A Practical Look at Volatility in Financial Time Series

MATH 287C - Advanced Time Series Analysis
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Outline

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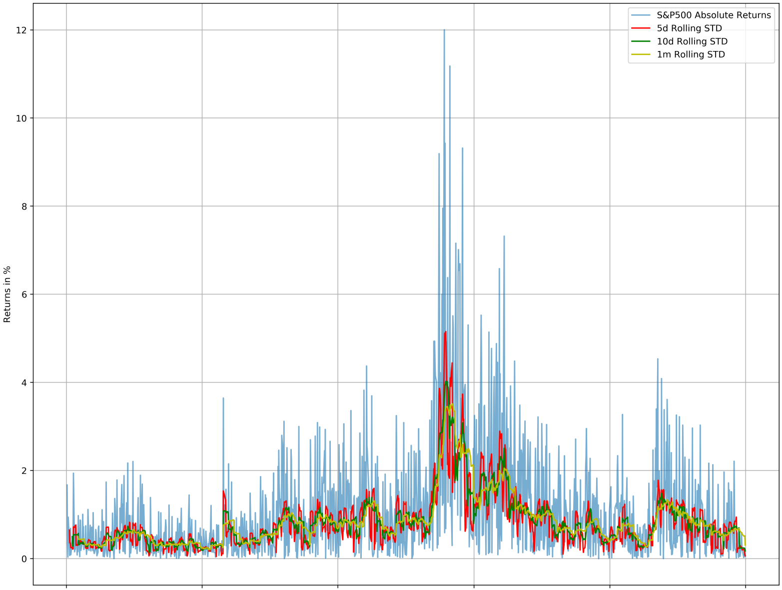
What is Volatility?

- Volatility is a measure of price variability over some period of time
- Typically described by the standard deviation σ of the return series $\{X_t\}$
- Volatility is peculiar in that we know it exists, but in some sense we can't really measure it
- Bachelier (1900) showed that $\{X_t\} \sim$ iid. $N(0,1)$, but this is only good for a first order approximation

Naive Measure - Realized Volatility



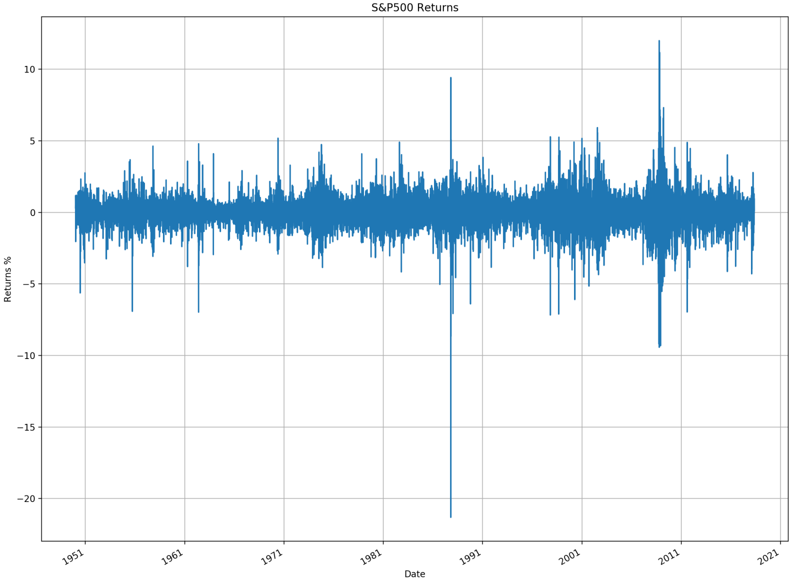
Stylized Facts

Further analysis of $\{X_t\}$ reveals other kinds of structure that cannot be explained by the gaussian assumption.

In particular, the return series displays the following distinctive behavior:

1. $\{X_t\}$ is heavy-tailed, much more so than the Gaussian white noise
2. Although $\{X_t\}$ is uncorrelated, the series $\{X_t^2\}$ is highly correlated
3. The changes in $\{X_t\}$ tend to be clustered, large changes tend to be followed by large changes and vice v
4. Effects are asymmetric, bad news results in larger downward price moves than positive news does to upward price moves

S&P500 Daily Returns (1950-2018)



GARCH

The Generalized ARCH (GARCH) model of Bollerslev (1986) and it's variants are extremely popular (albeit imperfect) methods to model volatility.

GARCH(p,q) model can be expressed as:

$$X_t = \sigma_t \varepsilon_t, \qquad \varepsilon_t \sim N(0,1)$$

where

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 + \sum_{j=1}^{q} \alpha_j X_{t-j}^2$$

For the purposes of this talk, we'll focus on GARCH(1,1) models where $\sigma_t^2 = \alpha_0 + \beta_1 \sigma_{t-1}^2 + \alpha_1 X_{t-1}^2$

Outline

1. What is Volatility?
2. Normalizing and Variance Stabilizing (NoVaS) Transformation
3. Volatility Prediction
4. A Simple Volatility Trading Strategy

NoVaS Transformation (Politis 2007)

The NoVaS Transformation is defined as

$$W_{t,a} = \frac{X_t}{\sqrt{\alpha s_{t-1}^2 + a_0 X_t^2 + \sum_{i=1}^{p} a_i X_{t-i}^2}}$$

for $t = p+1, p+2, \ldots, n$

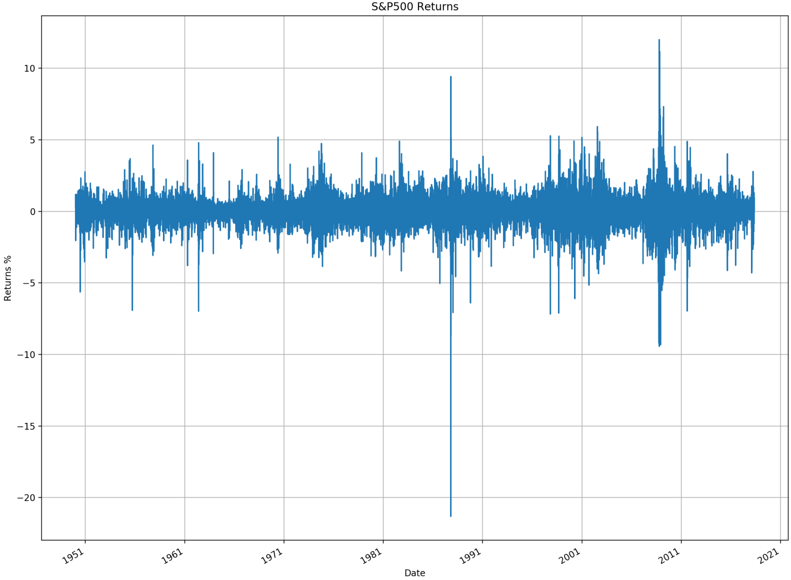
It is a clever extension of the ARCH model where we include the value X_t in order to "studentize" the returns.

The order p and the vector of nonnegative parameters $(\alpha, a_0, \ldots, a_p)$ are chosen by the practitioner with the twin goals of normalization and variance-stabilization.

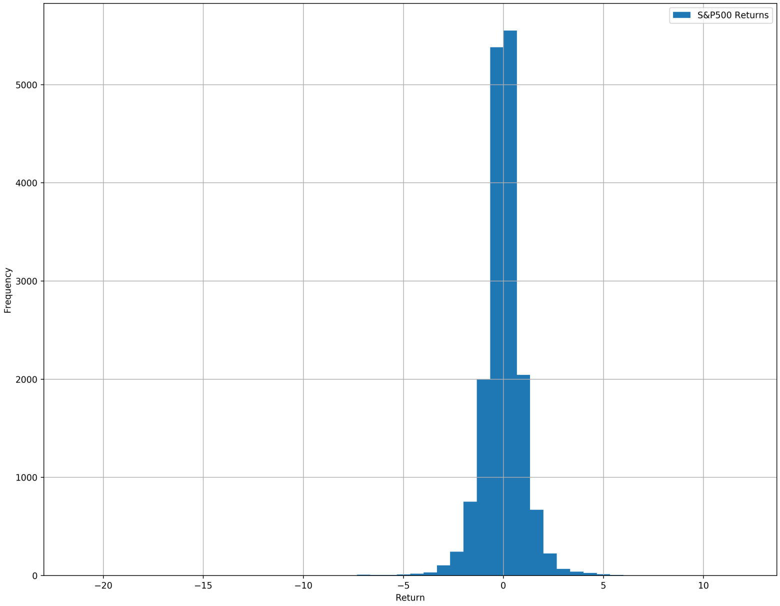
Algorithm for Simple NoVaS:

- Let $\alpha = 0$ and $a_i = \frac{1}{p+1}$ for all $0 \leq i \leq p$
- Pick p such that $|KURT_n(W_{t,p}^S)| \approx 3$

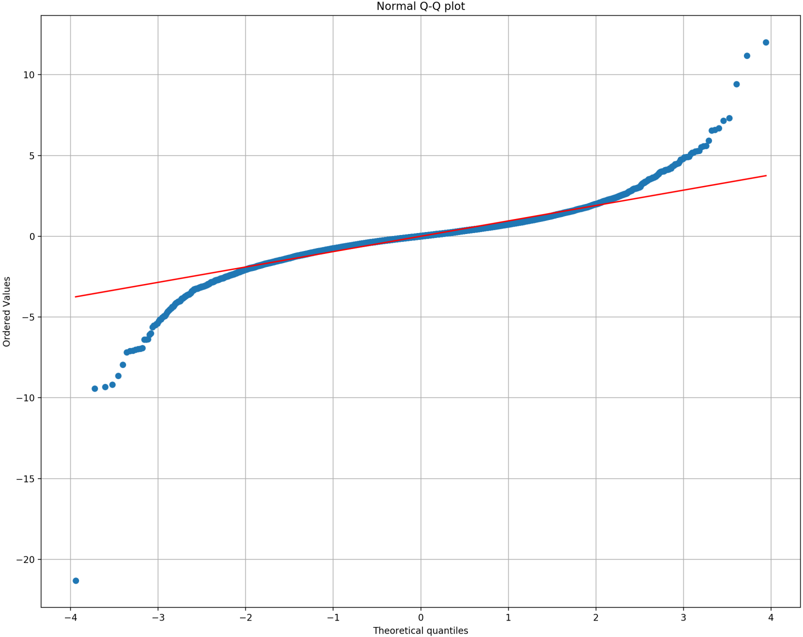
S&P500 Daily Returns (1950-2018)



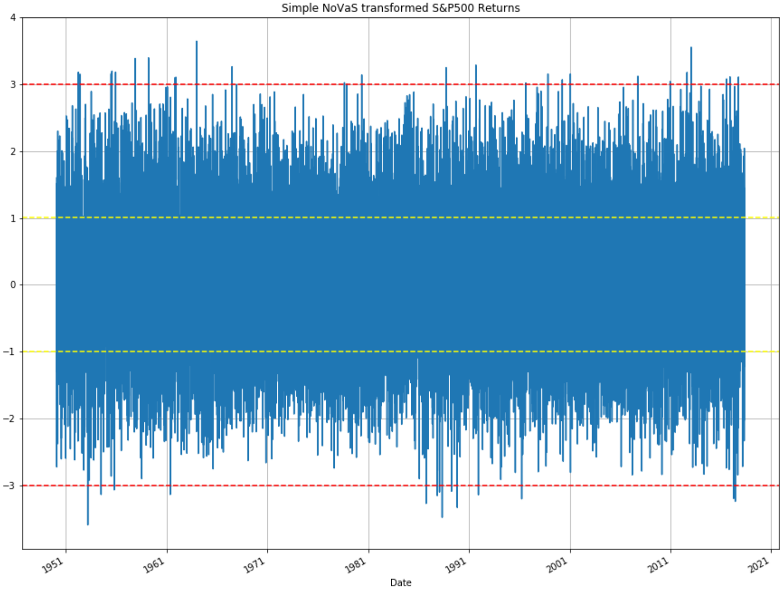
S&P500 Daily Returns Histogram



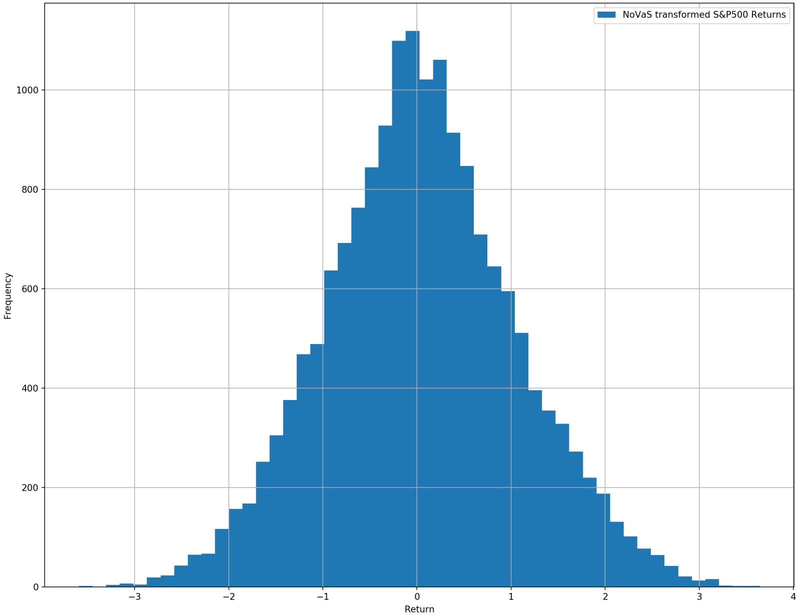
S&P500 Daily Returns Q-Q Plot



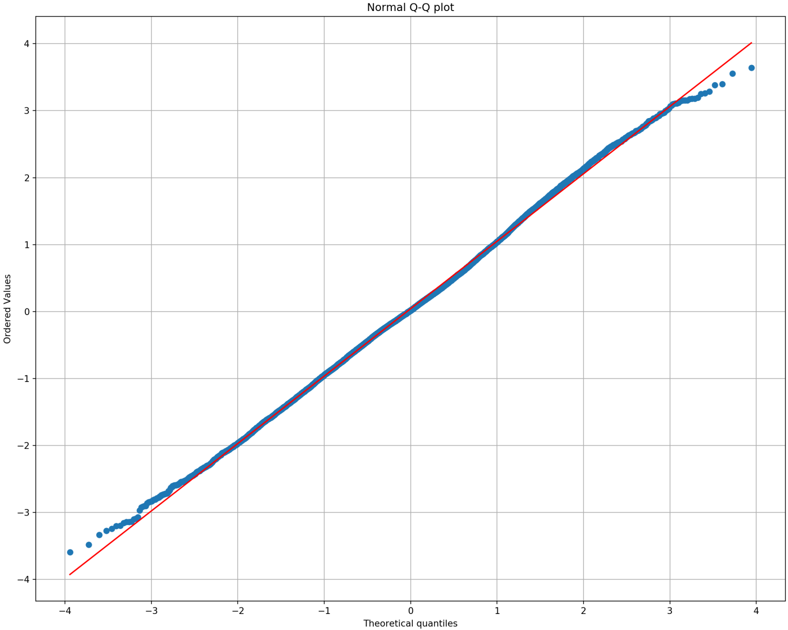
NoVaS Transformed S&P500 Daily Returns (p=16)



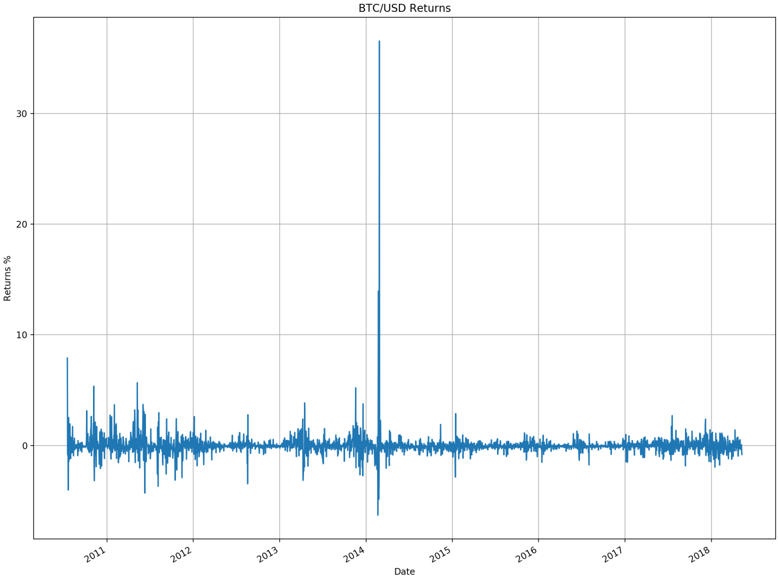
NoVaS Transformed S&P500 Histogram (p=16)



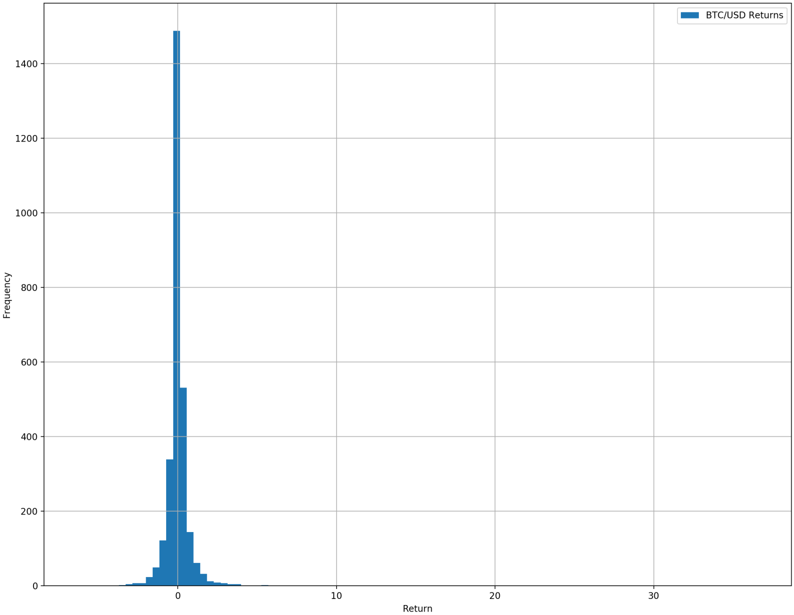
NoVaS Transformed S&P500 QQ-Plot (p=16)



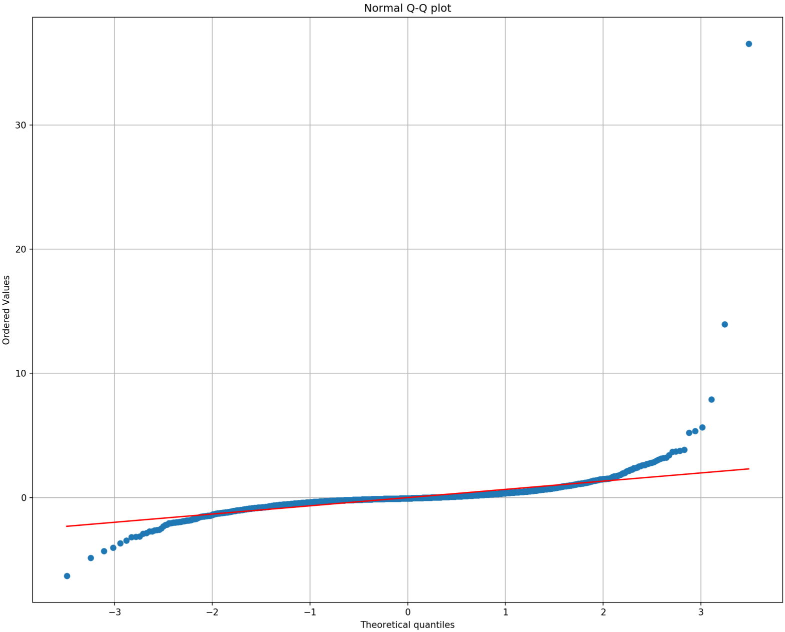
BTC/USD Daily Returns (2010-2018)



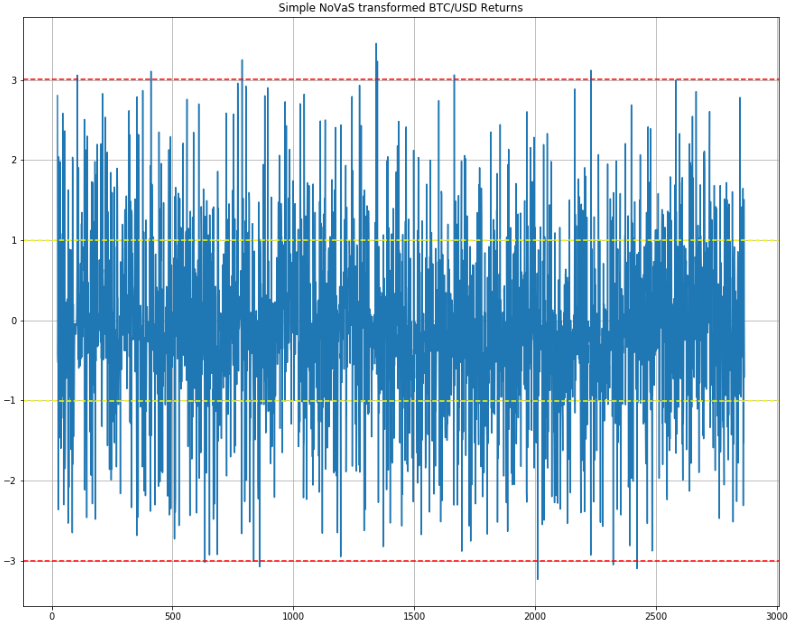
BTC/USD Daily Returns Histogram (2010-2018)



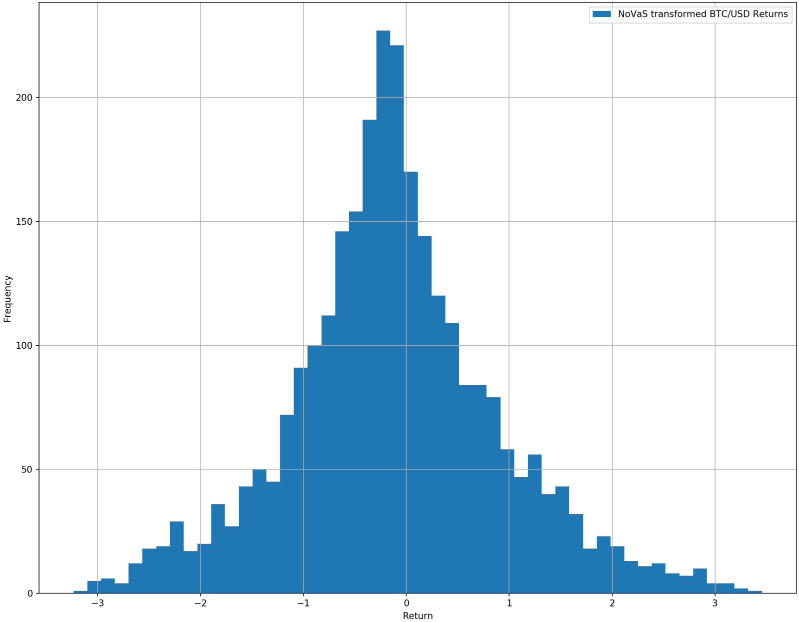
BTC/USD Daily Returns QQ-Plot (2010-2018)



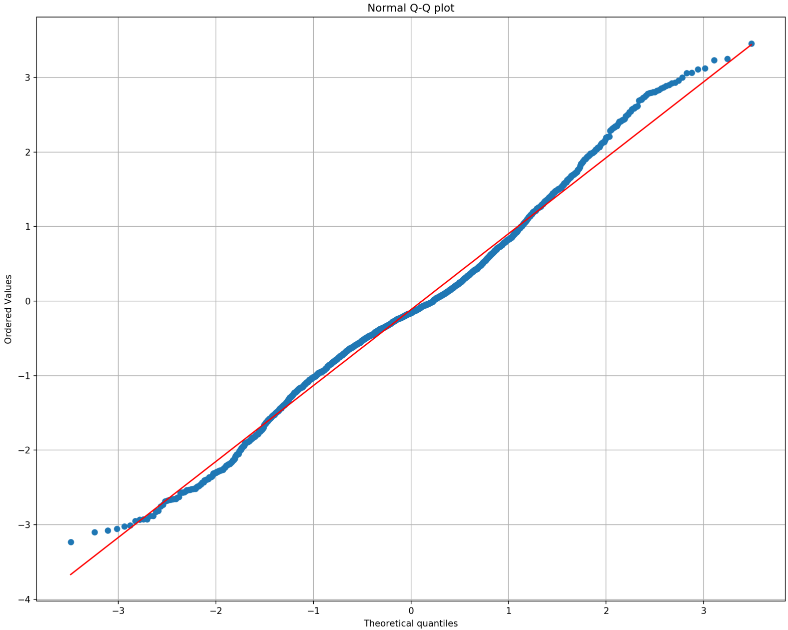
NoVaS Transformed BTC/USD Returns (p=12)



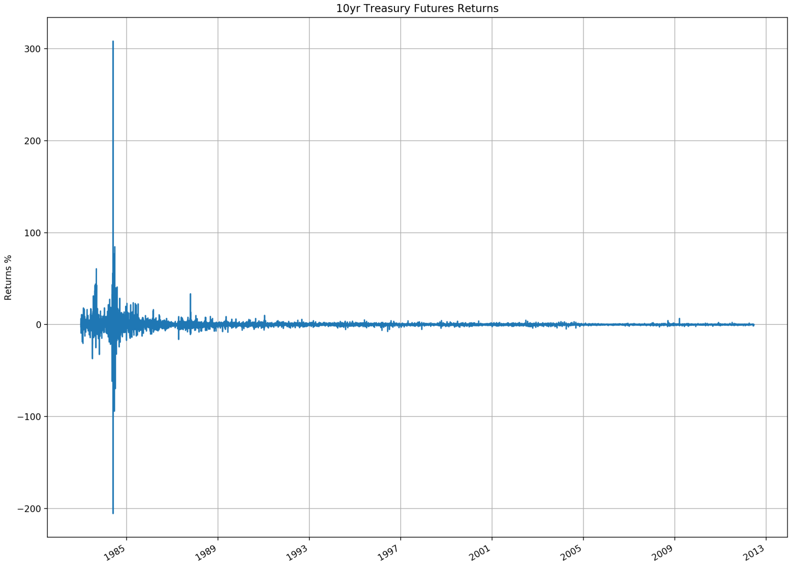
NoVaS Transformed BTC/USD Histogram (p=12)



NoVaS Transformed BTC/USD QQ-Plot (p=12)



5min Bar 10yr Treasury Futures (1983-2012)



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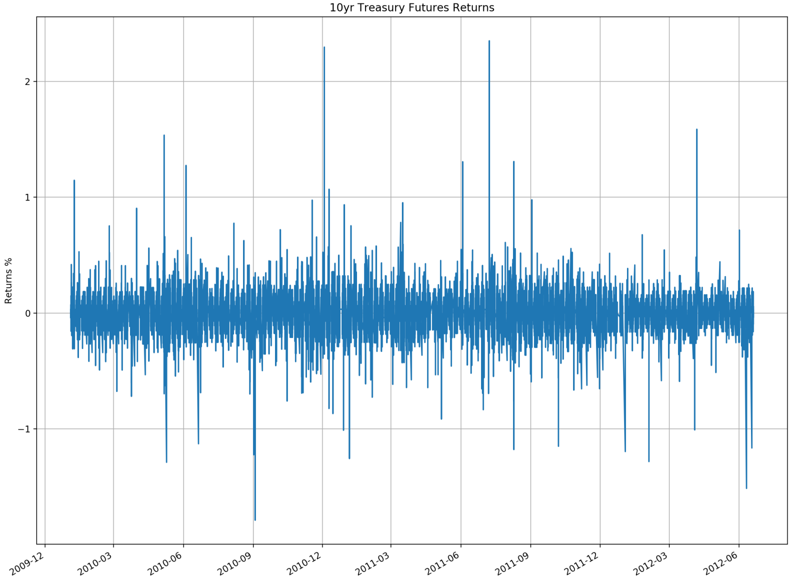

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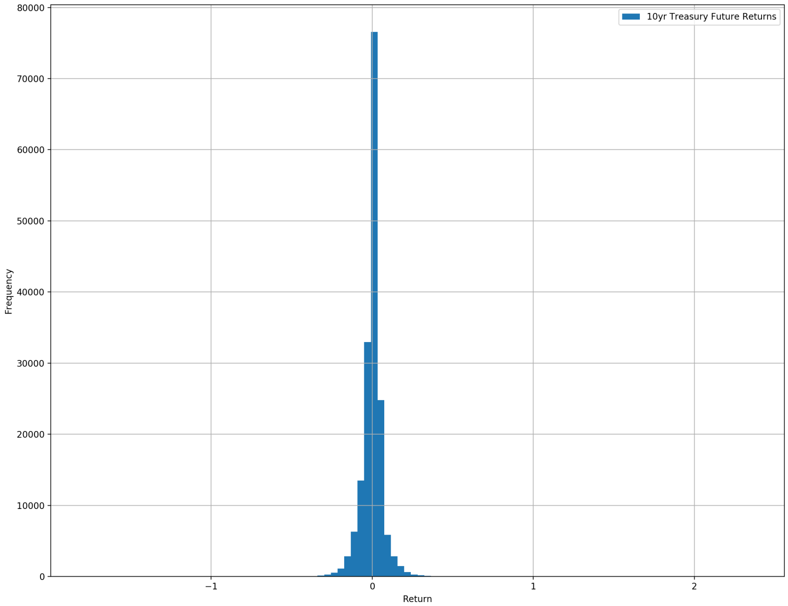

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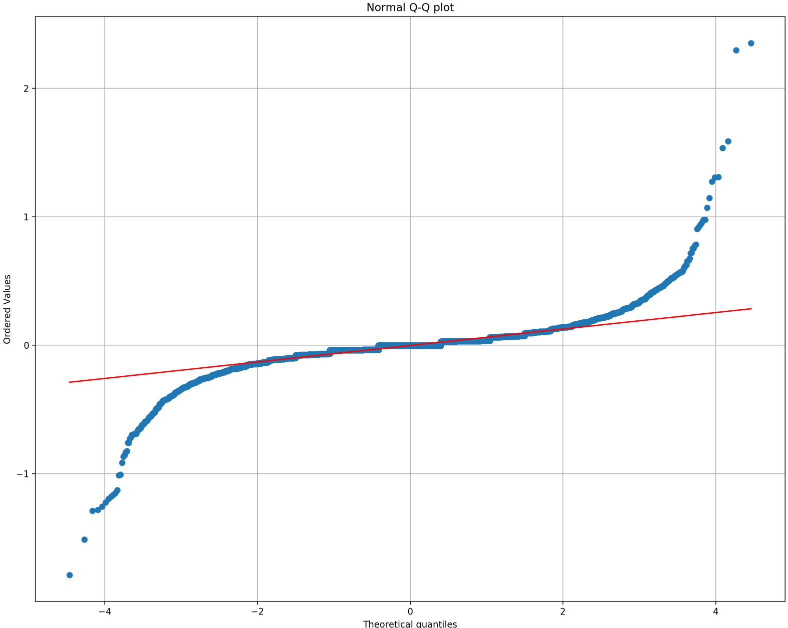

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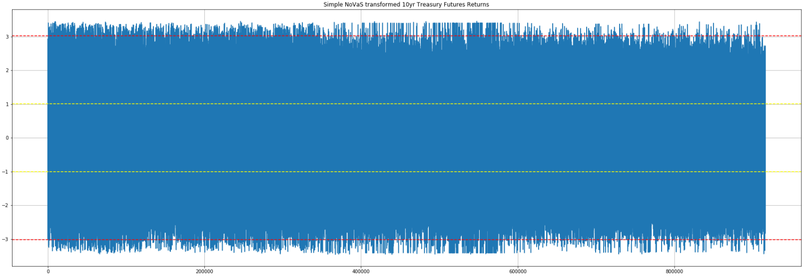
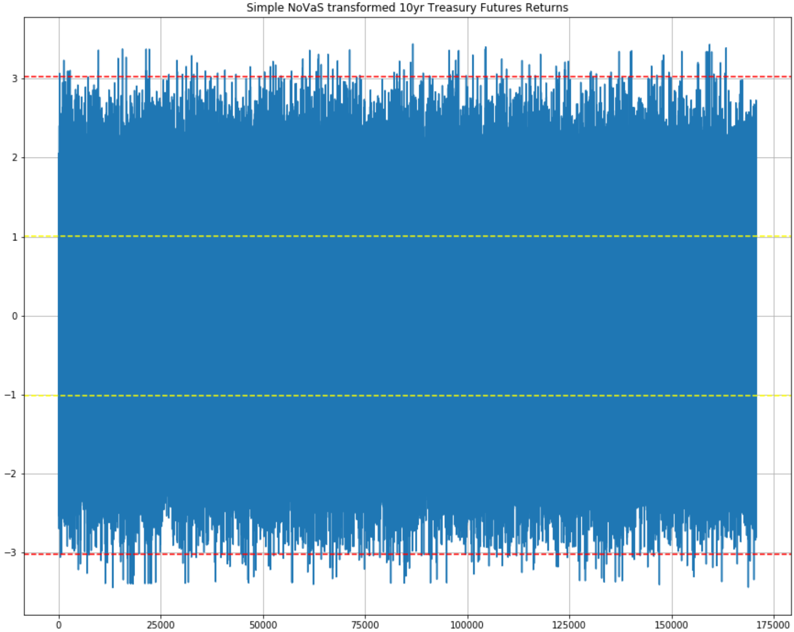
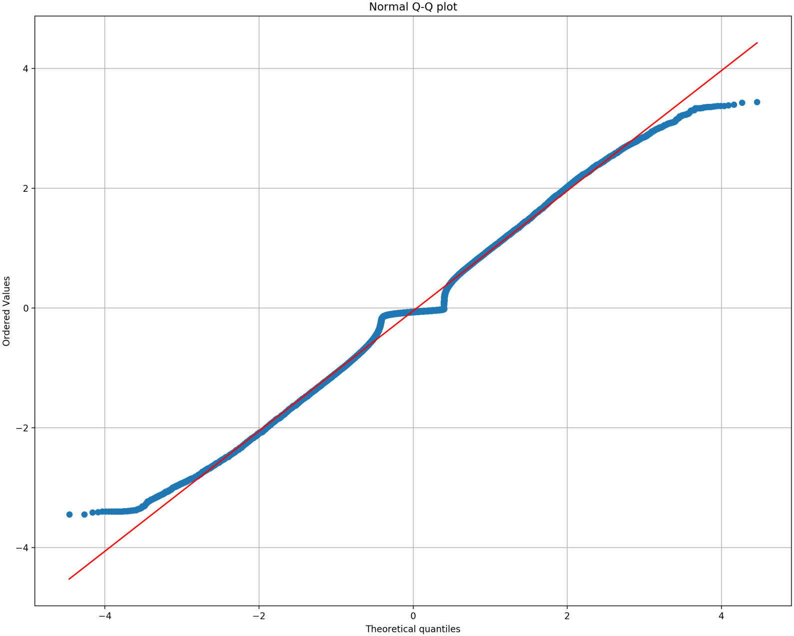


NoVaS 10yr Treasury Futures (2010-2012) (p=12)



NoVaS 10yr Treasury Futures (2010-2012) (p=12)



Outline

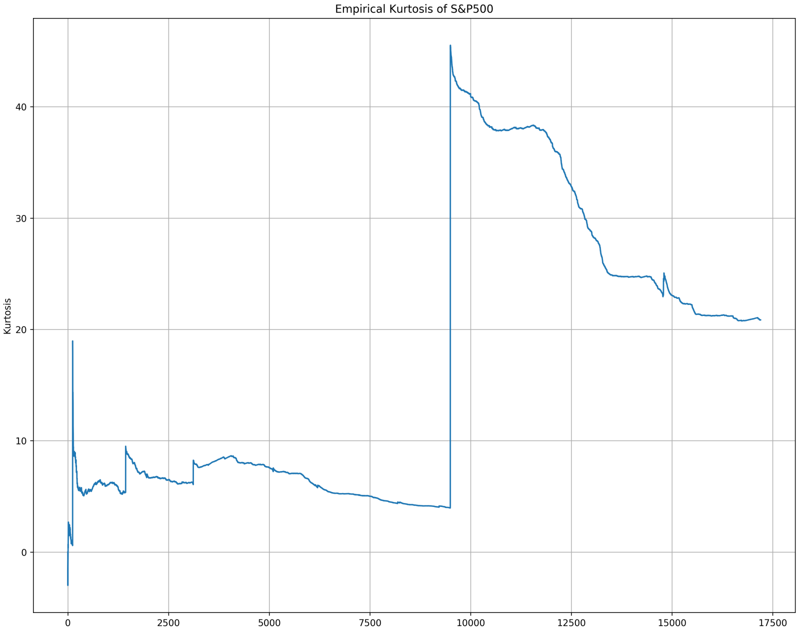
Volatility Prediction

- Forecasts of volatility are important when assessing and managing the risks of portfolios
- We focus on the problem of one-step ahead X_{t+1} prediction based on the observed past
- For our purposes, volatility prediction = predicting X_{t+1}^2
- Even though X_{t+1}^2 is a noisy proxy for $\mathbb{E}(X_{t+1}^2|\mathfrak{F}_{\mathfrak{n}})$, we'll see that in under some conditions NoVaS allows us to predict the latter
- Assuming more realistically that financial returns are locally stationary, we use a rolling window size of 250 days to calculate our forecasts

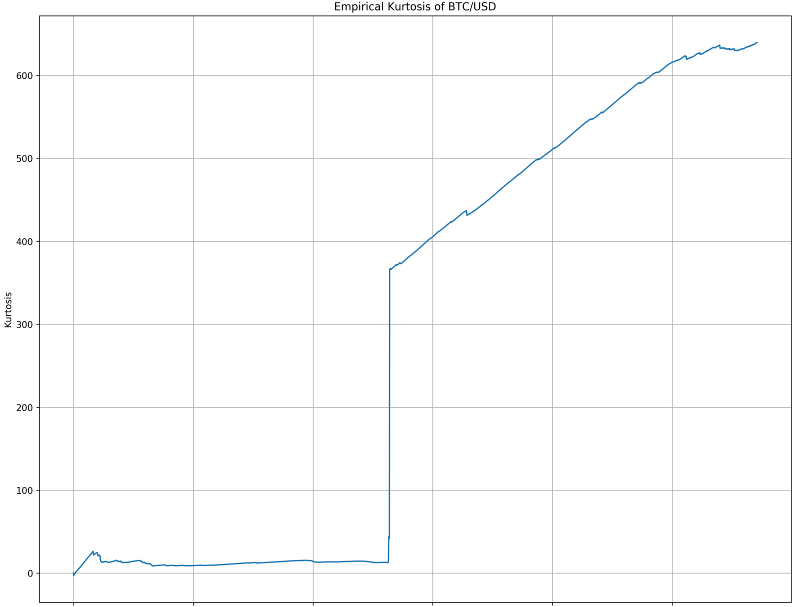
Which Loss Function? L_1 or L_2?

- To assess the accuracy of forecasts, we need to decide on a loss function to use
- The MSE is most commonly used, however note that $\mathbb{E}(Y_{n+1}^2 - \widehat{Y_{n+1}^2})^2$ is essentially a fourth moment
- Thus the unconditional MSE is infinite if the returns process has infinite kurtosis!
- We find that this indeed the case and so focus on the Mean Absolute Deviation (MAD) loss function
- Under the objective of L_1-optimal prediction, the optimal predictor is $Median(X_{n+1}^2|\mathfrak{F}_\mathfrak{n})$

Empirical Kurtosis Plot S&P500



Empirical Kurtosis Plot BTC



GARCH(1,1) Prediction

Recall the GARCH(1,1) model can be expressed as:

$$X_t = \sigma_t \varepsilon_t, \qquad \varepsilon_t \sim N(0,1)$$

where $\sigma_t^2 = \alpha_0 + \beta_1 \sigma_{t-1}^2 + \alpha_1 X_{t-1}^2$

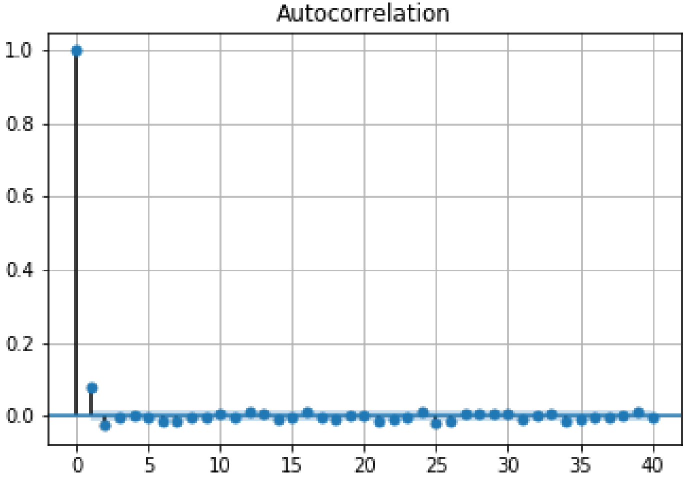
To perform one-step ahead prediction with GARCH(1,1) we:

- Estimate the parameters α_0, β_1 and α_1 using MLE
- Let $a = \frac{\alpha_0}{1-\beta_1}$ and $a_i = \alpha_1 \beta_1^{i-1}$ for $i = 1, 2, \ldots$

Then the L_1-optimal GARCH(1,1) predictor of X_{n+1}^2 is given by

$$Median(X_{n+1}^2 | \mathfrak{F}_{\mathfrak{n}}) = (a + \sum_{i=1}^{p} a_i X_{n+1-i}^2) Median(\varepsilon_{n+1}^2)$$

Special Case: Uncorrelated NoVaS Series $W_{t,a}$



NoVaS Prediction in Special Case

From the ACF plot on the previous slide, we can conclude that the NoVaS transformed series $\{W_{t,a}\}$ of the S&P500 returns appears to be uncorrelated.

As a result we can predict X_{n+1}^2 (under L_1 loss) by

$$X_{n+1}^2 = \widehat{\mu_2} A_n^2$$

where

$$\widehat{\mu_2} = median\{\frac{W_{t,a}^2}{1 - a_0 W_{t,a}^2}; t = p+1, p+2, \ldots, n\}$$

and

$$A_n^2 = \alpha s_{t-1}^2 + \sum_{i=1}^{p} a_i X_{t+1-i}^2$$

Bootstrap Prediction Intervals

In addition to calculating point estimates, we calculate prediction intervals.

Below is an outline of the procedure used for NoVaS (GARCH is almost identical):

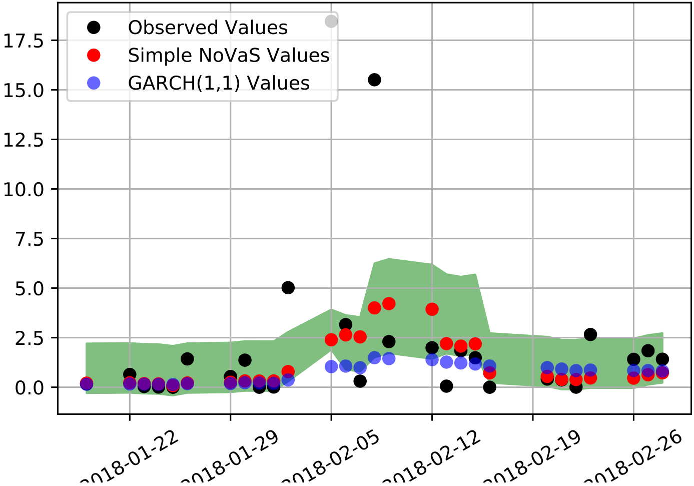
1. Use Simple NoVaS to obtain transformation $\{W_{t,a}\}$ and fitted parameter p
2. Calculate $\widehat{g(X_{n+1})}$ the point predictor of $g(X_{n+1})$
3. Resample randomly (with replacement) the transformed variables $\{W_{t,a}\}$ for $t = p+1, \ldots, n$ to create the pseudo-data $W^*_{p+1}, \ldots, W^*_{n-1}, W^*_n$ and W^*_{n+1}
4. Let $(X^*_1, \ldots, X^*_p)' = (X^*_{1+I}, \ldots, X^*_{p+I})'$ where I $\sim U\{0, n-p\}$

Bootstrap Prediction Intervals

5. Generate the bootstrap pseudo-data Y_t^* for $t = p+1, \ldots, n$
6. Based on the bootstrap data $Y_1^*, \ldots, Y_n^*$ re-estimate the NoVaS transformation, then calculate the bootstrap predictor $g\widehat{(Y_{n+1}^*)}$
7. Calculate the bootstrap future value Y_{n+1}^* and the bootstrap root: $g(Y_{n+1}^*) - g\widehat{(Y_{n+1}^*)}$
8. Repeat steps 3-7 B times - the B bootstrap root replicates are collected in the form of an empirical distribution whose α-quantile is denoted $q(\alpha)$
9. The $(1-\alpha)100\%$ equal-tailed prediction interval for $g(Y_{n+1})$ is given by

$$[g\widehat{(Y_{n+1})} + q(\alpha/2), g\widehat{(Y_{n+1})} + q(1-\alpha/2)]$$

S&P500 Feb 2018 One Step Ahead Prediction



Outline

Estimating the volatility $\mathbb{E}(X_{n+1}^2|\mathfrak{F}_\mathfrak{n})$

In order to implement our volatility trading strategy we'd ideally like an estimate of $\mathbb{E}(X_{n+1}^2|\mathfrak{F}_\mathfrak{n})$.

Fortunately, it is straightforward to do so under the case were the NoVaS series $W_{t,a}$ is uncorrelated and independent.

$$\mathbb{E}(X_{n+1}^2|\mathfrak{F}_\mathfrak{n}) = A_n^2 \mathbb{E}\left(\frac{W_{t,a}^2}{1 - a_0 W_{t,a}^2}\right)$$

a natural estimate is therefore

$$\frac{A_n^2}{n-p} \sum_{t=p+1}^{n} \left(\frac{W_{t,a}^2}{1 - a_0 W_{t,a}^2}\right)$$

Volatility Trading Strategy (Ahmad & Wilmott 2005)

Relevant Definitions:

- Implied Volatility (IV) - value calculated from an option price
- VIX - popular index which is a measure of the stock market's expectation volatility implied by S&P500 index options
- VIX(t) = IV(t) current implied volatility
- VXX - ETN that (imperfectly) tracks VIX index
- RV(t+1) - is the GARCH or NoVaS predicted realized volatility for next period
- Expect RV(t+1) to be better predictor of VIX(t+1) than VIX(t)

Strategy:

If RV(t+1) - VIX(t) > 0 then BUY VXX. Vice Versa

Strategy Results