	F	F	/	F
<i>e</i>	false			<i>e</i>	false	<i>e</i>	false	<i>e</i>	F	F	F	F	T
<i>mid</i>	true	<i>mid</i>	{ <i>f</i> , <i>e</i> }	<i>mid</i>	false	<i>mid</i>	false	<i>mid</i>	F	F	/	F	/
<i>b</i>	false			<i>b</i>	false	<i>b</i>	true	<i>b</i>	F	F	F	F	F



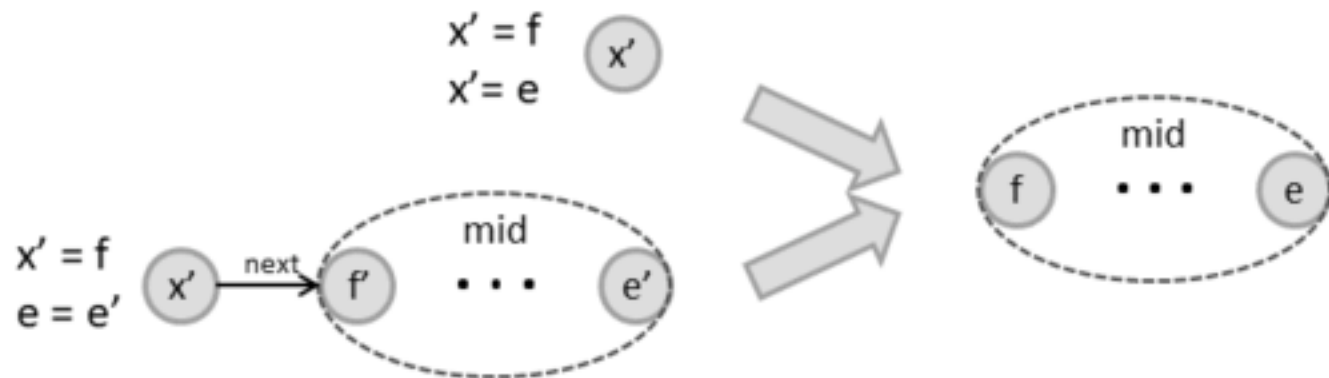
# Inductive insights with fold/unfold

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Unfold:

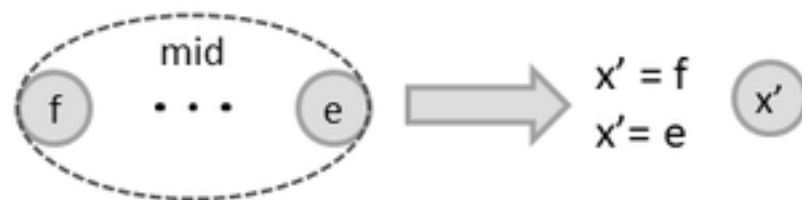


Fold:

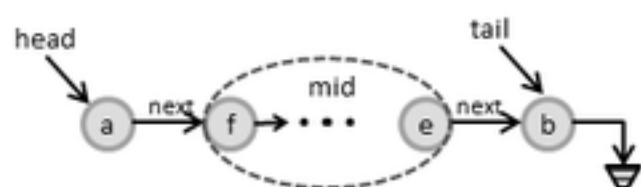




# Unfold

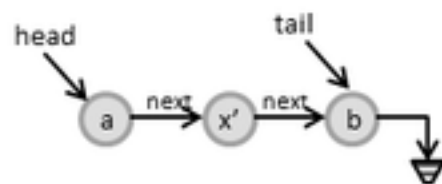


$sm$		$\mathcal{A}$		$head$		$tail$		$next$	$a$	$f$	$e$	$mid$	$b$
$a$	false			$a$	true	$a$	false	$a$	F	T	/	/	F
$f$	false			$f$	false	$f$	false	$f$	F	F	F	/	F
$e$	false			$e$	false	$e$	false	$e$	F	F	F	F	T
$mid$	true		$\{f, e\}$	$mid$	false	$mid$	false	$mid$	F	F	/	F	/
$b$	false			$b$	false	$b$	true	$b$	F	F	F	F	F

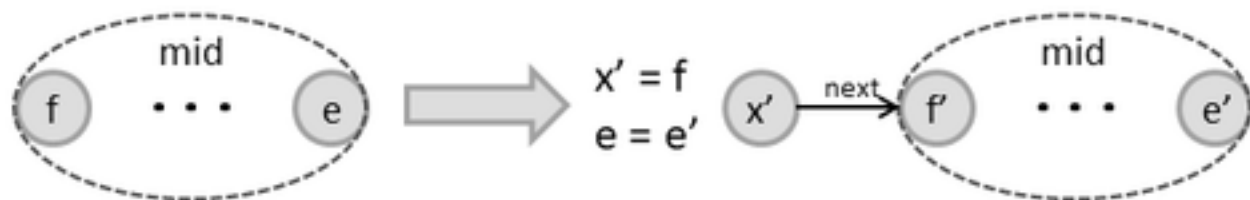


Unfold(head.next)

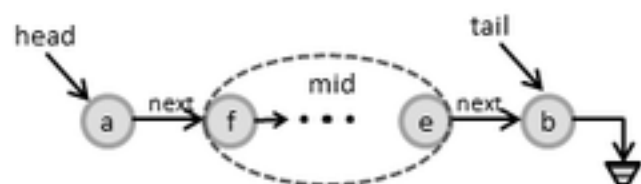
$sm$		$\mathcal{A}$		$head$		$tail$		$next$	$a$	$x'$	$b$
$a$	false			$a$	true	$a$	false	$a$	F	T	F
$x'$	false			$x'$	false	$x'$	false	$x'$	F	F	T
$b$	false			$b$	false	$b$	true	$b$	F	F	F



# Unfold

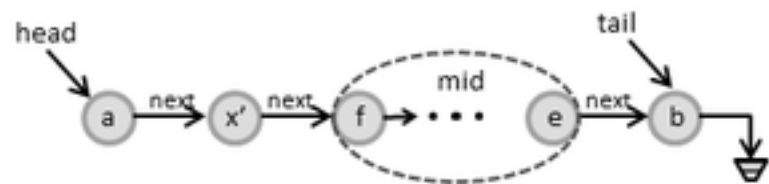


$sm$		$\mathcal{A}$		$head$		$tail$		$next$	$a$	$f$	$e$	$mid$	$b$
$a$	false			$a$	true	$a$	false	$a$	F	T	/	/	F
$f$	false			$f$	false	$f$	false	$f$	F	F	F	/	F
$e$	false			$e$	false	$e$	false	$e$	F	F	F	F	T
$mid$	true		$\{f, e\}$	$mid$	false	$mid$	false	$mid$	F	F	/	F	/
$b$	false			$b$	false	$b$	true	$b$	F	F	F	F	F

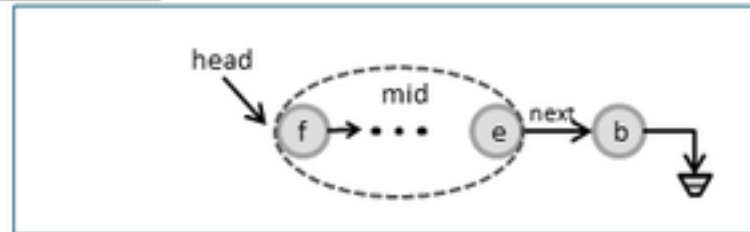


Unfold( $head.next$ )

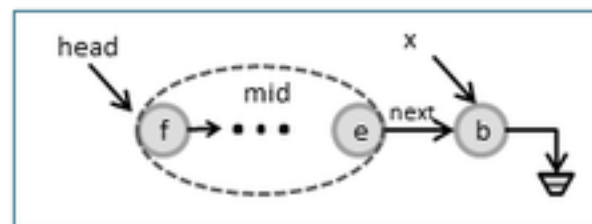
$sm$		$\mathcal{A}$		$head$		$tail$		$nex$	$a$	$f$	$e$	$x'$	$mid$	$b$
$a$	false			$a$	true	$a$	false	$a$	F	F	F	T	F	F
$f$	false			$f$	false	$f$	false	$f$	F	F	F	F	/	F
$e$	false			$e$	false	$e$	false	$e$	F	F	F	F	F	T
$x'$	false			$x'$	false	$x'$	false	$x'$	F	T	/	F	/	F
$mid$	true		$\{f, e\}$	$mid$	false	$mid$	false	$mid$	F	F	/	F	F	/
$b$	false			$b$	false	$b$	true	$b$	F	F	F	F	F	F



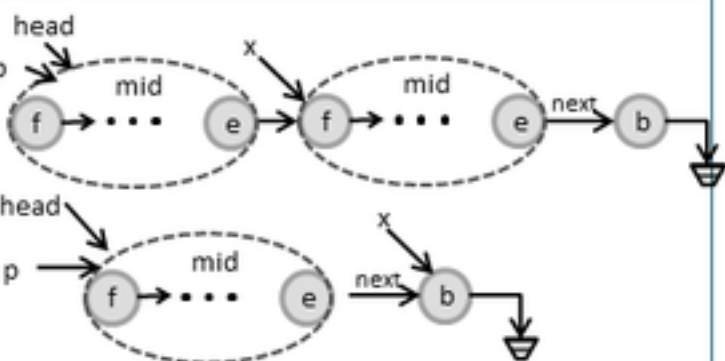
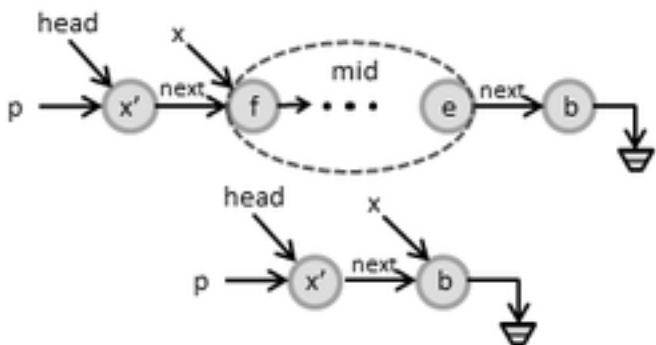
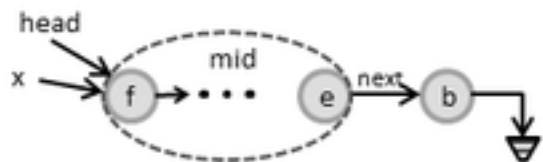
# Example



```
x = head;  
while (x.next != null) {  
    unfold(x);  
    x = x.next;  
    fold(x);  
}
```



# Example



x = head;

while (x.next != null) {

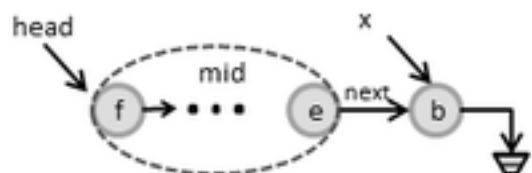
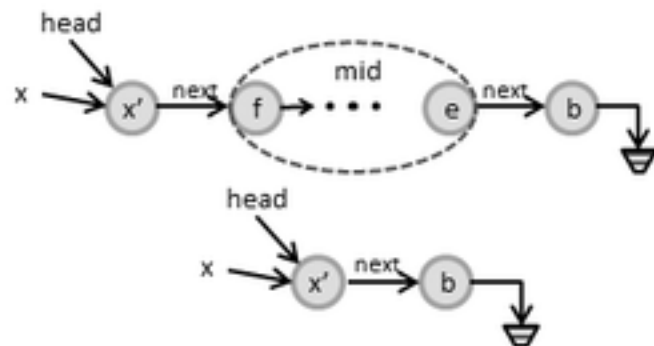
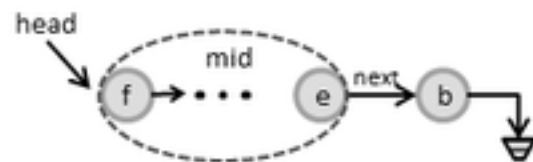
  unfold(x);

  p = x;

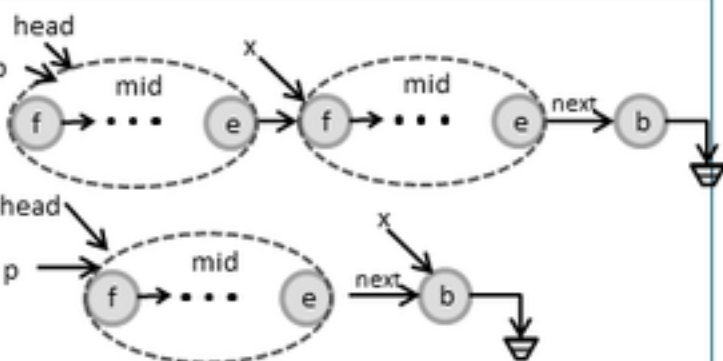
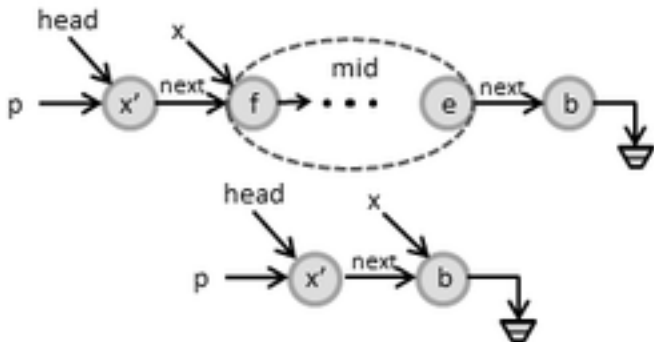
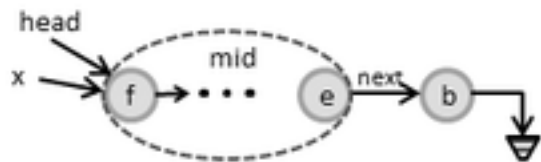
  x = x.next;

  fold(p);

}



# Example



```
x = head;
```

```
while (x.next != null) {
```

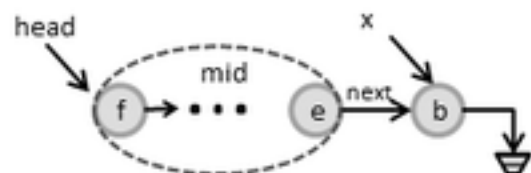
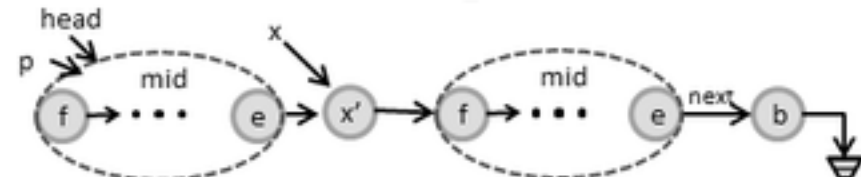
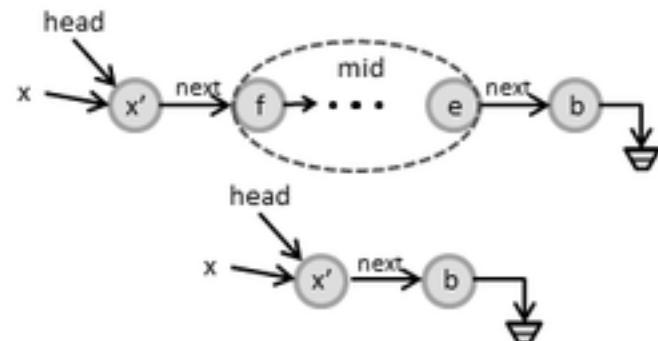
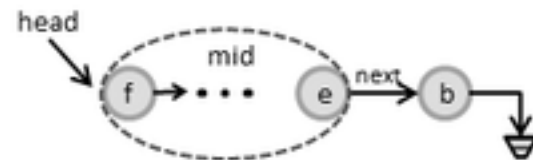
```
    unfold(x);
```

```
    p = x;
```

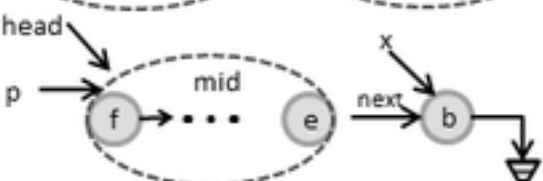
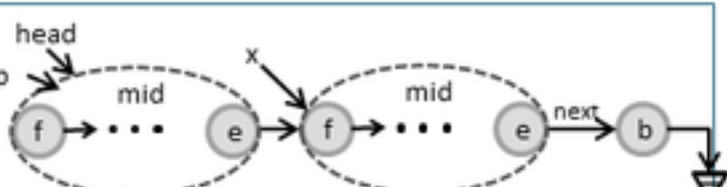
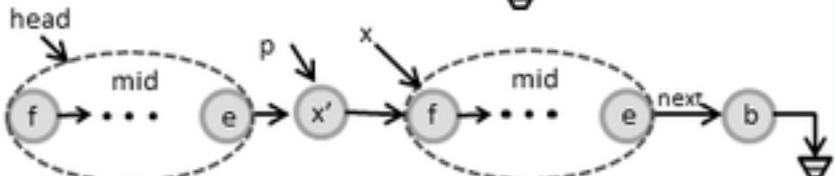
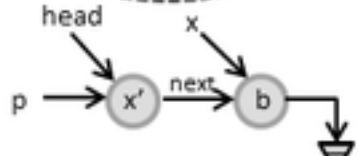
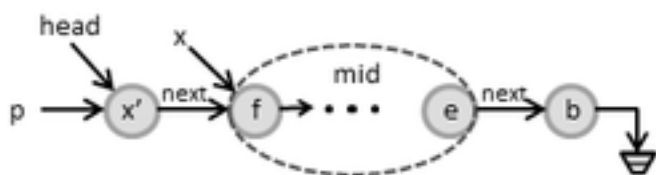
```
    x = x.next;
```

```
    fold(p);
```

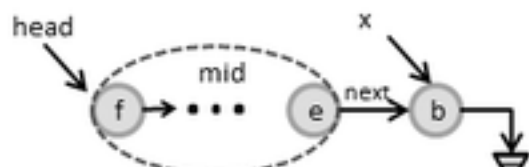
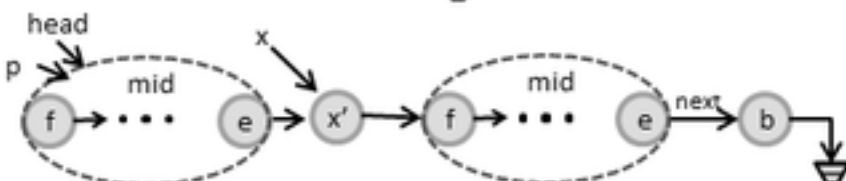
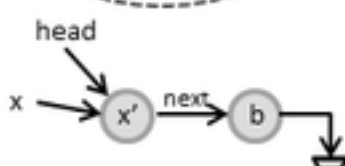
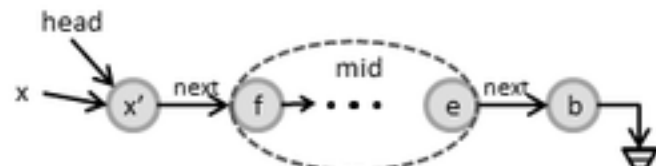
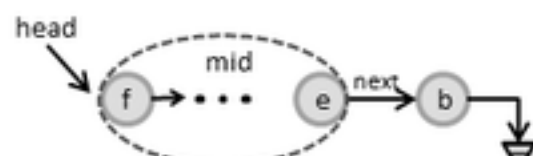
```
}
```



# Example



```
x = head;  
while (x.next != null) {  
  unfold(x);  
  p = x;  
  x = x.next;  
  fold(p);  
}
```



# Look Sketch

---

```
void llReverse(Node head)
{
    ?? /*1*/
    while (?? /*p*/)
    {
        ?? /*2*/
    }
    ?? /*3*/
}
```

# Look Sketch

---

```
void llReverse(Node head)
{
    cstmt* /*1*/
    while (cond /*p*/)
    {
        cstmt* /*2*/
    }
    cstmt* /*3*/
}
```



# Conditional Statements

---

$\text{var}(.ptr?) \text{ op } \text{var}(.ptr?) \mid \text{null}$



$\text{cstmt} : \text{if}(\text{COND}) \text{ then } \text{STMT}$

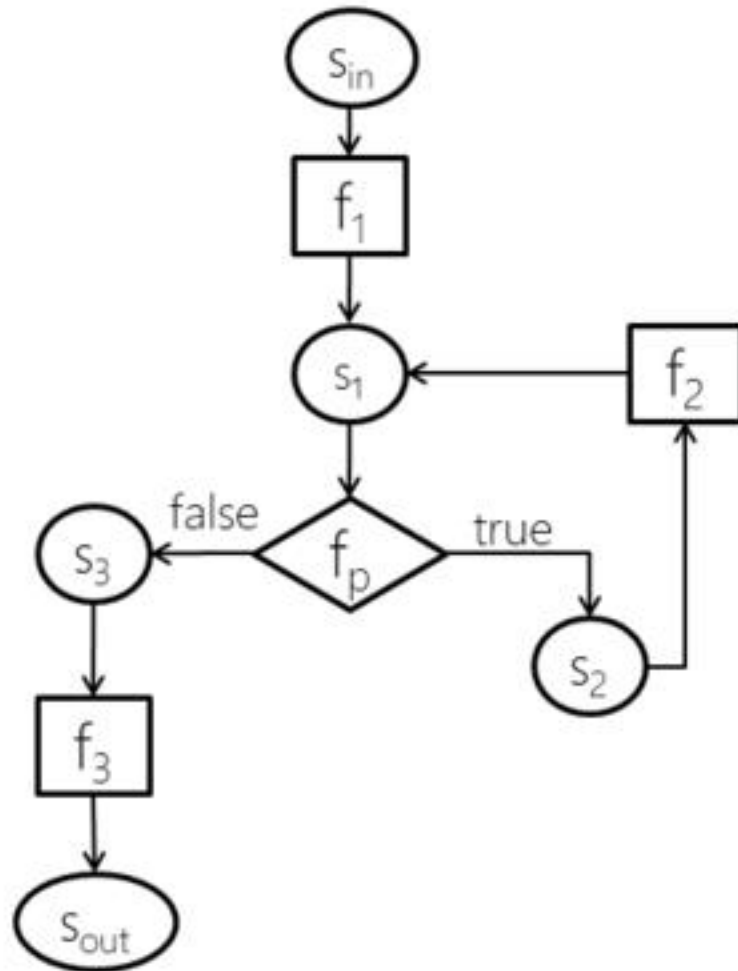


$\text{var}(.ptr?) = \text{var}(.ptr?)$

$\text{unfold/fold } \text{var}$

# Data flow equations

---



$$s_1 = f_1(s_{in}) \cup f_2(s_2)$$

$$s_2 = f_p(s_1)$$

$$s_3 = \bar{f}_p(s_1)$$

$$s_{out} = f_3(s_3)$$

# Today

- Synthesizing data-structure manipulation from storyboards
  - Rishabh Singh, Armando Solar-Lezama
- Absynthe: Abstract Interpretation-Guided Synthesis
  - Sankha Narayan Guria, Jeff Foster, David Van Horn

# Example

arg0

	id	valueA
0	255	1141
1	91	1130
2	347	830
⋮	⋮	⋮
8	225	638
9	257	616

arg1

	id	valueB
0	255	1231
1	91	1170
2	5247	954
⋮	⋮	⋮
12	211	575
13	25	530



arg2

"valueA  $\neq$  valueB"

	id	valueA	valueB
0	255	1141	1231
1	91	1130	1170
2	347	830	870
5	159	715	734
8	225	638	644

Types and column labels are a potential good abstraction

{"id", "valueA", "valueB"} x DataFrame

# Types Abstract Interpreter

```
class PyTypeInterp
```

Parameter to Absynthe  
for a class of problems

**Pandas data frame merge**


```
# left.merge(right, opts)  
df1.merge(df2, on = ['id'])
```

```
end
```

# Types Abstract Interpreter

```
class PyTypeInterp
  def self.pd_merge(left, right, opt)
    if left ⊆ DataFrame &&
      right ⊆ DataFrame &&
      opt ⊆ { on: Array<String>}
      DataFrame
    end
  end
end
```

## Pandas data frame query



```
# df.query(pred)
df.query('valueA > 10')
```

```
end
```

# Columns Abstract Interpreter

```
class ColNameInterp
```

```
end
```

## Pandas data frame merge

```
df1.merge(df2, on = ['id'])
```

Final data frame is union of both



# Columns Abstract Interpreter

```
class ColNameInterp  
  
    def self.pd_merge(left, right, opt)  
        left ∪ right  
    end  
  
end
```

## Pandas data frame query

`df.query('valueA > 10')`

Final data frame has same columns



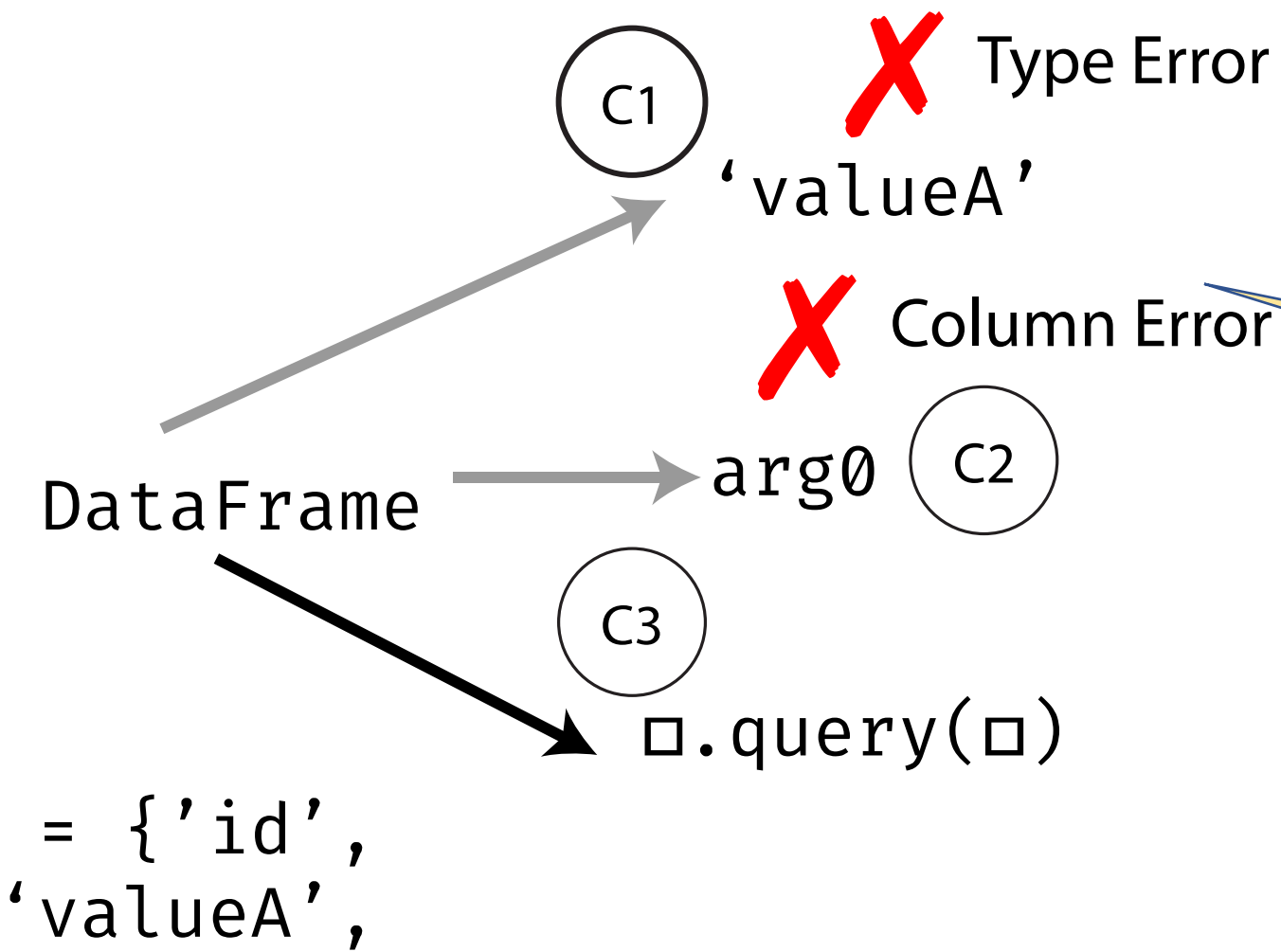


Starting candidate derived  
from the synthesis goal

C0

□: Col x DataFrame

```
Col = {'id',  
       'valueA',  
       'valueB'}
```



Concrete values not in abstract  
domain never synthesized

Partial programs are evaluated through the abstract interpreter

C4

$\square$ .query(arg0)

**X** Type Error

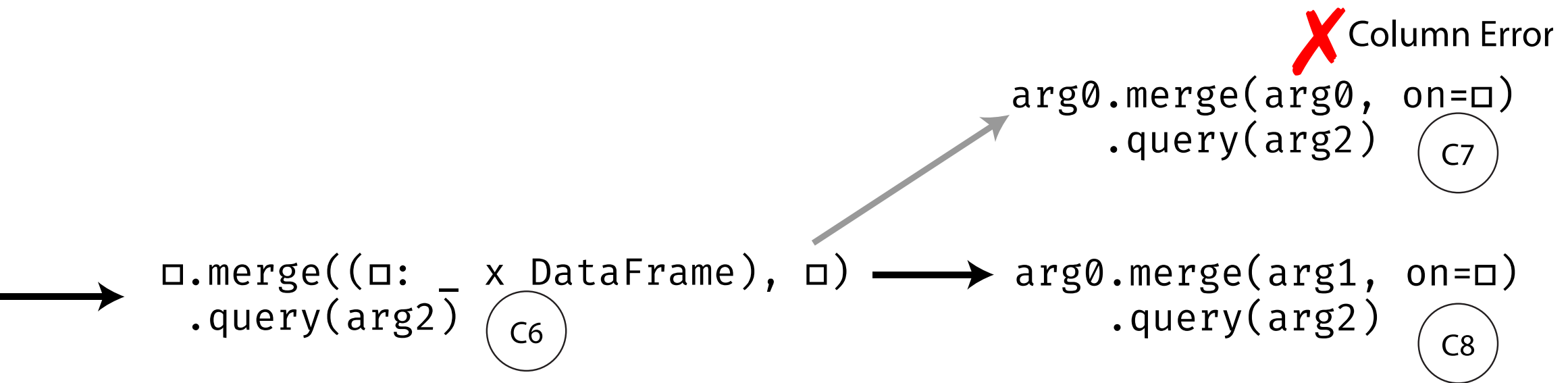
Abstract values for holes are inferred

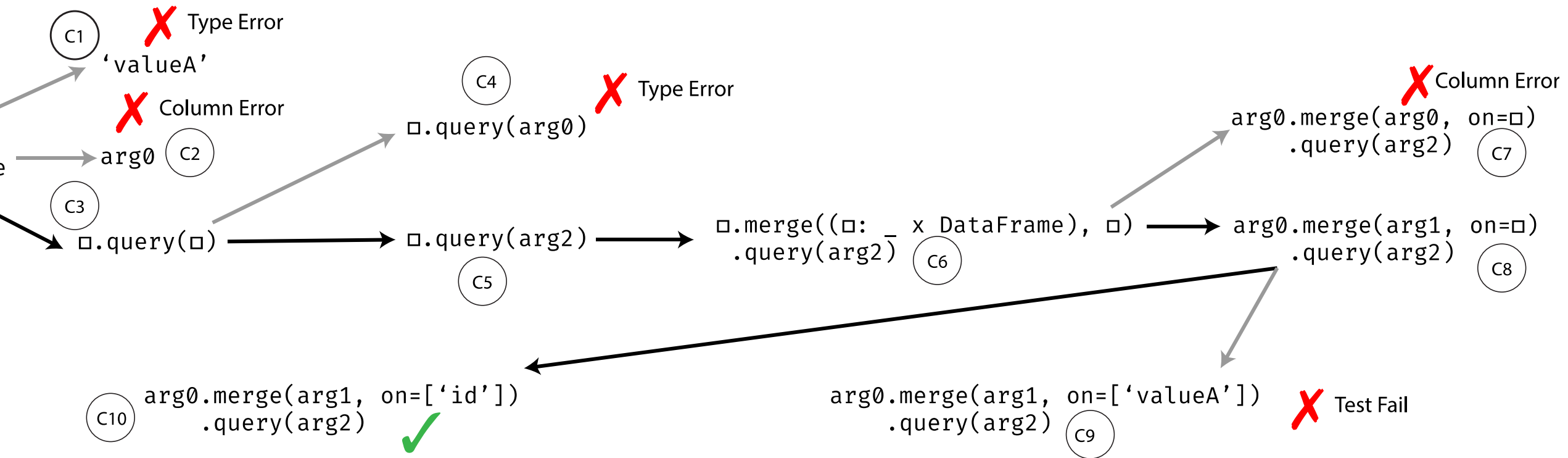
$\square$ .query(arg2)

C5

$\square$ .merge(( $\square$ :  $\square$ .query(arg2) x DataFrame),  $\square$ )

C6






Correct solution!

# Searching for Programs

Synthesize a term:

$\blacksquare_1.\text{query}(\blacksquare_2)$

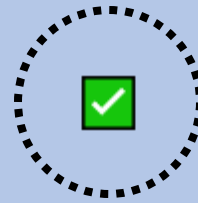
such that it satisfies a  
synthesis goal 

Semantics of  $\blacksquare_1.\text{query}(\blacksquare_2)$

`arg0.query("id != 0")`

`arg0.query( $\blacksquare_3$ ).query( $\blacksquare_2$ )`

`arg0.query( $\blacksquare_2$ )`



# Searching for Programs

Synthesize a term:

`arg0.query(■2)`

○ = Synthesis goal

Assign something  
to ■<sub>2</sub>

`arg0.query(■2)`

Reason over partial  
programs



# Inferring abstract values

## Finite abstract domains:

Types: Int, Str, DataFrame

## Infinite abstract domains:

### Solver-aided:

String Length: Linear integer arithmetic

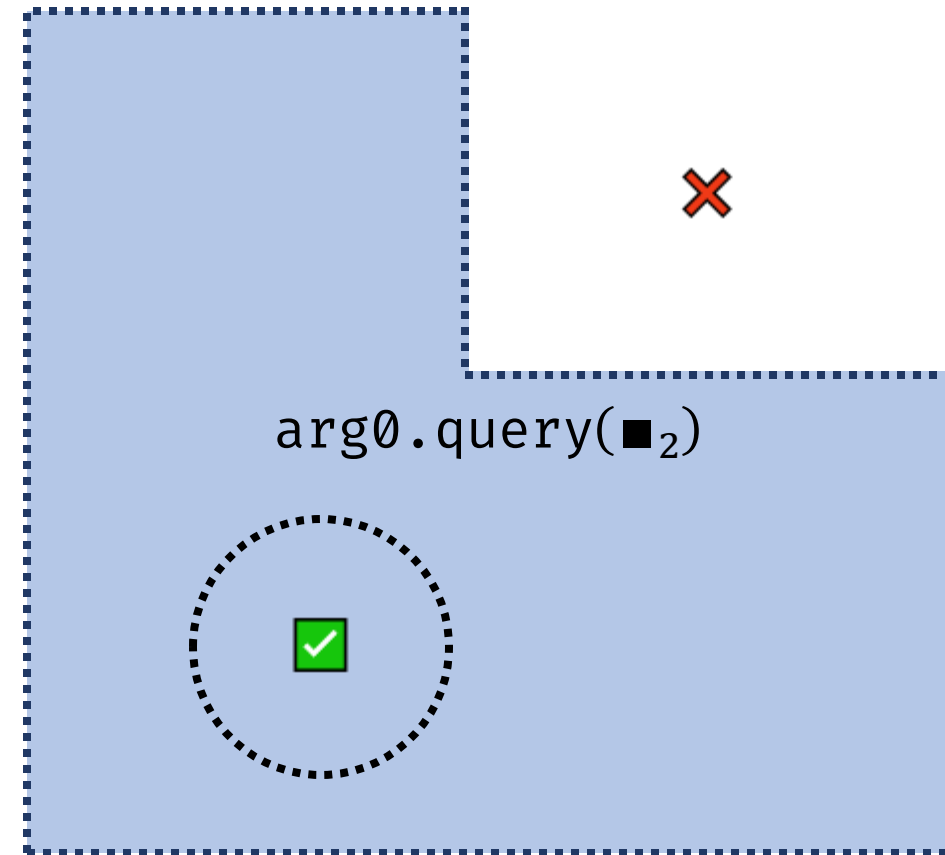
### Other:

Data frame columns

Enumerate through  
valid abstract values

Keep 1 hole symbolic  
and solve for it

Fall back to term  
enumeration





# Absynthe: Abstract Interpretation-Guided Synthesis

- Abstract domains are good at pruning search space
- Framework uses abstract interpreters as a parameter to guide search
- Abstractions for holes are inferred from abstract semantics
- Solves AutoPandas with simple abstract semantics without GPUs



<https://github.com/ngsankha/absynthe>