#5: Top-down Propagation

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EECS 700: Introduction to Program Synthesis

The problem statement

Behavioral constraints = examples

```
[1,4,7,2,0,6,9,2,5]  →  [1,2,4,7,0]
[0] → [0]
[5,1] → [1,5,0]
```

Search strategy?

Structural constraints = grammar

```
L ::= sort(L)  |  L[N..N]
   |  L + L  |  [N]  |  x
N ::= find(L,N)  |  0
```

Enumerative search

=

Explicit / Exhaustive Search

Idea: Enumerate programs from the grammar one by one and test them on the examples

```
L ::= sort(L)   |
      L[N..N]   |
      L + L     |
      [N]       |
      x
N ::= find(L,N) |
      0
```

bottom-up

```
x   0
sort(x)   x[0..0]   x + x   [0]
find(x,0)
sort(sort(x))   sort(x[0..0])
sort(x + x)   sort([0])
x[0..find(x,0)]   ...
```

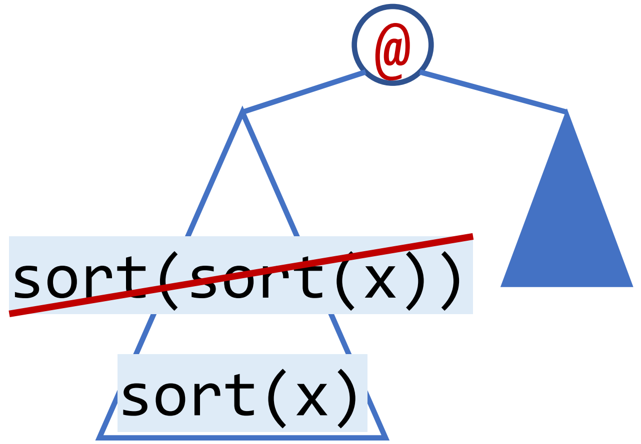
top-down

```
L
x   sort(L)   L[N..N]   L + L   [N]

sort(x)   sort(sort(L))   sort([N])
sort(L[N..N])   sort(L + L)
x[N..N]   (sort L)[N..N]   ...
```

When can we discard a subprogram?

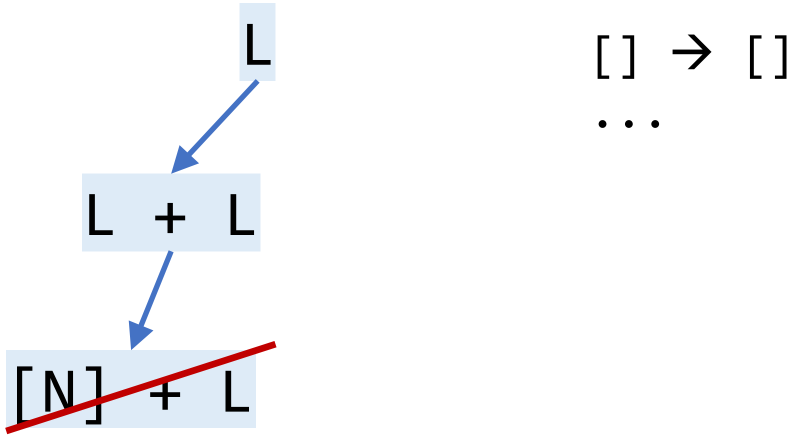
redundant



Equivalence reduction

(also: symmetry breaking)

infeasible



Top-down propagation

Top-down search: reminder

generates a lot of incomplete terms
only discards complete terms

need to reject hopeless programs early!

iter 0: `L`

iter 1: `x` `L[N..N]`

iter 2: `L[N..N]`

iter 3: `x[N..N]` `L[N..N][N..N]`

iter 4: `x[0..N]` `…)..N]` `L[N..N][N..N]`

iter 5: `x[0..0]` `x[0.. find(L,N)]` `x[find(L,N)..N]` `…`

iter 6: `x[0.. find(L,N)]` `x[find(L,N)..N]` `…` `…`

iter 7: `x[0.. find(x,N)]` `x[0.. find(L[N..N],N)]` `…` `…` `…`

iter 8: `x[0.. find(x,0)]` `x[0.. find(x,find(L,N))]` `…` `…` `…` `…`

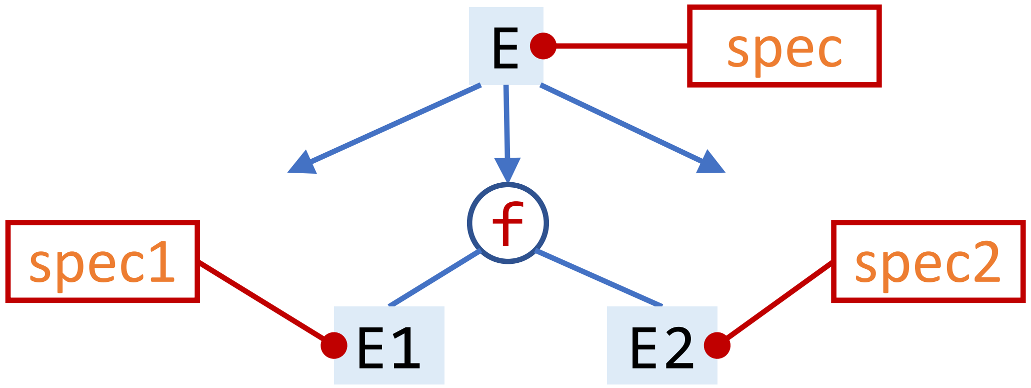
iter 9:

```
L ::= L[N..N]   |
      x
N ::= find(L,N) |
      0
```

`[[1,4,0,6] → [1,4]]`

Top-down propagation

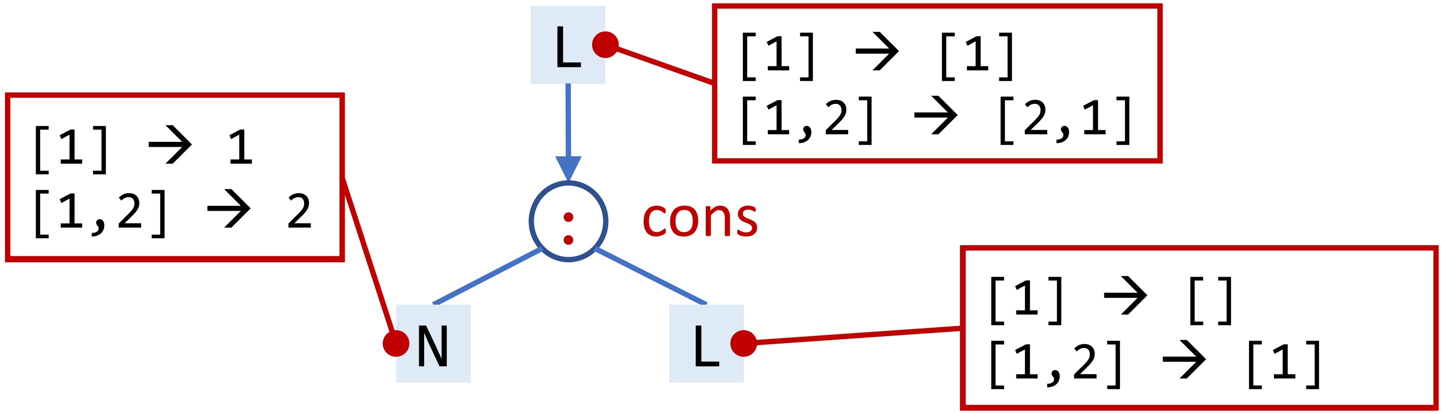
- **Idea:** once we pick the production, infer specs for subprograms



- If spec1 = ⊥ or spec2 = ⊥ discard f(E1,E2)!
- For now: spec = examples

When is TDP possible?

Depends on f!



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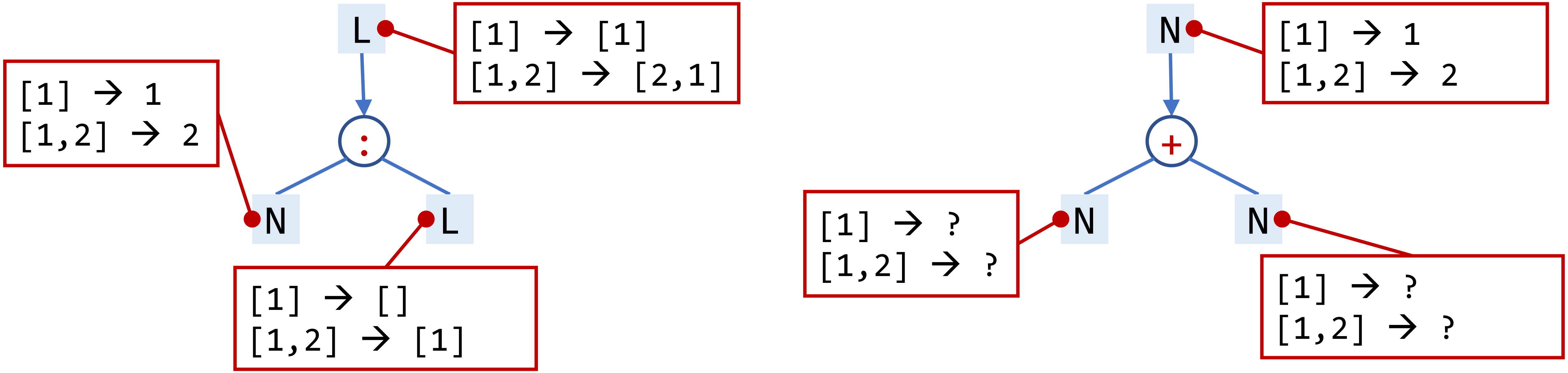
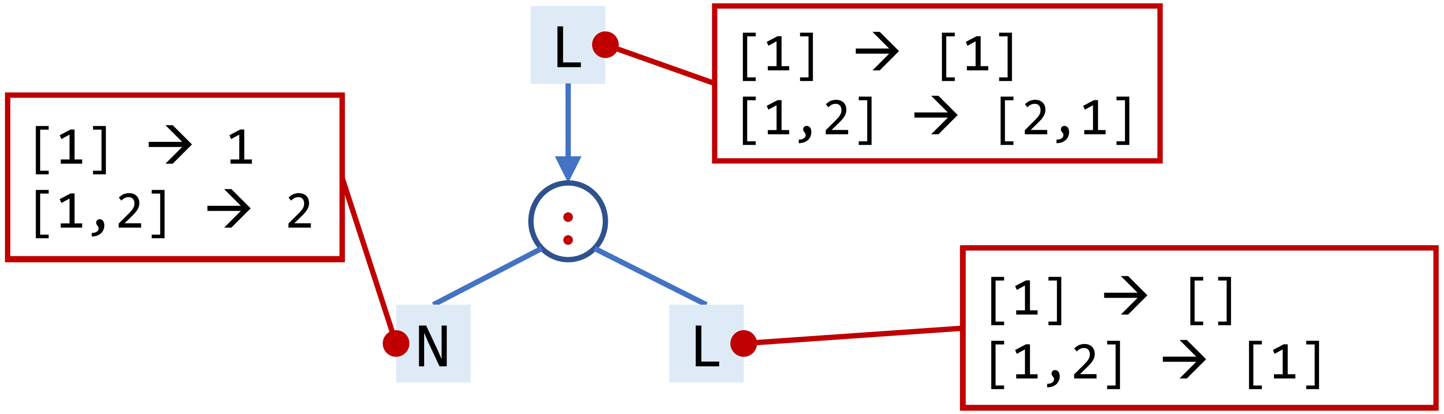


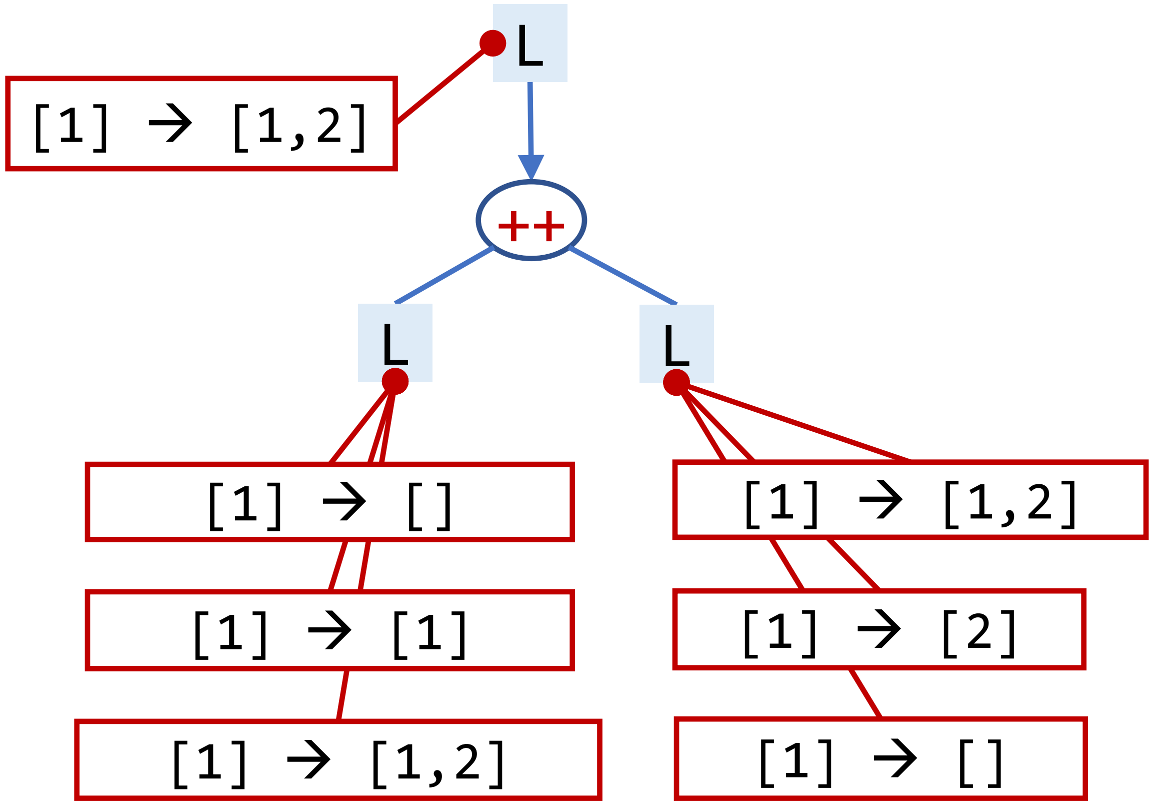
When is TDP possible?

- Depends on f!



- Works when the function is injective!
- **Q:** when would we infer ⊥? **A:** If at least one of the outputs is [ ]!

Something in between?

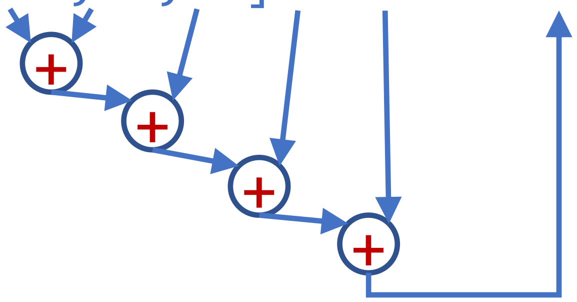


Works when the function has a "small inverse"

- or just the output examples have a small inverse

λ²: TDP for list combinators

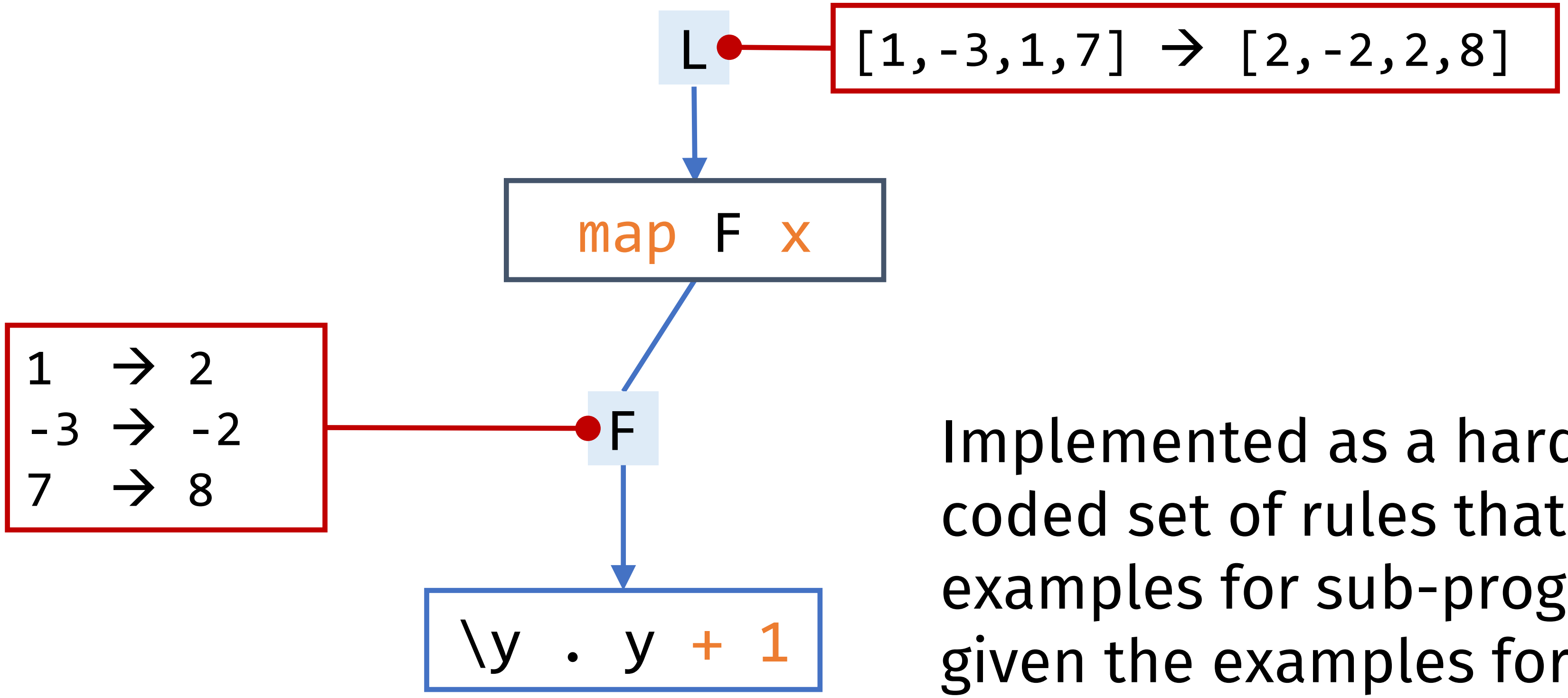
- `map f x` `map (\y . y + 1) [1, -3, 1, 7] → [2, -2, 2, 8]`
- `filter f x` `filter (\y . y > 0) [1, -3, 1, 7] → [1, 1, 7]`
- `fold f acc x` `fold (\y z . y + z) 0 [1, -3, 1, 7] → 6`



`fold (\y z . y + z) 0 [] → 0`

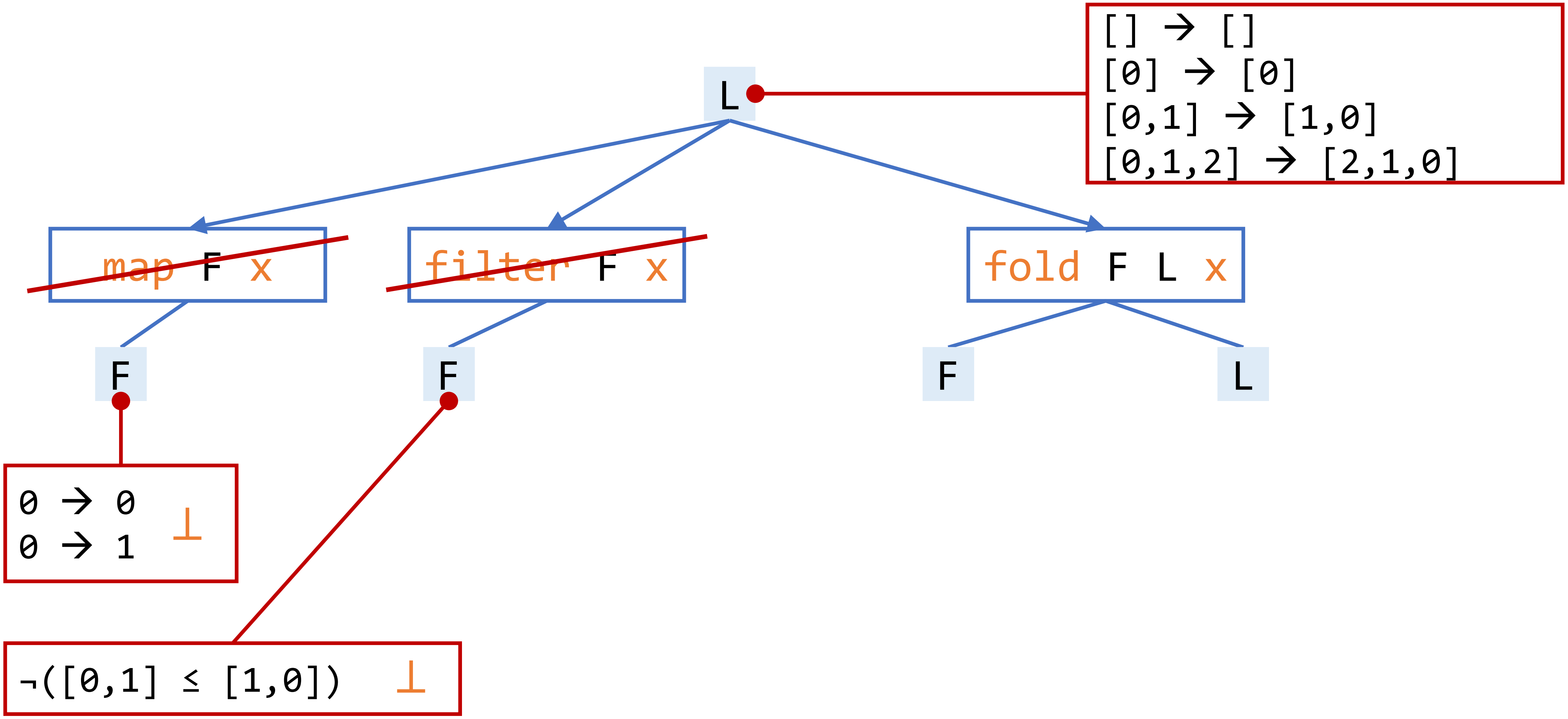
[Feser, Chaudhuri, Dillig ‘15]

λ²: TDP for list combinators

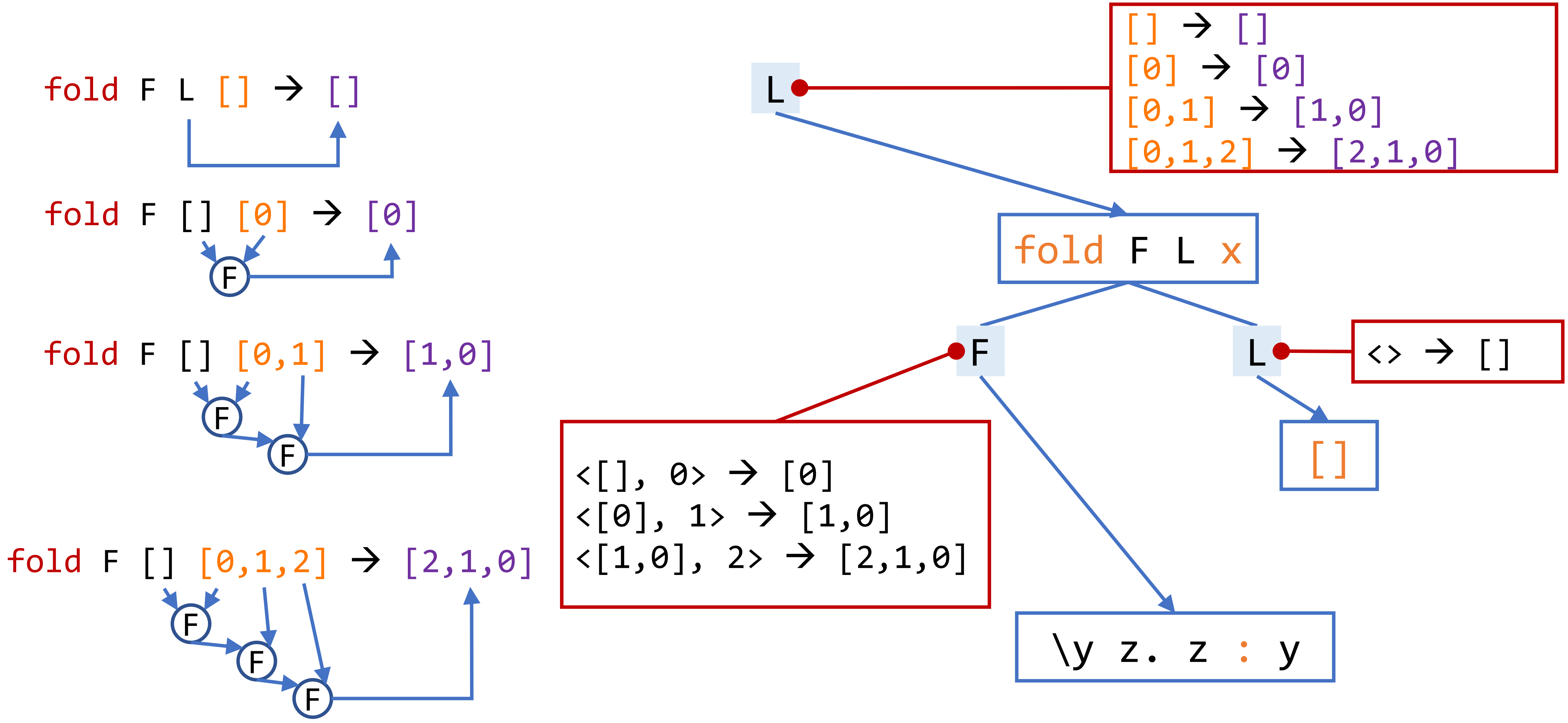


Implemented as a hard-coded set of rules that derive examples for sub-program(s) given the examples for the whole program and the combinator

λ²: TDP for list combinators



λ²: TDP for list combinators



Condition abduction

- Smart way to synthesize conditionals
- Used in many tools (under different names):
  - **FlashFill** [Gulwani ‘11]
  - **Escher** [Albarghouthi et al. ‘13]
  - **Leon** [Kneuss et al. ‘13]
  - **Synquid** [Polikarpova et al. ‘16]
  - **EUSolver** [Alur et al. ‘17]
- In fact, an instance of TDP!

Condition abduction