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PHY 482

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Research Summary

Magnetic Confinement Fusion

There are several approaches to developing sustainable fusion, but this paper will focus on Magnetic Confinement Fusion (MCF), specifically inside of tokamak reactors. Tokamaks are the most developed style of fusion reactor, and with ITER being developed, the most relevant as well. Tokamaks are also based on the simplest physical and technical parameters out of all the fusion reactors making it possible for this paper to develop a detailed model. The MHD involved is a mess to look at and will be kept to high level with the detailed model being dedicated to the magnetic confinement.

Tokamaks are a style of stead state MCF reactor that uses a set of toroidal and poloidal electromagnetics to create a magnetic bottle torus that contains fusing plasma. The magnetic containment of the plasma is vital to achieving fusion, because the two most import physical parameters for fusion, temperature and particle density are dependent on it (I.R. Lindemuth and R. E. Siemon). That is, the cross section of fusion is dependent on the number of particles in a given volume and the average energy of the particles, dependent of temperature. Mathematically, this is given by the Maxwellian averaged cross section, σv which is a function of temperature multiplied by the number density of each particle species in the fusion fuel.

$$\frac{dn_1}{dt} = \frac{dn_2}{dt} = -\overline{\sigma v} n_1 n_2,$$

The plasma is primarily heated by a solenoid running through the middle of the reactor torus. The solenoid will induce a current into the plasma which will increase the temperature due to ohmic heating (L.A. Artsimovich). Ohmic heating is caused by the internal resistance of plasma. Like a wire, when current runs through it, it will heat up. It works best when the plasma is cold and dense with relatively high resistance, but drops off as the plasma heats up, becoming less dense and less resistive. This will heat the plasma up to a temperature where it can fuse, but problems in containment arise. The plasma pressure fighting against the magnetic bottle increases with temperature and attempts to drop the number density of the fusing particles. The current in the electromagnets creating the magnetic bottle is adjusted to contain the plasma, but due to more complicated MHD interactions, containment is still broken after a few seconds of fusion.

Excluding the MHD interactions a simple model can be built for a single charged particle inside the magnetic bottle of a tokamak reactor. For simplicity, the torus is going to be assumed to be perfectly axially symmetric. The set of poloidal and toroidal magnets create a helical field for a charged particle to follow along with the electric field produced by the solenoid.

[Second attempt at making a model for particle motion]

By changing the current through the central solenoid, the magnetic field is also being changed, which will create an electric field that induces a current to oppose the change in magnet field (Lenz's law). This induced current in the plasma will be in the opposite toroidal direction as the current going through the solenoid. And now we have.

F = q(E + V X B) (Lorentz force)

The particles will be driven in the toroidal direction by qE for as long as the solenoid takes to get a steady state current. Once the solenoid has a steady state current through it, there will be no change in flux in the magnetic field and will cease to induce an electric field. The solenoid is there to give the give the particles an initial kick to start moving around the reactor and heat the plasma.

Let us consider the motion of a single particle in a uniform magnetic field. We will look at the motion right after the solenoid gives its kick, saying that the particle has an initial velocity and the electric field is zero. The derivation for the motion is the following (Collins 2019):

$$\mathbf{B} = B_z\hat{\mathbf{z}} \qquad \mathbf{E} = \mathbf{0}$$

Given the electric field is zero, the Lorentz force can be written

$$\frac{d\mathbf{v}}{dt} = \frac{q\mathbf{v} \times \mathbf{B}}{m}$$

The Lorentz force can then be split into components

$$\dot{v}_x\hat{\mathbf{x}} + \dot{v}_y\hat{\mathbf{y}} + \dot{v}_z\hat{\mathbf{z}} = \frac{qv_yB_z\hat{\mathbf{x}} - qv_xB_z\hat{\mathbf{y}}}{m}$$

$$\dot{v}_x = \frac{qv_yB_z}{m} \qquad \dot{v}_y = -\frac{qv_xB_z}{m}$$

Importantly, the component of force along the magnetic field is zero, meaning the particle is free to move along the field.

$$\dot{v}_z = 0$$

A derivative of the Lorentz force components can then be taken to form a set of differential equations that describes the motion.

$$\dot{v}_x = \frac{qv_yB_z}{m} \qquad \dot{v}_y = -\frac{qv_xB_z}{m}$$

$$\ddot{v}_x = \frac{q\dot{v}_yB_z}{m} \qquad \ddot{v}_y = -\frac{q\dot{v}_xB_z}{m}$$

$$\ddot{v}_x = -\left(\frac{qB_z}{m}\right)^2 v_x \qquad \ddot{v}_y = -\left(\frac{qB_z}{m}\right)^2 v_y$$

The solution to the differential equations takes the form of sin and cos, with the velocity in the y direction having a sign dependency on the charge of the particle.

$$v_x = v_\perp \cos\left(\frac{|q|B_z}{m}t + \phi_0\right) \qquad v_y = \mp v_\perp \sin\left(\frac{|q|B_z}{m}t + \phi_0\right)$$

$$v_\perp = \sqrt{(v_x^2 + v_y^2)}$$

Integrating the solution yields the position of the particle.

$$x = \frac{mv_\perp}{|q|B_z}\sin\left(\frac{|q|B_z}{m}t + \phi_0\right) + x_0 \qquad y = \pm\frac{mv_\perp}{|q|B_z}\cos\left(\frac{|q|B_z}{m}t + \phi_0\right) + y_0$$

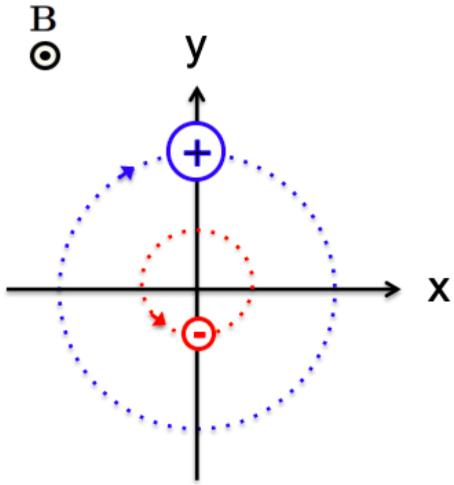
The equations are then condensed by defining the Larmor radius, how far away the particle circles form the magnetic field line it is follow, and the Cyclotron frequency, how many rotations around the magnetic field line the particle takes per second.

$$r_L \equiv \frac{mv_\perp}{|q|B}$$ **Larmor radius**

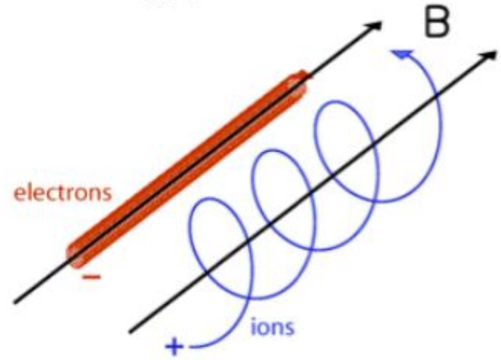
$$\omega_c \equiv \frac{|q|B}{m}$$ **Cyclotron frequency**

$$x = r_L \sin\left(\omega_c t + \phi_0\right) + x_0 \qquad y = \pm r_L \cos\left(\omega_c t + \phi_0\right) + y_0$$

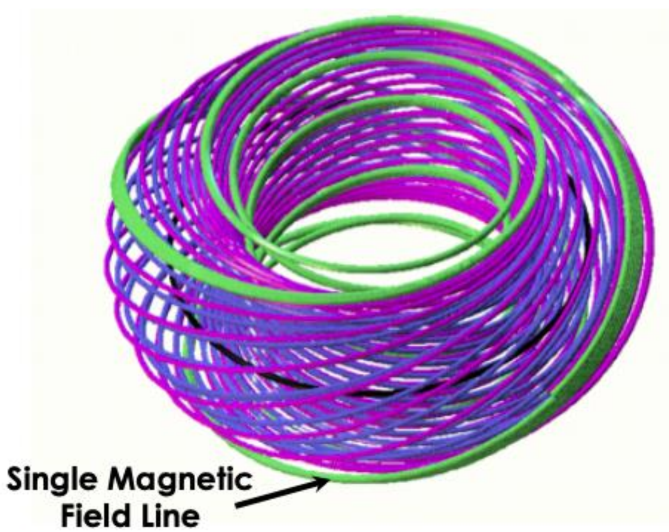
The equations of motion tell us that the particles move in a helical path, translating along the path of the magnetic field with rotation perpendicular to it.



$$r_L = \frac{mv_\perp}{eB}$$



Tokamaks exploit the fact that particles are confined perpendicular to magnetic field lines to achieve confinement with toroidal geometry. As long as the radius of the torus is sufficiently larger than the Larmor radius of the traveling particles, they approximately just follow the magnetic field lines.



Unfortunately, this is not the entire story of tokamak confinement. Assuming there is no electric field gets a model that works decently well but misses a key element in confinement. If there were no electric field, a helical magnetic field would not be necessary, a toroidal field would do the job. In short, an additional electric field is externally supplied to the plasma to keep the particles moving as well as keeping the plasma at a fusible temperature.

This electric field gives the moving particles a drift velocity that needs to be counter acted with a helical field. This drift velocity can be derived from the next simplest model, assuming there is a constant electric field (Collins 2019).

$$\mathbf{E} = E_x \hat{\mathbf{x}} \qquad \mathbf{B} = B_z \hat{\mathbf{z}}$$

The derivation works out similar to the one above, but an extra factor comes out of it.

$$\dot{v}_x = \frac{q}{m}\left(v_y B_z + E_x\right)$$

$$v_y = \mp v_\perp \sin\left(\frac{|q|B_z}{m}t + \phi_0\right) - \frac{E_x}{B_z}$$

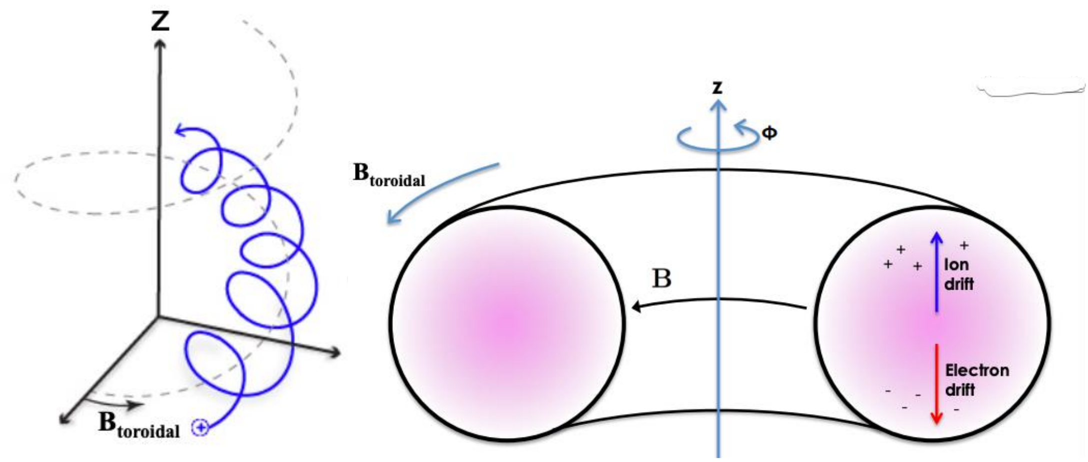
The extra term drifts the particles guiding center in the –y direction. This drift can be more generally expressed as

$$\mathbf{v_E} = \frac{\mathbf{E} \times \mathbf{B}}{B^2}$$

Which can be further generalized to describe any drift on the particles which can be caused by any force perpendicular to the magnetic field. These forces include gravity and the centrifugal force.

$$\mathbf{v_d} = \frac{1}{q}\frac{\mathbf{F} \times \mathbf{B}}{B^2}$$

The combined drift forces cause the particles to move along curved field lines that will make them tend upward or downward depending on charge.



This is what makes the poloidal component of the magnetic field necessary. The particles in the plasma want to drift upward and downward out of confinement. By continuously overturning them, the drift can be nullified. When the Ions are shifted to the bottom of the reactor, they will drift upward towards the center which will counter act the upward drift when they are shifted to the top. Vice versa for the electrons.