the fibonacci sequence is defined as follows:
$F_0 = 0;\ F_1 = 1;\ F_n + F_{n+1} = F_{n+2}$ for $n > 1$

We can write a power series:

$$f(x) = \sum_{n=0}^{\infty} F_n x^n$$

It's coefficients look like this: 0 1 1 2 3 5 8

Coefficient for:

$f(x)-x$	0	0	1	2	3	5	8
$x f(x)$	0	0	1	1	2	3	5
$x^2 f(x)$	0	0	0	1	1	2	3

$$f(x) - x = x f(x) + x^2 f(x)$$

so $$f(x) = \frac{x}{1 - x - x^2}$$

Minor technical point: This will only converge if:

$$|x| < \frac{\sqrt{5}-1}{2}$$

But we will only evaluate at 0

then we can take partial fractions

$$\left(\frac{S_0}{x - r_0} + \frac{S_1}{x - r_1}\right)$$

$$r_0, r_1 = \frac{-1 \pm \sqrt{5}}{2},\quad S_0, S_1 = \frac{-5 \pm \sqrt{5}}{10}$$

It is known that the nth derivative of $\frac{r}{x-s}$ is $(-1)^n n! \frac{r}{(x-s)^{n+1}}$

Recall Maclaurin series: $\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$

Since we broke it down into 2 simple fractions, now we can plug it into the formula $F_n = -\left(\frac{S_0}{r_0^{\,n+1}} + \frac{S_1}{r_1^{\,n+1}}\right)$

~~This is a formula~~

$$F_n = -\left[\frac{\left(\frac{-5+\sqrt{5}}{10}\right)}{\left(\frac{-1+\sqrt{5}}{2}\right)^{n+1}} + \frac{\frac{-5-\sqrt{5}}{10}}{\left(\frac{-1-\sqrt{5}}{2}\right)^{n+1}}\right]$$

First, replacing their reciprocals it with the familiar ϕ ~~r~~ and ψ
while: $\phi = \frac{1+\sqrt{5}}{2}$; $\psi = \frac{1-\sqrt{5}}{2}$

$$F_n = -\left(\left(\frac{-5+\sqrt{5}}{10}\right)\phi^{n+1} + \left(\frac{-5-\sqrt{5}}{10}\right)\psi^{n+1}\right)$$

APERO

Next Replacing to the power of n+1 with to the power of n

$$F_n = -\left(\left(\frac{-5+\sqrt{5}}{10}\phi\right)\phi^n + \left(\frac{-5-\sqrt{5}}{10}\psi\right)\psi^n\right)$$

$$F_n = \left(\left(\frac{1}{\sqrt{5}}\right)\phi^n + \left(\frac{-1}{\sqrt{5}}\right)\psi^n\right)$$

$$F_n = \frac{1}{\sqrt{5}}\left(\phi^n - \psi^n\right)$$