Lecture 12

Hypothesis Testing

Outline

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- 9-3 Large Sample Mean Test
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9-1 Introduction

- Hypothesis testing is a decision-making process for evaluating claims about a population.
- We must define the population under study,
- state the particular hypotheses that will be investigated,
- give the significance level,
- select a sample from the population,
- collect the data,
- perform the calculations required for the statistical test,
- reach a conclusion.

9-2 Steps in Hypothesis Testing

- A **Statistical hypothesis** is a conjecture about a population parameter. This conjecture may or may not be true.
- The **null hypothesis**, symbolized by H_0, is a statistical hypothesis that states that there is no difference between a parameter and a specific value or that there is no difference between two parameters.

9-2 Steps in Hypothesis Testing

- The **alternative hypothesis**, symbolized by H_1, is a statistical hypothesis that states a specific difference between a parameter and a specific value or states that there is a difference between two parameters.

9-2 Steps in Hypothesis Testing - Example

- A medical researcher is interested in finding out whether a new medication will have any undesirable side effects. The researcher is particularly concerned with the pulse rate of the patients who take the medication.

9-2 Steps in Hypothesis Testing - Example

- What are the hypotheses to test whether the pulse rate will be different from the mean pulse rate of 82 beats per minute?
- H_0: $\mu = 82$ H_1: $\mu \neq 82$
- This is a two-tailed test.

9-2 Steps in Hypothesis Testing - Example

- A chemist invents an additive to increase the life of an automobile battery. If the mean lifetime of the battery is 36 months, then his hypotheses are
- H_0: $\mu \leq 36$ H_1: $\mu > 36$
- This is a right-tailed test.

9-2 Steps in Hypothesis Testing - Example

- A contractor wishes to lower heating bills by using a special type of insulation in houses. If the average of the monthly heating bills is $78, her hypotheses about heating costs will be
- H_0: $\mu \geq \$78$ H_0: $\mu < \$78$
- This is a **left-tailed** test.

9-2 Steps in Hypothesis Testing

- A **statistical test** uses the data obtained from a sample to make a decision about whether or not the null hypothesis should be rejected.

9-2 Steps in Hypothesis Testing

- The numerical value obtained from a statistical test is called the **test value**.
- In the hypothesis-testing situation, there are four possible outcomes.

9-2 Steps in Hypothesis Testing

- In reality, the null hypothesis may or may not be true, and a decision is made to reject or not to reject it on the basis of the data obtained from a sample.

9-2 Steps in Hypothesis Testing

	H_0 True	H_0 False
Reject H_0	Error Type I	Correct decision
Do not reject H_0	Correct decision	Error Type II

9-2 Steps in Hypothesis Testing

- A **type I error** occurs if one rejects the null hypothesis when it is true.
- A **type II error** occurs if one does not reject the null hypothesis when it is false.

9-2 Steps in Hypothesis Testing

- The **level of significance** is the maximum probability of committing a type I error. This probability is symbolized by α (Greek letter alpha). That is, $P(\text{type I error}) = \alpha$.
- $P(\text{type II error}) = \beta$ (Greek letter beta).

9-2 Steps in Hypothesis Testing

- Typical significance levels are: 0.10, 0.05, and 0.01.
- For example, when $\alpha = 0.10$, there is a 10% chance of rejecting a true null hypothesis.

9-2 Steps in Hypothesis Testing

- The **critical value(s)** separates the critical region from the noncritical region.
- The symbol for critical value is C.V.

9-2 Steps in Hypothesis Testing

- The **critical** or **rejection region** is the range of values of the test value that indicates that there is a significant difference and that the null hypothesis should be rejected.

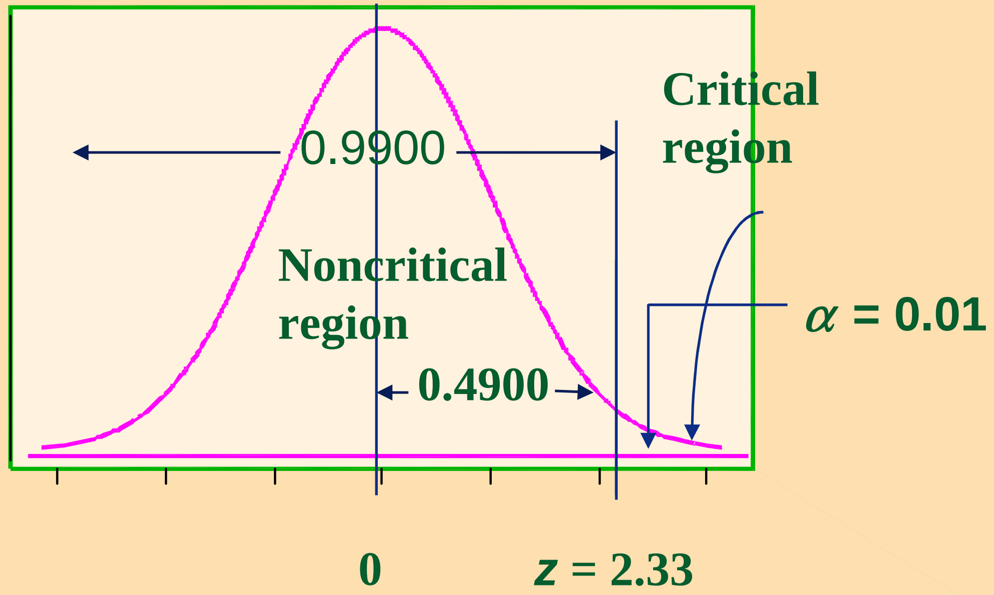
9-2 Steps in Hypothesis Testing

- The **noncritical** or **nonrejection region** is the range of values of the test value that indicates that the difference was probably due to chance and that the null hypothesis should not be rejected.

9-2 Steps in Hypothesis Testing

- A **one-tailed test (right or left)** indicates that the null hypothesis should be rejected when the test value is in the critical region on one side of the mean.

9-2 Finding the Critical Value for $\alpha = 0.01$ (Right-Tailed Test)



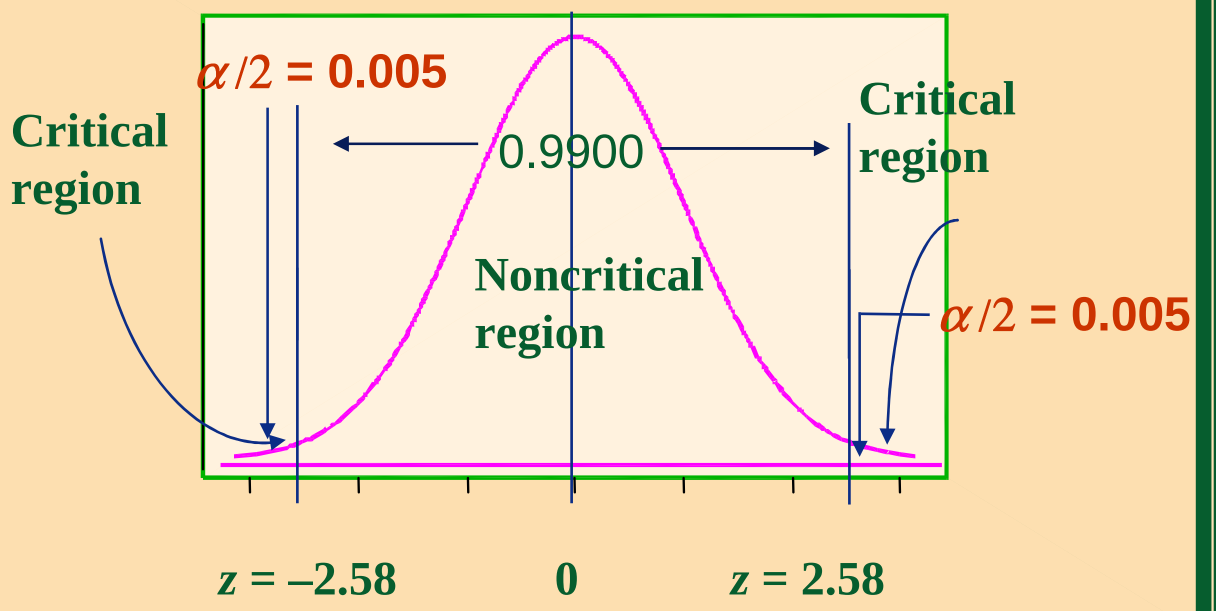
9-2 Finding the Critical Value for $\alpha = 0.01$ (Left-Tailed Test)

- **For a left-tailed test when $\alpha = 0.01$, the critical value will be –2.33 and the critical region will be to the left of –2.33.**

9-2 Finding the Critical Value for $\alpha = 0.01$ (Two-Tailed Test)

- In a **two-tailed** test, the null hypothesis should be rejected when the test value is in either of the two critical regions.

9-2 Finding the Critical Value for $\alpha = 0.01$ (Two-Tailed Test)



9-3 Large Sample Mean Test (*z* test)

- The *z* test is a statistical test for the mean of a population. It can be used when $n \geq 30$, or when the population is normally distributed and σ is known.
- The formula for the *z* test is given on the next slide.

9-3 Large Sample Mean Test

$$z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

where

$\bar{X}$ = *sample mean*

μ = *hypothesized population mean*

σ = *population deviation*

n = *sample size*

Five steps for hypothesis-testing:

- State the hypotheses and identify the claim.
- Find the critical value(s).
- Compute the test value.
- Make the decision to reject or not reject the null hypothesis.
- Summarize the results.

9-3 Large Sample Mean Test - Example

- A researcher reports that the average salary of assistant professors is more than \$42,000. A sample of 30 assistant professors has a mean salary of \$43,260. At $\alpha = 0.05$, test the claim that assistant professors earn more than \$42,000 a year. The standard deviation of the population is \$5230.

9-3 Large Sample Mean Test - Example

- **Step 1:** State the hypotheses and identify the claim.
- H_0: $\mu \leq \$42,000$ H_1: $\mu > \$42,000$ (claim)
- **Step 2:** Find the critical value. Since $\alpha = 0.05$ and the test is a right-tailed test, the critical value is $z = +1.65$.
- **Step 3:** Compute the test value.

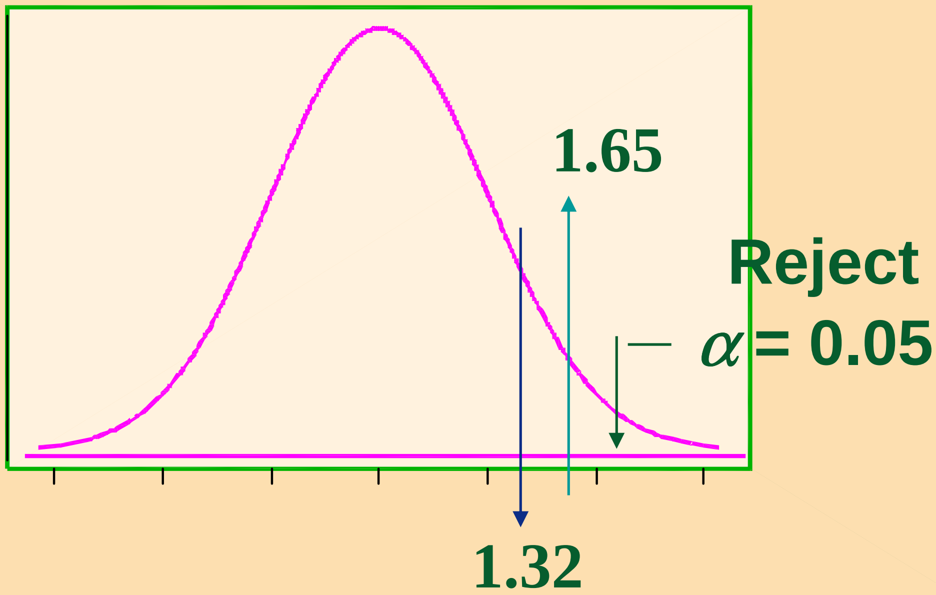
9-3 Large Sample Mean Test - Example

- **Step 3:** $z = [43{,}260 - 42{,}000]/[5230/\sqrt{30}] = 1.32$.
- **Step 4:** Make the decision. Since the test value, +1.32, is less than the critical value, +1.65, and not in the critical region, the decision is "Do not reject the null hypothesis."

9-3 Large Sample Mean Test - Example

- **Step 5:** Summarize the results. There is not enough evidence to support the claim that assistant professors earn more on average than $42,000 a year.
- See the next slide for the figure.

9-3 Large Sample Mean Test - Example



9-3 Large Sample Mean Test - Example

- A national magazine claims that the average college student watches less television than the general public. The national average is 29.4 hours per week, with a standard deviation of 2 hours. A sample of 30 college students has a mean of 27 hours. Is there enough evidence to support the claim at $\alpha = 0.01$?

9-3 Large Sample Mean Test - Example

- **Step 1:** State the hypotheses and identify the claim.
- H_0: $\mu \geq 29.4$ H_1: $\mu < 29.4$ (claim)
- **Step 2:** Find the critical value. Since $\alpha = 0.01$ and the test is a left-tailed test, the critical value is $z = -2.33$.
- **Step 3:** Compute the test value.

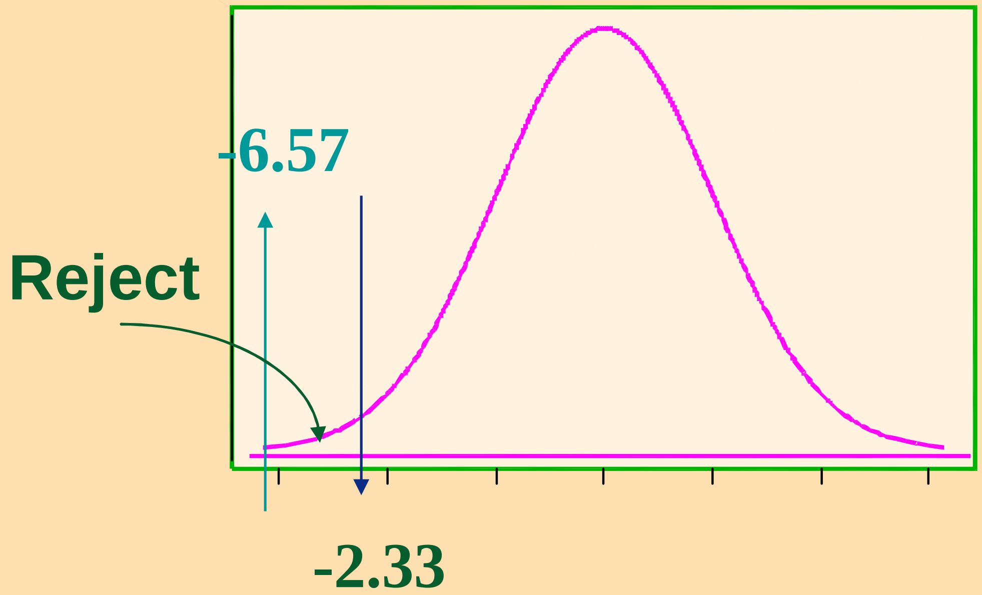
9-3 Large Sample Mean Test - Example

- **Step 3:** $z = [27 - 29.4]/[2/\sqrt{30}] = -6.57$.
- **Step 4:** Make the decision. Since the test value, –6.57, falls in the critical region, the decision is to reject the null hypothesis.

9-3 Large Sample Mean Test - Example

- **Step 5:** Summarize the results. There is enough evidence to support the claim that college students watch less television than the general public.
- See the next slide for the figure.

9-3 Large Sample Mean Test - Example



9-3 Large Sample Mean Test - Example

- The Medical Rehabilitation Education Foundation reports that the average cost of rehabilitation for stroke victims is $24,672. To see if the average cost of rehabilitation is different at a large hospital, a researcher selected a random sample of 35 stroke victims and found that the average cost of their rehabilitation is $25,226.

9-3 Large Sample Mean Test - Example

- The standard deviation of the population is $3,251. At $\alpha = 0.01$, can it be concluded that the average cost at a large hospital is different from $24,672?
- **Step 1:** State the hypotheses and identify the claim.
- H_0: $\mu = \$24,672$ H_1: $\mu \neq \$24,672$ (claim)

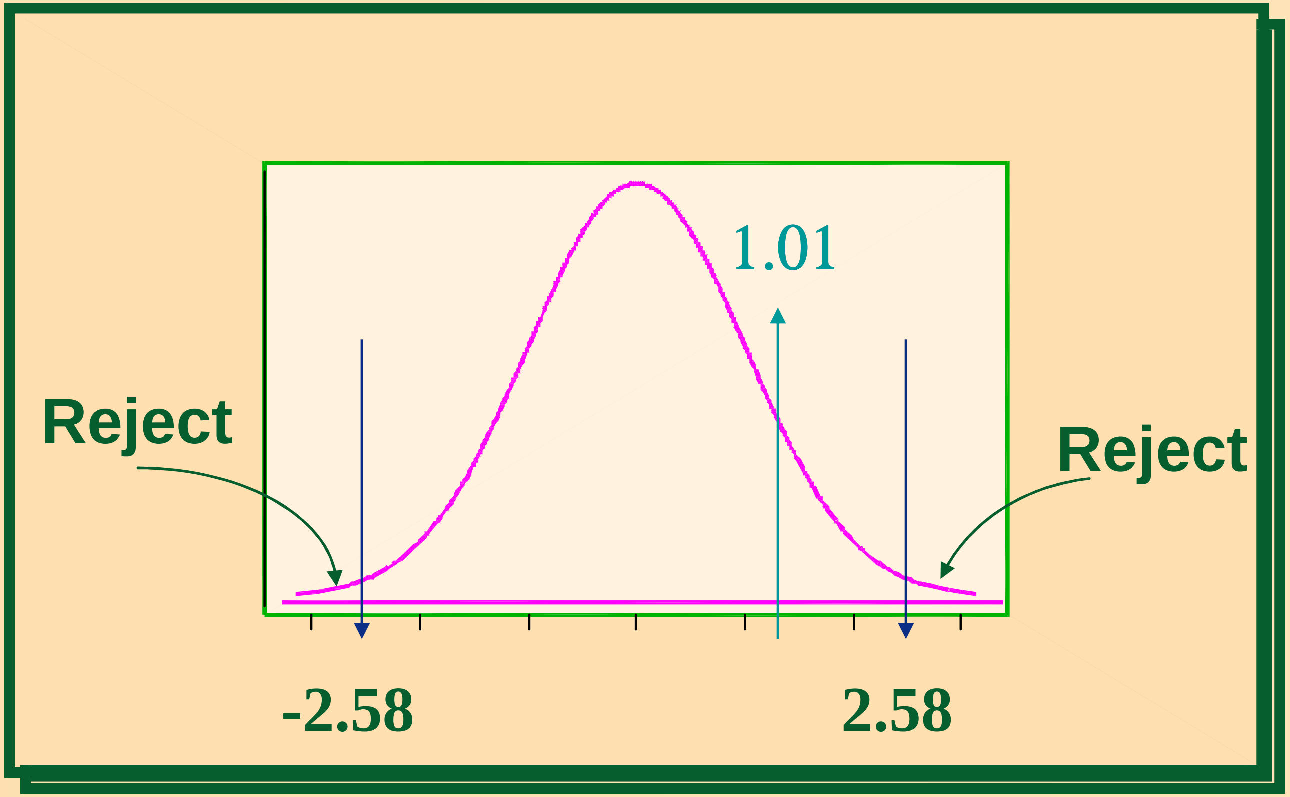
9-3 Large Sample Mean Test - Example

- **Step 2:** Find the critical values. Since $\alpha = 0.01$ and the test is a two-tailed test, the critical values are $z = -2.58$ and $+2.58$.
- **Step 3:** Compute the test value.
- **Step 3:** $z = [25{,}226 - 24{,}672]/[3{,}251/\sqrt{35}] = 1.01$.

9-3 Large Sample Mean Test - Example

- **Step 4:** Make the decision. Do not reject the null hypothesis, since the test value falls in the noncritical region.
- **Step 5:** Summarize the results. There is not enough evidence to support the claim that the average cost of rehabilitation at the large hospital is different from $24,672.

9-3 Large Sample Mean Test - Example



9-3 *P*-Values

- **Besides listing an α value, many computer statistical packages give a *P*-value for hypothesis tests.**
- **The *P*-value is the actual probability of getting the sample mean value or a more extreme sample mean value in the direction of the alternative hypothesis (> or <) if the null hypothesis is true.**

9-3 *P*-Values

- **The *P*-value is the actual area under the standard normal distribution curve (or other curve, depending on what statistical test is being used) representing the probability of a particular sample mean or a more extreme sample mean occurring if the null hypothesis is true.**

9-3 *P*-Values – decision rule

- **If *P*-value $\leq \alpha$, reject the null hypothesis.**
- **If *P*-value $> \alpha$, do not reject the null hypothesis.**

9-3 *P*-Values - Example

- A researcher wishes to test the claim that the average age of lifeguards in Ocean City is greater than 24 years. She selects a sample of 36 guards and finds the mean of the sample to be 24.7 years, with a standard deviation of 2 years. Is there evidence to support the claim at $\alpha = 0.05$? Find the *P*-value.

9-3 *P*-Values - Example

- **Step 1:** **State the hypotheses and identify the claim.**
- H_0: $\mu \le 24$ H_1: $\mu > 24$ (claim)
- **Step 2:** **Compute the test value.**

$$z = \frac{24.7 - 24}{2/\sqrt{36}} = 2.10$$

9-3 *P*-Values - Example

- **Step 3:** Using Table E in Appendix C, find the corresponding area under the normal distribution for $z = 2.10$. It is 0.4821
- **Step 4:** Subtract this value for the area from 0.5000 to find the area in the right tail.

  $$0.5000 – 0.4821 = 0.0179$$

  Hence the *P*-value is 0.0179.

9-3 *P*-Values - Example

- **Step 5:** **Make the decision. Since the *P*-value is less than 0.05, the decision is to reject the null hypothesis.**
- **Step 6:** **Summarize the results. There is enough evidence to support the claim that the average age of lifeguards in Ocean City is greater than 24 years.**

9-3 *P*-Values - Example

- A researcher claims that the average wind speed in a certain city is 8 miles per hour. A sample of 32 days has an average wind speed of 8.2 miles per hour. The standard deviation of the sample is 0.6 mile per hour. At $\alpha = 0.05$, is there enough evidence to reject the claim? Use the *P*-value method.

9-3 *P*-Values - Example

- **Step 1:** State the hypotheses and identify the claim.
- H_0: $\mu = 8$ (claim) H_1: $\mu \neq 8$
- **Step 2:** Compute the test value.

$$z = \frac{8.2 - 8}{0.6/\sqrt{32}} = 1.89$$

9-3 *P*-Values - Example

- **Step 3:** Using table E, find the corresponding area for $z = 1.89$. It is 0.4706.
- **Step 4:** Subtract the value from 0.5000.

  $0.5000 – 0.4706 = 0.0294$

9-3 *P*-Values - Example

- **Step 5:** Make the decision: Since this test is two-tailed, the value 0.0294 must be doubled; 2(0.0294) = 0.0588. Hence, the decision is not to reject the null hypothesis, since the *P*-value is greater than 0.05.
- **Step 6:** Summarize the results. There is not enough evidence to reject the claim that the average wind speed is 8 miles per hour.

9-4 Small Sample Mean Test (*t* test)

- When the population standard deviation is unknown and $n < 30$, the z test is inappropriate for testing hypotheses involving means.
- The *t* test is used in this case.
- Properties for the t distribution are given in Chapter 8.

9-4 Small Sample Mean Test - Formula for *t* test

$$t = \frac{\bar{X} - \mu}{s/\sqrt{n}}$$

where

$\bar{X}$ = sample mean

μ = hypothesized population mean

s = sample standard deviation

n = sample size

degrees of freedom $= n - 1$

9-4 Small Sample Mean Test - Example

- **A job placement director claims that the average starting salary for nurses is \$24,000. A sample of 10 nurses has a mean of \$23,450 and a standard deviation of \$400. Is there enough evidence to reject the director's claim at $\alpha = 0.05$?**

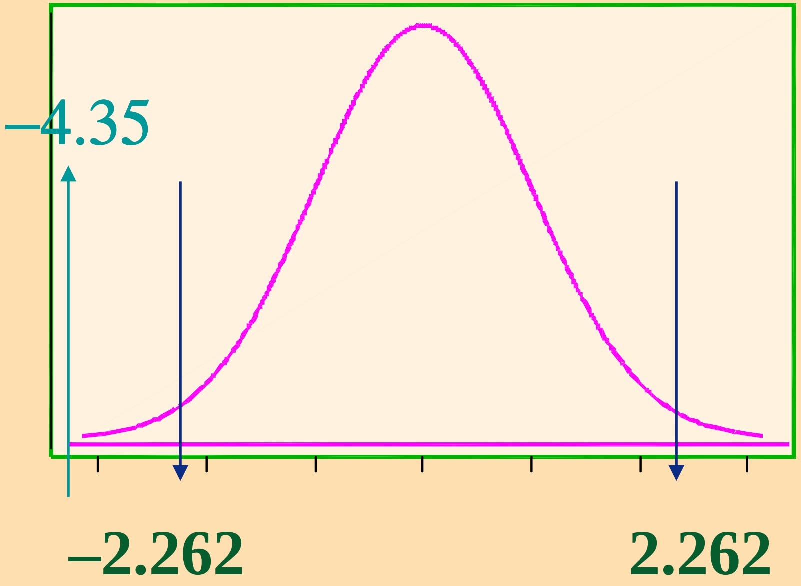
9-4 Small Sample Mean Test - Example

- **Step 1:** State the hypotheses and identify the claim.
- H_0: $\mu = \$24{,}000$ (claim) H_1: $\mu \neq \$24{,}000$
- **Step 2:** Find the critical value. Since $\alpha = 0.05$ and the test is a two-tailed test, the critical values are $t = -2.262$ and $+2.262$ with d.f. = 9.

9-4 Small Sample Mean Test - Example

- **Step 3:** Compute the test value.
  $t = [23{,}450 - 24{,}000]/[400/\sqrt{10}] = -4.35.$
- **Step 4:** Reject the null hypothesis, since $-4.35 < -2.262$.
- **Step 5:** There is enough evidence to reject the claim that the starting salary of nurses is $24,000.

9-3 Small Sample Mean Test - Example



9-7 Confidence Intervals and Hypothesis Testing

- **When the null hypothesis is rejected in a hypothesis-testing, the confidence interval for the mean using the same level of significance will not contain the hypothesized mean.**
- **Likewise, when the null hypothesis is not rejected, the confidence interval computed using the same level of significance will contain the hypothesized mean.**

9-7 Confidence Intervals and Hypothesis Testing - Example

- Sugar is packed in 5-pound bags. An inspector suspects the bags may not contain 5 pounds. A sample of 50 bags produces a mean of 4.6 pounds and a standard deviation of 0.7 pound. Is there enough evidence to conclude that the bags do not contain 5 pounds as stated, at $\alpha = 0.05$? Also, find the 95% confidence interval of the true mean.

9-7 Confidence Intervals and Hypothesis Testing - Example

- H_0: $\mu = 5$ H_1: $\mu \neq 5$ (claim)
- The critical values are +1.96 and – 1.96
- The test value is $z = \dfrac{4.6 - 5.00}{0.7/\sqrt{50}} = -4.04$
- Since – 4.04 < –1.96, the null hypothesis is rejected.
- There is enough evidence to support the claim that the bags do not weigh 5 pounds.

9-7 Confidence Intervals and Hypothesis Testing - Example

- The 95% confidence interval for the mean is given by

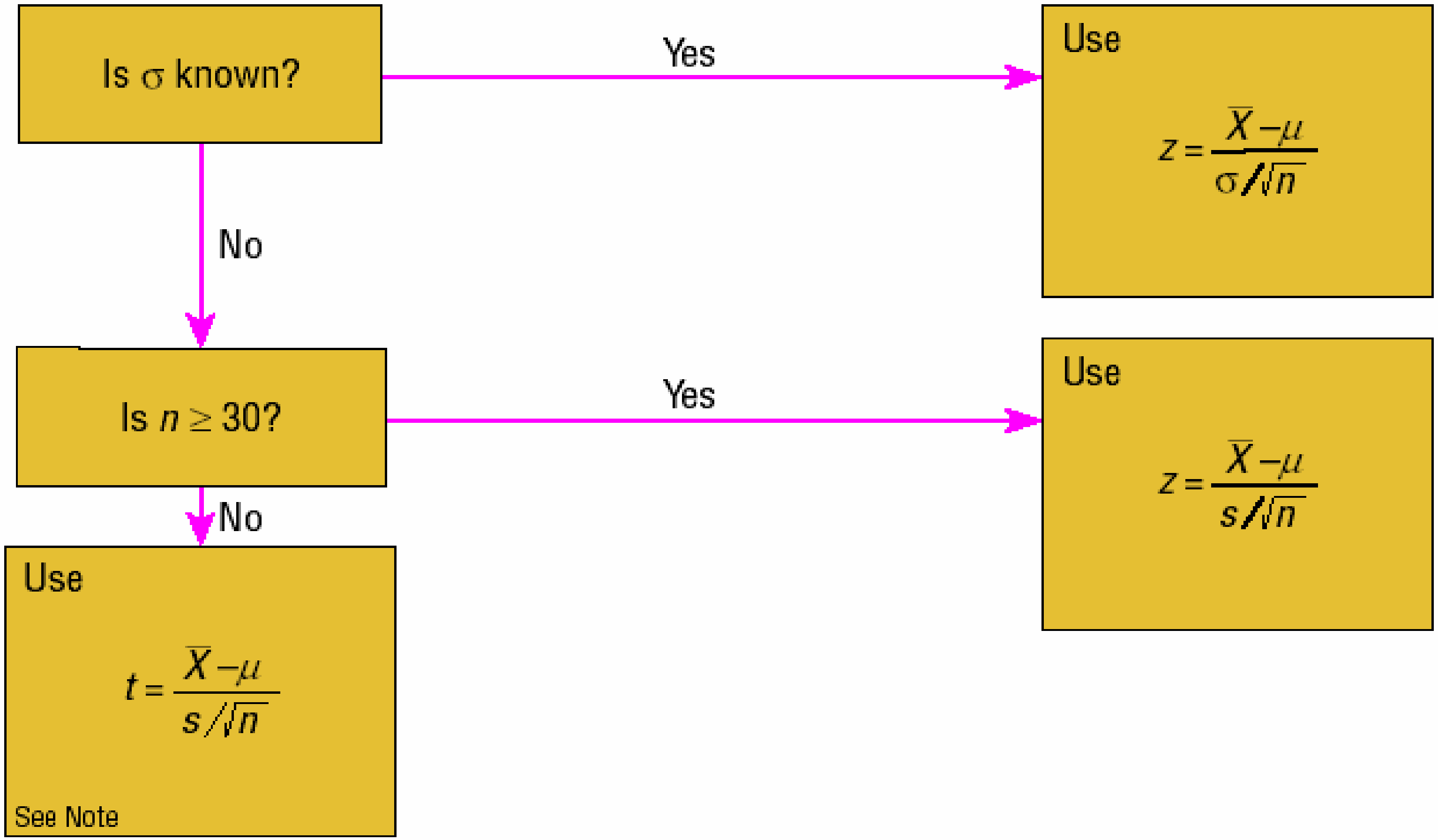
$$\bar{X} - z_{\alpha/2} \cdot \frac{s}{\sqrt{n}} < \mu < \bar{X} + z_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

$$4.6 - (1.96)\left(\frac{0.7}{\sqrt{50}}\right) < \mu < 4.6 + (1.96)\left(\frac{0.7}{\sqrt{50}}\right)$$

$$4.406 < \mu < 4.794$$

- Notice that the 95% confidence interval of μ does not contain the hypothesized value $\mu = 5$. Hence, there is agreement between the hypothesis test and the confidence interval.

Using the *z* or *t* test



Note: With d.f. = $n - 1$, and the population must be approximately normally distributed.