A Practical Look at Volatility in Financial Time Series

MATH 287C - Advanced Time Series Analysis
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June 4th, 2018

Outline

1. What is Volatility?

2. Normalizing and Variance Stabilizing (NoVaS) Transformation

3. Volatility Prediction

4. A Simple Volatility Trading Strategy

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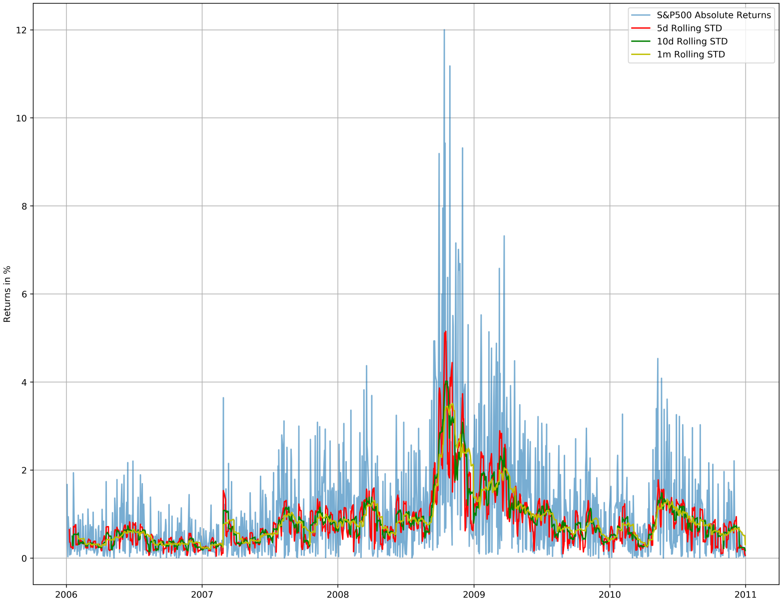
What is Volatility?

- Volatility is a measure of price variability over some period of time
- Typically described by the standard deviation σ of the return series $\{X_t\}$
- Volatility is peculiar in that we know it exists, but in some sense we can't really measure it
- Bachelier (1900) showed that $\{X_t\} \sim$ iid. $N(0, 1)$, but this is only good for a first order approximation

Naive Measure - Realized Volatility



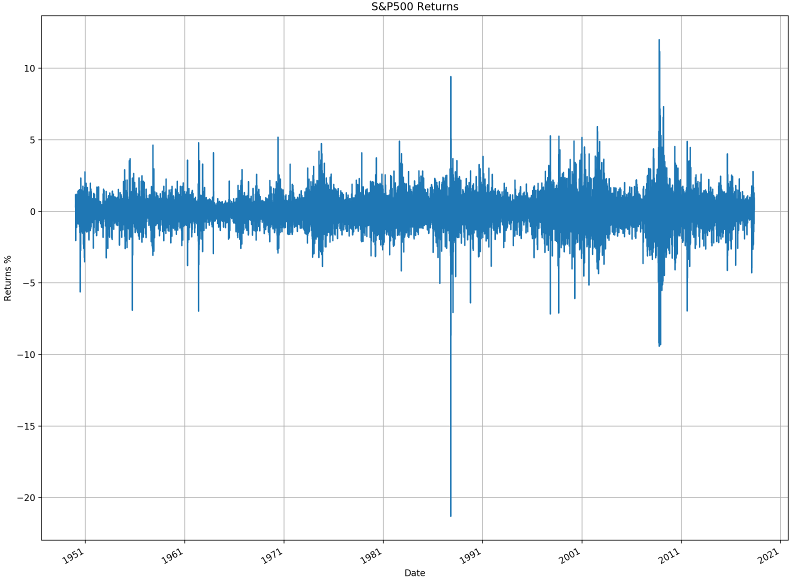
Stylized Facts

Further analysis of $\{X_t\}$ reveals other kinds of structure that cannot be explained by the gaussian assumption.

In particular, the return series displays the following distinctive behavior:

1. $\{X_t\}$ is heavy-tailed, much more so than the Gaussian white noise
2. Although $\{X_t\}$ is uncorrelated, the series $\{X_t^2\}$ is highly correlated
3. The changes in $\{X_t\}$ tend to be clustered, large changes tend to be followed by large changes and vice v
4. Effects are asymmetric, bad news results in larger downward price moves than positive news does to upward price moves

SP500 Daily Returns (1950-2018)



GARCH

The Generalized ARCH (GARCH) model of Bollerslev (1986) and it's variants are extremely popular (albeit imperfect) methods to model volatility.

GARCH(p,q) model can be expressed as:

$$X_t = \sigma_t \varepsilon_t, \qquad \varepsilon_t \sim N(0,1)$$

where

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 + \sum_{j=1}^{q} \alpha_j X_{t-j}^2$$

For the purposes of this talk, we'll focus on GARCH(1,1) models where $\sigma_t^2 = \alpha_0 + \beta_1 \sigma_{t-1}^2 + \alpha_1 X_{t-1}^2$

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NoVaS Transformation (Politis 2007)

The NoVaS Transformation is defined as

$$W_{t,a} = \frac{X_t}{\sqrt{\alpha s_{t-1}^2 + a_0 X_t^2 + \sum_{i=1}^{p} a_i X_{t-i}^2}}$$

for $t = p+1, p+2, \ldots, n$

It is a clever extension of the ARCH model where we include the value X_t in order to "studentize" the returns.

The order p and the vector of nonnegative parameters $(\alpha, a_0, \ldots, a_p)$ are chosen by the practitioner with the twin goals of normalization and variance-stabilization.

Algorithm for Simple NoVaS:

- Let $\alpha = 0$ and $a_i = \frac{1}{p+1}$ for all $0 \leq i \leq p$
- Pick p such that $|KURT_n(W_{t,p}^S)| \approx 3$

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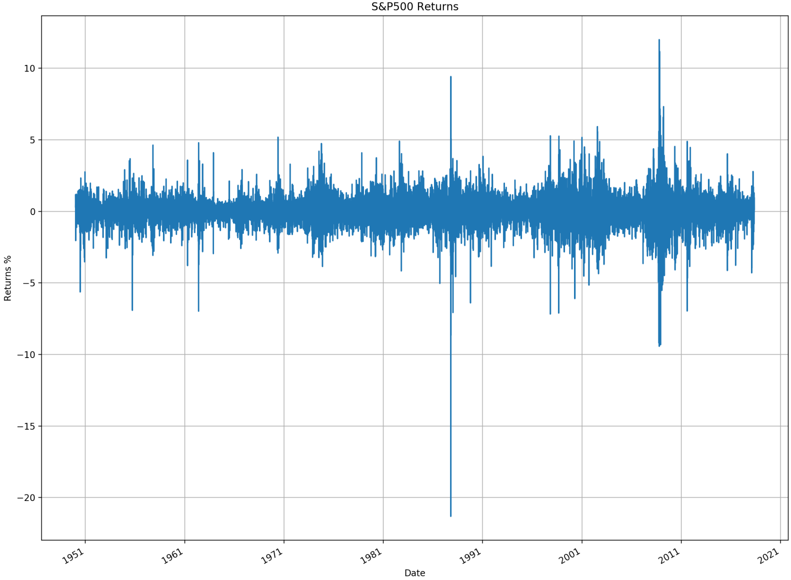
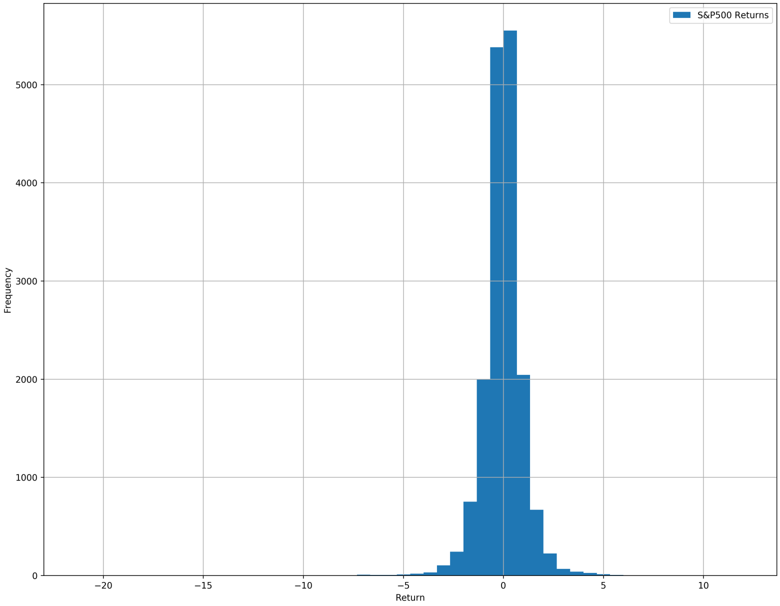
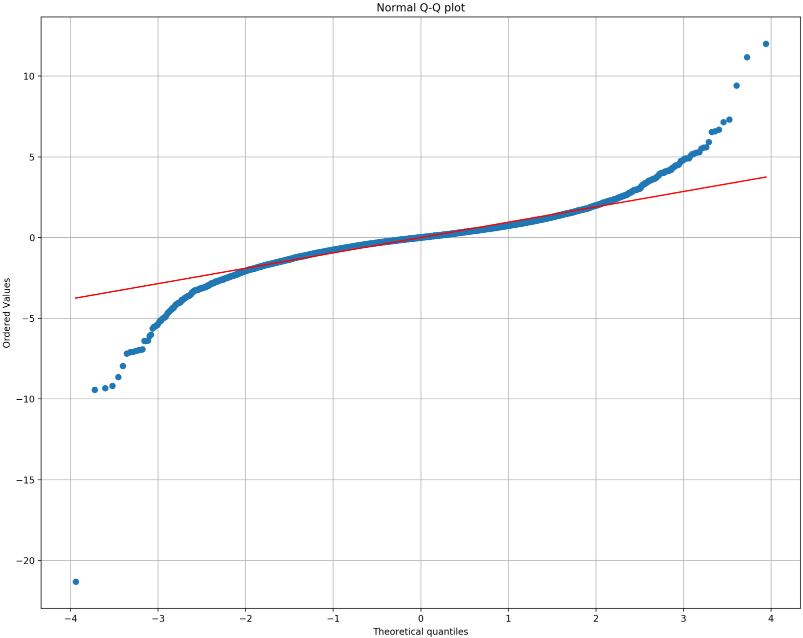


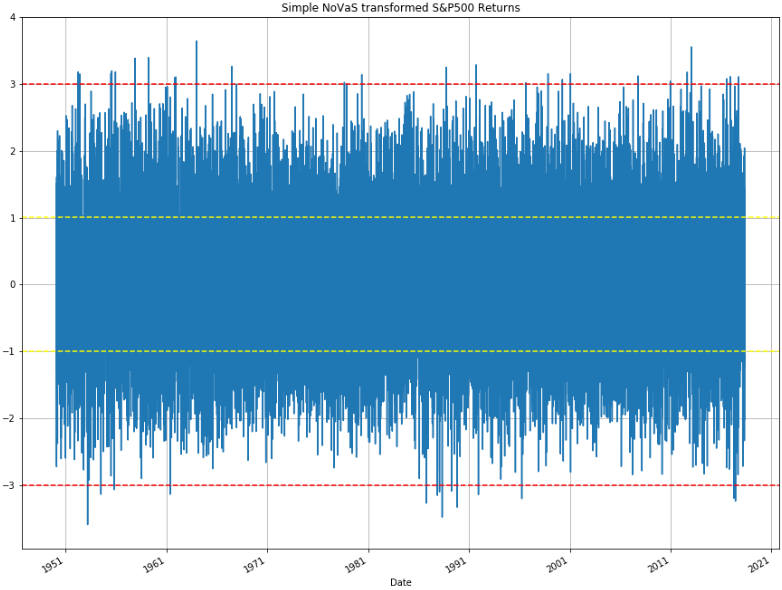
SP500 Daily Returns Histogram



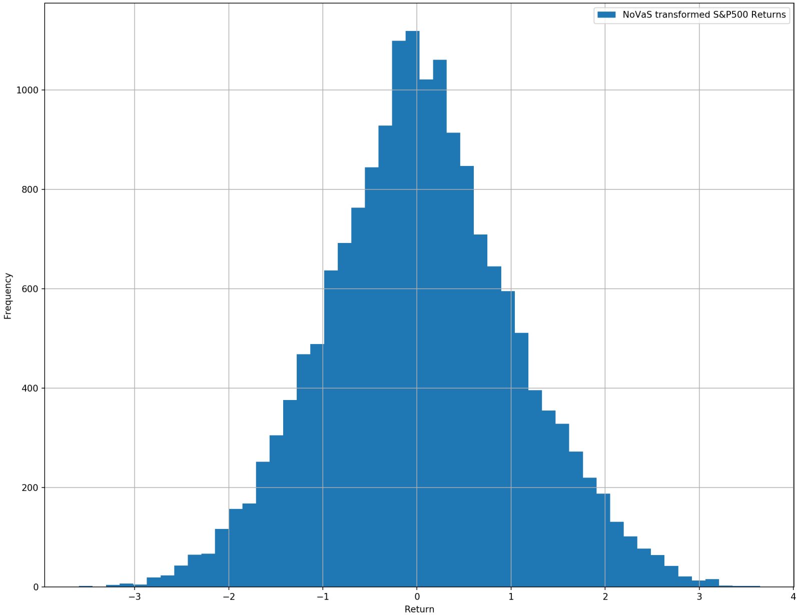
SP500 Daily Returns Q-Q Plot



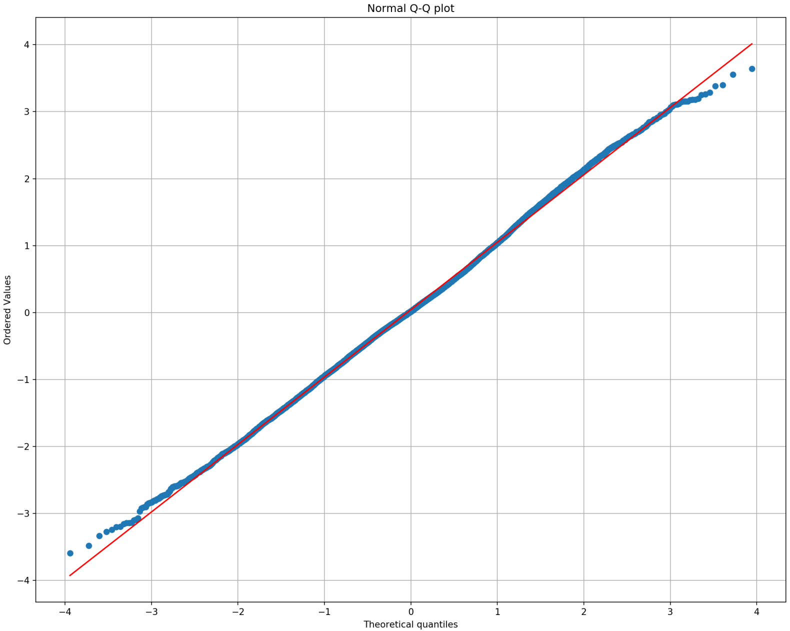
NoVaS Transformed SP500 Daily Returns (p=16)



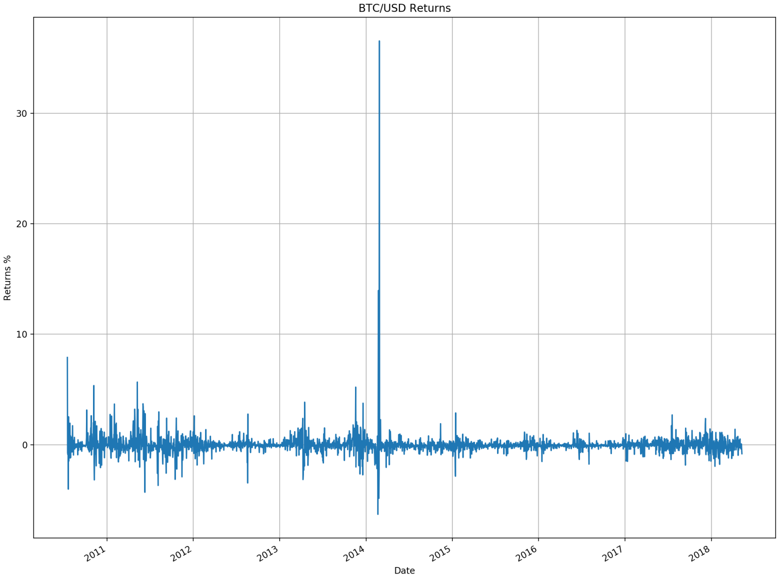
NoVaS Transformed SP500 Histogram (p=16)



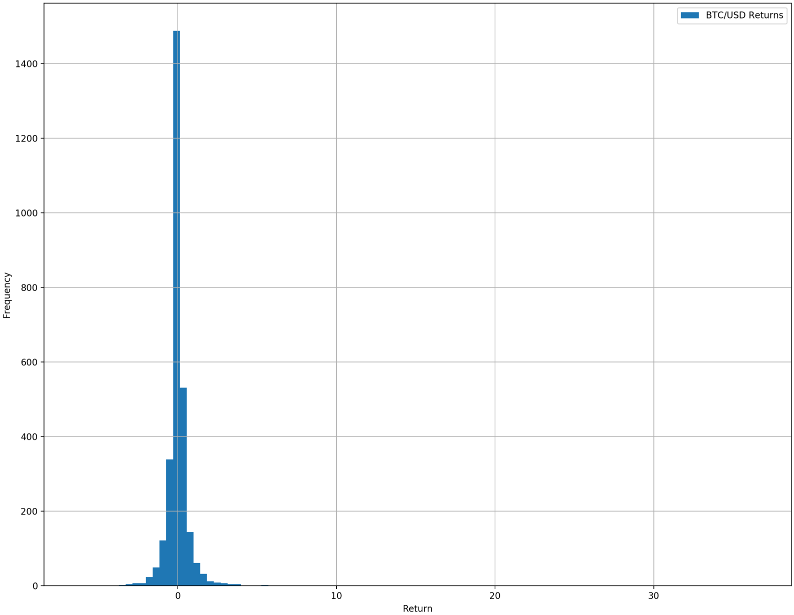
NoVaS Transformed SP500 QQ-Plot (p=16)



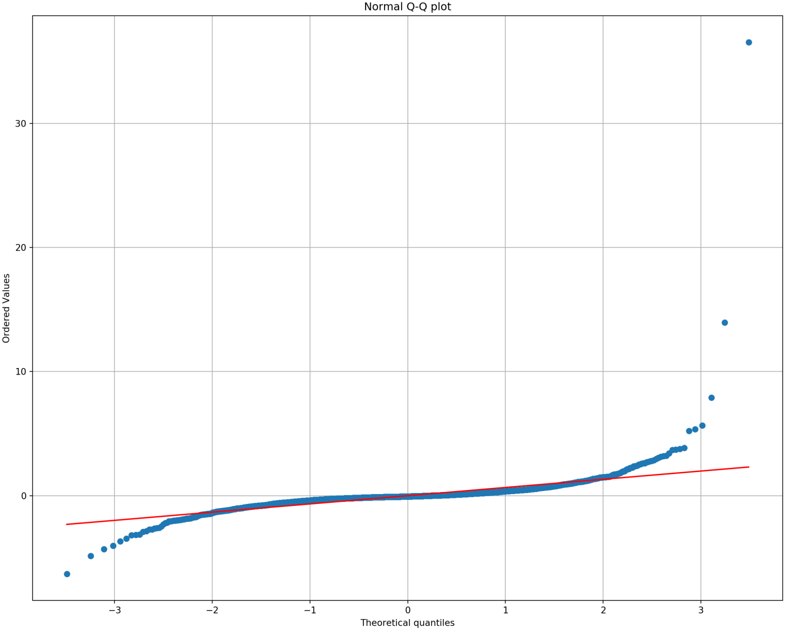
BTC/USD Daily Returns (2010-2018)



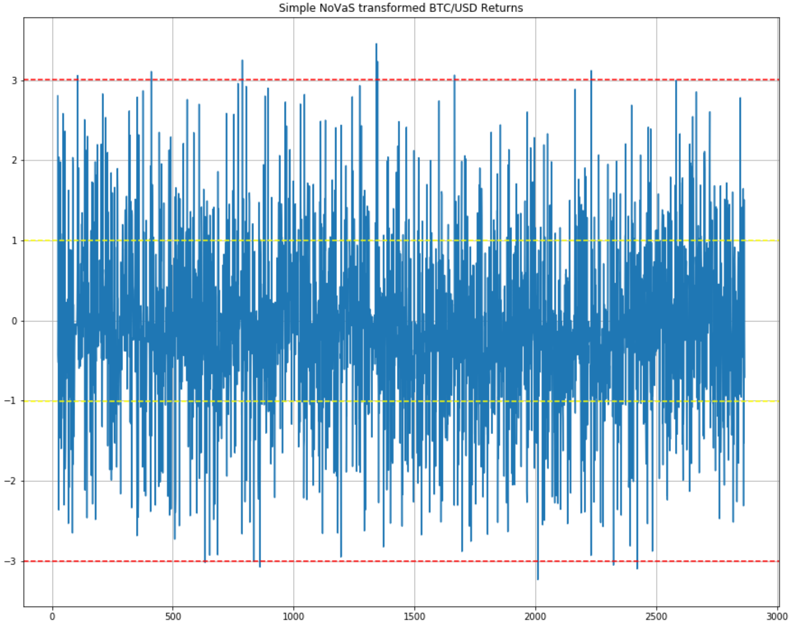
BTC/USD Daily Returns Histogram (2010-2018)



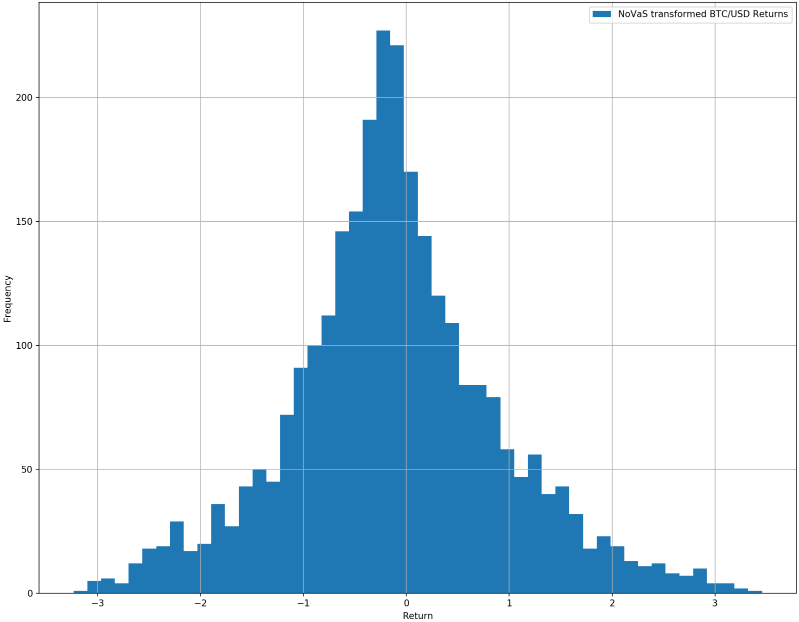
BTC/USD Daily Returns QQ-Plot (2010-2018)



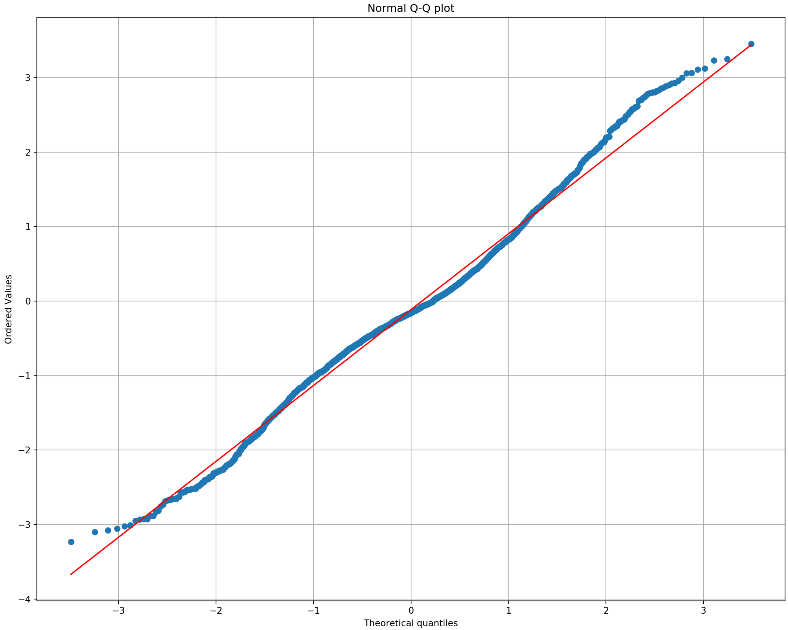
NoVaS Transformed BTC/USD Returns (p=12)



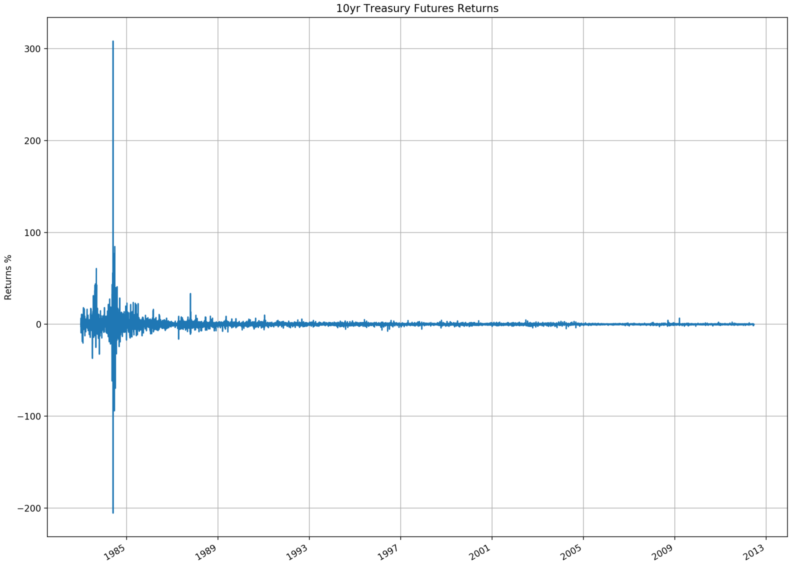
NoVaS Transformed BTC/USD Histogram (p=12)



NoVaS Transformed BTC/USD QQ-Plot (p=12)



5min Bar 10yr Treasury Futures (1983-2012)



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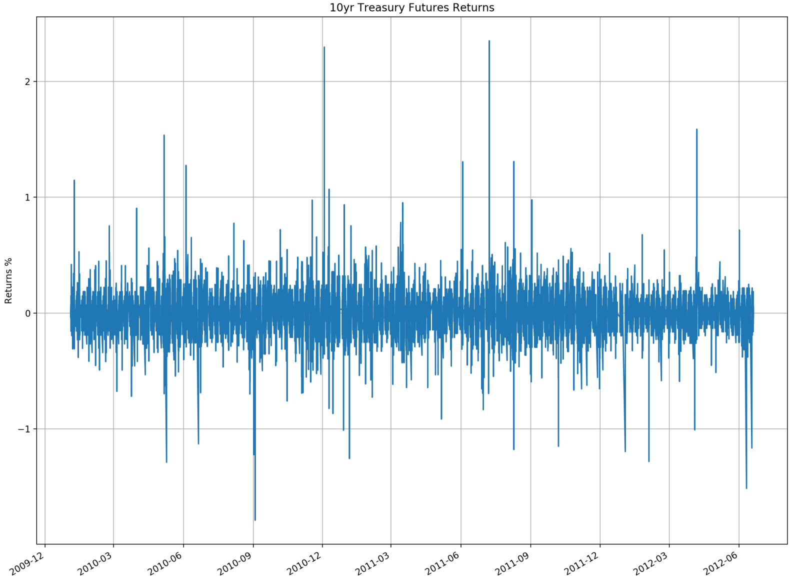

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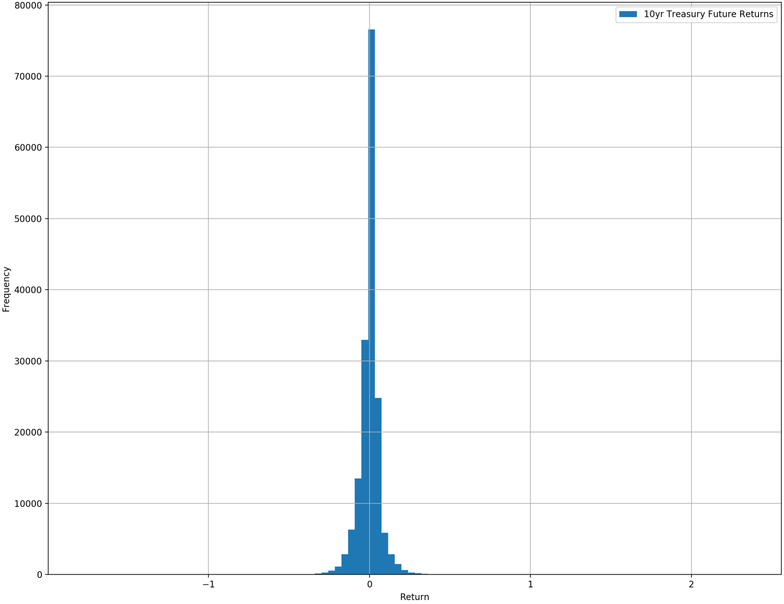

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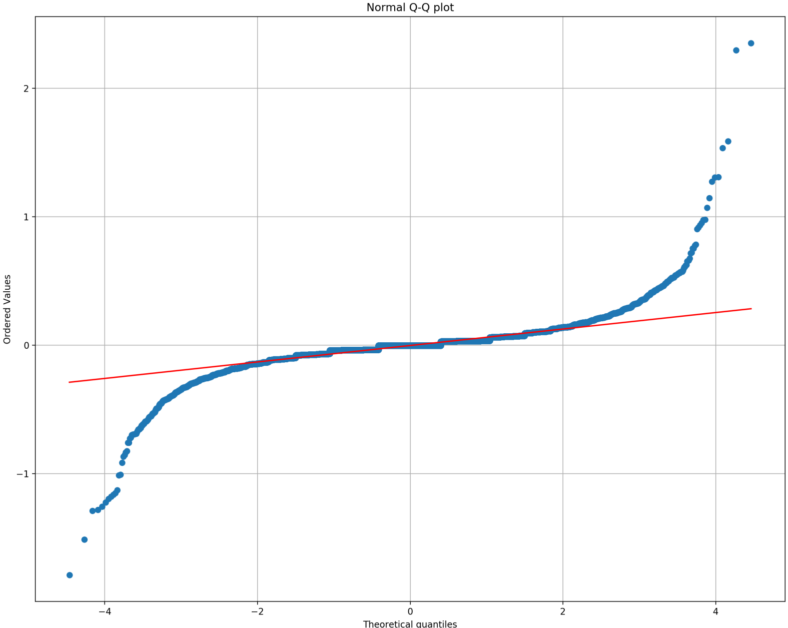

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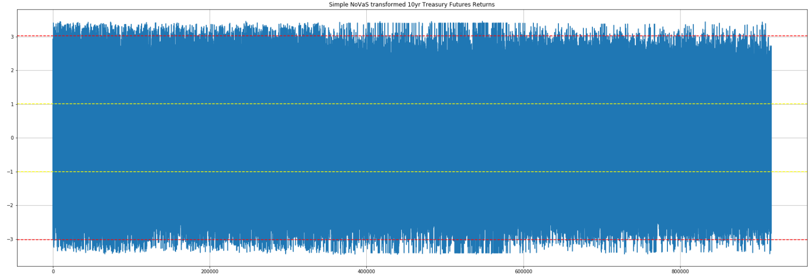

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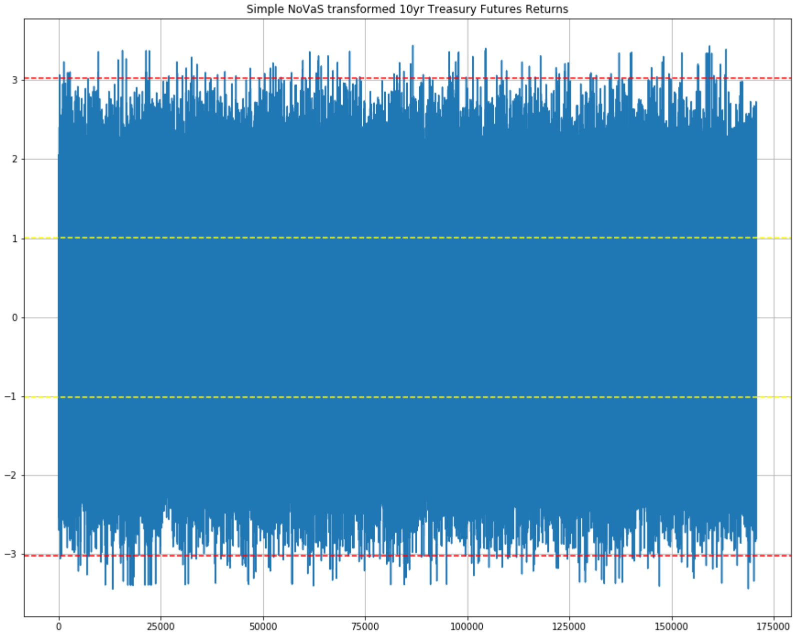

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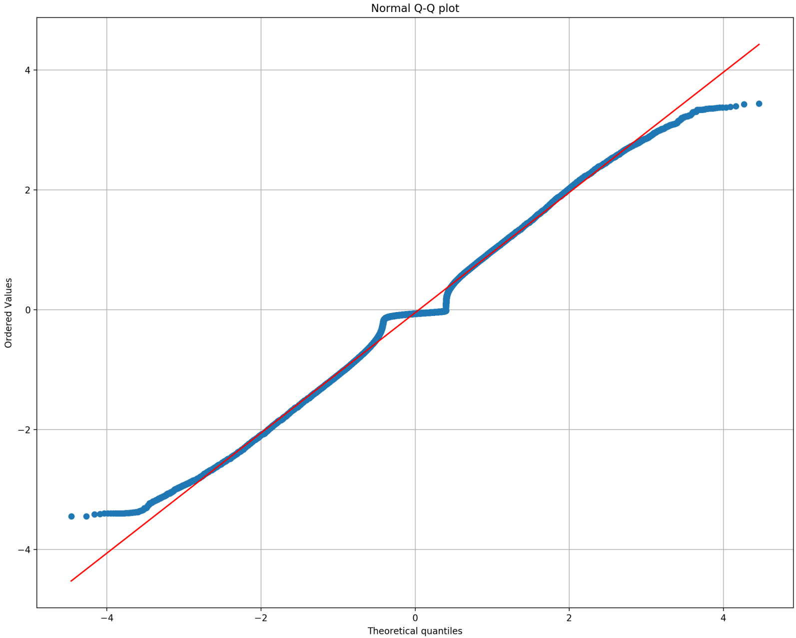


Outline

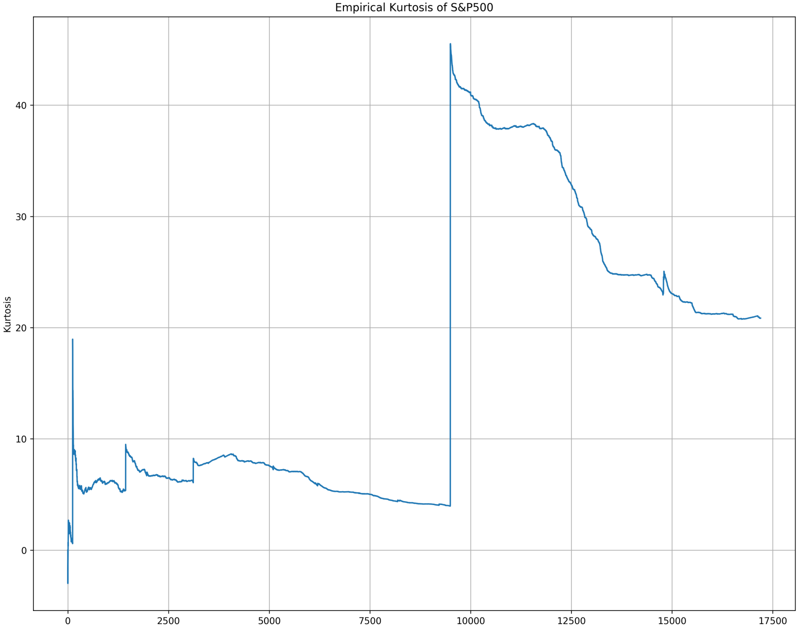
Volatility Prediction

- Forecasts of volatility are important when assessing and managing the risks of portfolios
- We focus on the problem of one-step ahead X_{t+1} prediction based on the observed past
- For our purposes, volatility prediction = predicting X_{t+1}^2
- Even though X_{t+1}^2 is a noisy proxy for $\mathbb{E}(X_{t+1}^2|\mathfrak{F}_\mathfrak{n})$, we'll see that in under some conditions NoVaS allows us to predict the latter
- Assuming more realistically that financial returns are locally stationary, we use a rolling window size of 250 days to calculate our forecasts

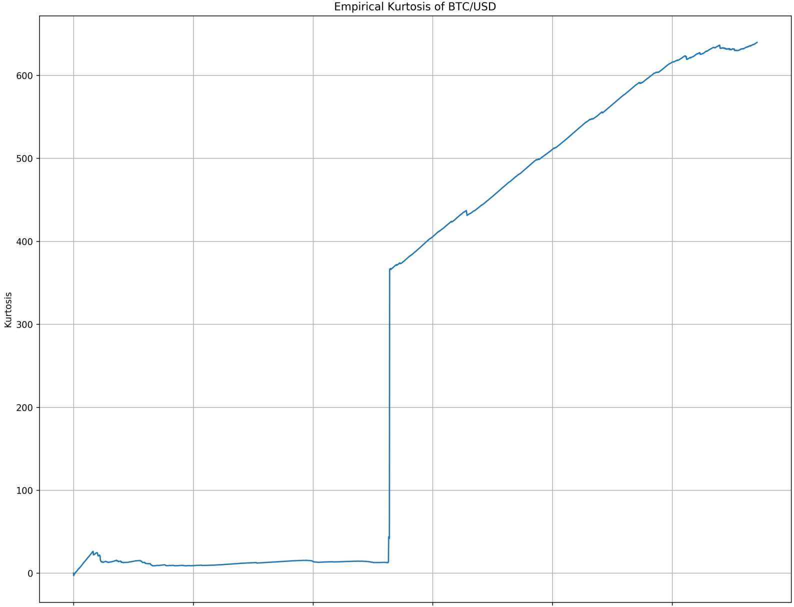
Which Loss Function? L_1 or L_2?

- To assess the accuracy of forecasts, we need to decide on a loss function to use
- The MSE is most commonly used, however note that $\mathbb{E}(Y_{n+1}^2 - \widehat{Y_{n+1}^2})^2$ is essentially a fourth moment
- Thus the unconditional MSE is infinite if the returns process has infinite kurtosis!
- We find that this indeed the case and so focus on the Mean Absolute Deviation (MAD) loss function
- Under the objective of L_1-optimal prediction, the optimal predictor is $Median(X_{n+1}^2|\mathfrak{F}_\mathfrak{n})$

Empirical Kurtosis Plot SP500



Empirical Kurtosis Plot BTC



Prediction Intervals

INCLUDE PREDICTION EQUATION FOR NOVAS AND GARCH

Steps for deriving prediction intervals - same for GARCH and NoVaS

1. Use simple NoVaS to obtain transformed data $\{W_{t,a}$ for $t = p+1, \ldots, n\}$ that are assumed to be approximately i.i.d. Let p, α and a_i denote the fitted NoVaS parameters
2. Calculate $\widehat{g(Y_{n+1})}$ the point predictor of $g(y_{n+1})$ as the median of the set etc.
3. Main Bootstrap Loop
   3.1 Resample randomly (with replacement) the transformed variables $\{W_{t,a}$ for $t = p+1, \ldots, n\}$ to create the pseudo-data $W^*_{p+1}, \ldots, W^*_{n-1}, W^*_n$ and W^*_{n+1}
   3.2 Let $(Y^*_1, \ldots, Y^*_p)' = (Y^*_{1+I}, \ldots, Y^*_{p+I})'$ where I is generated as a discrete random variable uniform on the values $0, 1, \ldots, n-p$
   3.3 Generate the bootstrap pseudo-data Y^*_t for $t = p+1, \ldots, n$ using equation (10.17)

$$Y^*_t = \frac{W_t}{\sqrt{1 - a_0 W^{*2}_t}} \sqrt{\sum_{i=1}^{p} a_i Y^{*2}_{t-i}}$$

SP500 Feb 2018 One Step Ahead Prediction

Plot predicting SP500 Feb 2018 Volatility spike, along with prediction intervals Shows that Simple NoVaS is better than GARCH(1,1)

Outline

Estimating the volatility $\mathbb{E}(Y_{n+1}^2|\mathfrak{F}_\mathfrak{n})$

Under case I, i.e. after empirically showing that the $W_{t,a}$ variables are (approximately) uncorrelated and hence independent, it is straightforward to construct a Model-free estimate of the conditional expectation $\mathbb{E}(Y_{n+1}^2|\mathfrak{F}_\mathfrak{n})$.

$$\mathbb{E}(Y_{n+1}^2|\mathfrak{F}_\mathfrak{n}) = A_n^2 \mathbb{E}\left(\frac{W_{t,a}^2}{1 - a_0 W_{t,a}^2}\right)$$

a natural estimate thereof is

$$\frac{A_n^2}{n-p} \sum_{t=p+1}^{n} \left(\frac{W_{t,a}^2}{1 - a_0 W_{t,a}^2}\right)$$

Talk about the conditions under which you can actually predict $sigma^2$, plot the ACF to confirm that transformed series is uncorrelated and independent.

RV(t+1)-IV(t)

We consider a very simple volatility trading strategy found in Ahmad Wilmott (2005)
Strategy:

- If RV(t+1) - VIX(t) $>$ 0 then BUY VXX. Vice Versa.
- RV(t+1) is the GARCH or NoVaS predicted realized volatility for next period
- Expect RV(t+1) to be better predictor of VIX(t+1) than VIX(t)
- VIX(t) = IV(t) is the current implied volatility

Strategy Results

Cumulative returns plot, legend contains CAGR and Sharpe Ratio