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T3 ________________	B	F3 ________________
T4 ________________		F4 ________________

2019
MCM/ICM
Summary Sheet

Urban economic vitality model

In order to build a suitable, reasonable and precise economic vitality model. Firstly, we chose Beijing as our research object and collected the related data in Beijing over the past four decades. Then we used Pearson coefficient and Chi-square test to identify the relationship between the various factors that may affect economic vitality. In addition, according to the relationship, we established enterprises vitality model and used genetic algorithm to find the most suitable value to further discuss how to improve economic vitality in the first question.

In the second question, we still chose Beijing as our research object and we used grey model to predict the result and we compared the prediction and actual value to get the short-term and long-term effects of economic policies.

In the third question, we considered the regional economic vitality from two angles which are inherent economic vitality and economic impact from surrounding cities. In order to figure out inherent economic vitality, we regarded some citiesâĂŹ index as input to establish a neural network model. In addition, we used economic vitality data from official statistics over the years as samples to train our neural network. Then considering the surrounding citiesâĂŹ impact, according to economic radiation principle, we calculated the variance between the economic vitality ranking given by the given economic radiation parameter value and the official standard economic ranking to obtain a nonlinear equation about the parameters of the economic radiation impact. Regarding the minimum value of the equation as the goal, to get the optimal economic radiation parameter. Trough the economic radiation parameter and the results of neural network, we calculated the economic vitality

Keywords: Nonlinear regression, Neural Network Model, Grey Model, Pearson Coefficient, Chi-square Test, Genetic Algorithm.

Contents

1 Introduction

1.1 Problem Background

In order to improve comprehensive competitiveness, various regions (or cities or provinces) have launched many policies. Economic vitality is a key factor in determining comprehensive competitiveness. If inappropriate policies are adopted, it will instead lead to a decline in regional economic vitality and regional competitiveness. Therefore, It has become apparent that we have to pay attention to how to formulate a consistent economic policy according to regional conditions.

1.2 Our work

1. We establish a relational model of factors affecting regional economic vitality to determine whether these factors interact.
2. We establish a mathematical model which analyzes and measures the regional (or urban or provincial) economic vitality, and compare the economic vitality of multiple regions
3. We propose effective policies to increase regional economic vitality

2 Preparation of the Models

2.1 Assumptions

It is difficult to accurately simulate a regional economic vitality model because of the complex the factors affecting regional economic vitality. In order to simplify the problem and to hold the correctness of results at the same time, we make the assumptions below:

1. We ignore the relational economic impact between different regions. We assume that each region (or city or province) is an independent individual, and the economic driving effect of neighboring regions is little in model of ecterprises' s vatility .
2. We ignore the country's economic control policies on a region (or city or province).
3. A city's inherent economic vitality is x ,and its degree of being affected economically by the surrounding cities is Y. The city's final economic vitality is expressed as X+kY

2.2 Notations

The primary notations used in this paper are listed in **Table 1**. There can be some other notations to be described in other parts of the paper.

Table 1: Notations

Symbol	Definition
P	The population
n	The Quantity of Newlyestablished Enterprises
S	The Quantity of Surviving Enterprises
k	The survival rate of Enterprises
H	The Regional Enterprises's vitality
r	The Registered Capital of Enterprise Entity
β	Pearson Correlation Coefficient
f	Degreeof Freedom
$\hat{x}_0$	estimated value
γ	Scale factor
ϕ	sigmod function
E	cost function

3 Relational Model of factors

In this problem,We chose Beijing as our research object. In order to complete the relational model. We collected the quantity of new enterprises , the survival rate and registered capital over the past two decades in Beijing

3.1 Construction of relational factors Model

From a practical point of view, the regional economic vitality is mainly affected by the two factors.They are population and regional enterprises's vitality respectively. The vitality of regional enterprises may be affected by variety factors, such as the quantity of enterprises, the survival rate of enterprises and the registered capital of enterprises. In this section,it is discussed that the factors affecting economic vitality are relevant or not. Besides, we establish a **enterprises's vitality model** to further discuss relationship of factors affecting regional economic vitality.

3.2 Two ways to judge the Correlation

3.2.1 Pearson coefficient

In statistics, the Pearson correlation coefficient,which is also known as the Pearson product-moment correlation coefficient (PPMCC or PCCs), is used to measure the correlation between two variables X and Y ,with values between -1 and 1.The reasons that it can be used in judging the relevanat are Covariance is an indicator that reflects the degree of correlation between two random variables. If one variable becomes larger or smaller at the same time, the covariance of the two variables is positive.To better represent the correlation, we normalize the covariance and get the Pearson coefficient

3.2.2 Chi-square test

Chi-square test is mainly for categorical variables, and its application in statistical inference of categorical data includes: Chi-square test for comparison of two rates or two constituent ratios; Chi-square test for comparison of multiple rates or multiple constituent ratios; and categorical data Correlation analysis, etc. Its principle is the chi-square test is the degree of deviation between the actual observed value and the theoretically inferred value of the statistical sample. The degree of deviation between the actual observed value and the theoretically inferred value determines the size of the chi-squared value.

3.3 Relational model

To solve the first problem, we still need to build a regional enterprise vitality model. we think the surival rate and the quantity of enterprise are vital to enterprise vitality model. In addition, we have chosen Beijing as research object. So it is unnecessary to consider about geographical factors. And we will focus on the relationship between surival rate and other factors, in order to build a better enterprise vitality model.

3.3.1 Surival rate and Quantity of Enterprises

Through the investigation of the data, we define a variable: the survival rate of the enterprise (S)to represent the probability that the established enterprise can survive in the past. It can be expressed as:

$$k = \frac{S}{n} \tag{1}$$

We use **Pearson coefficient** to analyze the correlation between the survival rate of enterprises and the quantity of enterprises.

Firstly, we caculated their Pearson coefficient :

$$E(x) = \mu \tag{2}$$

$$E(x) = \nu \tag{3}$$

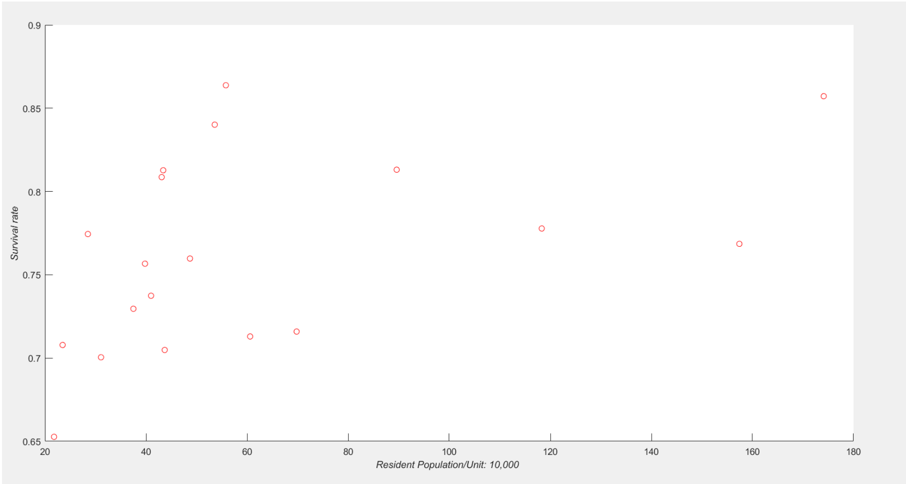

Figure 1: Enterprises's Quantity and Survival Rate

$$cov(X, Y) = E((x - \mu)(Y - \nu)) = E(XY) - \mu\nu \tag{4}$$

$$\beta_{x \cdot y} = \frac{cov(X, Y)}{\sigma_x \sigma_y} = \frac{E((x - \mu)(Y - \nu))}{\sigma_x \sigma_y} \tag{5}$$

Through these formulas,we got the Pearson coefficient which was 0.4539.However,according this result, it was hard to say they are relevant or not. Through analyzing **Figure 1** ,we found that the data seems take on an upward trend before about 90 and a downward trend after about 90. And this fluctuation can disturb Pearson coefficient. So we caculated the Pearson coefficient before 90 and after 90 respectively. Their results are 0.7769 and -0.9154. According to these reults, we can conclude that there is a relationship between Enterprises's Quantity and Survival Rate.

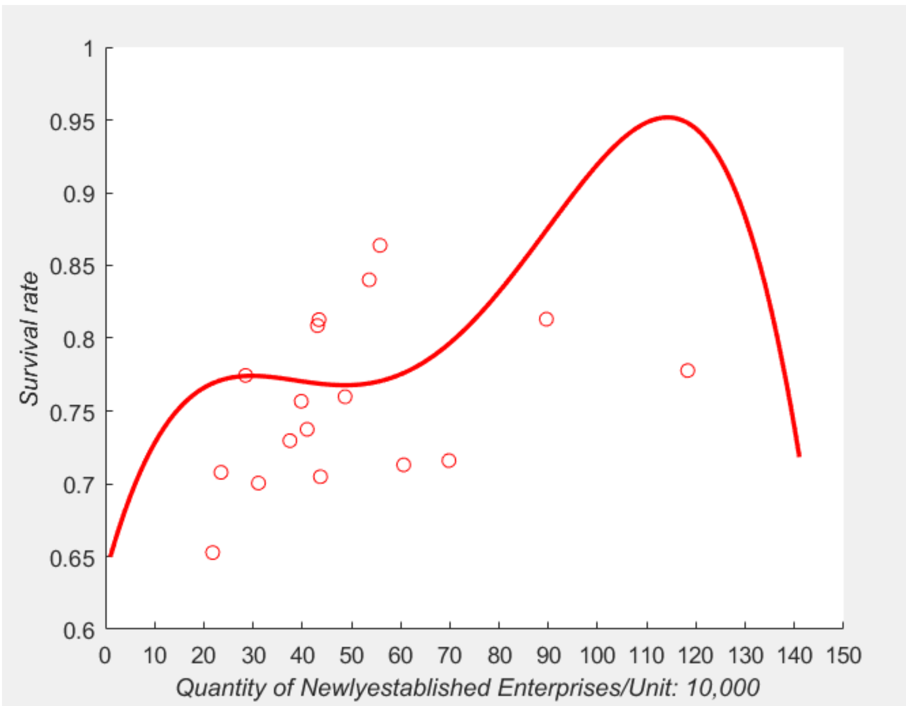

Figure 2: Enterprises's Quantity and Survival Rate

As disscussed, **k** and **n** are related, and we can get functions between the quantity of enterprises and the survival rate of enterprises:

$$k = -1.8935\times10^{-8}S^4+6.2997\times10^{-6}S^3-7.1149\times10^{-4}S^2+3.3223\times10^{-2}S+0.22249 \quad (6)$$

3.3.2 population quantity and surive rate

not decided city and city population

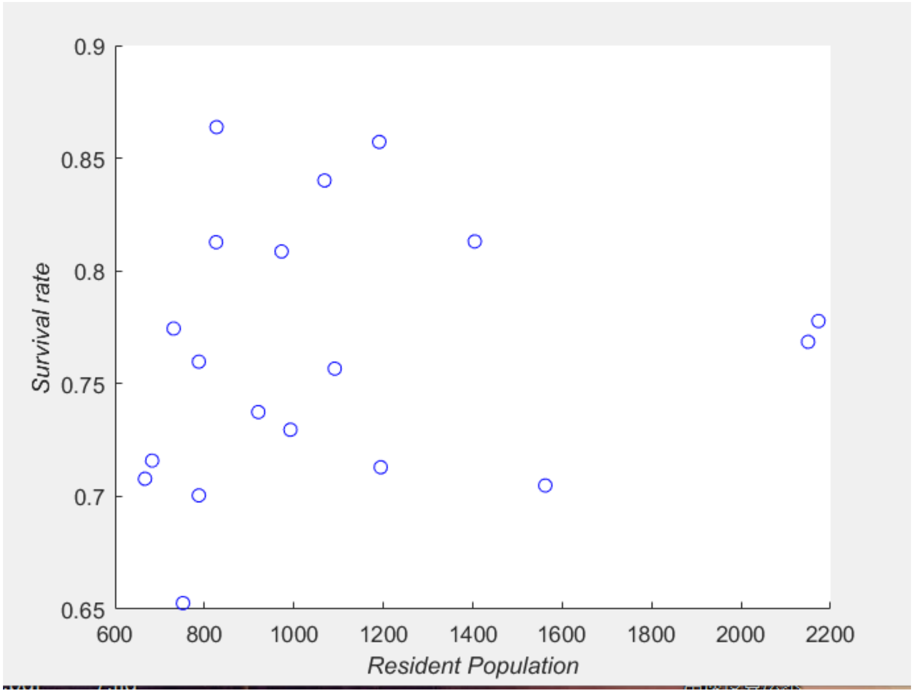

Figure 3: Regional population and Survival Rate of Enterprises

The Pearson coefficient between population quantity and surive is only 0.159. According to the figure and Pearson coefficient, we can judge they are not relevant.

3.3.3 registered capital and quantity of newly-established enterprise

In order to figure out whether registered capital and surival rate are relevant or not. We divided the quantity of newly-established enterprise into two parts. One is smaller than 80, the other on is larger than 80. And to judge whether the quantity of newly-established enterprise has a relationship whith registered capital.In this problem, the newly-established enterprise variable belongs to categorical variables so,we use Chi-square test to judge.
Firstly calculate Chi-Square Statistics:

$$\chi^2 = \sum_{i=1}^{\gamma}\sum_{j=1}^{\zeta}\frac{(\psi_{i,j}-E_{i,j})^2}{E_{i,j}} \quad (7)$$

Then calculate degrees of freedom:

$$f = (r-1)(c-1)df = (r-1)(c-1) \quad (8)$$

At the last, we can get the result that whether they are relevant through comparing the critical value andχ^2.

We presuppose *H0*: the registered capital and the number of new enterprises are independent Through the above statistical analysis of the data, we obtained significant values and chi-square:

$$\chi^2 = 0.4046 \tag{9}$$

$$p = 0.9821 \tag{10}$$

The degree of freedom is 4, and the significant value *p* is specified as$p(\chi^2 > 9.488)$=0.05 . However,the actual chi-square value is less than 9.488. Consequently,we accept the assumption H0, and draw the conclusion that the two are independent of each other.

3.4 economic vitality

3.4.1 enterprise vitality

In econometrics, regression analysis is used to study the relationship between the explanatory variables and the explanatory variables. In this section, we construct a linear regression model to simply measure the regional enterprises vitality. It can be expressed as:

$$H = k \times 100 + n \tag{11}$$

3.4.2 economic vitality change from enterprise vitality and population

Section 3.1 gives a model of the relationship between factors affecting regional economic vitality. We can draw the following conclusions:

- The size of the population is not directly related to the quantity of regional enterprises
- Enhancing the vitality of regional enterprises helps increase the vitality of the regional economy

Therefore, with attempt to prove regional economic vitality, effort needs to be made to reach the maximum of regional business vitality.

Referring to the survey data in Annex 1 and models built before, we have plotted **Figure 4** based on the general trend of population and economic vitality.

As shown in Figure 5, is can be seen that he cities with larger populations have more active economic development.

Figure ?? sketches these relationship.

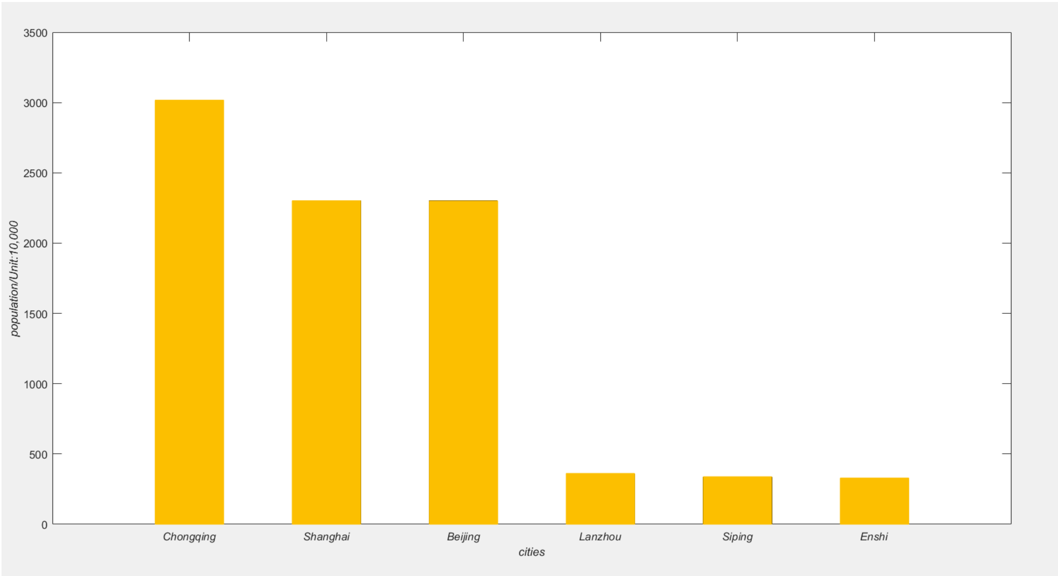

Figure 4: population and economic vitality

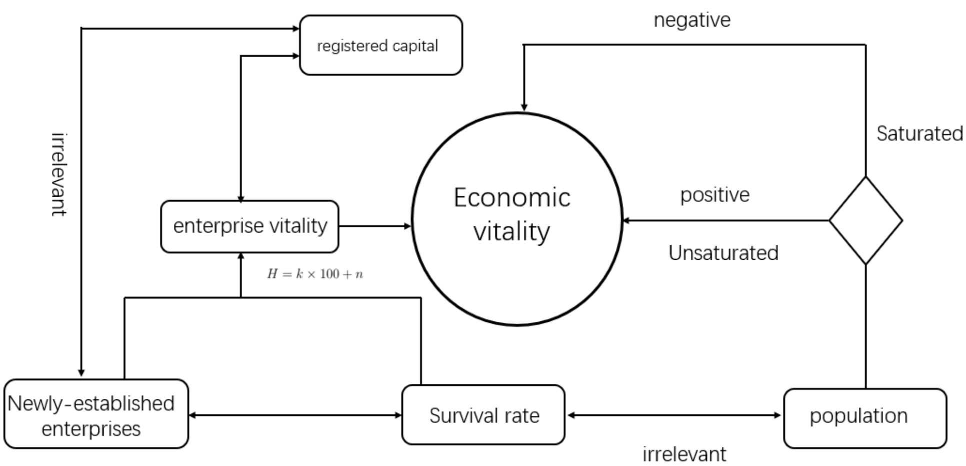

Figure 5: Relationship Model

3.5 how to improve economic vitality

Through section 3.4, we knew that if we want to improve economic vitality,we have to improve enterprise vitality and control popoulation. And we already fitted the function of enterprise vitality. So, in order to get the better enterprise vitality, we use **Genetic algorithm** to find the optimal solution. Our operations can be divided into four steps:

(1).**Initialization**:Independent variable(quantity of newly-established enterprises) is between 0 and 200.So we use eight-bit binary code to represent the quantity

(2).**Selection operation**:Our target is to get the max enterprise vitality. So, we according to the value of enterprise vitality to choose the better individual. If the value is lareger,it is more likely to be passed on to the next generation.

$$p_i = f_i \sum_{j=1}^{N} f_j \tag{12}$$

(3).**Cross operation**:Two pairs of chromosomes are exchanged with each other in some way according to the probability of crossover, thus forming two new individuals.It can generate much more kinds of individuals

(4).**Mutation operation**:Crossover operation refers to the exchange of some genes of two paired chromosomes according to the crossover probability in a certain way, thus forming two new individuals.

Finally, we found the optimal solution. When the quantity of newly-established enterprises is 100, the enterprise vitality can get its max, which is equal to **183.6**. However, the quantity of newly-established enterprises in Beijing had exceeded this number. Therefore, in order to improve enterprise vitality, we suggest the government to limit the quantity of newly-established enterprises until it reduces into about 100.

4 Effects of Economic Policies Transformation

Regions usually adopt economic policies for macroeconomic regulation. Economic policies have different effects on regional economic vitality in the short and long term. We chose Beijing and analyzed the impact of the economic policies introduced on the changes of economic vitality that followed. We believe that the impact of economic policies on regional economic vitality is expressed in the following ways: *population*, *regional GDP*, *regional GDP growth rate*, *number of new enterprises*, and *regional per capita GDP*. **Grey models** are based on a small amount of incomplete information to establish a grey differential prediction model to make a vague long-term description of the development law of things. In this section, we use the gray model to evaluate and predict the impact of economic policies through the above methods.

4.1 Construction of Grey Models

It is noticed that in 2005, the Beijing Municipal Government promoted several economic policies to stimulate economic vitality. Since 2005, Beijing has accelerated investment transfer, shifting fifty percents of government investment to the suburbs.Compared to last year, an sixty-one percents increase of investment was accomplished. The goverment arrange invested 500 million yuan in industry, actively promoted industrial restructuring, and increased support for modern manufacturing.Consequently, we chose **2005** as a key point.

We divide the collected data into eight sequences according to every five years. we recorded the data from 1970 to 2005 as matrix $A_7 \times_5$
(1).Caculate the average of every five years:

$$x_0(i) = \frac{1}{5}\sum_{j=1}^{5} a_{i,j} \qquad i = 1, 2, \cdots, 7 \tag{13}$$

(2).Acummulate generating operation :

$$x_1(k) = \sum_{i=1}^{k} x_0(i) \qquad i = 1, 2, \cdots, 7 \tag{14}$$

(3).Generate mean series:

$$z_1(k) = \alpha x_1(k) + (1-\alpha)x_1(k-1) \quad k = 2, 3, ..., 7 \quad \alpha = 0.4 \tag{15}$$

(4).Establishe grey Differential Equations:

$$x_0(k) + az_1(k) = b \qquad k = 2, 3, ..., 7 \tag{16}$$

(5).Corresponding **GM(1,1)**:

$$\frac{dx_1}{dt} + ax_1(t) = b \tag{17}$$

(6).Solve Differential Equations,get the value of a and b
(7).Discrete solutions for **GM(1,1)**:

$$\hat{x}_0(k) = [x_0(1) - \frac{b}{a}]exp(-\alpha(k-1)) + \frac{a}{b} \tag{18}$$

(8).Generate prediction model:

$$\hat{x}_0(k) = \hat{x}_1(k) - \hat{x}_1(k-1) \tag{19}$$

$$\gamma_j = \frac{\sum_{i=1}^{7}}{\sum_{i=1}^{6}\sum_{j=1}^{5} a_{ij}} \tag{20}$$

(9).let k =8,Get predictions

comment:*Equation 19 represents the average of the observed data from 2005 to 2009, and Equation 20 represents the proportion of statistical data from 2005 to 2009. Combining these two formulas, we can obtain the estimated values of the observed data for the five years separately.*

4.2 Analysis of Short Effects

Figure 6 − **8** give results of Grey model based on different factors. From 2006 to

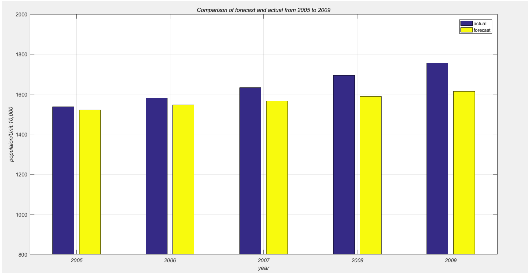

Figure 6: Comparison of predicted and estimated population

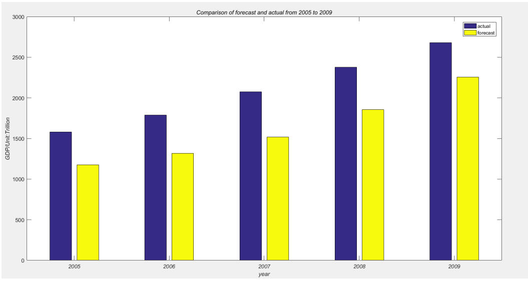

Figure 7: Comparison of predicted and estimated GDP

2007, Accordin to **Figure 6**, the gap between before and after the implementation of economic policy was not obvious. Similarly, from the data on **Figure 7**, after the transformation of economic policies, GDP has increased. However, there wa not significant improvement occurred. As shown in **Figure 8**, The predicted numbers of newly estab-

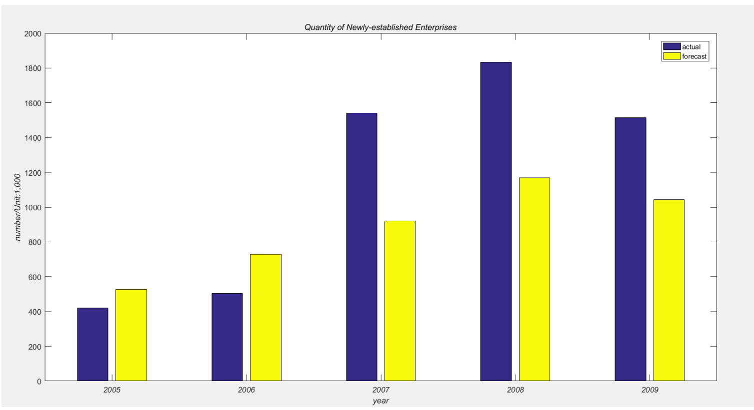

Figure 8: Comparison of predicted and estimated quantity of newly established enterprises

lished enterprises in 2006 and 2007 are higher than the actual values, indicating that the transformation of economic policies has not promoted entrepreneurship in past two years.

In general, many of the policies implemented in 2005 did not have effects on improvement of economic vitality in the short term. The overall situation of economic operation has not changed much.

4.3 Analysis of Long Effects

According to the data, in 2009, whether population or GDP, it can be seen that a clear gap existed between actual data and forecast data. Four or five years later, we can see that the growth rate of the number of new enterprises has increased significantly.

Truly, it is believed that transformation of economic policies have caused a short-term economic downturn; once new momentum is formed, it may lead to long-term continuous growth. Early twentieth century was the key node of economic development. In 2005, Beijing promoted economic policy reform and proposed to strengthen macroeconomic regulation and control. The reform did not stimulate the enthusiasm of actors in the society in the short term, and even a slight decline in corporate investment appeared.

The long-term effects of institutional innovation usually do not appear in a short period of time, which makes economic pressure greater in the short term. But this process helped to gradually bring out the opportunities and advantages that exist in the medium and long term. The government has increased investment in basic industries, encouraged corporate reforms, which cultivated long-term advantages.

5 economic vitality model

Measuring the economic vitality of a city is a complicated matter, In addition, it needs to consider the inherent economic level of a city itself, as well as the impact of surrounding cities and geographical locations.

Assumption: A city's inherent economic vitality is x ,and a cityâĂŹs degree of being affected economically by the surrounding cities is $\boldsymbol{Y}$. The city's final economic vitality is expressed as $\boldsymbol{X}+\boldsymbol{kY}$

5.1 inherent economic vitality

After reviewing *Index System of Urban Economic Vitality*, we knew that we can evaluate the inherent ecnonomic vitality from **Economic growth, attractiveness to capital, production factors, employment, quality of life of residents, innovative abilit,** etc. As a result, to build the right mode, we measured the inherent economic vitality of a city from several angles:Population, GDP, per capita income, urban innovation capacity.

5.1.1 BP neural network model

We estimated the inherent economic vitality of a city based on the BP neural network model, as shown in **Figure 9**:

We quantify the Population, GDP, per capita income, urban innovation capacity of a city and input them into the neural network as an input signal. Due to the limited

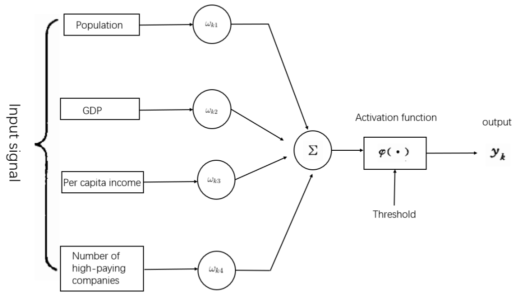

Figure 9: network construction

amount of data and in order to avoid overfitting, the number of hidden layers should not be over 4, Therefore we chose 2 layers.

We divide cities into three levels based on experience: "international first-tier cities", "domestic cities", and "small cities in china". Each group used two cities as the representative for fitting, which are"Beijing, Shenzheng", "Nanjing, Qingdao" and"Kunming, Zhengzhou". The city's inherent economic vitality and various indicators are obviously not purely linear combinations. In order to reduce the non-linearity of the given index and the inherent economic vitality, we choose the better-effect **sigmoid function** as the activation function.

$$\phi(\nu) = \frac{1}{1 + exp(-\alpha\nu)} \tag{21}$$

The output of each layer is:

$$H_j^s = \phi(\bar{h}_j^s) = \phi(\sum_{k=1}^{2} \bar{W}_{jk} I_k^s) \tag{22}$$

The final output of the two hidden layers is:

$$H = \phi(h) = \phi(\sum WI) \tag{23}$$

ϕ is the activation function, I is the input indicators, w is the weight of the layer. At the same time, we use the data collected by the city's inherent economic vitality level as a sample of the bp neural network to learn from the sample. we use:

$$E = \frac{\sum_i (t_i + O_i)^2}{2} \tag{24}$$

as a cost function to perform gradient descent. After the gradient drops to a prescribed threshold or the value of the loss function falls to a certain value, a model for judging the inherent economic vitality of a city is obtained. In order to fit the sample better, we set the loss function threshold to 0.000000001. The loss function and gradient descent are as follows:

5.2 economic impact from surrounding cities

According to economic principles, "urban agglomerations" are often formed around first-tier cities, and the central city radiates to surrounding cities, forming an economic

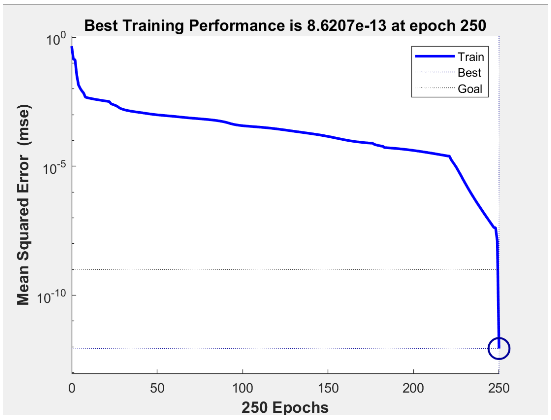

Figure 10: Cost Function

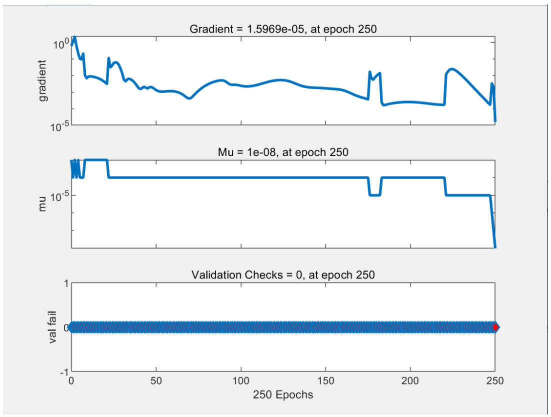

Figure 11: Gradient Discent Function

pulling effect. This is of great help to the economic vitality of surrounding cities, so when considering economic vitality, the economic radiation effect should be taken into account.

With reference to the geographical location of each city, we can classify the urban radiation effect into three regions: North China with Beijing as the center, East China with Shanghai as the center, South China with Guangzhou as the center. Economic centers as the radiation source. Radiation is performed within a range of 500 km. The relationships between the irradiated city and the radiation source are as follows: Reffering to date in Appendix 3, using the followed formula

$$R = \sum_{i=1}^{n} C_i C'_{ij} W_{ij} \tag{25}$$

, the economic radiation intensity of the three major cities is:Beijing=0.6689 SHanghai=0.9011 Guangzhou=0.6816

According to the formula of economic radiation field strength:$M = \frac{r}{l^2}$,we caculated the economic impact of every city in attachment 3,

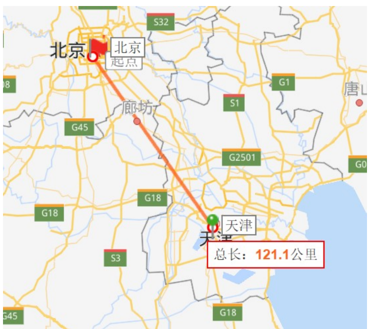

Figure 12: Beijing

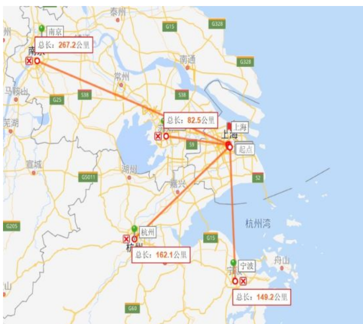

Figure 13: Shanghai

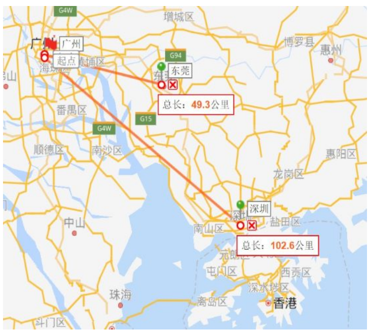

Figure 14: Guangzhou

Tianjing: 0.457×k
Dongwan: 2.72×k
Shengzhen: 0.642×k
Suzhou: 1.232×k
Ningbo: 0.405×k
Hangzhou: 0.329×k
Nanjing: 0.125×k

For the reason that k is an unknown parameter. We use bit variance:

$$LSD(A,B) = \frac{1}{n}\sum_{i=0}^{n-1}(i|A| - i|B|)^2 \tag{26}$$

to construct a function about *k*:

$$\frac{1}{n}\sum_{i=1}^{n}(\frac{1}{2}(\sum_{j=i(i\neq j)}^{n}\frac{(A_i - A_j) + k(B_i - B_j)}{|(A_i - A_j) + k(B_i - B_j)|} + 1) - Ci)^2 \tag{27}$$

Then, use bit variance :

$$LSD(A,B) = \frac{1}{n}\sum_{i=0}^{n-1}(i|A| - i|B|)^2 \tag{28}$$

to construct a function about *k*

5.3 economic vitality

Using the formula:X+kY,we can get two parts of economic vitality ranking, which are added in Appendix 2. Combining these two parts, we can get the final ranking as follows in **Figure 15**:

6 Conclusion

In this study,Firstly,we build a model of enterprise' s vitality to discuss the relationship It directly reflects the relationship between the nquantity of enterprises and the vitality of regional enterprises' s vatility.Then we construct Grey Model to evaluate effects of ecnomic transformation in short term and long term. Finally, we construct the Regional Economic Vitality Model through Network Method to analyze and mearsure the regional economic vitality.

6.1 Strategy

Through the construction of above models, we can roughly establish indicators to measure regional (or urban)economic vitality. The economic vitality model we have

1	Shenzhen	98. 2482
2	Guangzhou	96. 7657
3	Beijing	96. 6839
4	Shanghai	96. 6829
5	Wuhan	96. 6789
6	Tianjin	96. 6572
7	Suzhou	96. 5604
8	Qingdao	94. 4535
9	Dongguan	94. 084
10	Nanjing	92. 5636
11	Hangzhou	92. 3456
12	Chongqing	91. 7869
13	Zhengzhou	91. 4704
14	Xian	91. 0368
15	Chengdu	90. 6859
16	Ningbo	90. 1538
17	Changsha	88. 6444
18	Shenyang	86. 1672
19	Kunming	82. 7269

Figure 15: Final Economic Vitality Ranking

established mainly depends on the factors, such as regional population, the number of regional enterprises, and the survival rate of regional enterprises. Therefore, in order to increase the economic vitality as much as possible, we hope that the above factors can exert the best effect.

Due to the mutual influence of economic vitality models in different regions, when specifying strategies to stimulate economic vitality, we also need to consider the economic development of surrounding cities to ensure balanced regional development and long-term economic stability.

Different methods may show different ranks of effect. We suggest that government can choose and adjust the strategy based on the rules below:

Rule I *Regional economic vitality can be increased by attracting talents and increasing the population. However, as the population approaches saturation, excessive population will place a burden on urban development, restrict enterprises' investment in capital, and slow down the growth of regional economy.*

Rule II *Increasing the number of companies can help stimulate economic vitality. Encouraging innovation and entrepreneurship in the region and increasing corporate dividends will help increase employment opportunities and enhance the vitality of regional enterprises. But at the same time, it leads to low survival rates. How to regulate the number of companies and maintain market stability is the key point.*

Rule III *Regional economy is conducive to resource adjustment and economic symbiosis. Promoting the economic development of surrounding cities is also a strategy to adjust the pace of economic development in the region.*

6.2 Suggestions for Beijing's Economic Development

During the three decades of reform and opening up, Beijing's economy has undergone tremendous changes. As can be seen from Section 4, the long-term advantages have become increasingly apparent. As the capital, Beijing is the center of political, cultural and economic exchanges. Although the economic growth is fast, with the constraints of population and resource allocation, Beijing's economic development also faces severe challenges.

According to the conclusions drawn from Question 2, we provide the development proposal for Beijing from the following aspects:

- **Control the urban population**. An overly fast-growing population is burdening resource allocation. Introduce a centrifugal mechanism and strictly regulate labor to achieve the result that labor naturally flows out through market competition.
- **Increase investment in surrounding areas**. The focus of investment can be shifted to the suburbs or surrounding provinces and cities such as Hebei Province. This will alleviate the supply pressure in Beijing's city center and ease economic tension.
- **Encourage companies to move focus to surrounding cities**. Currently, as the quantity of enterprises in Beijing is oversaturated, encouraging large enterprises to relocate their centers can have the effect of restoring market balance.

7 Strength and Weakness

7.1 Strengths

- The model in section **5** can determine an measure indicator of regional economic vitality.For the reason that the impact of surrounding cities is included, the data is more accurate. Through calculation, we can compare the economic vitality of different cities and give a ranking.
- We can simply calculate the number of companies in a certain region when economic vitality is highest. Therefore, the model can assist in formulating strategies to regulate economic markets and promote economic development.
- Our model is highly practical and flexible.For example,we used genetic algorithm to find the most suitable value and neural network to caculate the economic vitality.So people can change their functions or input of network according to their needs, and our model can stay work rightly.

- We used offical data ,such as offical rank ,to train our neural network. So finally, we can get very accurate results.

7.2 Weaknesses

- The model measures the economic vitality of different regions. However, the influencing factors are too complicated. we cannot be sure that economic development will not be interfered by other policy factors in the realization of a certain policy.
- Some parameters in our model (such as the thermal conductivity between two materials) are adjusted several times in the MATLAB for there is no research that give these parameters certain values. The value of these parameters in our model may have some deprivation from the real value.
- Due to the difficulty of data surveys in recent years, the data we choose may not be representative, and there will be some deviations from the current development situation.

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Apendix 1: Population and Economic Vitality

排名	城市或地区	人口（万人）	
		2010年	2018年
1	重庆市	3017.42	3101.79 ↑
2	上海市	2301.91	2423.78↑
3	北京市	2300.24	2154.2 ↓
4	成都市	1404.76	1633 ↑
5	天津市	1293.82	1559.60↑
6	广州市	1270.08	1449.84↑（2017年）
7↑	深圳市	1035.79	1302.66↑
8↑	武汉市	910	1108↑

327	海南州	44.17
328	迪庆州	40.00
329	克拉玛依市	39.10
330	玉树州	37.84
331	山南地区	32.90
332	海北州	27.33
333	黄南州	25.67
334	嘉峪关市	23.19
335	阿拉善盟	23.13
336	杨凌示范区	20.12
337	林芝地区	19.51
338	果洛州	18.17
339	阿里地区	9.55

Apendix 2 : Rankings

1	Shanghai	96.6829
2	Shenzhen	96.9642
3	Beijing	96.6839
4	Guangzhou	96.7657
5	Chongqing	91.7869
6	Chengdu	90.6859
7	Nanjing	92.3136
8	Hangzhou	91.6876
9	Suzhou	95.0964
10	Tianjin	95.7432
11	Qingdao	94.4535
12	Dongguan	88.9916
13	Zhengzhou	91.4704
14	Wuhan	96.6789
15	Xian	91.0368
16	Ningbo	89.3438
17	Changsha	88.6444
18	Shenyang	86.1672
19	Kunming	82.7269

1	Dongguan	$2.72 \times k$
2	Suzhou	$1.232 \times k$
3	Shenzhen	$0.642 \times k$
4	Tianjin	$0.457 \times k$
5	Ningno	$0.405 \times k$
6	Hangzhou	$0.329 \times k$
7	Nanjing	$0.125 \times k$

Apendix 3

(一) 城市经济辐射力评价指标权重的赋值

本文采用层次分析法和直接赋权相结合的方法对城市经济辐射力指标体系各项指标进行赋权 (见表2)。

表2　中心城市经济辐射力指标体系、权重及2011年北京、上海、广州三大城市指标

一级指标	二级指标	北京	上海	广州
辐射源指标 0.6333)	GDP总量 (亿元) 0.20)	16 251.93	19 195.69	12 423.43
	第二产业比重 (%) 0.15)	23.09	41.30	36.84
	规模企业数 (个) 0.10)	3749	9962	4437
	城市人口数 (万人) 0.15)	1277.9	1419.4	814.6
	人均GDP (元) 0.15)	81 658	82 560	97 588
	土地面积 (平方公里) 0.10)	16 411	6340	7434
	人口密度 (人/平方公里) 0.15)	778.70	2238.74	1095.75
辐射通道指标 0.1062)	每天进出列车次数 0.20)	796	777	666
	每平方公里铁路里程公里数 0.15)	0.065	0.071	0.222
	每平方公里公路里程公里数 0.30)	1.30	1.91	1.21
	每平方公里高速里程公里数 0.15)	0.05	0.13	0.09
	人均民用汽车保有量 (辆) 0.10)	0.37	0.23	0.23
	固定及移动电话用户数 (万户) 0.10)	2575.90	2620.11	2566.93
辐射流指标 0.2605)	社会零售总额 (亿元) 0.20)	6900.32	6814.80	5243.00
	客流量 (万人) 0.25)	145 733	17 755	67 756
	货流量 (万吨) 0.25)	24 788	93 135	64 132
	利用FDI (万美元) 0.10)	705 447	1260 055	427 000
	进出口贸易额 (亿美元) 0.10)	3384.9	4374.36	1161.72
	在校大学生人数 (万人) 0.10)	57.9	51.13	89.61

数据来源:北京、上海、广州三市2012年国民经济和社会发展统计公报及2012年《中国城市统计年鉴》。