A Practical Look at Volatility in Financial Time Series

MATH 287C - Advanced Time Series Analysis

Nishant Gurnani

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Outline

1. What is Volatility?
2. Normalizing and Variance Stabilizing (NoVaS) Transformation
3. Volatility Prediction
4. A Simple Volatility Trading Strategy

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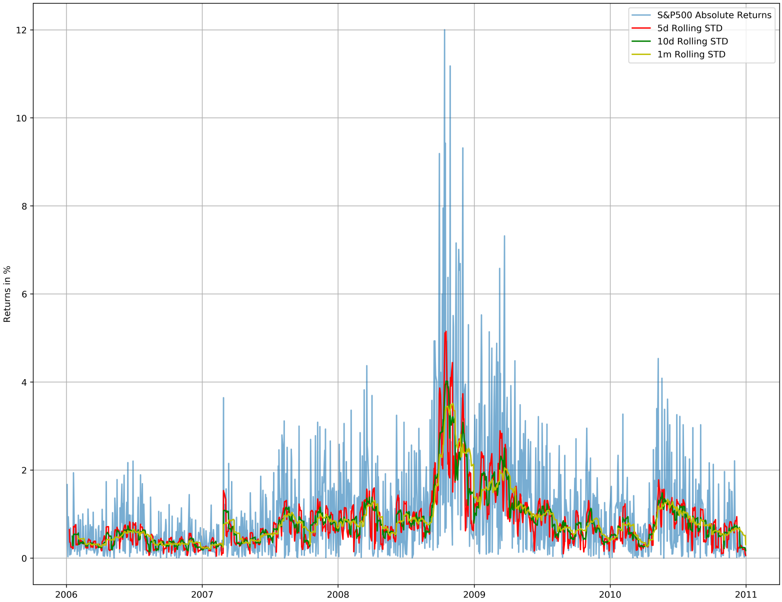
What is Volatility?

- Volatility is a measure of price variability over some period of time
- Typically described by the standard deviation σ of the return series $\{X_t\}$
- Volatility is peculiar in that we know it exists, but in some sense we can't really measure it
- Bachelier (1900) showed that $\{X_t\} \sim$ iid. $N(0,1)$, but this is only good for a first order approximation

Naive Measure - Realized Volatility



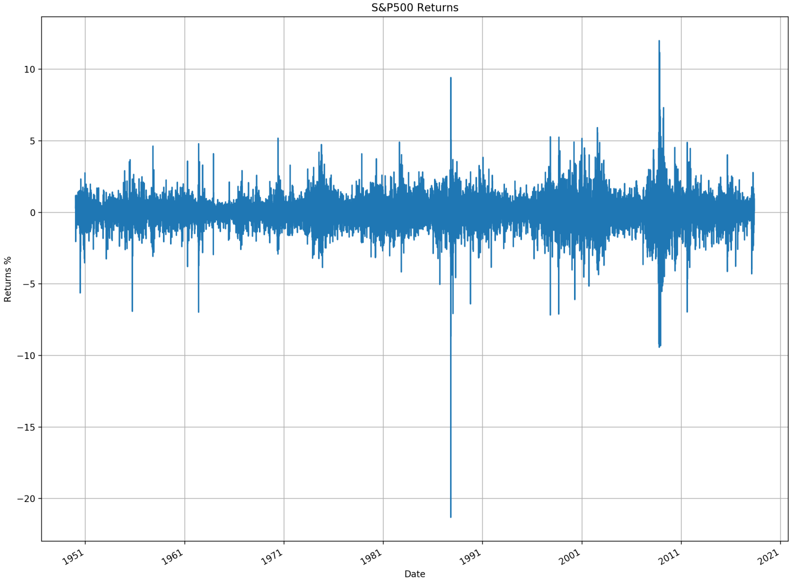
Stylized Facts

Further analysis of $\{X_t\}$ reveals other kinds of structure that cannot be explained by the gaussian assumption.

In particular, the return series displays the following distinctive behavior:

1. $\{X_t\}$ is heavy-tailed, much more so than the Gaussian white noise
2. Although $\{X_t\}$ is uncorrelated, the series $\{X_t^2\}$ is highly correlated
3. The changes in $\{X_t\}$ tend to be clustered, large changes tend to be followed by large changes and vice v
4. Effects are asymmetric, bad news results in larger downward price moves than positive news does to upward price moves

SP500 Daily Returns (1950-2018)



GARCH

The Generalized ARCH (GARCH) model of Bollerslev (1986) and it's variants are extremely popular (albeit imperfect) methods to model volatility.

GARCH(p,q) model can be expressed as:

$$X_t = \sigma_t \varepsilon_t, \qquad \varepsilon_t \sim N(0,1)$$

where

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 + \sum_{j=1}^{q} \alpha_j X_{t-j}^2$$

For the purposes of this talk, we'll focus on GARCH(1,1) models where $\sigma_t^2 = \alpha_0 + \beta_1 \sigma_{t-1}^2 + \alpha_1 X_{t-1}^2$

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NoVaS Transformation (Politis 2007)

The NoVaS Transformation is defined as

$$W_{t,a} = \frac{X_t}{\sqrt{\alpha s_{t-1}^2 + a_0 X_t^2 + \sum_{i=1}^{p} a_i X_{t-i}^2}}$$

for $t = p+1, p+2, \ldots, n$

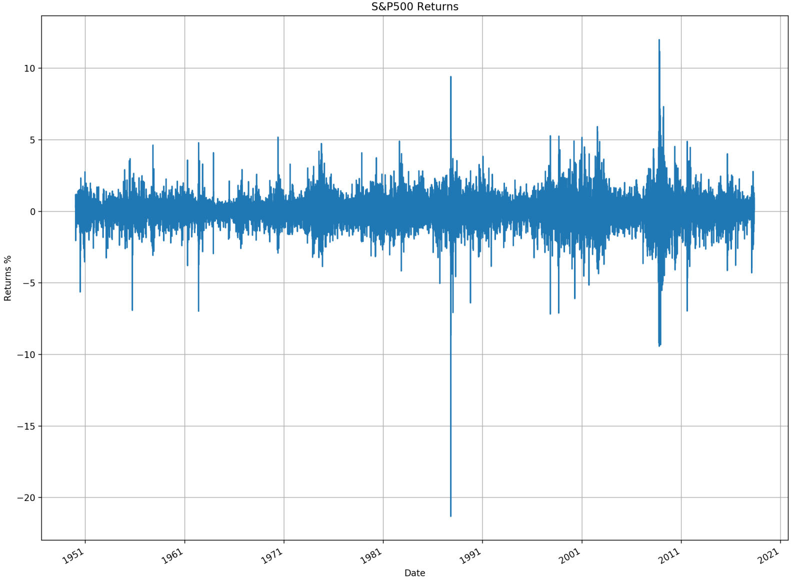
It is a clever extension of the ARCH model where we include the value X_t in order to "studentize" the returns.

The order p and the vector of nonnegative parameters $(\alpha, a_0, \ldots, a_p)$ are chosen by the practitioner with the twin goals of normalization and variance-stabilization.

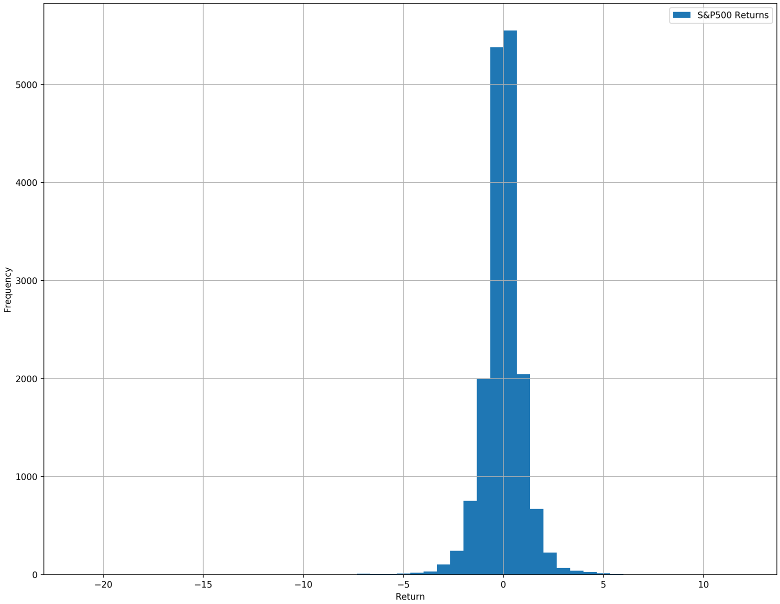
Algorithm for Simple NoVaS:

- Let $\alpha = 0$ and $a_i = \frac{1}{p+1}$ for all $0 \leq i \leq p$
- Pick p such that $|KURT_n(W_{t,p}^S)| \approx 3$

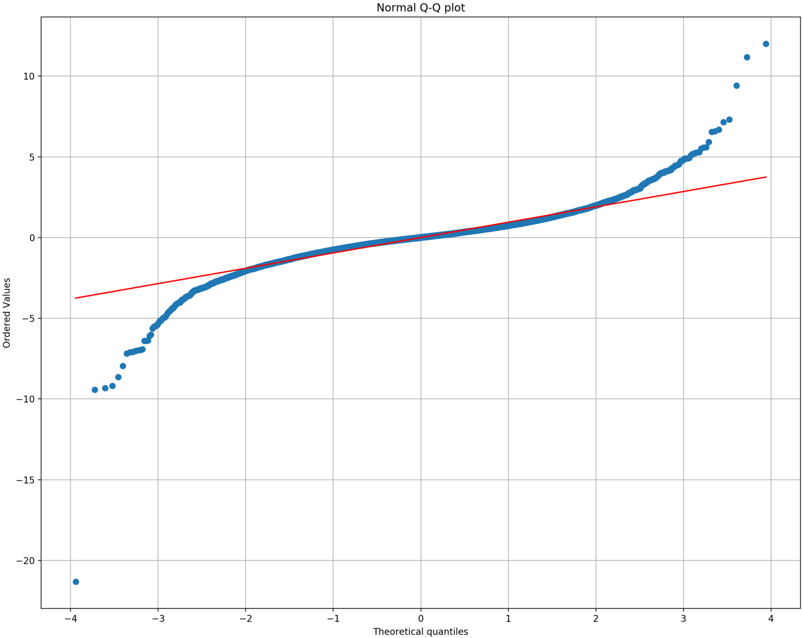
SP500 Daily Returns (1950-2018)



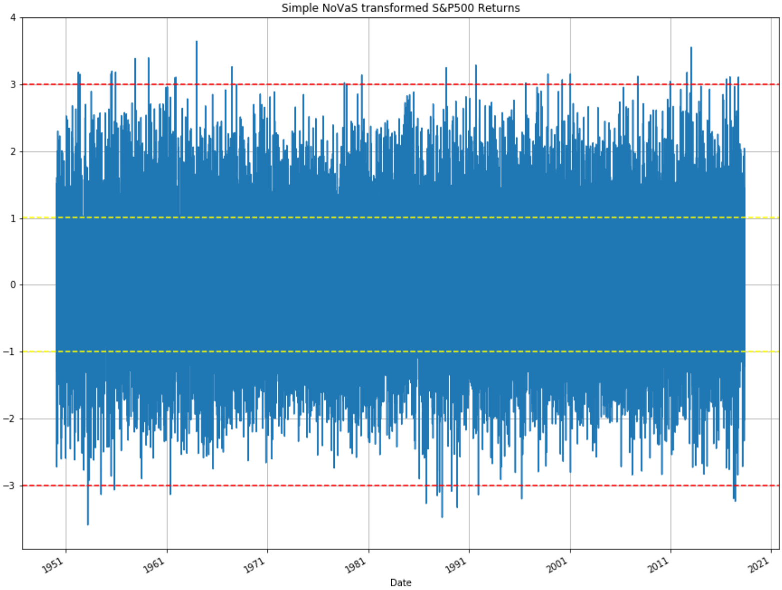
SP500 Daily Returns Histogram



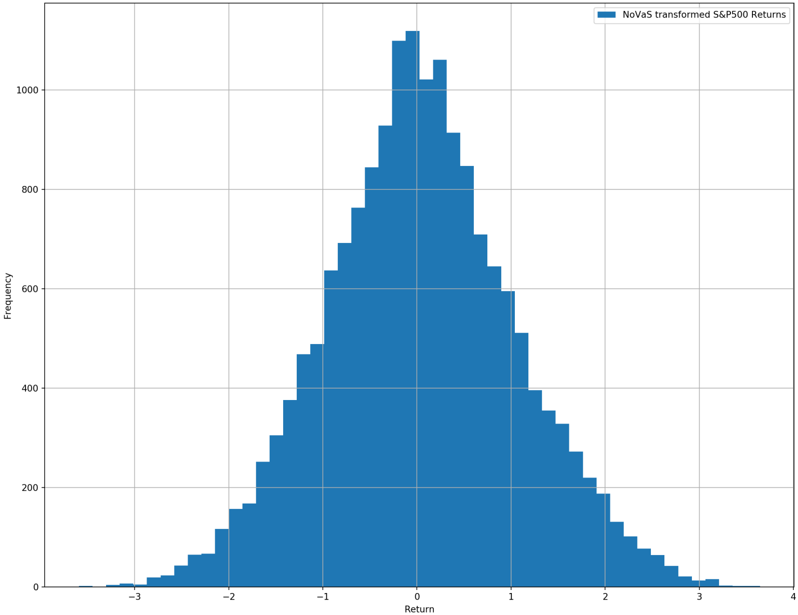
SP500 Daily Returns Q-Q Plot



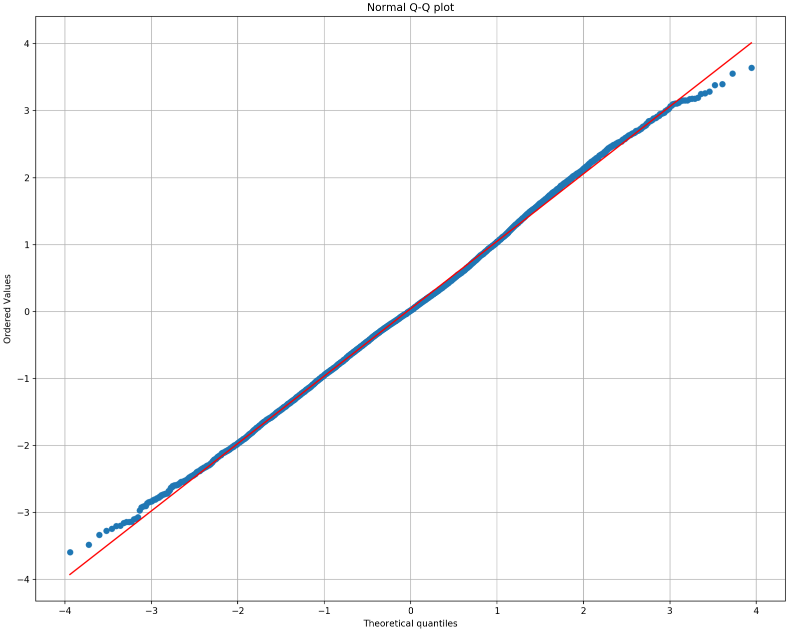
NoVaS Transformed SP500 Daily Returns (p=16)



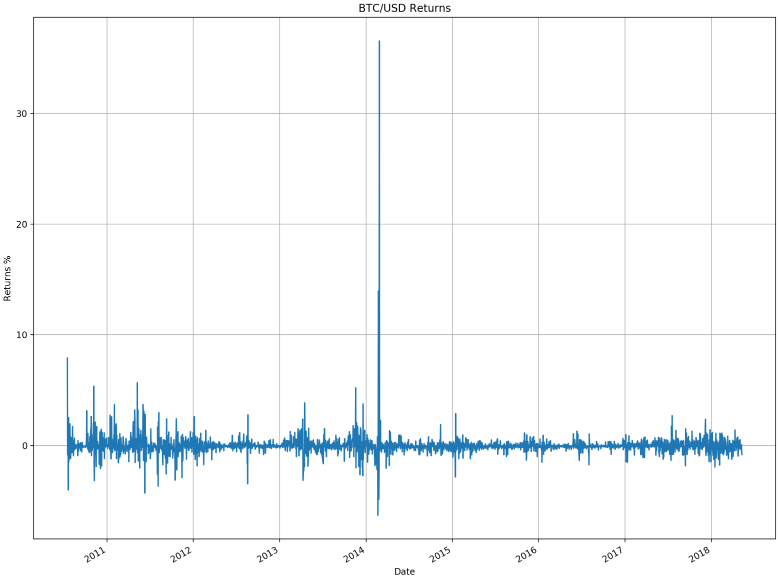
NoVaS Transformed SP500 Histogram (p=16)



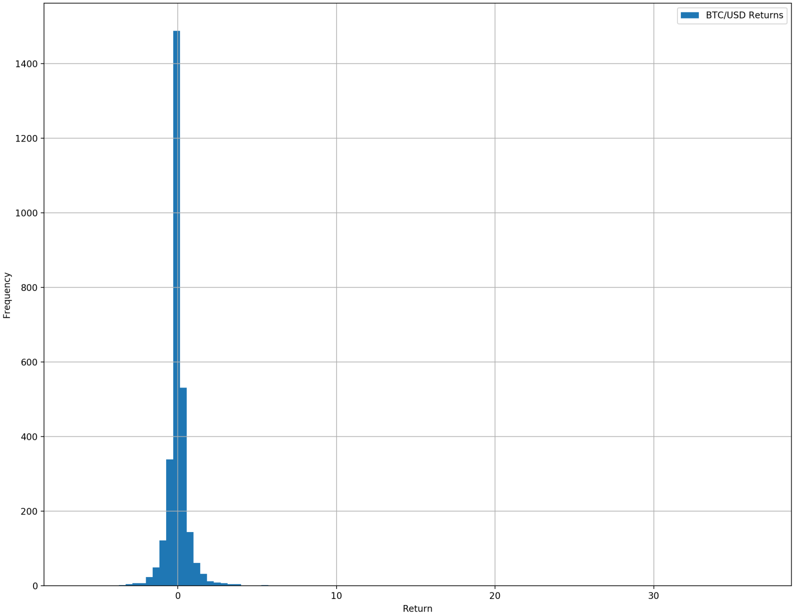
NoVaS Transformed SP500 QQ-Plot (p=16)



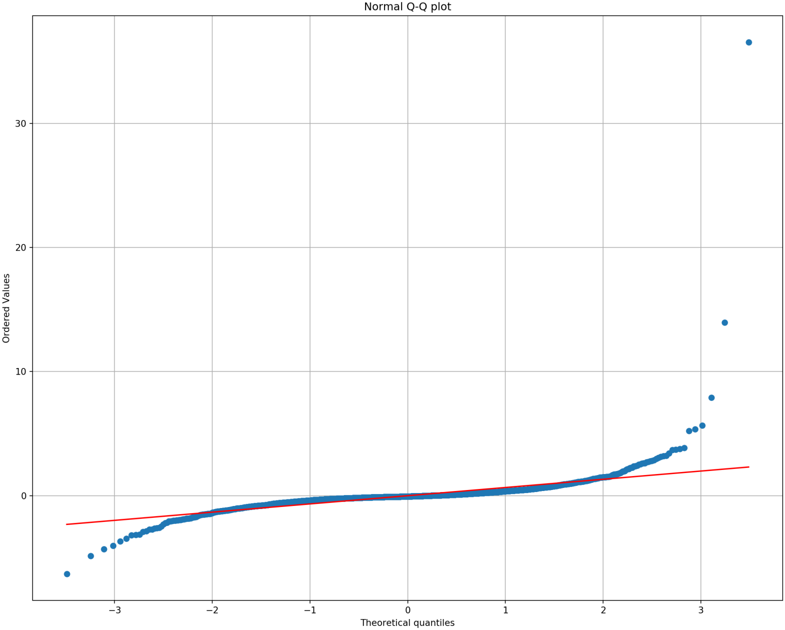
BTC/USD Daily Returns (2010-2018)



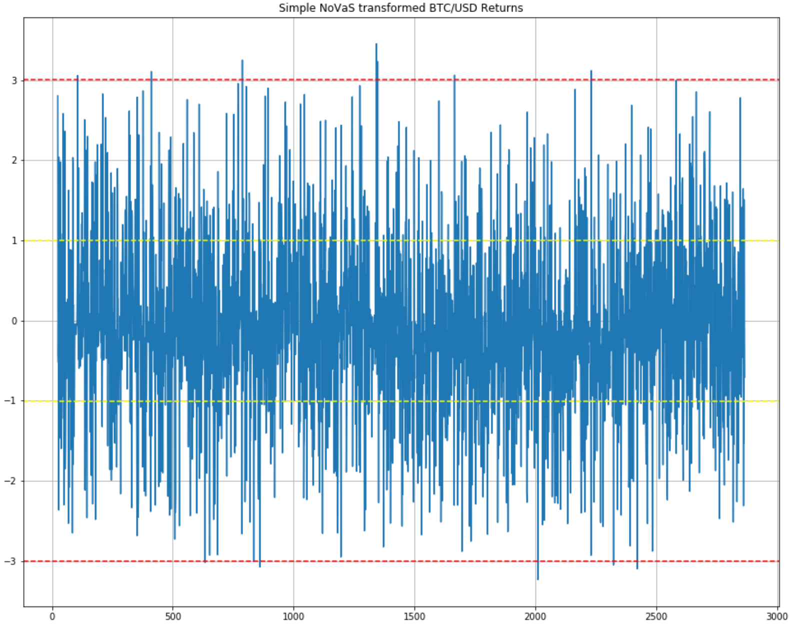
BTC/USD Daily Returns Histogram (2010-2018)



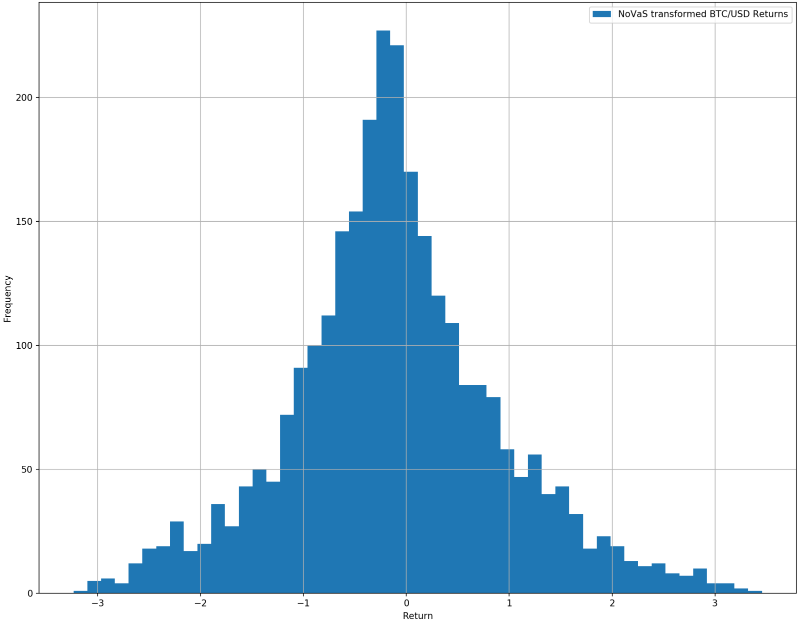
BTC/USD Daily Returns QQ-Plot (2010-2018)



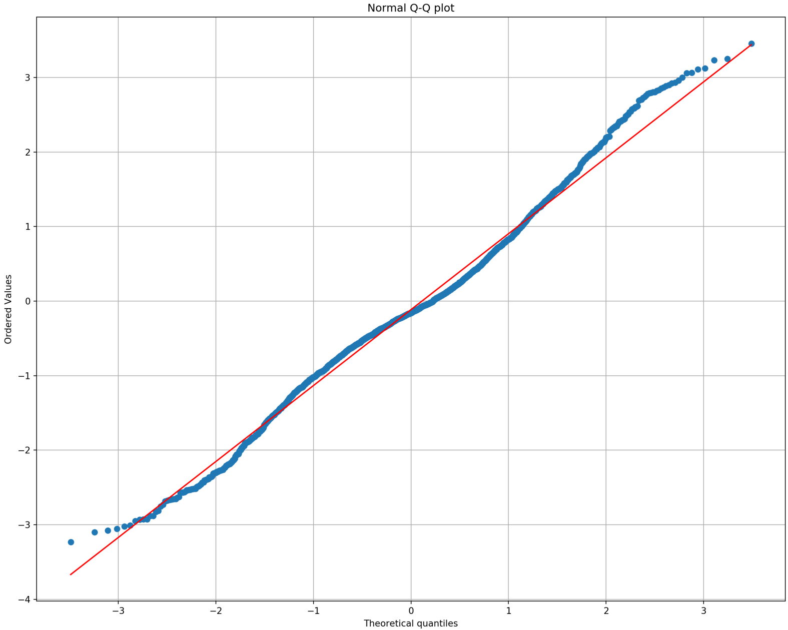
NoVaS Transformed BTC/USD Returns (p=12)



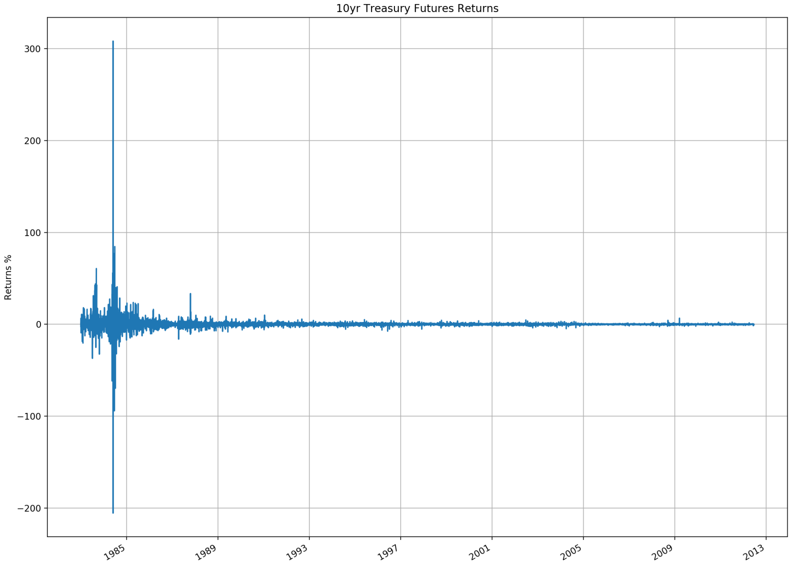
NoVaS Transformed BTC/USD Histogram (p=12)



NoVaS Transformed BTC/USD QQ-Plot (p=12)



5min Bar 10yr Treasury Futures (1983-2012)



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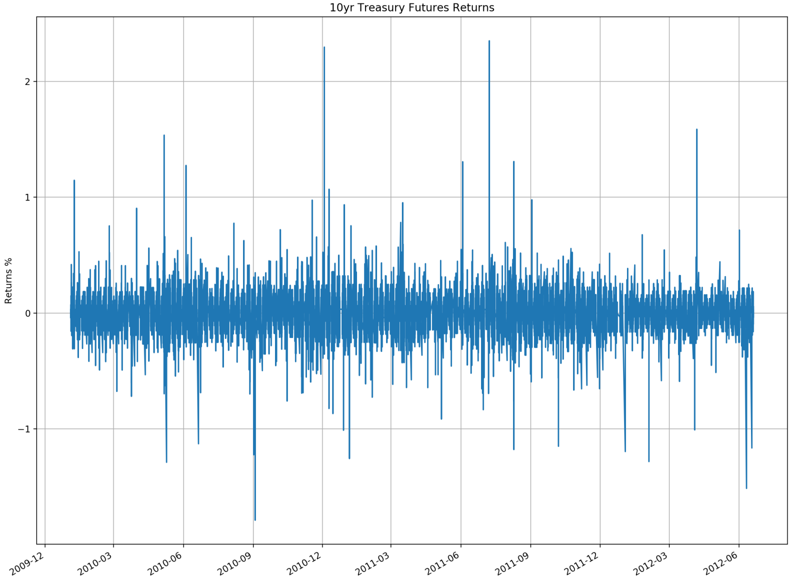

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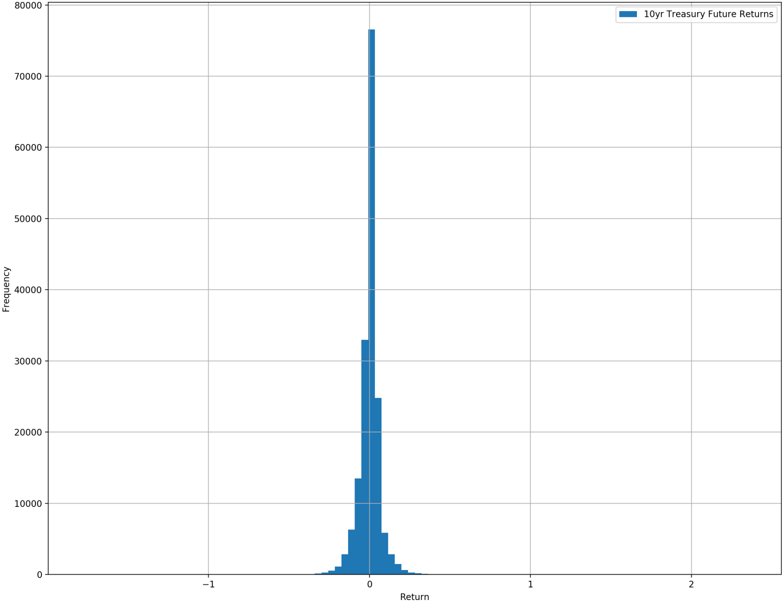

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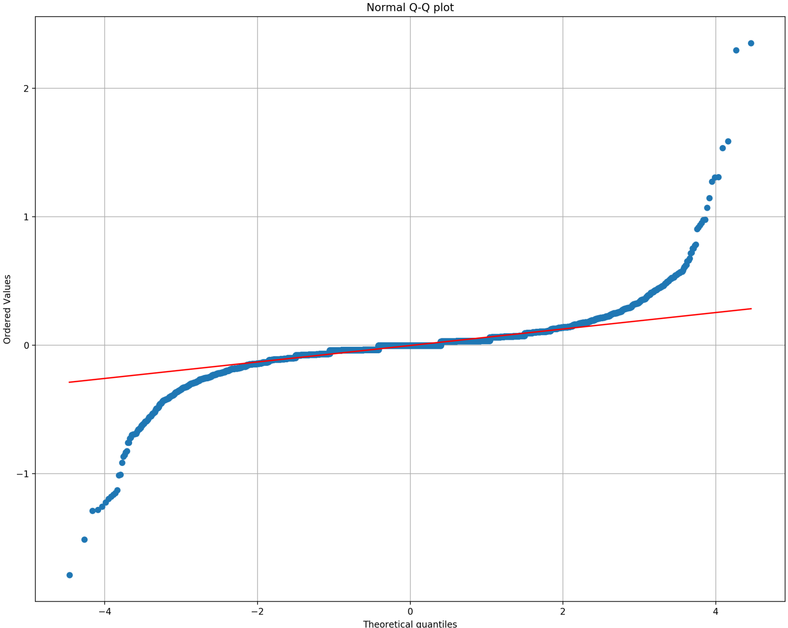

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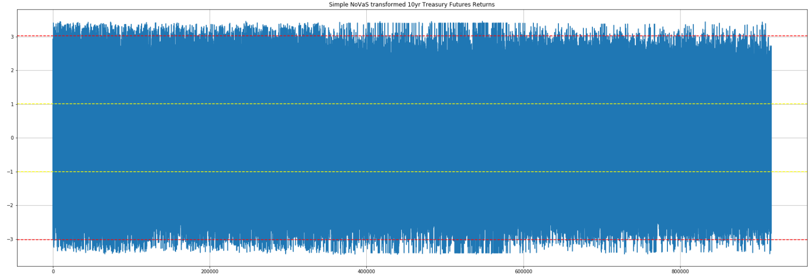

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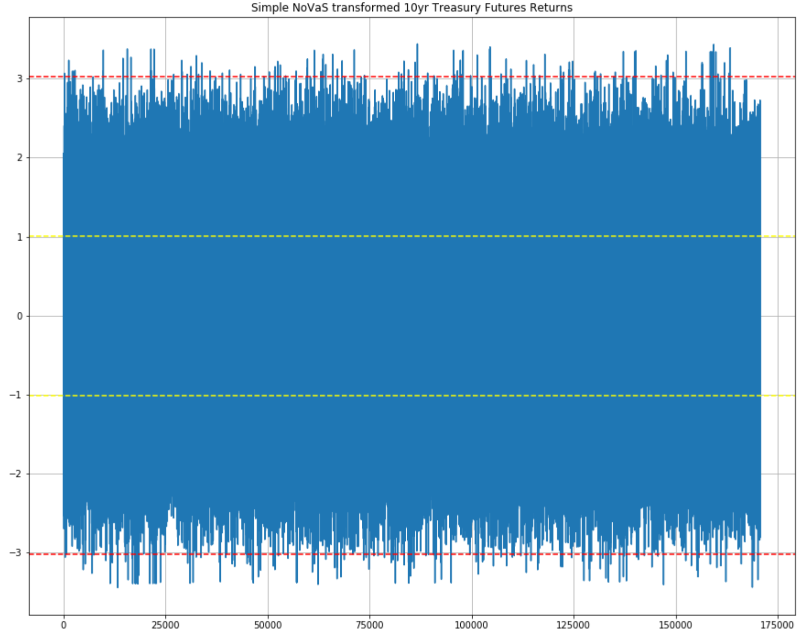

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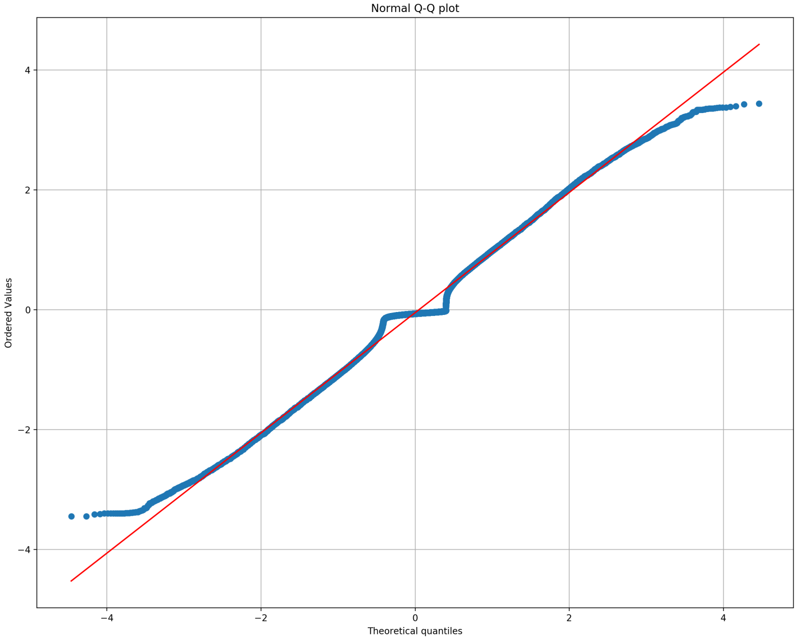

Outline

Volatility Prediction

We focus on the problem of one-step ahead prediction
Volatility prediction = prediction of X_t^2
Prediction of X_{n+1}^2 based on the observed past
Define the volatility prediction problem.
Outline that you're using squared returns Y_t^2 as a noisy proxy for Volatility
What loss function should you use?
Will use a window size of 250 days which is approx. 1 year

Infinite Kurtosis?

Do financial returns have infinite kurtosis? If this is the case you, predicting under L2 is incorrect. Instead you should L1 loss where the median is optimal

Infinite Kurtosis Plot SP500

Infinite Kurtosis Plot BTC

Infinite Kurtosis Plot Treasury Futures

Prediction Intervals

Steps for deriving prediction intervals - same for GARCH and NoVaS

1. Use simple NoVaS to obtain transformed data $\{W_{t,a}$ for $t = p+1, \ldots, n\}$ that are assumed to be approximately i.i.d. Let p, α and a_i denote the fitted NoVaS parameters
2. Calculate $\widehat{g(Y_{n+1})}$ the point predictor of $g(y_{n+1})$ as the median of the set etc.
3. Main Bootstrap Loop
   3.1 Resample randomly (with replacement) the transformed variables $\{W_{t,a}$ for $t = p+1, \ldots, n\}$ to create the pseudo-data $W^*_{p+1}, \ldots, W^*_{n-1}, W^*_n$ and W^*_{n+1}
   3.2 Let $(Y^*_1, \ldots, Y^*_p)' = (Y^*_{1+I}, \ldots, Y^*_{p+I})'$ where I is generated as a discrete random variable uniform on the values $0, 1, \ldots, n-p$
   3.3 Generate the bootstrap pseudo-data Y^*_t for $t = p+1, \ldots, n$ using equation (10.17)

$$Y^*_t = \frac{W_t}{\sqrt{1 - a_0 W^{*2}_t}} \sqrt{\sum_{i=1}^{p} a_i Y^{*2}_{t-i}}$$

for $t = p+1, \ldots, n$

SP500 Feb 2018 One Step Ahead Prediction

Plot predicting SP500 Feb 2018 Volatility spike, along with prediction intervals Shows that Simple NoVaS is better than GARCH(1,1)

Outline

Estimating the volatility $\mathbb{E}(Y_{n+1}^2|\mathfrak{F}_\mathfrak{n})$

Under case I, i.e. after empirically showing that the $W_{t,a}$ variables are (approximately) uncorrelated and hence independent, it is straightforward to construct a Model-free estimate of the conditional expectation $\mathbb{E}(Y_{n+1}^2|\mathfrak{F}_\mathfrak{n})$.

$$\mathbb{E}(Y_{n+1}^2|\mathfrak{F}_\mathfrak{n}) = A_n^2 \mathbb{E}\left(\frac{W_{t,a}^2}{1 - a_0 W_{t,a}^2}\right)$$

a natural estimate thereof is

$$\frac{A_n^2}{n-p} \sum_{t=p+1}^{n} \left(\frac{W_{t,a}^2}{1 - a_0 W_{t,a}^2}\right)$$

Talk about the conditions under which you can actually predict $sigma^2$, plot the ACF to confirm that transformed series is uncorrelated and independent.

RV(t+1)-IV(t)

We consider a very simple volatility trading strategy found in Ahmad Wilmott (2005)
Strategy:

- If RV(t+1) - VIX(t) $>$ 0 then BUY VXX. Vice Versa.
- RV(t+1) is the GARCH or NoVaS predicted realized volatility for next period
- Expect RV(t+1) to be better predictor of VIX(t+1) than VIX(t)
- VIX(t) $=$ IV(t) is the current implied volatility

Strategy Results

Cumulative returns plot, legend contains CAGR and Sharpe Ratio